

Attitude estimation with Extended Kalman Filter

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Control Automation Lab



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- Introduction
- Model
- Preprocessing and implementation of EKF

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Scope of the laboratory

- Use the Kalman Filter to perform attitude estimation of a smartphone, using inertial platform measurements:
 - Gyroscope $\omega(k)$
 - Accelerometer $a(k)$
 - Magnetometer $m(k)$
- LPT: Access SAMSUNG smartphone sensors with the combination: *#0*#

Reference → Orientation Estimation using Smartphone Sensors – Linköping university

Reference → *Robotica: modellistica, simulazione e controllo*
Siciliano, Sciavicco, Villani, Oriolo

Frames

- We will use two coordinate systems: the **world frame** and the **sensor frame**
- These coordinate systems are related via **linear transformations**

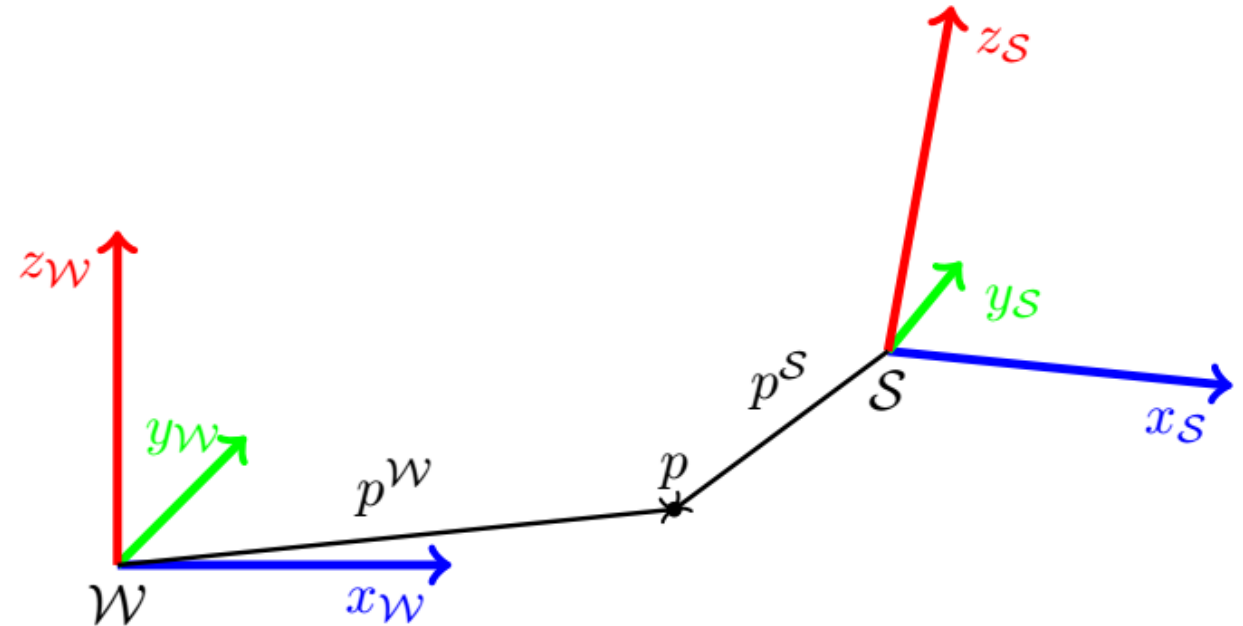
$$p^W = R^{W/S} p^S + t^{W/S}$$

where p^S is a point expressed in the sensor frame and p^W is the same point expressed in the world frame

- The relative rotation between the two frames is given by $R^{W/S}$, the **rotation matrix**
- Since we consider only rotation, the term $t^{W/S}$ is discarded

Frames

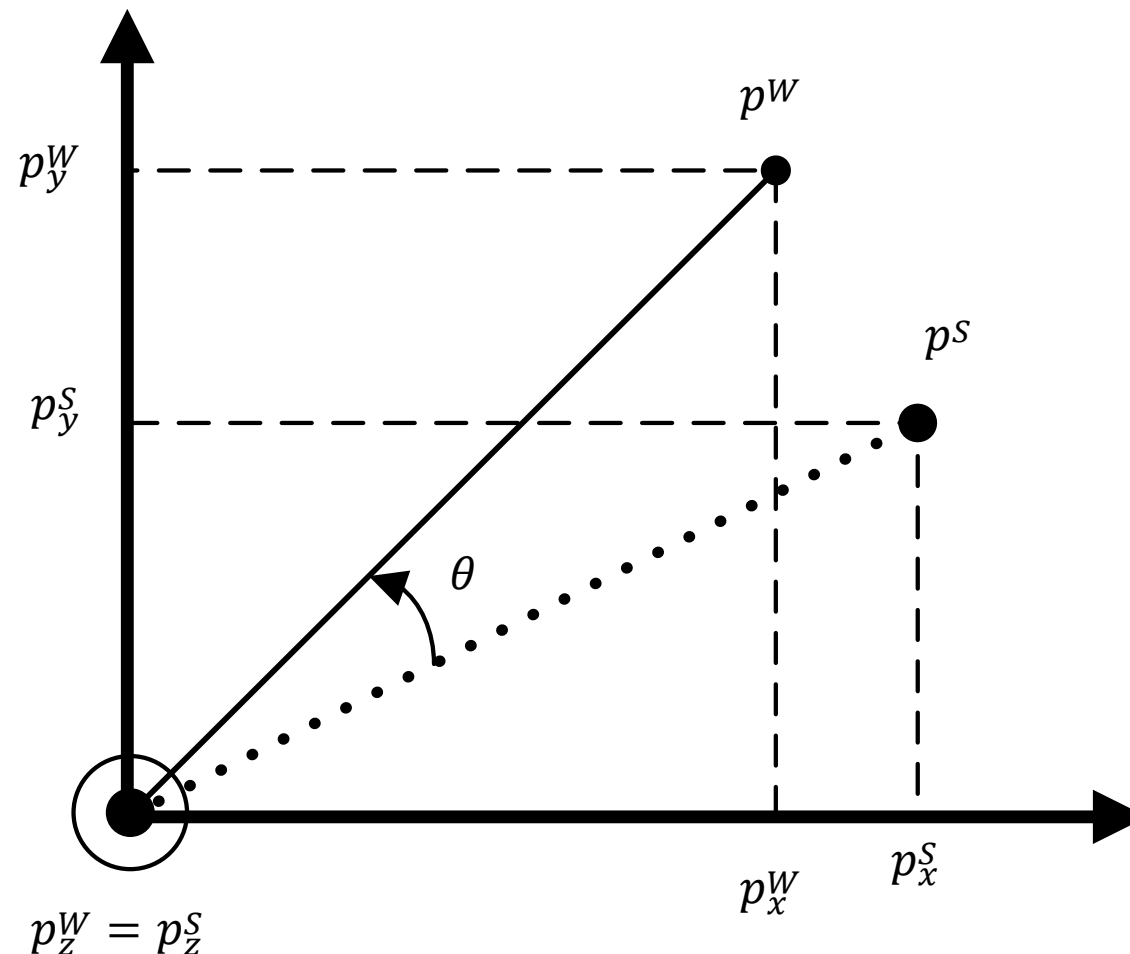
- The world frame is fixed
- The sensor frame rotate
- p^S and p^W express the point coordinates in the two frames
- The focus of this lab is to estimate the rotation $R^{W/S}$ based on the sensor measurements available from a smartphone
- By estimating $R^{W/S}$ we are able to obtain the object attitude in the fixed frame



Frames

A rotation matrix has 3 meanings:

1. Gives the orientation of one set of coordinates with respect to another
2. Represents a coordinates transformation that relates the coordinates of the same point in two different frames
3. Is the operator which rotates a vector in the same frame



Attitude representation

An object attitude can be expressed via rotation matrices (9 elements)

- The 9 elements are not independent because of orthogonality constraints → 3 independent parameters (3 angles)

Euler angles

$$R = R_z(\varphi)R_{y'}(\vartheta)R_{z''}(\psi) \quad \text{ZYZ representation (current frame)}$$

$$R = R_z(\varphi)R_y(\vartheta)R_x(\psi) \quad \text{RPY representation (fixed frame)}$$

PROBLEM: Singularities

Different position in the same have the same representation

Quaternions

- Non minimal representation (4 parameters) but with no singularities
- Represent a rotation of an angle α around the axis \hat{v} , $||\hat{v}|| = 1$

$$q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{1}{2}\alpha) \\ \sin(\frac{1}{2}\alpha) \begin{pmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \cos(\frac{1}{2}\alpha) \\ \sin(\frac{1}{2}\alpha)\hat{v} \end{pmatrix} \quad q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

- We have that q and $-q$ represents the same rotation. So the representation of a rotation must be chosen according to the specific application

Rotation matrix corresponding to a quaternion

$$R = Q(q) = \begin{pmatrix} 2q_0^2 - 1 + 2q_1^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 2q_0^2 - 1 + 2q_2^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 2q_0^2 - 1 + 2q_3^2 \end{pmatrix}$$

Time derivative of a quaternion

$$\dot{q} = \frac{1}{2}S(\omega)q = \frac{1}{2} \begin{pmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{pmatrix} q$$

Discretize the derivative $\dot{q} = \frac{q^{(k+1)} - q^{(k)}}{T} \Rightarrow$

$$q_{k+1} = \left(I + \frac{1}{2}S(\omega_k)T \right) q_k$$

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Simple model

$$x_k = q_k, \quad u_k = \omega_k, \quad y_k = \left(y_k^{a,T} \quad y_k^{m,T} \right)^T$$

- **The unknown states** are the quaternion components. We want to estimate them in order to obtain the object attitude which we do not measure (4-d vector)
- **Input:** gyroscope measurements (3-d vector)
- **Outputs:** accelerometers and magnetometer measurements (6-d vector)

Model state space equations

$$\begin{cases} q(k+1) = [I_4 + \frac{1}{2} S(w(k)) \cdot T] \cdot q(k) + e^q(k) \\ y^a(k) = Q^T(q(k)) \cdot g^o + e^a(k) \\ y^a(k) = Q^T(q(k)) \cdot m^o + e^a(k) \end{cases}$$

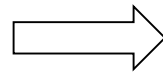
$$g^o = \begin{pmatrix} 0 \\ 0 \\ 9.81 \end{pmatrix}$$

$$m^o = \begin{pmatrix} 0 & \sqrt{m_x^2 + m_y^2} & m_z \end{pmatrix}^T$$

Linearization – state space matrix $F_{4 \times 4}$

$$q(k+1) = q(k) + \frac{1}{2}T \cdot \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \cdot \begin{bmatrix} q_1(k) \\ q_2(k) \\ q_3(k) \end{bmatrix}$$

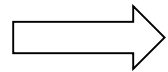
$$F = \begin{bmatrix} \partial f_1 / \partial x_1 & \dots & \partial f_1 / \partial x_n \\ \vdots & & \vdots \\ \partial f_n / \partial x_1 & \dots & \partial f_n / \partial x_n \end{bmatrix}_{\substack{x=\bar{x} \\ u=\bar{u}}}$$



$$F = \begin{bmatrix} 1 & -\frac{1}{2}T \cdot \omega_x & -\frac{1}{2}T \cdot \omega_y & -\frac{1}{2}T \cdot \omega_z \\ \frac{1}{2}T \cdot \omega_x & 1 & \frac{1}{2}T \cdot \omega_z & -\frac{1}{2}T \cdot \omega_y \\ \frac{1}{2}T \cdot \omega_y & -\frac{1}{2}T \cdot \omega_z & 1 & \frac{1}{2}T \cdot \omega_x \\ \frac{1}{2}T \cdot \omega_z & \frac{1}{2}T \cdot \omega_y & -\frac{1}{2}T \cdot \omega_x & 1 \end{bmatrix}$$

Linearization – input matrix $G_{4 \times 3}$

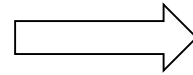
$$G = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_3} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} & \frac{\partial f_3}{\partial u_3} \\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} & \frac{\partial f_4}{\partial u_3} \end{bmatrix}$$



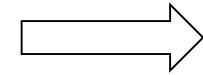
$$G = \begin{bmatrix} -\frac{1}{2}T \cdot q_1 & -\frac{1}{2}T \cdot q_2 & -\frac{1}{2}T \cdot q_3 \\ \frac{1}{2}T \cdot q_0 & -\frac{1}{2}T \cdot q_3 & \frac{1}{2}T \cdot q_2 \\ \frac{1}{2}T \cdot q_3 & \frac{1}{2}T \cdot q_0 & -\frac{1}{2}T \cdot q_1 \\ -\frac{1}{2}T \cdot q_2 & \frac{1}{2}T \cdot q_1 & \frac{1}{2}T \cdot q_0 \end{bmatrix}$$

Linearization – Output matrix $\mathbf{H}_{6 \times 4}$

$$\mathbf{H} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_4} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_6}{\partial x_1} & \cdots & \frac{\partial g_6}{\partial x_4} \end{bmatrix}$$



$$m_{xy} = \sqrt{m_x^2 + m_y^2}$$



$$\mathbf{H} = \begin{bmatrix} 9.81 \cdot (-2q_2) & 9.81 \cdot 2q_3 & 9.81 \cdot (-2q_0) & 9.81 \cdot 2q_1 \\ 9.81 \cdot 2q_1 & 9.81 \cdot 2q_0 & 9.81 \cdot 2q_3 & 9.81 \cdot 2q_2 \\ 9.81 \cdot 4q_0 & 0 & 0 & 9.81 \cdot 4q_3 \\ m_{xy} \cdot 2q_3 + m_z \cdot (-2q_2) & m_{xy} \cdot 2q_2 + m_z \cdot 2q_3 & m_{xy} \cdot 2q_1 + m_z \cdot (-2q_0) & m_{xy} \cdot 2q_0 + m_z \cdot 2q_1 \\ m_{xy} \cdot 4q_0 + m_z \cdot 2q_1 & m_z \cdot 2q_0 & m_{xy} \cdot 4q_2 + m_z \cdot 2q_3 & m_z \cdot 2q_2 \\ m_{xy} \cdot (-2q_1) + m_z \cdot 4q_0 & m_{xy} \cdot (-2q_0) & m_{xy} \cdot 2q_3 & m_{xy} \cdot 2q_2 + m_z \cdot 4q_3 \end{bmatrix}$$

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Steps

1. Check sensors frames on the smarthpone
2. Acquire static measurement to compensate for biases in the sensors
3. Calibrate magnetometer
4. Implement the code