# Different applications of CATE + ASH

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#### Abstract

I look at the performance of different options of CATE and compare them against the performance of MOUTHWASH. In terms of estimating  $\pi_0$ , if control genes are not known, it looks like the robust regression version of CATE using PCA to estimate the latent variables and then using ASH to estimate  $\pi_0$  works the best when the sample size is at least 20. In terms of AUC, any of the CATE + ASH methods will work. MOUTHWASH always underestimates  $\pi_0$ .

## 1 Methods

For CATE, I varied three parameters.

- 1. The factor analysis method: Either quasi-maximimum likelihood ("ml"), PCA ("pc"), or an early stopping method I haven't read about but is an option ("esa").
- 2. Whether the p-values are calibrated using maximum absolute deviation (TRUE) or not (FALSE). This only matters for the qualue methods and shouldn't affect the ASH methods.
- 3. Whether we used the robust-regression version of CATE ("rr") or the negative controls verison of CATE ("nc") using half of the null genes as the negative controls.

For each setting in CATE, I performed two methods. The first method consisted of a two-step procedure:

- 1. Estimate  $\hat{\beta}_{[2,i]}$  and it's corresponding standard error  $\hat{s}_i$ .
- 2. Run ASH on  $\hat{\beta}_{[2,i]}$  and  $\hat{s}_i$ .

The second method was to use the p-values output by CATE.

I always ran CATE on  $\log(COUNTS + 1)$ .

The ASH methods provide an estimate of  $\pi_0$ . I obtained an estimate of  $\pi_0$  from the p-values by the qvalue package in R [Storey, 2002].

The number of hidden confounders was estimated using the methods of Buja and Eyuboglu [1992] implemented in the num.sv() function in the sva package in R. CATE doesn't work sometimes when there is only one confounder, so I set the minimum number to 2 confounders.

For MOUTHWASH:

- I used PCA to esitmate the hidden confounders as this is the option that works best in CATE.
- I used the mixture of normals version with the same regularization and grid-size choices as in the ashr package.

## 2 Simulation Study

I ran through 100 repetitions of generating data from GTEX lung data under the following parameter conditions:

- $n \in \{10, 20, 40\},\$
- p = 1000,
- $\pi_0 \in \{0.5, 0.9\},\$
- $\sigma_{log2} \in \{1, 5\}.$

I extracted the most expressed p genes (excluding the top 5 expressed genes) from the GTEX lung data and n samples are chosen at random. Half of these samples are randomly given the "treatment" label 1, the other half given the "treatment" label 0. Of the p genes,  $\pi_0 p$  were chosen to be non-null. Signal was added by the Poisson-thinning approach in Mengyin's code with a mean log2-fold change of 0 and a standard deviation log2-fold change of  $\sigma_{log2}$ . That is

$$A_1, \dots, A_{p/2} \sim N(0, \sigma_{log2}^2)$$
 (1)

$$B_i = 2^{A_i} \text{ for } i = 1, \dots, p/2.$$
 (2)

If  $A_i > 0$  then we replace  $Y_{[1:(n/2),i]}$  with  $Binom(Y_{[j,i]}, 1/B_i)$  for j = 1, ..., n/2. If  $A_i < 0$  then we replace  $Y_{[(n/2+1):n,i]}$  with  $Binom(Y_{[j,i]}, B_i)$  for j = n/2 + 1, ..., n.

For each iteration, I calculated two things:

- 1. The AUC using either the lfdrs or p-values.
- 2. The estimates of  $\pi_0$ .

## 3 Results

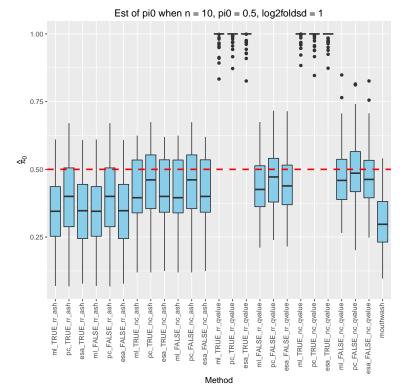
For the frequentist procedures, I used the vector of p-values as the predictions and I used the vector of lfdr's from the ASH-like procedures for prediction. These were used to create ROC curves and calculate AUCs. In general, ASH procedures performed better than just using the p-values and using negative controls worked better than the robust regression version. MOUTHWASH does OK in terms of AUC — sometimes the best (even compared to the methods that use negative controls), sometimes among the top.

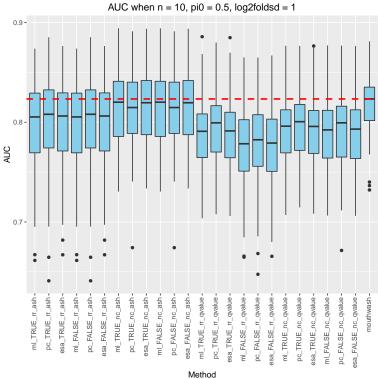
From the p-values, I used the qvalue package [Storey, 2002] to estimate  $\pi_0$ . Estimates of  $\pi_0$  are given from ashr for the ASH-like methods.

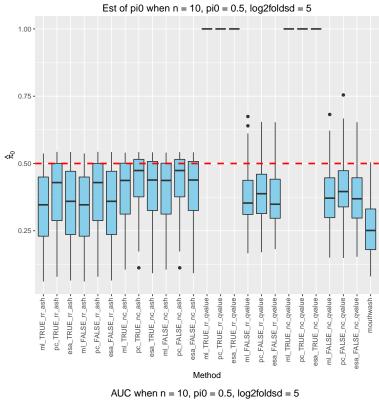
For a sample size of 10 (5 vs 5), none of the methods perform well at estimating  $\pi_0$ . But when the sample size is 20 or 40, negative control version of CATE + ASH performs remarkably well. Interestingly, the robust regression version of CATE + ASH works almost as well when we use PCA to estimate the hidden confounders instead of the "ML" or "ESA" options in CATE.

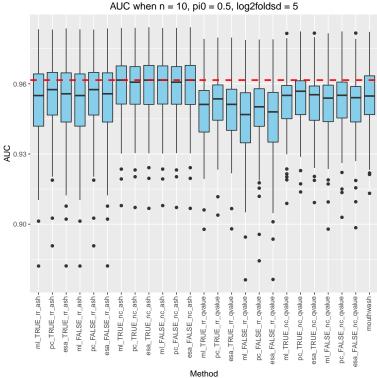
MOUTHWASH always underestimates  $\pi_0$ .

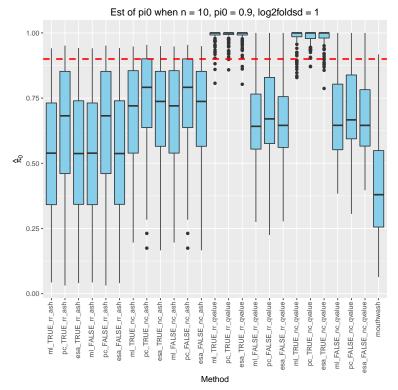
Inserting the calibrated p-values of CATE into qvalue always resulted in very high estimates of  $\pi_0$ .

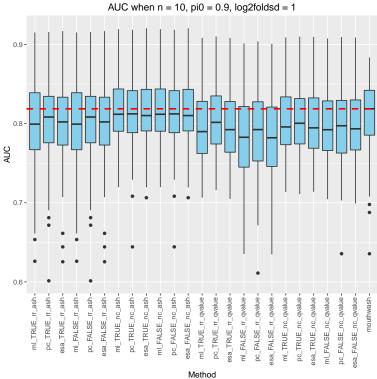


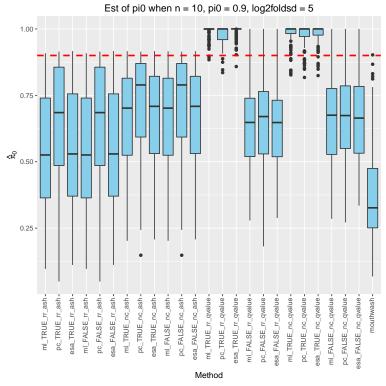


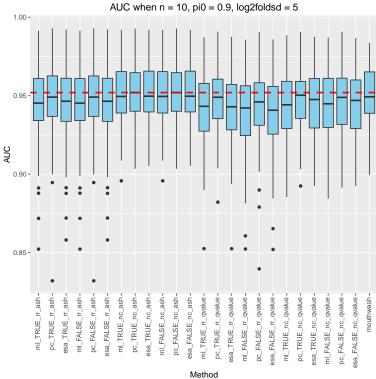


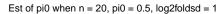


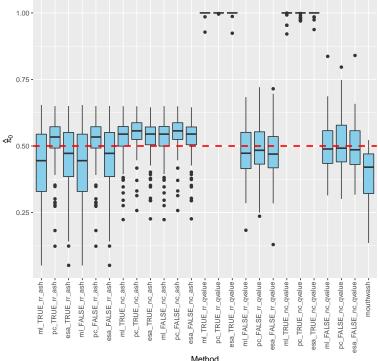




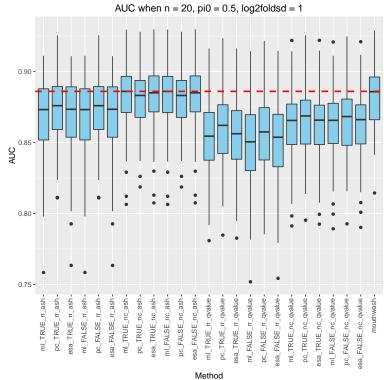


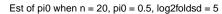


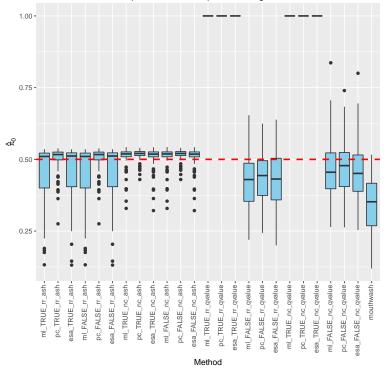




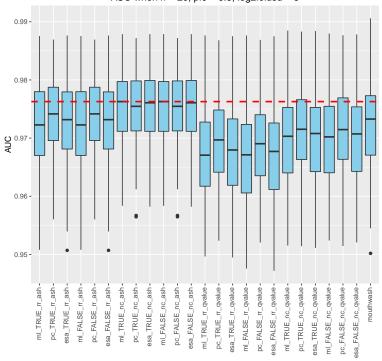
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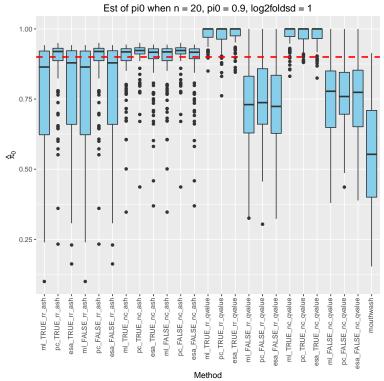


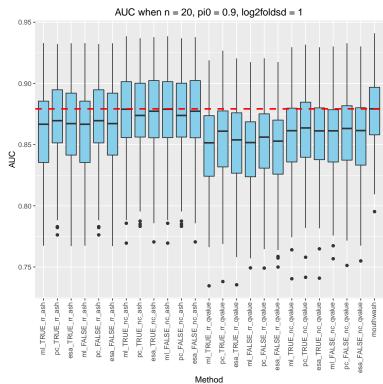


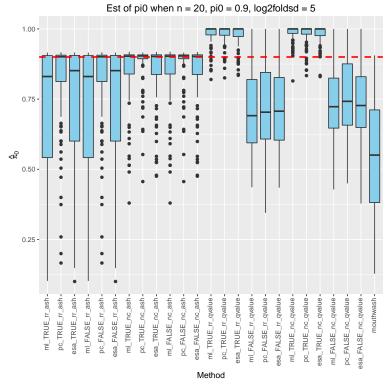


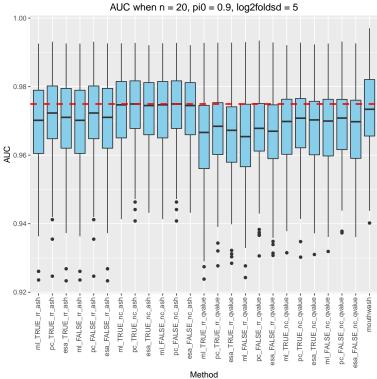
### AUC when n = 20, pi0 = 0.5, log2foldsd = 5

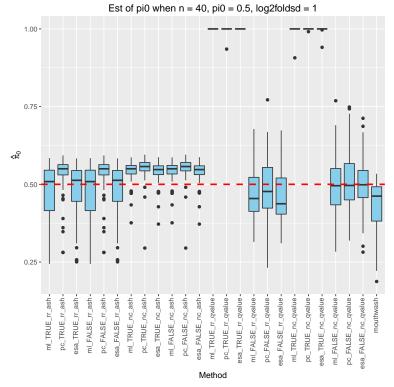


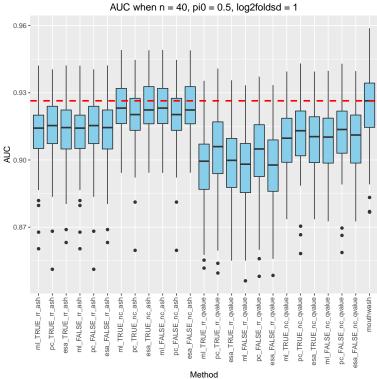


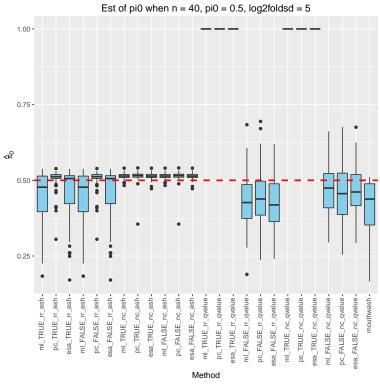


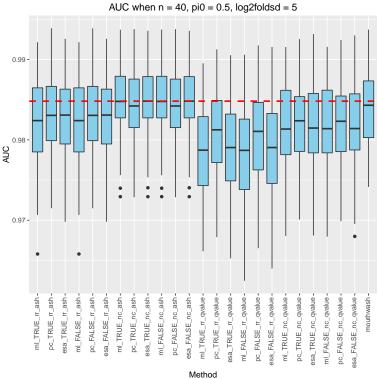


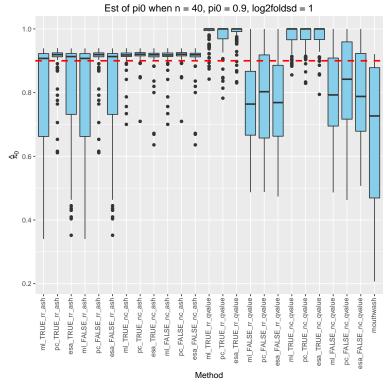


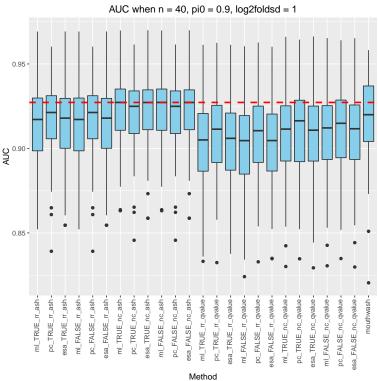


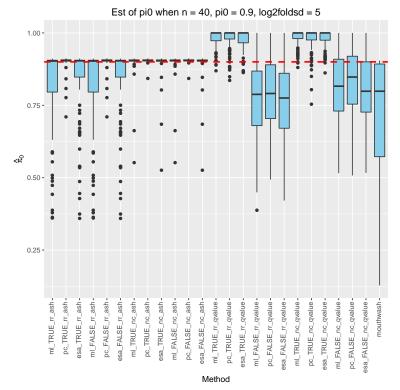


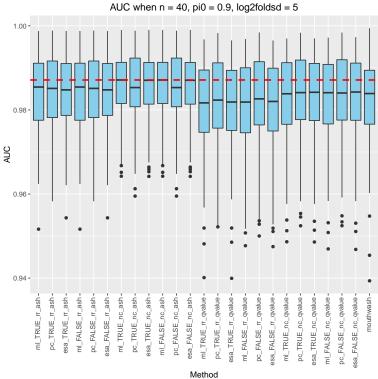












# References

Andreas Buja and Nermin Eyuboglu. Remarks on parallel analysis.  $Multivariate\ behavioral\ research,\ 27(4):509–540,\ 1992.$ 

John D Storey. A direct approach to false discovery rates. *Journal of the Royal Statistical Society:* Series B (Statistical Methodology), 64(3):479–498, 2002.