Means and Standard Deviations of T-statistics in All Null Setting

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Abstract

Under the all-null setting, I calculate t-statistics from different versions of RUV4. I plot the standard deviations and means of the t-statistics.

Simulations

I take subsets of the GTEX muscle data with $p = 1000$, randomly assign half of the observations to be "treatments" and the other half to be "controls". I then run RUV4, giving it half of the observations as negative controls and obtain its t-statistics. I do this in four ways:

1. The t-statistics output from RUV4, which uses OLS to estimate the confounders.
2. RUV4 using GLS to estimate the confounders.
3. RUV4 with variance inflation but no ad-hoc multiplication by $n/(n-p-q)$.
4. RUV4 with variance inflation AND ad-hoc multiplication by $n/(n-p-q)$.

I then calculate the means and standard deviations of the t-statistics.

Results

Above, method 3 results in standard deviations near 1. The no-inflation t-statistics standard deviations have a mean close to the average theoretical standard deviations (red horizontal lines). I calculated these average theoretical standard deviations by taking

$$df = n - k - q \tag{1}$$

$$sd = \sqrt{df/(df-2)}, \tag{2}$$

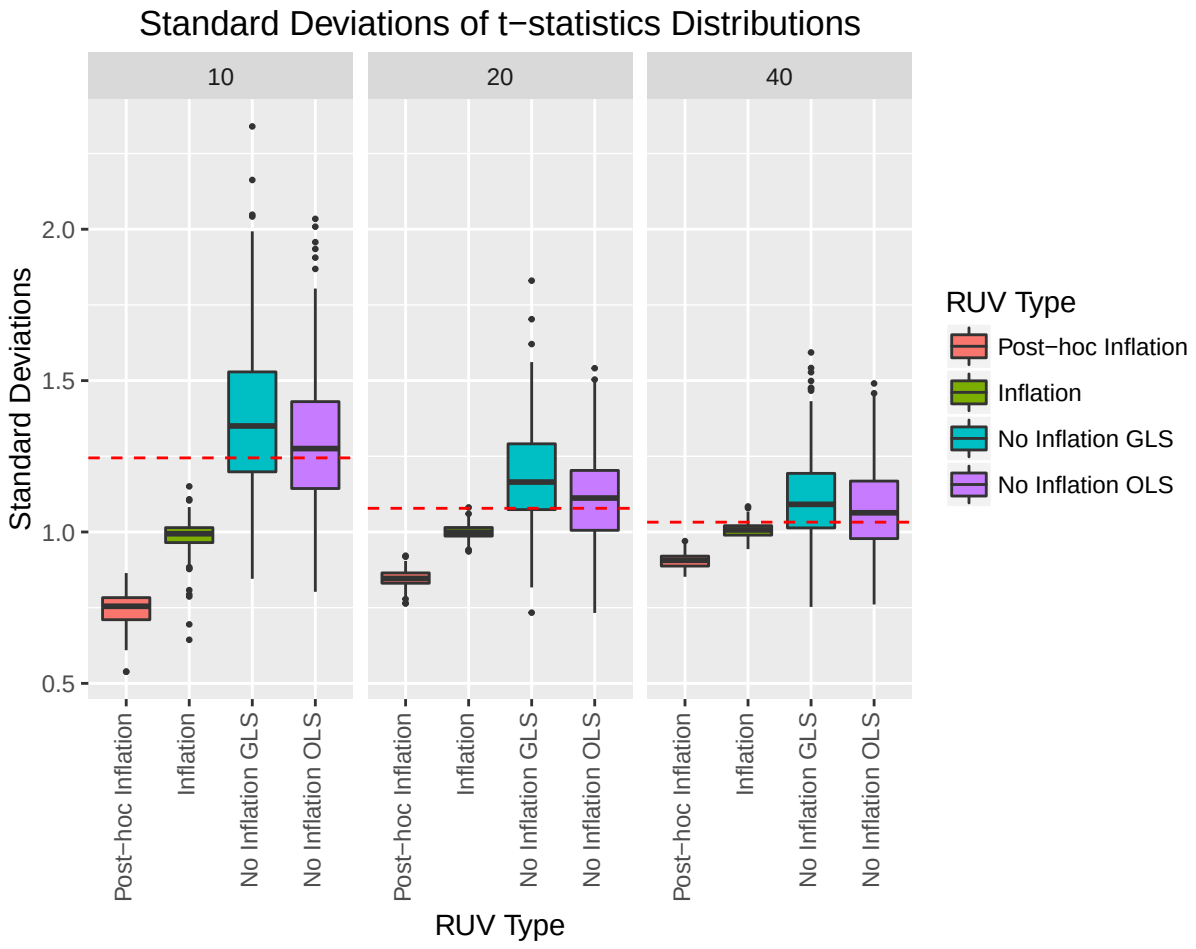
then averaging these standard deviations. Here, $k = 2$ (number of covariates), n is the sample size, and q is the estimated number of hidden confounders calculated using `num.sv`.

However, the standard deviations of the standard deviations of the t-statistics using non-inflated variance is much larger than the t-statistics using inflated variance.

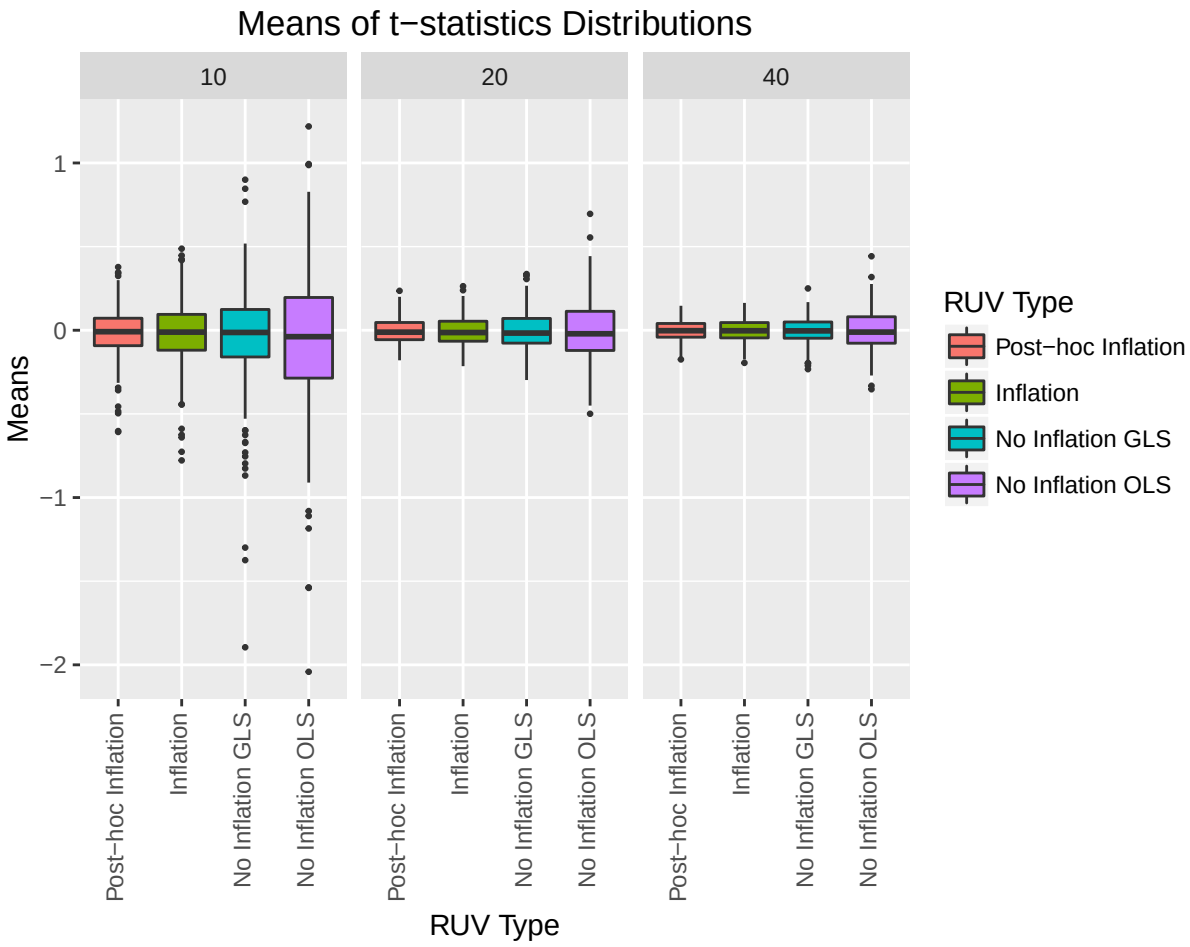
The variance inflation parameter was estimated assuming a normal distribution. If assuming a t-distribution, I suspect the t-values would be pretty close to the red line.

```
library(ggplot2)
library(reshape2)

itermax <- 200
Nsamp_seq <- c(10, 20, 40)
```

```
par_vals <- expand.grid(list(1:itermax, Nsamp_seq))


nullmat <- read.csv("null_mat.csv")
names(nullmat) <- c("inp_mean", "inp_sd", "i_mean", "i_sd", "s_mean",
                    "s_sd", "scale", "adhoc_mult")
nullmat$sgls_mean <- nullmat$inp_mean * sqrt(nullmat$scale)
nullmat$sgls_sd <- nullmat$inp_sd * sqrt(nullmat$scale)

nullmat$Nsamp <- par_vals$Var2

numsv <- nullmat$Nsamp - nullmat$Nsamp / nullmat$adhoc_mult^2 -  2
nu <- nullmat$Nsamp - numsv - 2
nullmat$tsd <- sqrt(nu / (nu - 2))
dummy_df <- aggregate(tsd ~ Nsamp, data = nullmat, FUN = mean)
names(dummy_df) <- c("Nsamp", "mean_sd")

sd_mat <- melt(nullmat, measure.vars = c("inp_sd", "i_sd", "sgls_sd", "s_sd"),
               id.vars = "Nsamp")

ggplot(data = sd_mat, mapping = aes(y = value,
                                    x = factor(variable,
                                               labels = c("Post-hoc Inflation",
                                                          "Inflation",
                                                          "No Inflation GLS",
                                                          "No Inflation OLS")),
                                    fill = factor(variable,
                                               labels = c("Post-hoc Inflation",
                                                          "Inflation",
                                                          "No Inflation GLS",
                                                          "No Inflation OLS")))) +
    facet_grid(.~Nsamp) +
    geom_boxplot(outlier.size = 0.5) +
    ggtitle("Standard Deviations of t-statistics Distributions") +
    guides(fill = guide_legend(title = "RUV Type")) +
    geom_hline(data = dummy_df, aes(yintercept = mean_sd), col = 2, lty = 2) +
    theme(axis.text.x = element_text(angle = 90, hjust = 1, vjust = 0.3)) +
    xlab("RUV Type") + ylab("Standard Deviations")
```




```
mean_mat <- melt(nullmat, measure.vars = c("inp_mean", "i_mean", "sgls_mean", "s_mean"),
                 id.vars = "Nsamp")
ggplot(data = mean_mat, mapping = aes(y = value,
                                      x = factor(variable,
                                                 labels = c("Post-hoc Inflation",
                                                            "Inflation",
                                                            "No Inflation GLS",
                                                            "No Inflation OLS")),
                                      fill = factor(variable,
                                                    labels = c("Post-hoc Inflation",
                                                               "Inflation",
                                                               "No Inflation GLS",
                                                               "No Inflation OLS")))) +
    facet_grid(.~Nsamp) +
    geom_boxplot(outlier.size = 0.5) +
    ggtitle("Means of t-statistics Distributions") +
    guides(fill = guide_legend(title = "RUV Type"))  +
    theme(axis.text.x = element_text(angle = 90, hjust = 1, vjust = 0.3)) +
    xlab("RUV Type") + ylab("Means")
```


Means of t−statistics Distributions
10
20
40
1
0
−1
−2
Means
Post−hoc Inflation
Inflation
No Inflation GLS
No Inflation OLS
RUV Type
RUV Type
Post−hoc Inflation
Inflation
No Inflation GLS
No Inflation OLS