# SUCCOTASH when $\alpha$ and $\tau$ are known, but $\Sigma$ is "estimated".

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**Abstract**

Here, I compare SUCCOTASH to LEAPP when the assumed $\Sigma$ is a noisy version of the true $\Sigma$.

## 1 Model Description

$$Y_{p\times 1} = \beta_{p\times 1} + \alpha_{p\times k} Z_{k\times 1} + E_{p\times 1}, \tag{1}$$

such that

- $\alpha$ is known.
- $E \sim N_p(0, \Sigma)$, $\Sigma = \text{diag}\{\sigma_1^2, \ldots, \sigma_p^2\}$.

## 2 Procedure

- $p = 100$,
- $k \in \{5, 10, 50\}$,
- $\beta_j \sim N(0, \tau_k^2)$ w.p. $\pi_k$,
- $\tau_k^2 = 0, 1, 100$ for $k = 0, 1, 2$ when we have a three mixture and $\tau_k = 0, 100$ for $k = 0, 1$ when we have a two mixture model,
- $\pi \in \{(0.5, 0.5), (0.9, 0.1), (1, 0, 0), (0.9, 0.1, 0), (0.9, 0, 0.1), (0.5, 0.5, 0), (0.5, 0, 0.5), (0.5, 0.25, 0.25)\}$
- $Z_j \overset{i.i.d.}{\sim} N(0, 1)$,
- $\alpha_{ij} \overset{i.i.d.}{\sim} N(0, 1)$,
- 400 iterations for each $\pi$ by $k$ combination, sampling a new $Z$ and $\alpha$ at each iteration.
- The true $\Sigma$ I used to generate the data is $\Sigma = I_p$.
- I drew $\hat{\Sigma} = \text{diag}\{\hat{\sigma}_1, \ldots, \hat{\sigma}_p\}$ from $\hat{\sigma}_i \sim \sigma_i^2 \chi_5^2/5$. This roughly corresponds to having a sample size of 7 (if we are just estimating a slope and intercept). I used this $\hat{\Sigma}$ when estimating $\beta$ and $Z$ in (1). But again, $\sigma_i^2 = 1$ for all $i = 1, \ldots, p$.

- I did not regularize the estimates of $\pi$ for SUCCOTASH.
- At each iteration, I calculated the Sum of Squared Errors (SSE) for the posterior means under SUCCOTASH, and the estimates of $\beta$ given by the second step of LEAPP.
- I also calculated the SSE when using just $Y$ to estimate $\beta$ (called OLS in Figures and Tables below).
- I also calculated $\hat{\pi}_0$ given by SUCCOTASH and LEAPP at each iteration.

- LEAPP uses an $L_1$ penalty, so I called its $\hat{\pi}_0$ to just be the proportion of elements of $\beta$ it sets to 0.

## 3 Results

SUCCOTASH beats LEAPP in every scenario (Table 1). It also seems to estimate $\pi_0$ fairly comparably to the case when $\Sigma$ is known (Table 2). There is only one scenario studied where there is a noticable difference. When $k = 50$ and $\pi = (0.5, 0.5)$ with $\tau^2 = (0, 100)$, the mean SUCCOTASH estimate of $\pi_0$ is 0.52 when $\Sigma$ is known. But with the "estimated" $\Sigma$, the mean SUCCOTASH estimate of $\pi_0$ is 0.65.

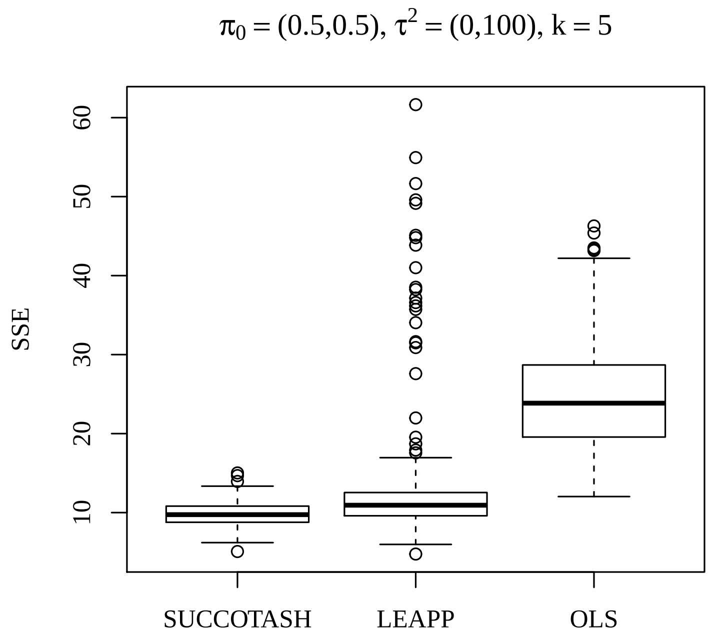
| $k$ | $\pi_0$ | $\pi_1$ | $\pi_2$ | SUC | LEAPP | OLS |
|---|---|---|---|---|---|---|
| 5 | .5 | - | .5 | 9.8 | 12.5 | 24.6 |
| 5 | .9 | - | .1 | 6.1 | 6.4 | 25.0 |
| 5 | 1 | 0 | 0 | 1.6 | 4.0 | 24.8 |
| 5 | .9 | .1 | 0 | 3.7 | 5.2 | 24.3 |
| 5 | .9 | 0 | .1 | 6.7 | 7.1 | 25.2 |
| 5 | .5 | .5 | 0 | 5.9 | 8.0 | 24.6 |
| 5 | .5 | 0 | .5 | 10.3 | 12.4 | 25.2 |
| 5 | .5 | .25 | .25 | 7.5 | 8.5 | 23.7 |
| 10 | .5 | - | .5 | 11.2 | 23.9 | 32.7 |
| 10 | .9 | - | .1 | 5.8 | 6.2 | 32.8 |
| 10 | 1 | 0 | 0 | 0.9 | 3.1 | 32.9 |
| 10 | .9 | .1 | 0 | 3.6 | 5.3 | 33.4 |
| 10 | .9 | 0 | .1 | 6.1 | 6.5 | 33.3 |
| 10 | .5 | .5 | 0 | 6.3 | 8.6 | 33.4 |
| 10 | .5 | 0 | .5 | 10.8 | 22.6 | 32.7 |
| 10 | .5 | .25 | .25 | 9.4 | 12.3 | 33.0 |
| 50 | .5 | - | .5 | 54.1 | 99.1 | 71.7 |
| 50 | .9 | - | .1 | 10.0 | 99.8 | 71.5 |
| 50 | 1 | 0 | 0 | 0.7 | 101.0 | 71.1 |
| 50 | .9 | .1 | 0 | 3.3 | 98.8 | 71.0 |
| 50 | .9 | 0 | .1 | 7.4 | 99.2 | 71.2 |
| 50 | .5 | .5 | 0 | 7.8 | 98.4 | 71.3 |
| 50 | .5 | 0 | .5 | 54.2 | 99.4 | 70.8 |
| 50 | .5 | .25 | .25 | 25.8 | 99.7 | 71.9 |

Table 1: Average Sum of Squared Errors for SUCCOTASH, LEAPP, and OLS at given $k$ and $\pi$ values.

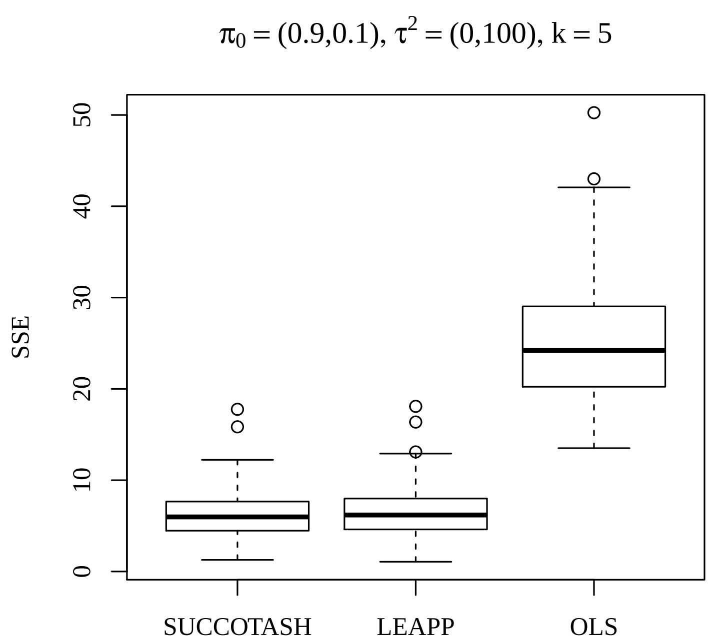
| $k$ | $\pi_0$ | $\pi_1$ | $\pi_2$ | SUC | SUC Sig Known | LEAPP |
|---|---|---|---|---|---|---|
| 5 | .5 | - | .5 | .50 | .51 | .60 |
| 5 | .9 | - | .1 | .89 | .90 | .92 |
| 5 | 1 | 0 | 0 | 1.00 | .98 | .99 |
| 5 | .9 | .1 | 0 | .95 | .92 | .99 |
| 5 | .9 | 0 | .1 | .88 | .86 | .92 |
| 5 | .5 | .5 | 0 | .60 | .57 | .96 |
| 5 | .5 | 0 | .5 | .49 | .48 | .61 |
| 5 | .5 | .25 | .25 | .64 | .57 | .78 |
| 10 | .5 | - | .5 | .51 | .52 | .54 |
| 10 | .9 | - | .1 | .89 | .91 | .91 |
| 10 | 1 | 0 | 0 | 1.00 | .98 | .99 |
| 10 | .9 | .1 | 0 | .97 | .94 | .98 |
| 10 | .9 | 0 | .1 | .89 | .88 | .91 |
| 10 | .5 | .5 | 0 | .63 | .65 | .96 |
| 10 | .5 | 0 | .5 | .51 | .50 | .55 |
| 10 | .5 | .25 | .25 | .67 | .61 | .76 |
| 50 | .5 | - | .5 | .65 | .52 | .16 |
| 50 | .9 | - | .1 | .92 | .90 | .22 |
| 50 | 1 | 0 | 0 | 1.00 | 1.00 | .20 |
| 50 | .9 | .1 | 0 | 1.00 | 1.00 | .19 |
| 50 | .9 | 0 | .1 | .92 | .93 | .20 |
| 50 | .5 | .5 | 0 | .99 | 1.00 | .22 |
| 50 | .5 | 0 | .5 | .67 | .68 | .18 |
| 50 | .5 | .25 | .25 | .79 | .81 | .19 |

Table 2: Mean $\hat{\pi}_0$ for SUCCOTASH and LEAPP at given $k$ and $\pi$ values. Also included are mean $\pi_0$ estimates when $\Sigma$ is known.

## 4 SSE Plots

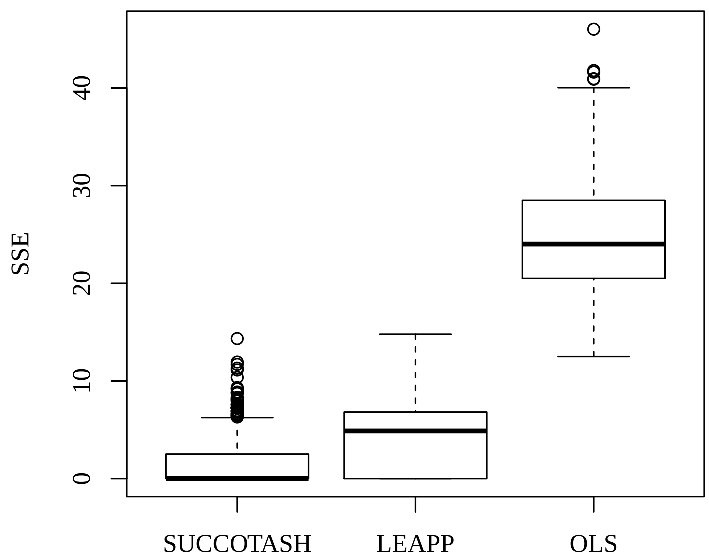


$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 5$
SSE
60
50
40
30
20
10
SUCCOTASH
LEAPP
OLS

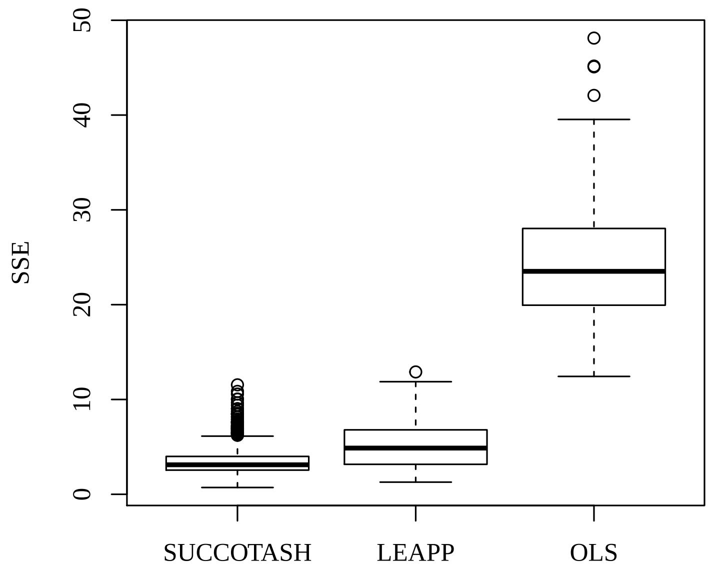



$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 5$
SSE
50
40
30
20
10
0
SUCCOTASH
LEAPP
OLS

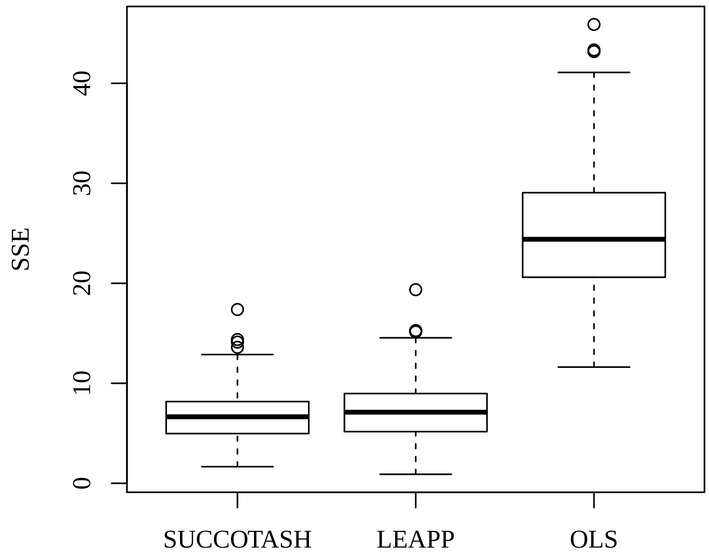
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 5$



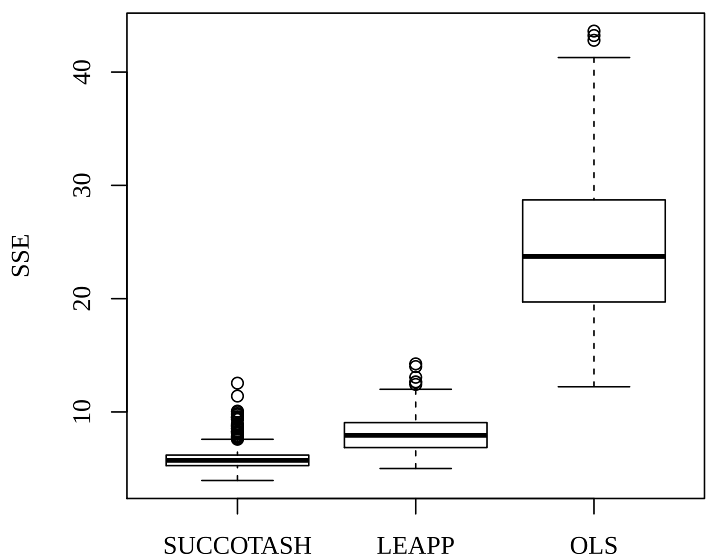

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 5$



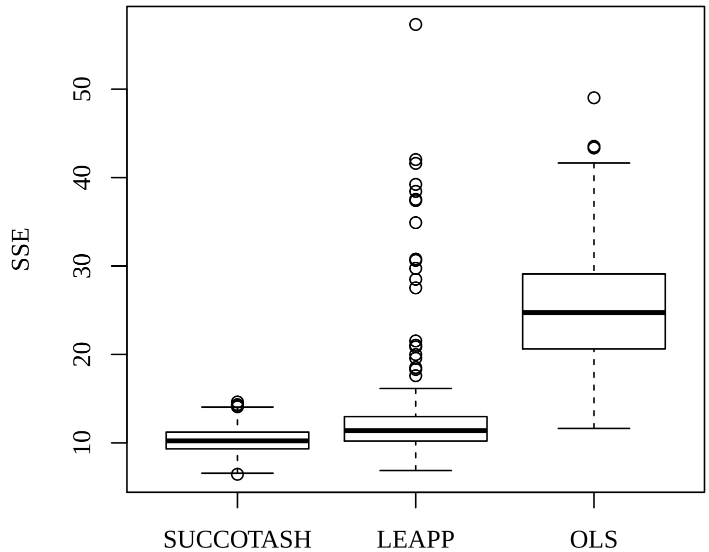
$\pi_0 = (0.9,0,0.1),\ \tau^2 = (0,1,100),\ k = 5$




$\pi_0 = (0.5,0.5,0),\ \tau^2 = (0,1,100),\ k = 5$



$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 5$

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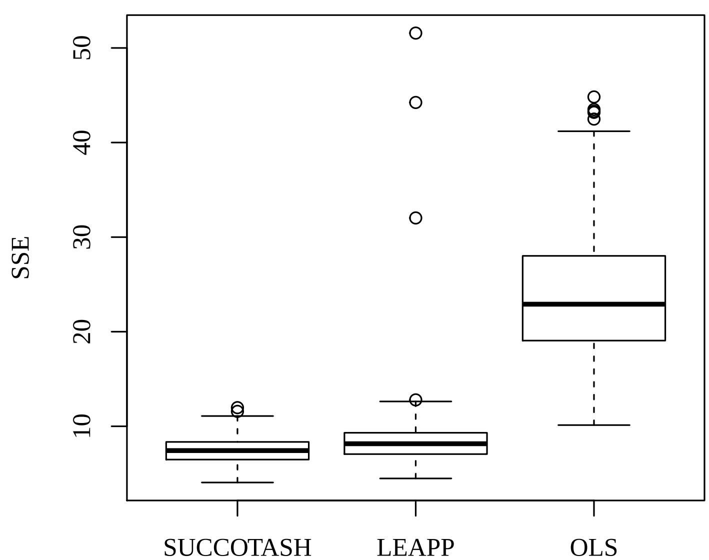
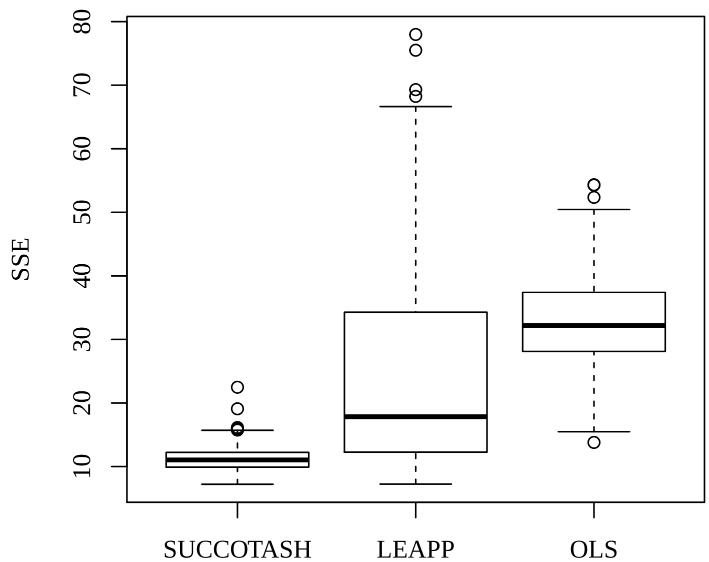
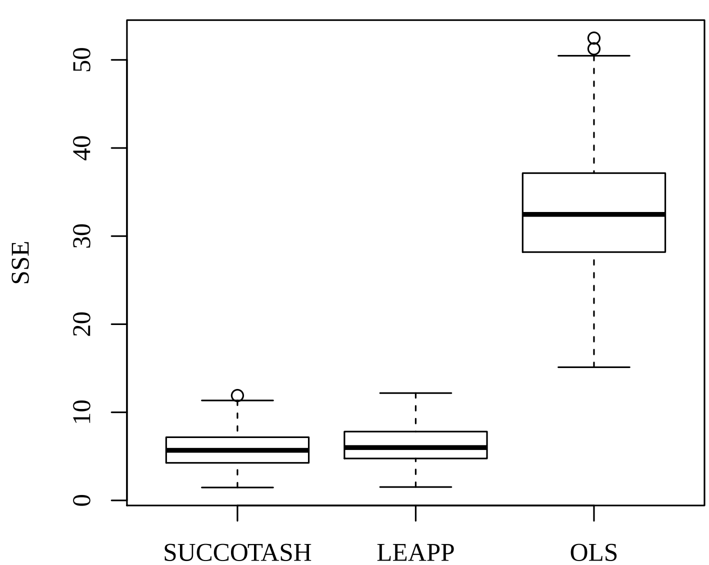
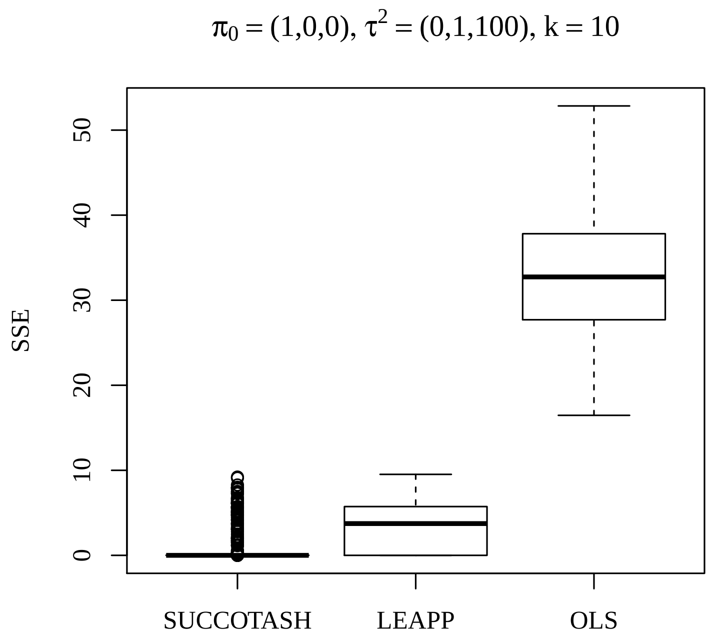


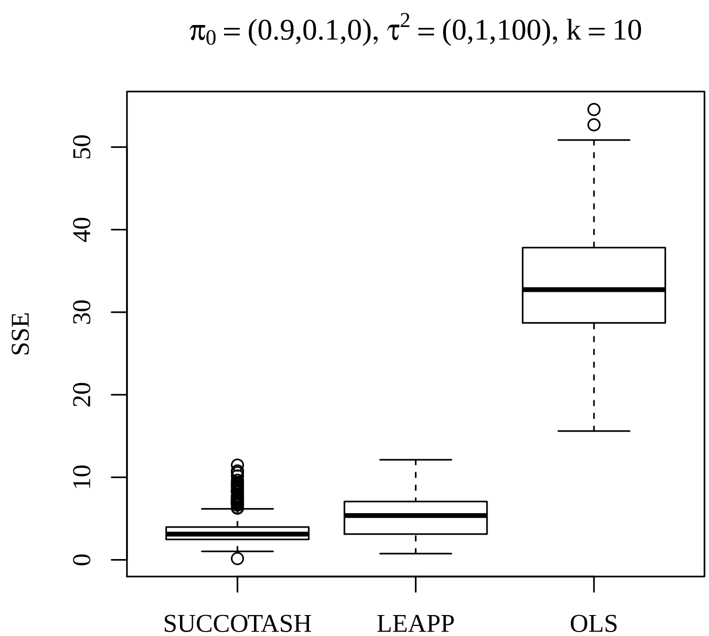
$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, k = 10

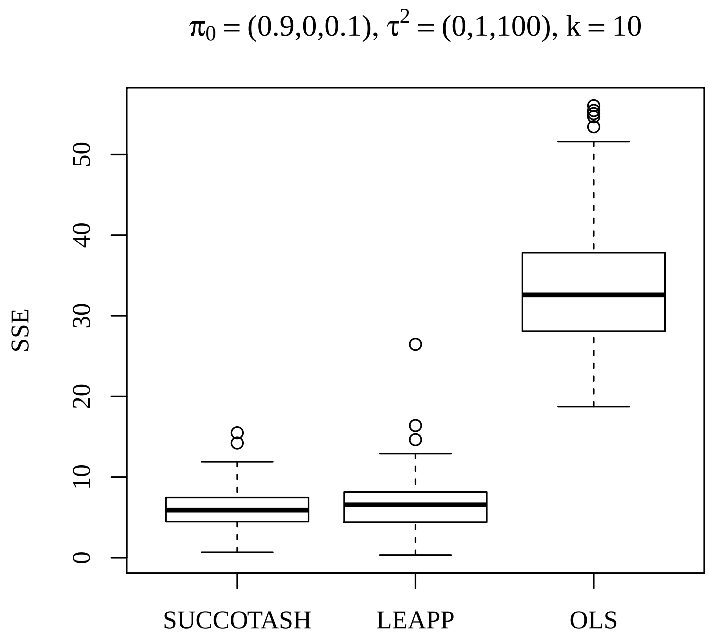


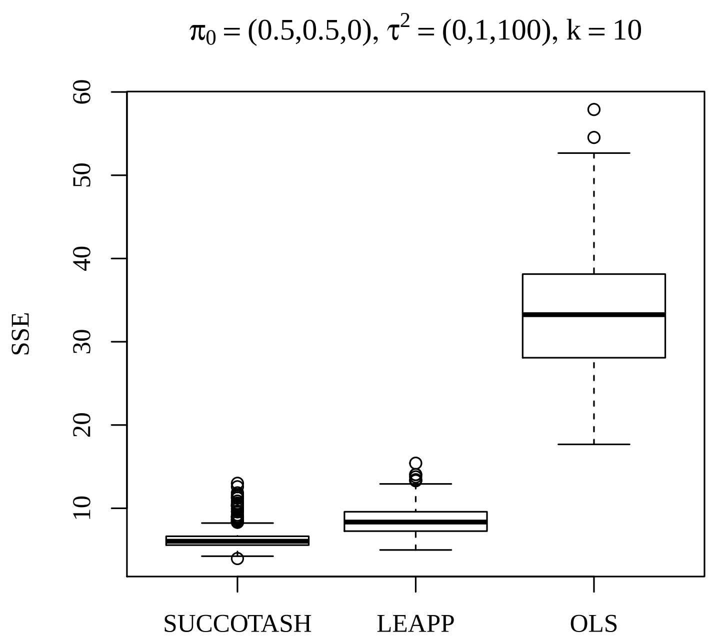

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, k = 10

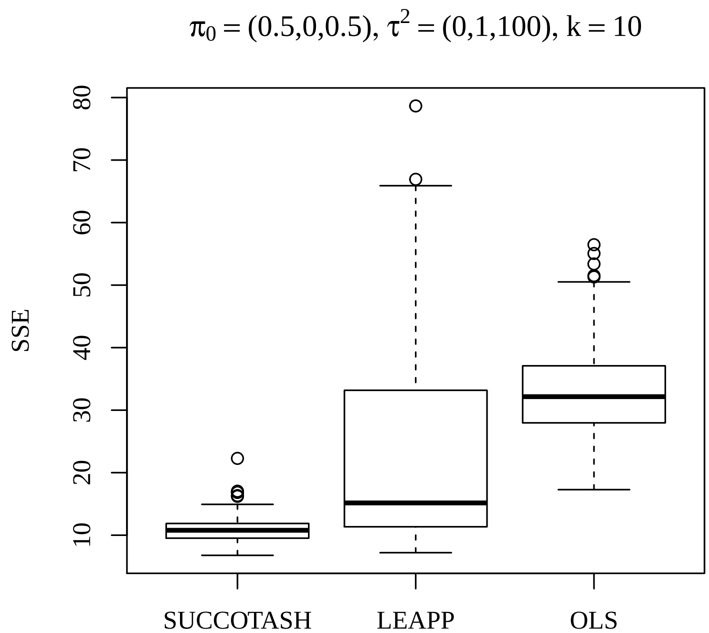


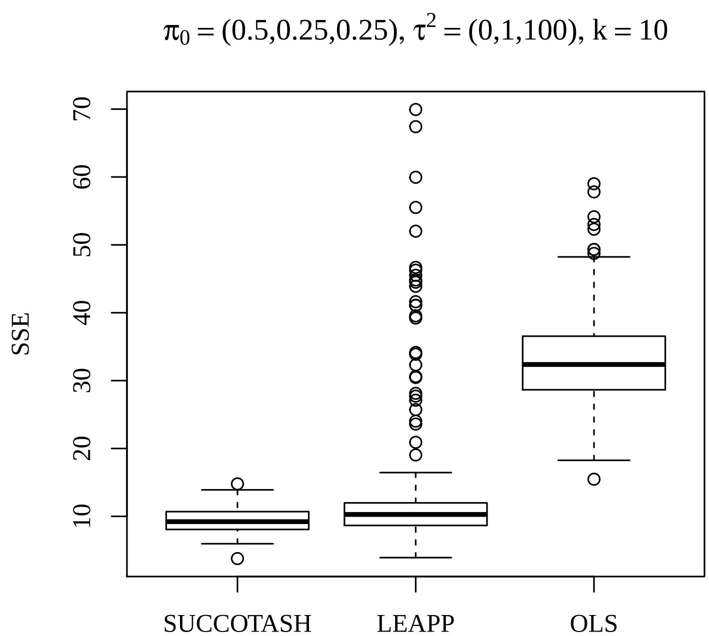

π0 = (1,0,0), τ2 = (0,1,100), k = 10
SSE
0
10
20
30
40
50
SUCCOTASH
LEAPP
OLS


π0 = (0.9,0.1,0), τ2 = (0,1,100), k = 10
SSE
0
10
20
30
40
50
SUCCOTASH
LEAPP
OLS

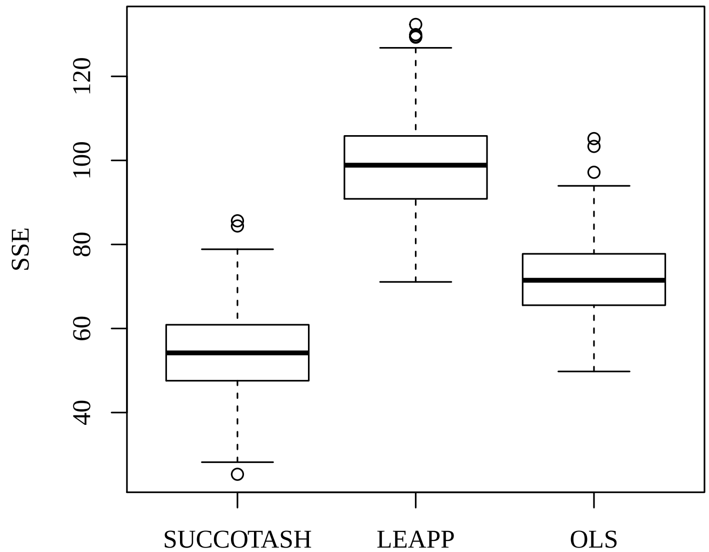

$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
0
10
20
30
40
50
SUCCOTASH
LEAPP
OLS


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
10
20
30
40
50
60
SUCCOTASH
LEAPP
OLS

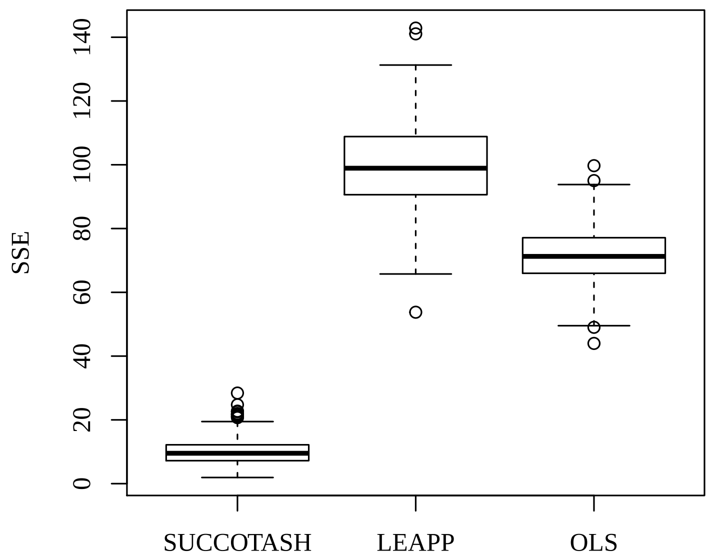

$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, k = 10
SSE
10
20
30
40
50
60
70
80
SUCCOTASH
LEAPP
OLS


$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, k = 10
SSE
10
20
30
40
50
60
70
SUCCOTASH
LEAPP
OLS

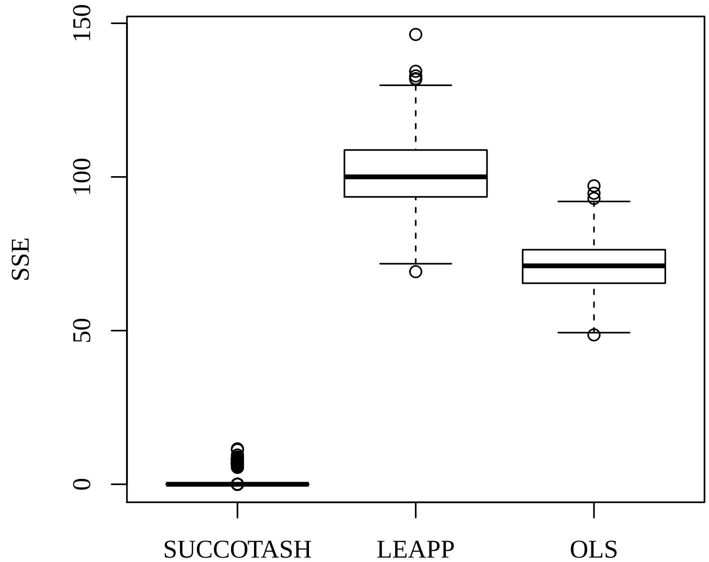
$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 50$



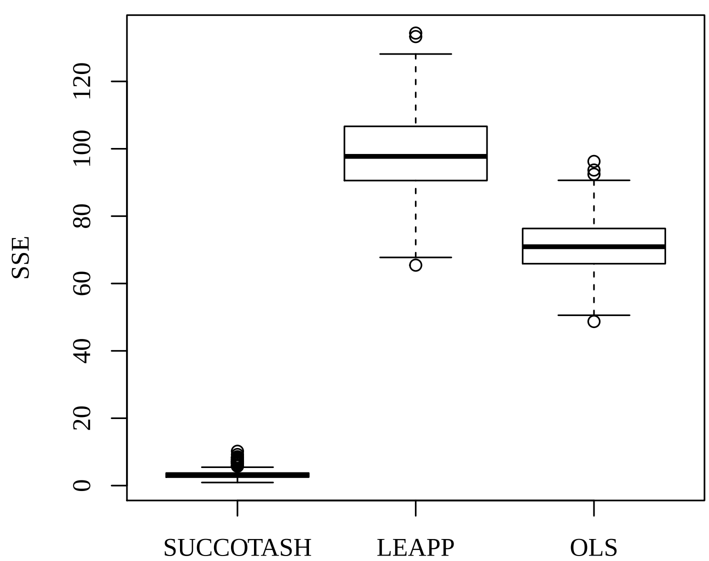

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 50$



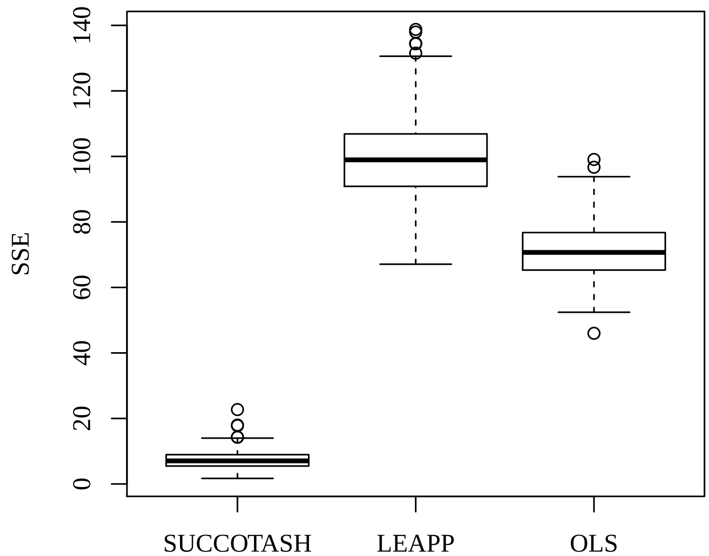
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 50$



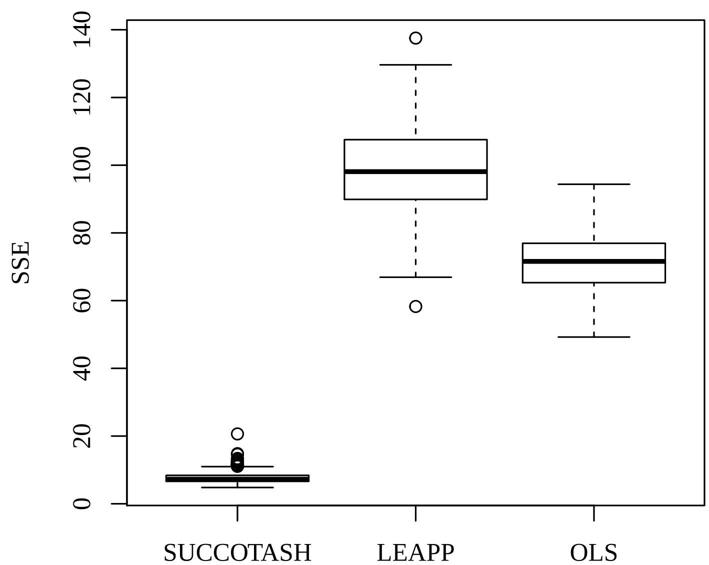

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 50$



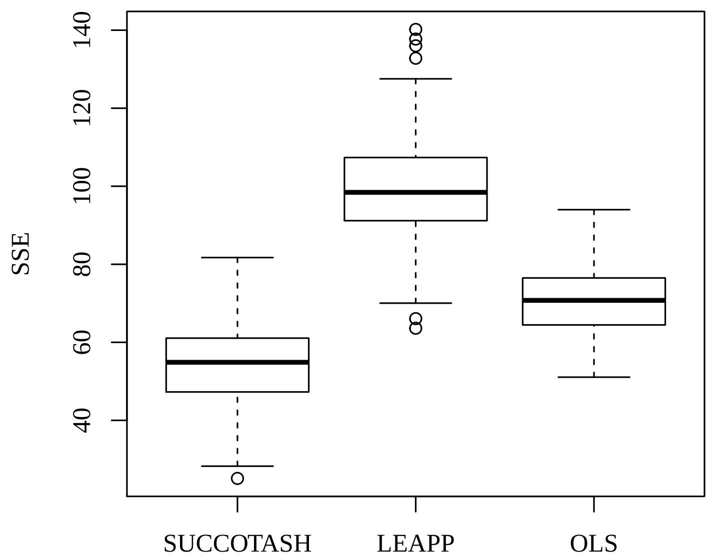
$\pi_0 = (0.9,0,0.1),\ \tau^2 = (0,1,100),\ k = 50$



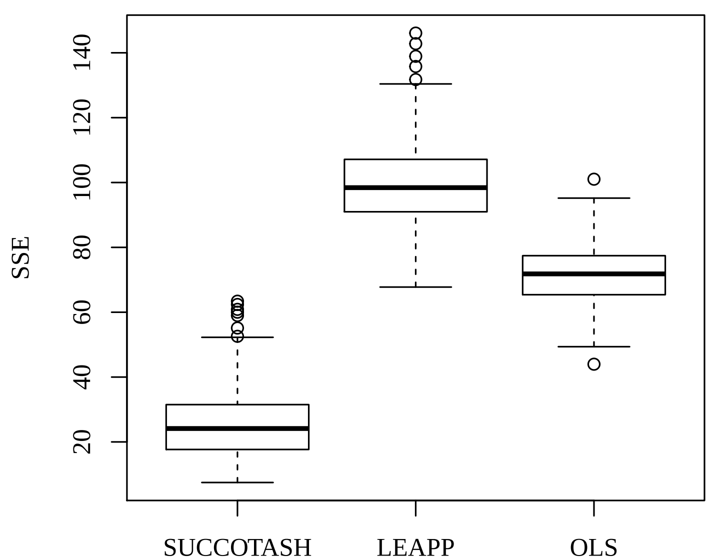

$\pi_0 = (0.5,0.5,0),\ \tau^2 = (0,1,100),\ k = 50$



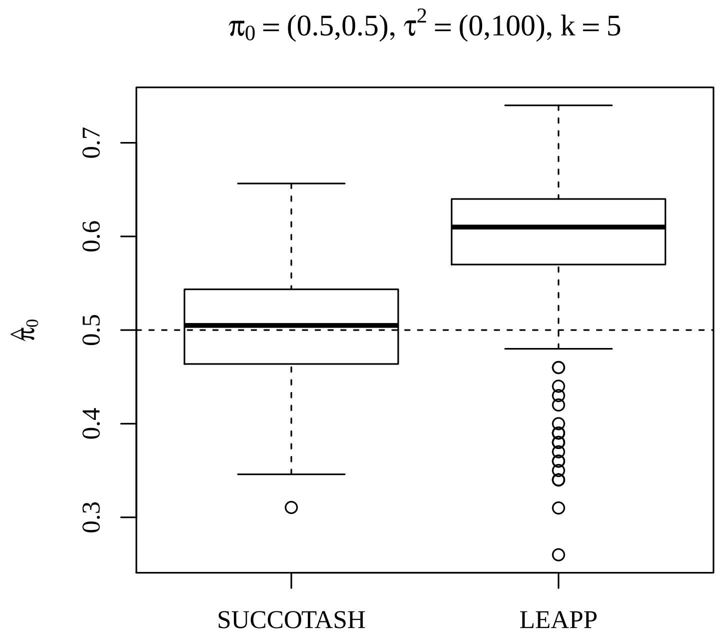
$\pi_0 = (0.5, 0, 0.5)$, $\tau^2 = (0, 1, 100)$, $k = 50$

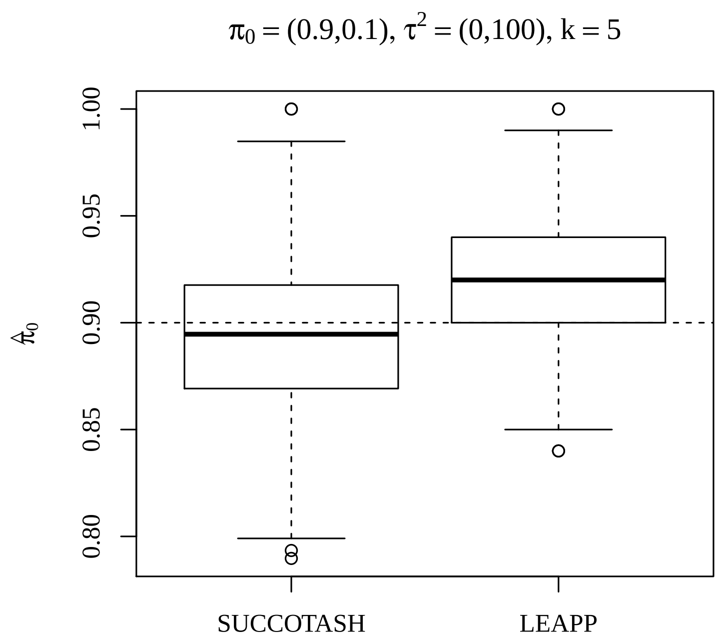


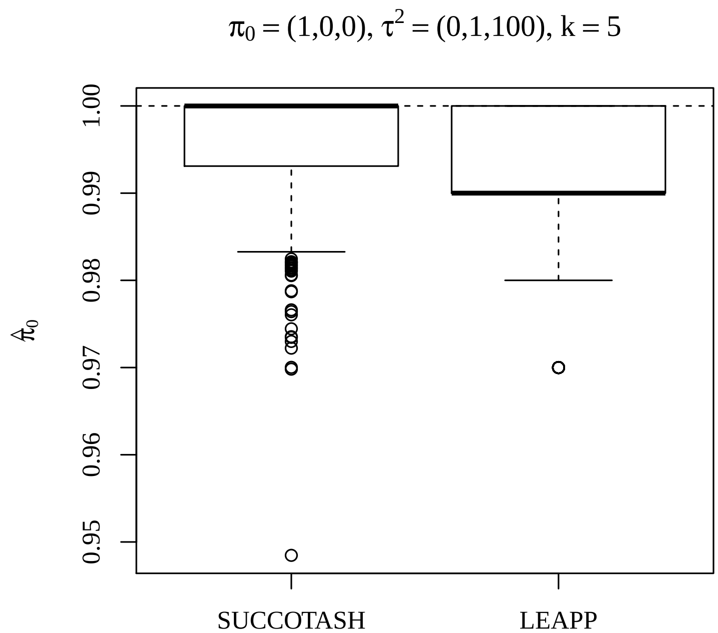

$\pi_0 = (0.5, 0.25, 0.25)$, $\tau^2 = (0, 1, 100)$, $k = 50$

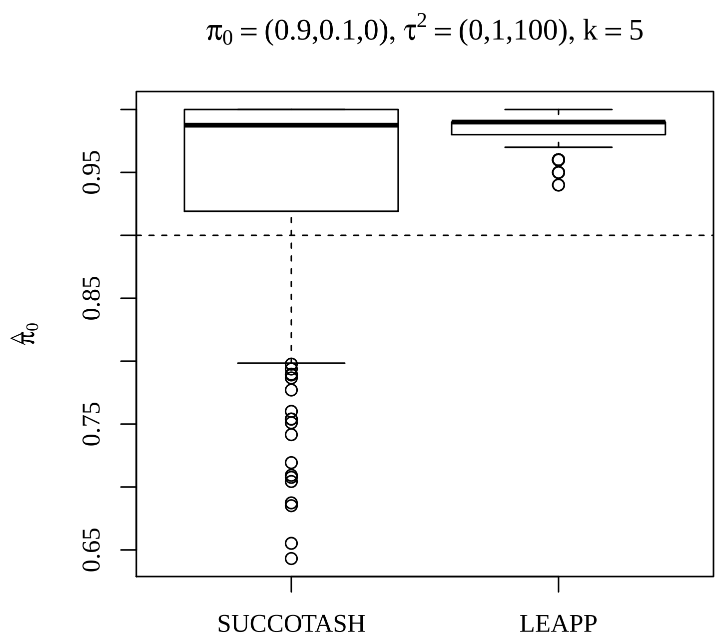


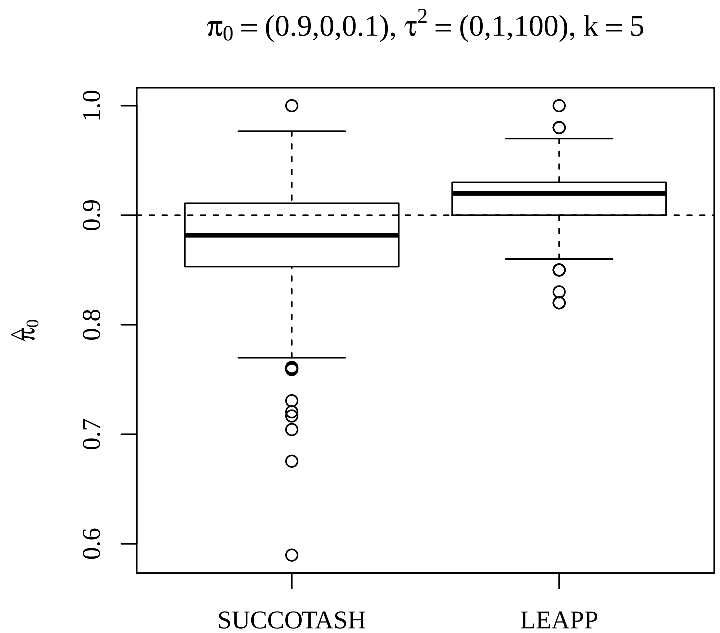
## 5 $\hat{\pi}_0$ Plots



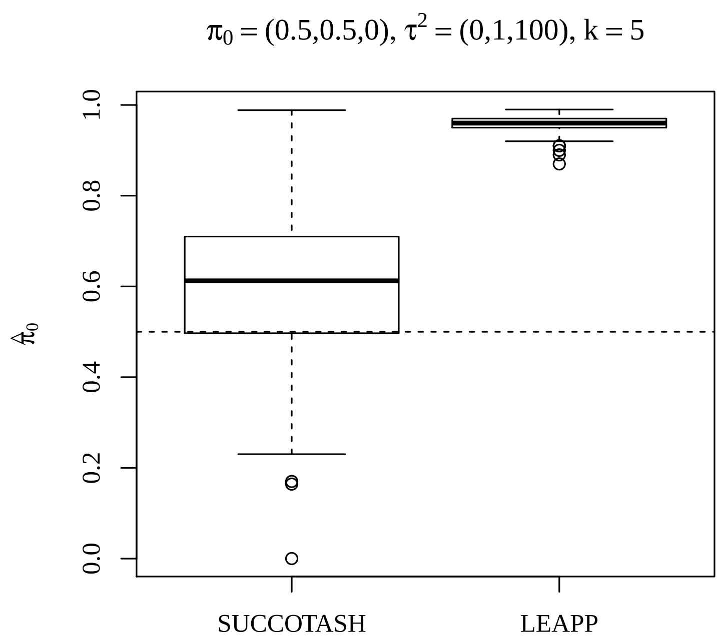




$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
1.00
0.99
0.98
0.97
0.96
0.95
SUCCOTASH
LEAPP

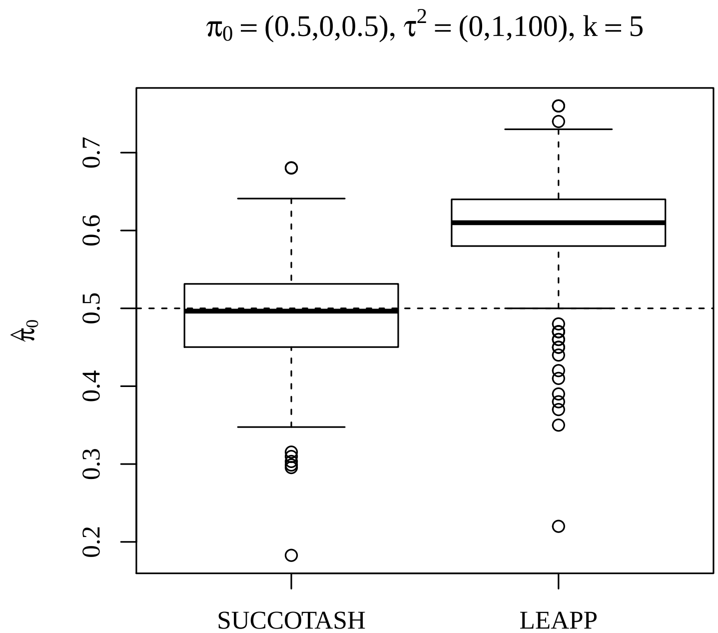

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
0.95
0.85
0.75
0.65
SUCCOTASH
LEAPP

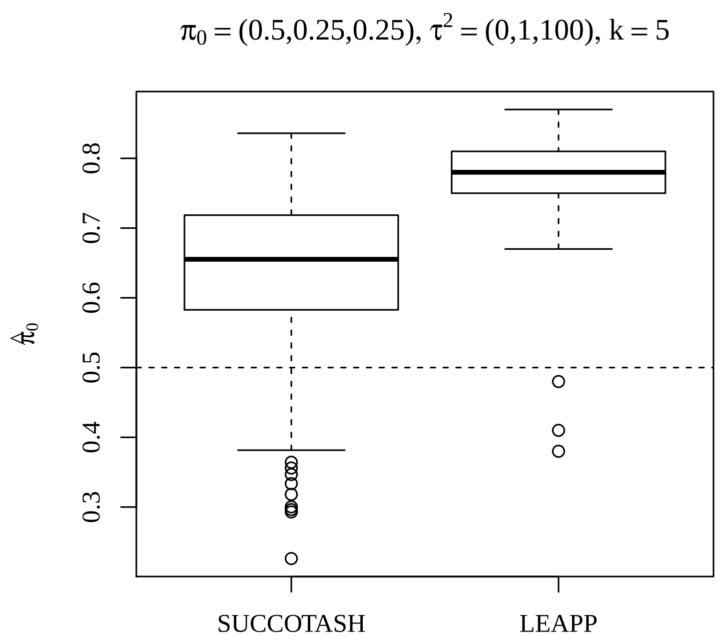


$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
1.0
0.9
0.8
0.7
0.6
SUCCOTASH
LEAPP

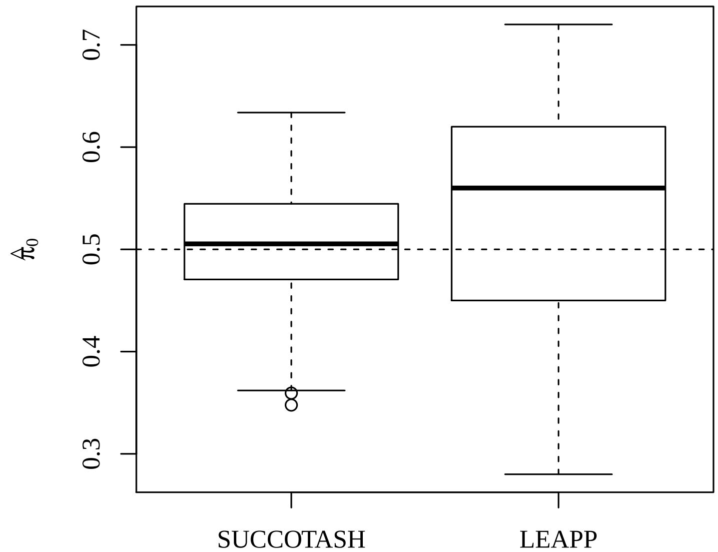



$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
1.0
0.8
0.6
0.4
0.2
0.0
SUCCOTASH
LEAPP

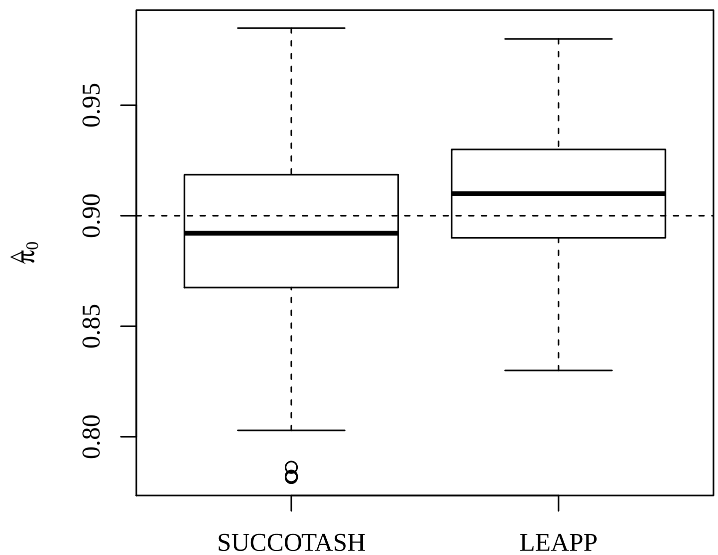

$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, k = 5
0.2
0.3
0.4
0.5
0.6
0.7
$\hat{\pi}_0$
SUCCOTASH
LEAPP

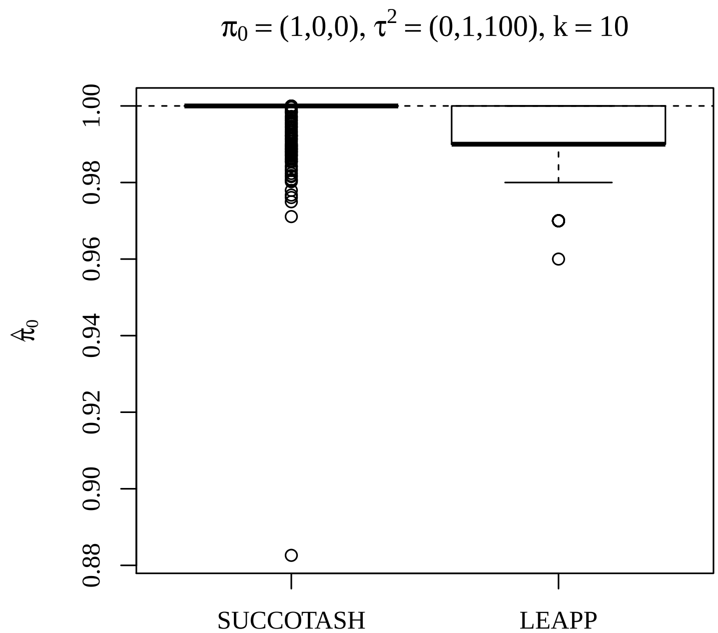

$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, k = 5
0.3
0.4
0.5
0.6
0.7
0.8
$\hat{\pi}_0$
SUCCOTASH
LEAPP

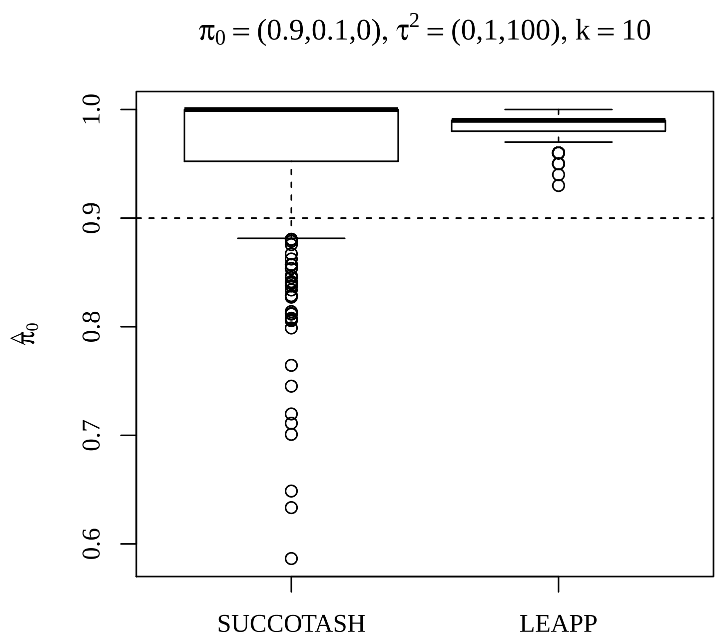
$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 10$

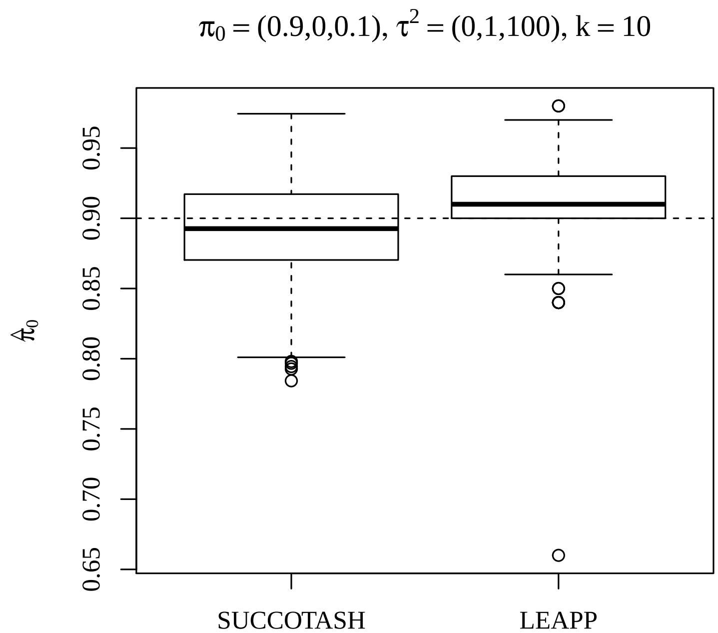


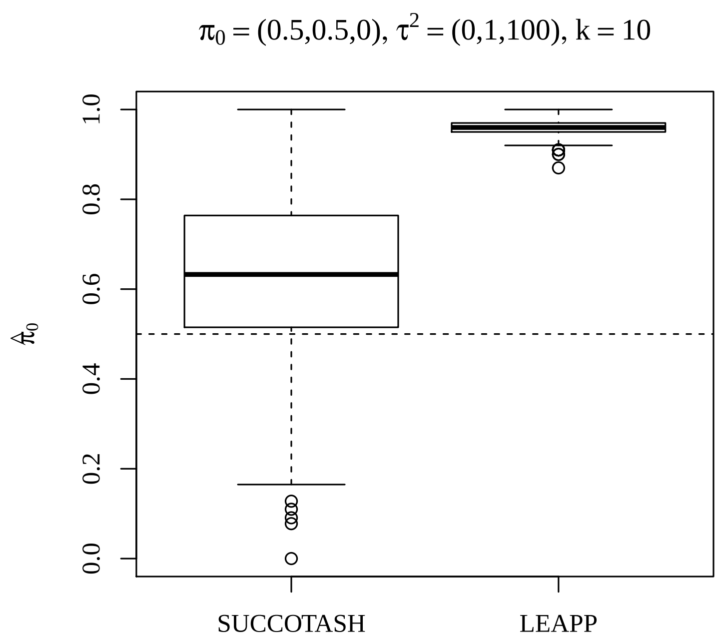

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 10$

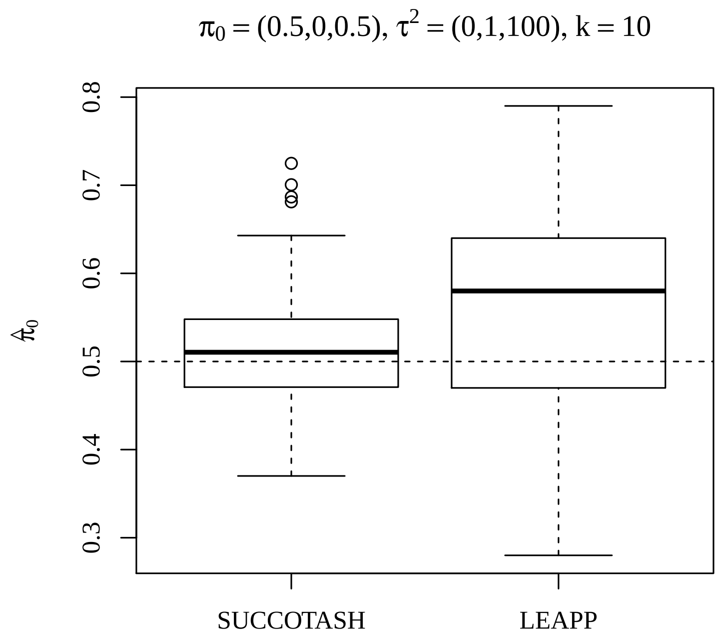


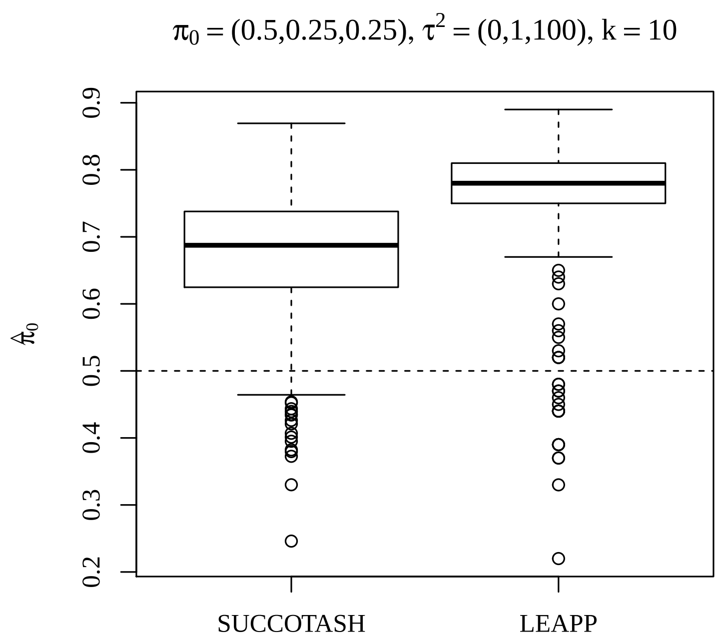

$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
1.00
0.98
0.96
0.94
0.92
0.90
0.88
SUCCOTASH
LEAPP


$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
1.0
0.9
0.8
0.7
0.6
SUCCOTASH
LEAPP

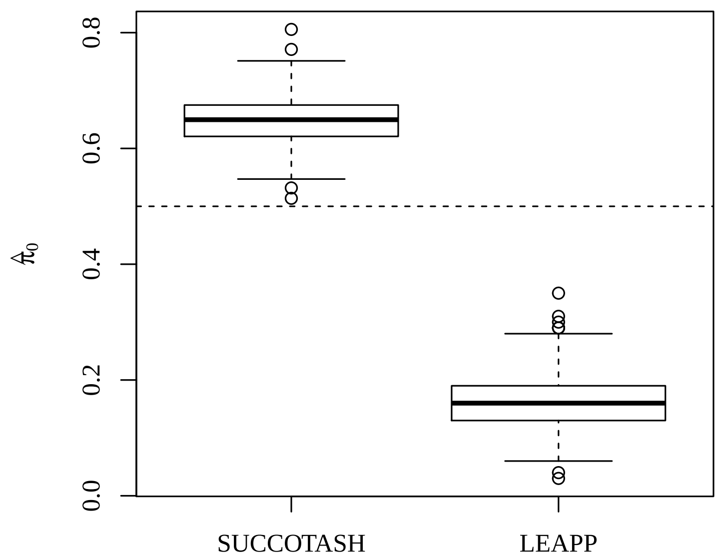

$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
0.65
0.70
0.75
0.80
0.85
0.90
0.95
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
0.0
0.2
0.4
0.6
0.8
1.0
SUCCOTASH
LEAPP

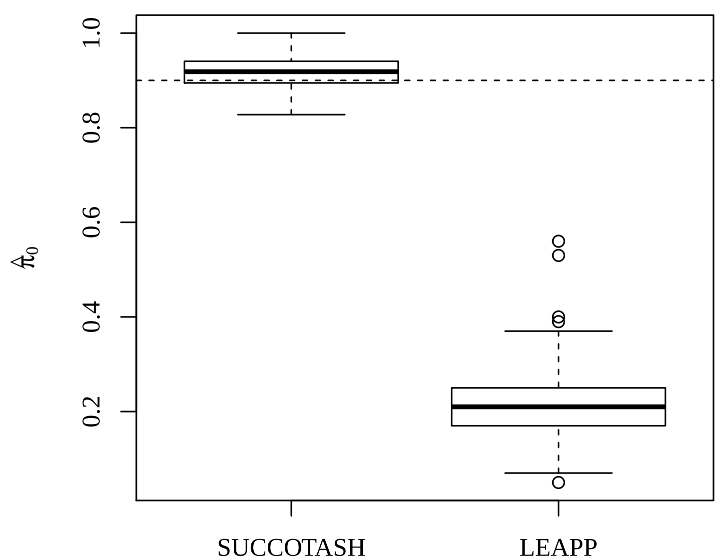

π0 = (0.5,0,0.5), τ2 = (0,1,100), k = 10
0.3
0.4
0.5
0.6
0.7
0.8
SUCCOTASH
LEAPP

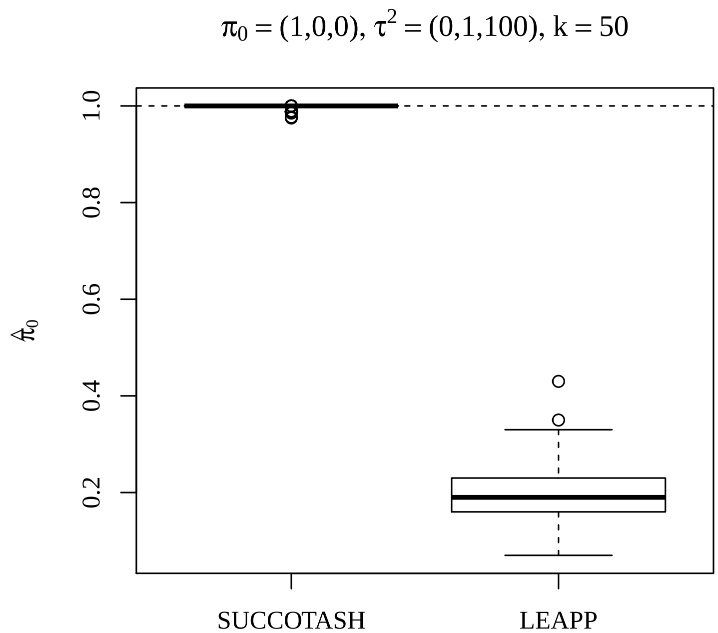

π0 = (0.5,0.25,0.25), τ2 = (0,1,100), k = 10
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
SUCCOTASH
LEAPP

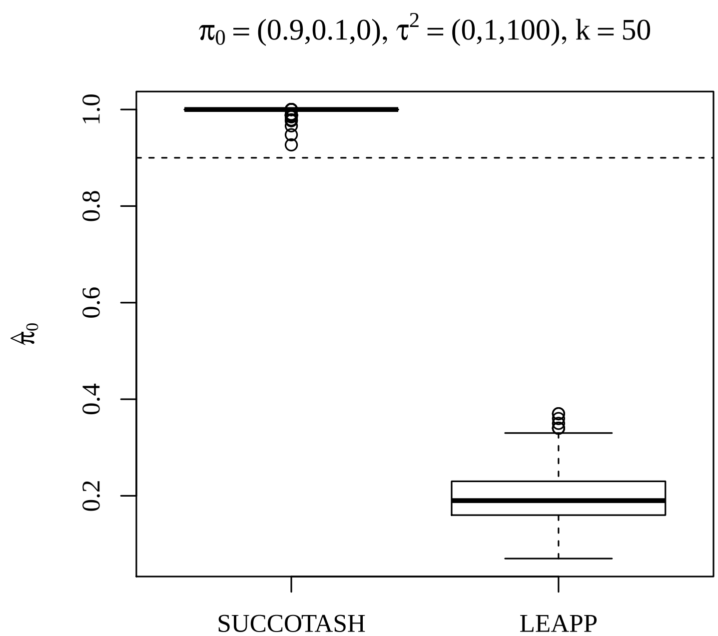
$\pi_0 = (0.5, 0.5)$, $\tau^2 = (0, 100)$, $k = 50$

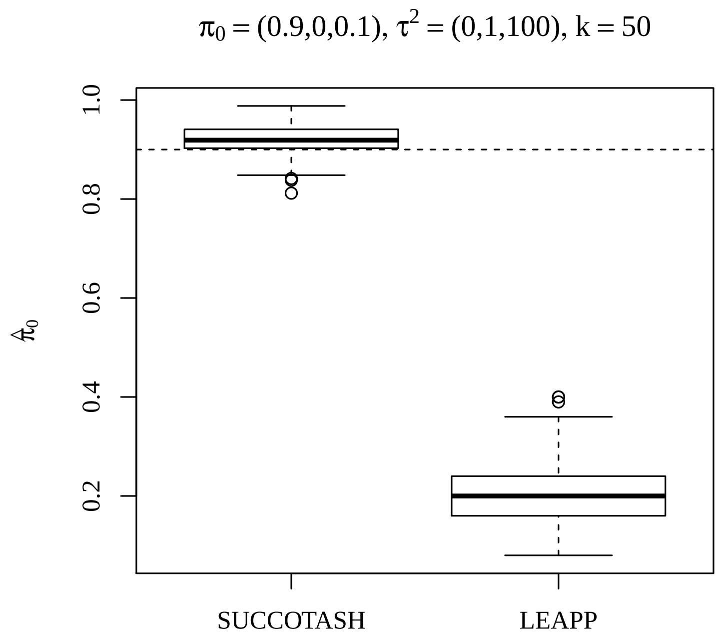


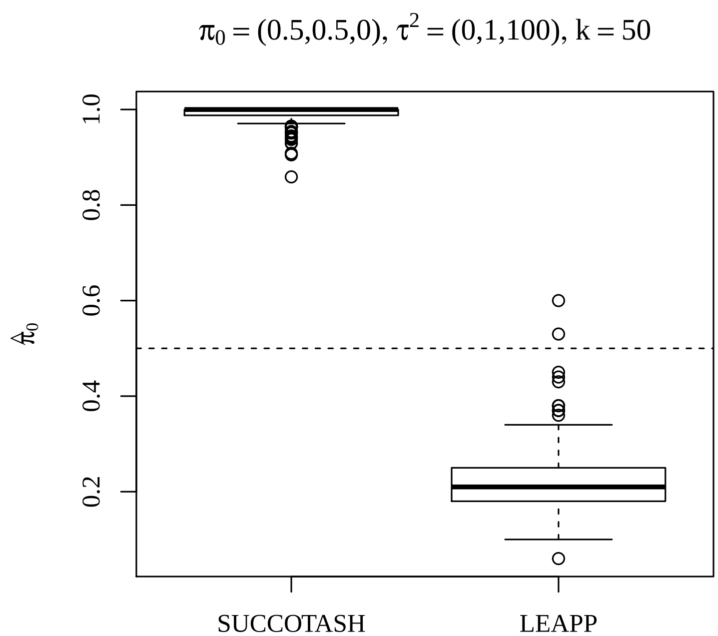

$\pi_0 = (0.9, 0.1)$, $\tau^2 = (0, 100)$, $k = 50$

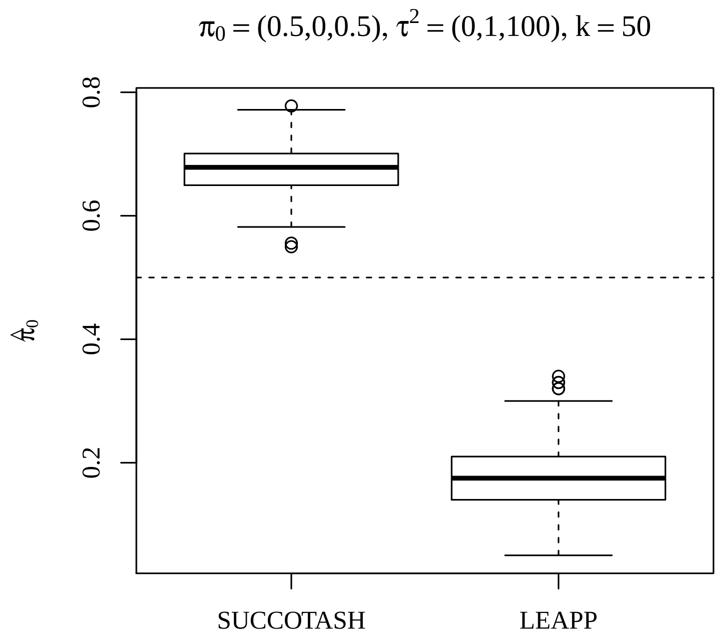



π0 = (1,0,0), τ2 = (0,1,100), k = 50
1.0
0.8
0.6
0.4
0.2
π̂0
SUCCOTASH
LEAPP

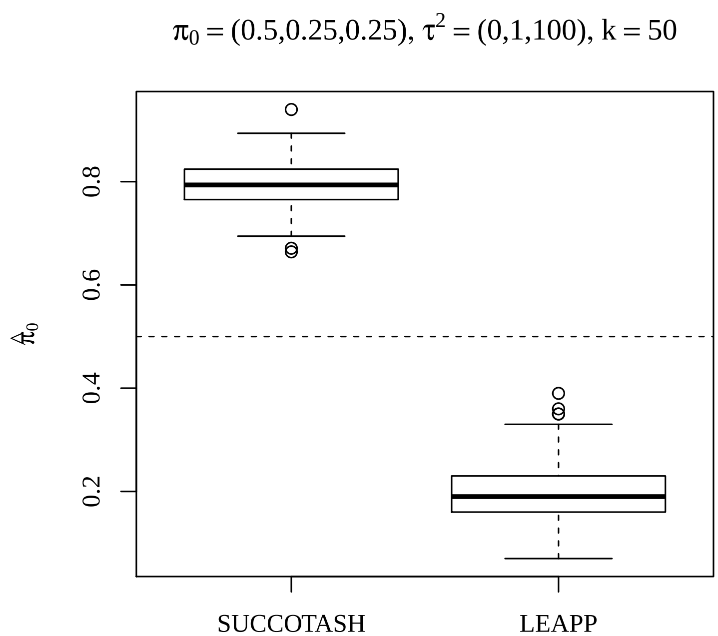

π0 = (0.9,0.1,0), τ2 = (0,1,100), k = 50
1.0
0.8
0.6
0.4
0.2
π̂0
SUCCOTASH
LEAPP


$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 50$
$\hat{\pi}_0$
1.0
0.8
0.6
0.4
0.2
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\hat{\pi}_0$
1.0
0.8
0.6
0.4
0.2
SUCCOTASH
LEAPP



π0 = (0.5,0,0.5), τ2 = (0,1,100), k = 50
0.8
0.6
0.4
0.2
π̂0
SUCCOTASH
LEAPP

π0 = (0.5,0.25,0.25), τ2 = (0,1,100), k = 50
0.8
0.6
0.4
0.2
π̂0
SUCCOTASH
LEAPP