

# Competitors when Non-null, Varying Sample Size, log2-fold standard deviation, and $\pi_0$ .

David Gerard

February 10, 2016

## Abstract

I compare MOUTHWASH to various competitors. I look at AUC and estimates of  $\pi_0$ . I also look at the Kendall's Tau between the p-values or lfdr's for the different methods. This file runs the same analysis as varyNsampNullpiLog2sd.pdf but with smaller sample sizes and smaller log2sd.

## 1 Competitors

For each competitor, I performed two methods. The first method consisted of a two-step procedure:

1. Estimate  $\hat{\beta}_{[2,i]}$  and its corresponding standard error  $\hat{s}_i$ .
2. Run ASH on  $\hat{\beta}_{[2,i]}$  and  $\hat{s}_i$ .

The second method was to just calculate a normal (or t, where appropriate) p-values from  $\hat{\beta}_{[2,i]}/\hat{s}_i$ .

The ASH methods provide an estimate of  $\pi_0$ . I obtained an estimate of  $\pi_0$  from the p-values by the `qvalue` package in R [Storey, 2002]. In some cases for the quasi-binomial methods, the largest p-values were less than 0.9 and `qvalue` would return an error (because it uses the largest p-values to estimate the proportion of nulls). For these, I used the upper quartile of p-values to estimate the proportion of nulls. Maybe a bad idea.

The methods that Mengyin and I have coded to get  $\hat{\beta}_{[2,i]}$  and  $\hat{s}_i$  were

- VOOM [Law et al., 2014].
- RUVseq [Risso et al., 2014] followed by VOOM [Law et al., 2014] with the estimated confounding factors. Half of the factors were used as control genes. RUVseq is RUV2 on the  $\log(\text{counts} + 1)$  matrix.
- SVASEq [Leek, 2014] followed by VOOM [Law et al., 2014] with the estimated confounding factors. SVASEq is SVA on the  $\log(\text{counts} + 1)$  matrix.
- Quasi-binomial glm.
- RUVseq + quasi-binomial glm.
- SVASEq + quasi-binomial glm.
- MYRNA, which is just a quasi-binomial glm using the 75th percentile of the samples' counts as covariates [Langmead et al., 2010].
- MYRNA offset, which is just a quasi-binomial glm using the 75th percentile of the samples' counts as offsets [Langmead et al., 2010].
- DESeq2glm [Love et al., 2014].
- The robust regression version of CATE [Wang et al., 2015] on the  $\log(\text{counts} + 1)$ .

- The negative controls version of CATE [Wang et al., 2015] on the  $\log(\text{counts} + 1)$ .
- SVA [Leek and Storey, 2007] with the number of confounders estimated using the method of Buja and Eyuboglu [1992] on the  $\log(\text{counts} + 1)$ , followed by OLS.
- RUV2 [Gagnon-Bartsch et al., 2013] with 50% of the observations being control genes with the number of confounders estimated using the method of Buja and Eyuboglu [1992] on the  $\log(\text{counts} + 1)$ , followed by OLS.
- OLS on the  $\log(\text{counts} + 1)$ .
- The ridge-regression version of LEAPP [Sun et al., 2012] on the  $\log(\text{counts} + 1)$ .
- The soft-thresholding version of LEAPP [Sun et al., 2012] on the  $\log(\text{counts} + 1)$ .

Notes:

- “VOOM” means using VOOM [Law et al., 2014] to find weights for each observations, then fitting a linear model using LIMMA [Smyth, 2005].
- LEAPP does not easily provide standard errors, so I excluded it from the ASH analysis. But I still use it for the `qvalue` analysis.
- EdgeR was giving me trouble, so I excluded it.

The factor analysis part of MOUTHWASH was done with the quasi-mle approach of Bai et al. [2012] with the number of hidden confounders using the methods of Buja and Eyuboglu [1992] implemented in the `num.sv()` function in the `sva` package in R.

In summary, there are 31 methods that I compared in estimating  $\pi_0$  and in their AUC:

1. MOUTHWASH
2. voom + ASH
3. Quasi-binomial GLM + ASH
4. MyrnaQB + ASH
5. Myrnaoffqb + ASH
6. RUVseq + voom + ASH
7. SVaseq + voom + ASH
8. RUVseq + Quasi-binomial GLM + ASH
9. SVaseq + Quasi-binomial GLM + ASH
10. DESeq2glm + ASH
11. OLS on  $\log(\text{counts} + 1)$  + ASH
12. RUV2 on  $\log(\text{counts} + 1)$  + ASH
13. SVA on  $\log(\text{counts} + 1)$  + ASH
14. Robust Regression Cate + ASH
15. Negative Control CATE + ASH
16. voom + `qvalue`
17. Quasi-binomial GLM + `qvalue`
18. Myrnaqb + `qvalue`
19. Myrnaoffqb + `qvalue`
20. RUVseq + voom + `qvalue`
21. SVaseq + voom + `qvalue`
22. RUVseq + Quasi-binomial GLM + `qvalue`
23. SVaseq + Quasi-binomial GLM + `qvalue`
24. DESeq2glm + `qvalue`
25. OLS on  $\log(\text{counts} + 1)$  + `qvalue`
26. RUV2 on  $\log(\text{counts} + 1)$  + `qvalue`

27. SVA on  $\log(\text{counts} + 1) + \text{qvalue}$
28. Robust Regression CATE + **qvalue**
29. Negative Control CATE + **qvalue**
30. Soft-thresholding version of LEAPP+ **qvalue**
31. Ridge version of LEAPP+ **qvalue**

## 2 Simulation Study

I ran through 100 repetitions of generating data from GTEX lung data under the following parameter conditions:

- $n \in \{3, 5\}$ ,
- $p = 1000$ ,
- $\pi_0 \in \{0.5, 0.9\}$ ,
- $\sigma_{\log 2} \in \{1, 5\}$ .

I extracted the most expressed  $p$  genes (excluding the top 5 expressed genes) from the GTEX lung data and  $n$  samples are chosen at random. Half of these samples are randomly given the “treatment” label 1, the other half given the “treatment” label 0. Of the  $p$  genes,  $\pi_0 p$  were chosen to be non-null. Signal was added by the Poisson-thinning approach in Mengyin’s code with a mean  $\log 2$ -fold change of 0 and a standard deviation  $\log 2$ -fold change of  $\sigma_{\log 2}$ . That is

$$A_1, \dots, A_{p/2} \sim N(0, \sigma_{\log 2}^2) \quad (1)$$

$$B_i = 2^{A_i} \text{ for } i = 1, \dots, p/2. \quad (2)$$

If  $A_i > 0$  then we replace  $Y_{[1:(n/2), i]}$  with  $\text{Binom}(Y_{[j, i]}, 1/B_i)$  for  $j = 1, \dots, n/2$ . If  $A_i < 0$  then we replace  $Y_{[(n/2+1):n, i]}$  with  $\text{Binom}(Y_{[j, i]}, B_i)$  for  $j = n/2 + 1, \dots, n$ .

For each iteration, I calculated three things:

1. The pairwise Kendall’s tau between the methods’ lfr’s or p-values.
2. The AUC using either the lfrs or p-values.
3. The estimates of  $\pi_0$ .

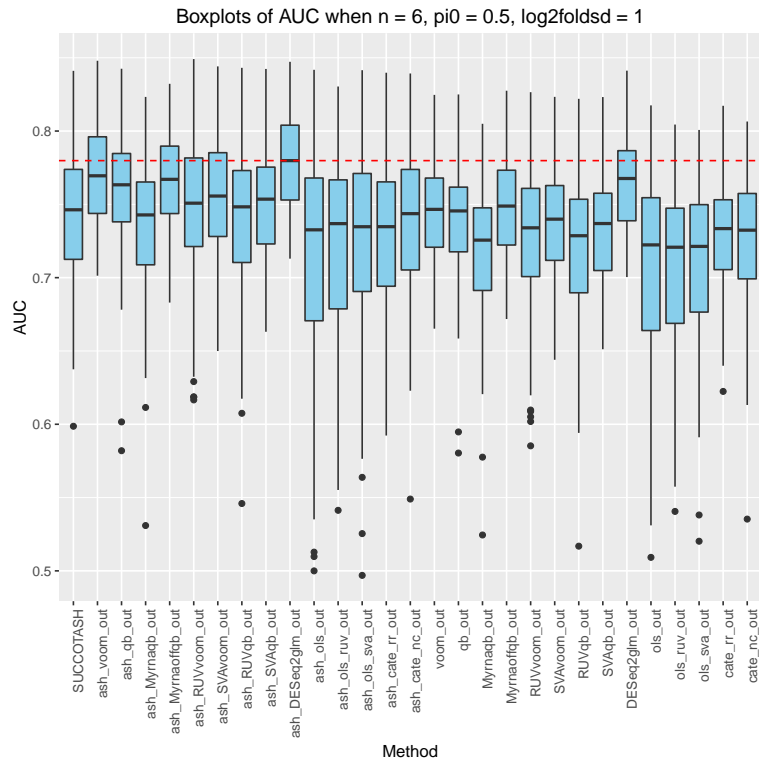
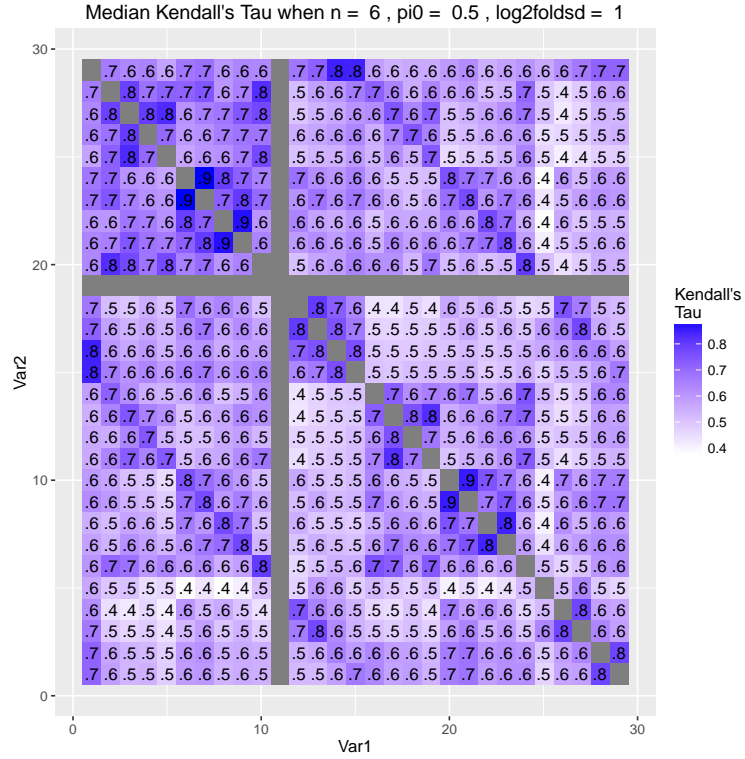
## 3 Results

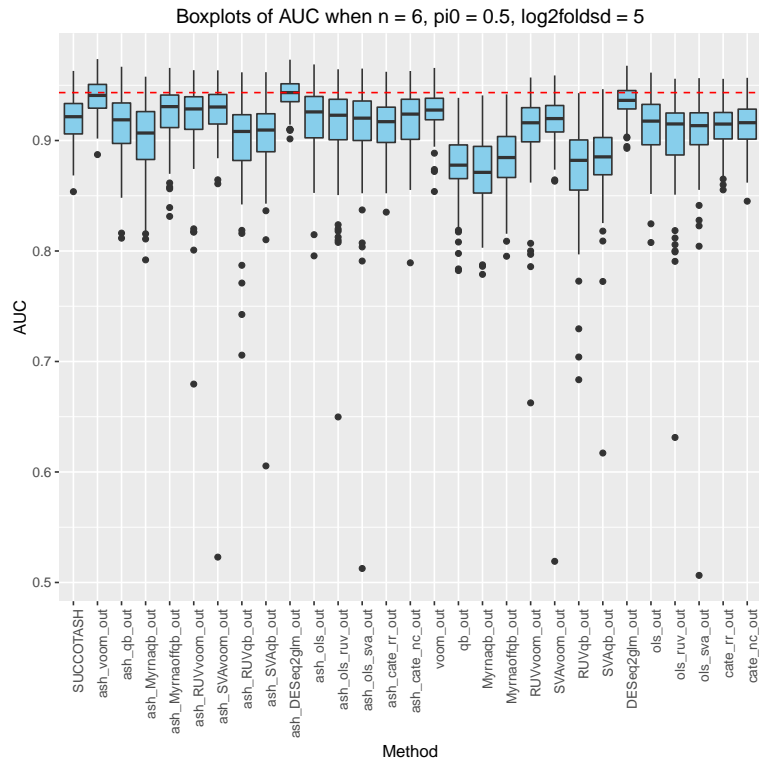
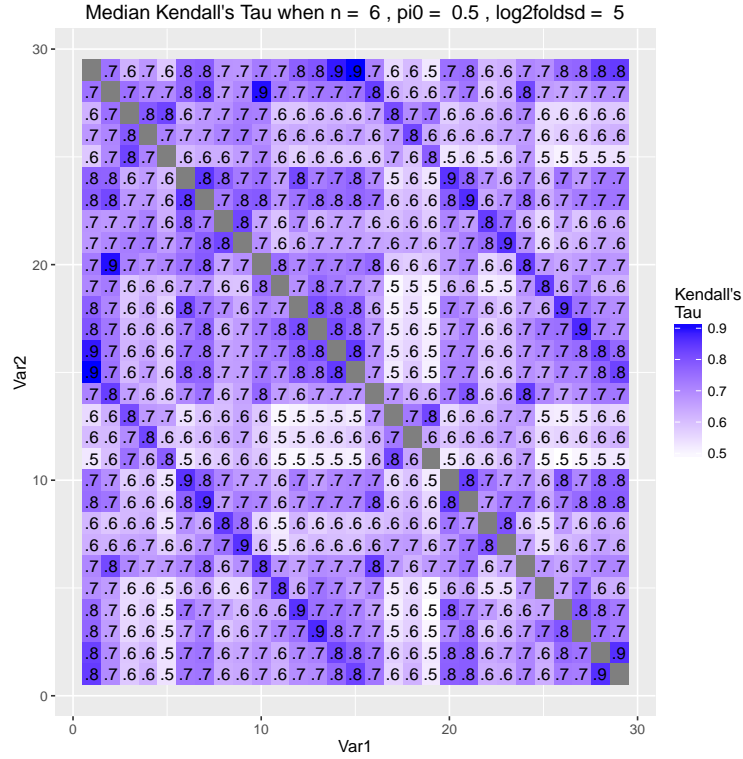
For the frequentist procedures, I used the vector of p-values as the predictions and I used the vector of lfr’s from the ASH-like procedures for prediction. These were used to create ROC curves and calculate AUCs. In general, the AUC’s were all very similar with the ash-like methods having slightly higher AUC. DESeq2glm (with or without ASH) is the winner when the sample size is 3 vs 3. The results are less clear for 5 vs 5.

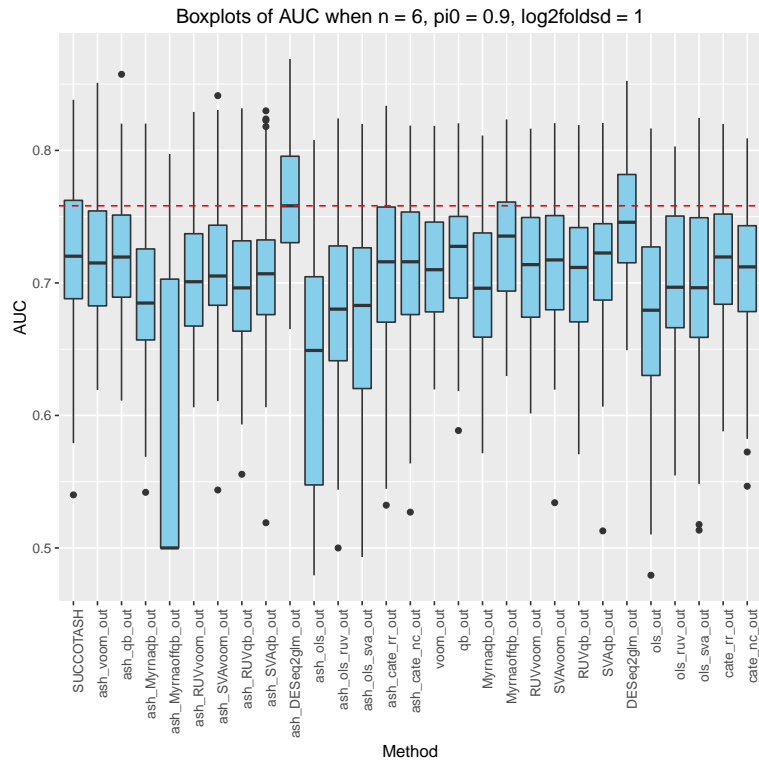
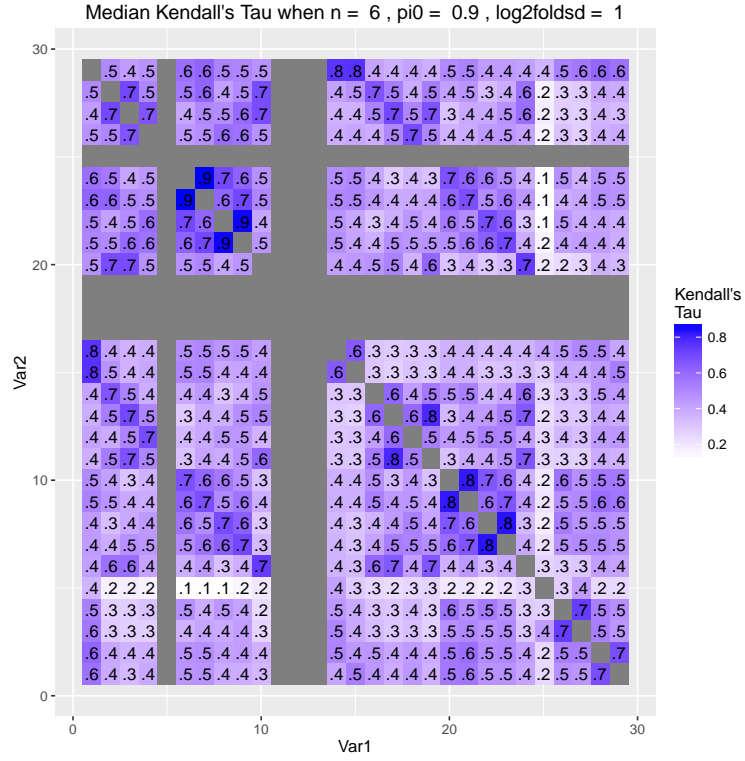
Graphical representations of the median Kendall’s taus are presented below. The median Kendall’s tau can get quite low between the separate groups — as small as 0.2. This indicates that for many datasets, the rankings can be quite different.

From the p-values, I used the **qvalue** package [Storey, 2002] to estimate  $\pi_0$ . Estimates of  $\pi_0$  are given from **ashr** for the ASH-like methods. MOUTHWASH (SUCCOTASH) performs the worst in estimating  $\pi_0$ , usually underestimating it. The ASH-like methods usually estimate  $\pi_0$  to be smaller

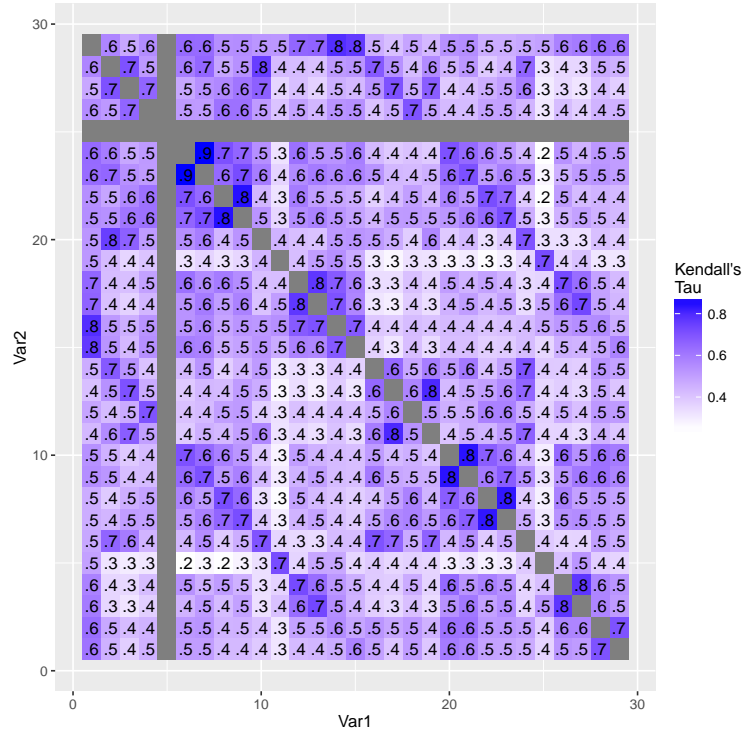
than their non-ASH counterparts. Notably, the negative controls version of CATE does not excel in the small sample size scenario.



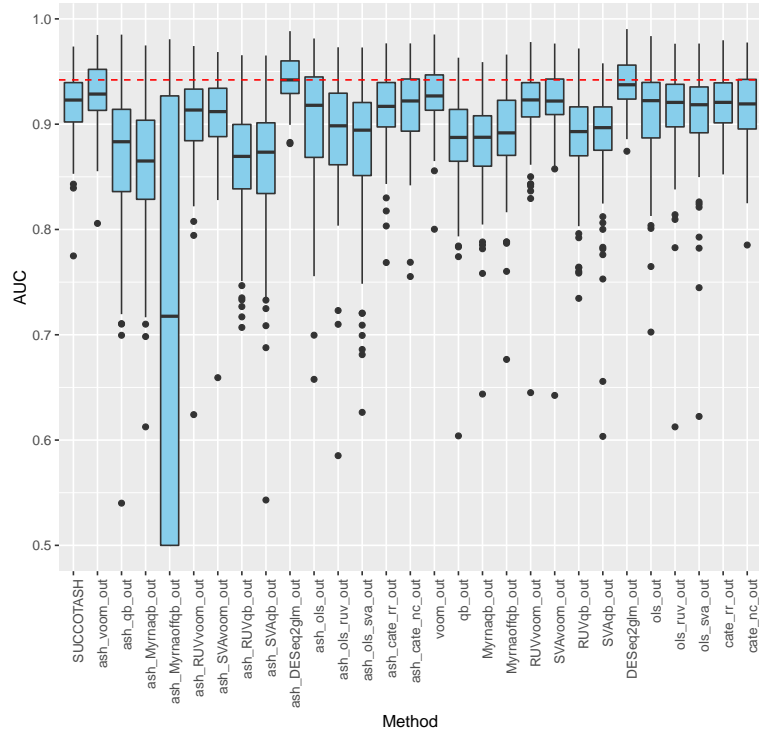




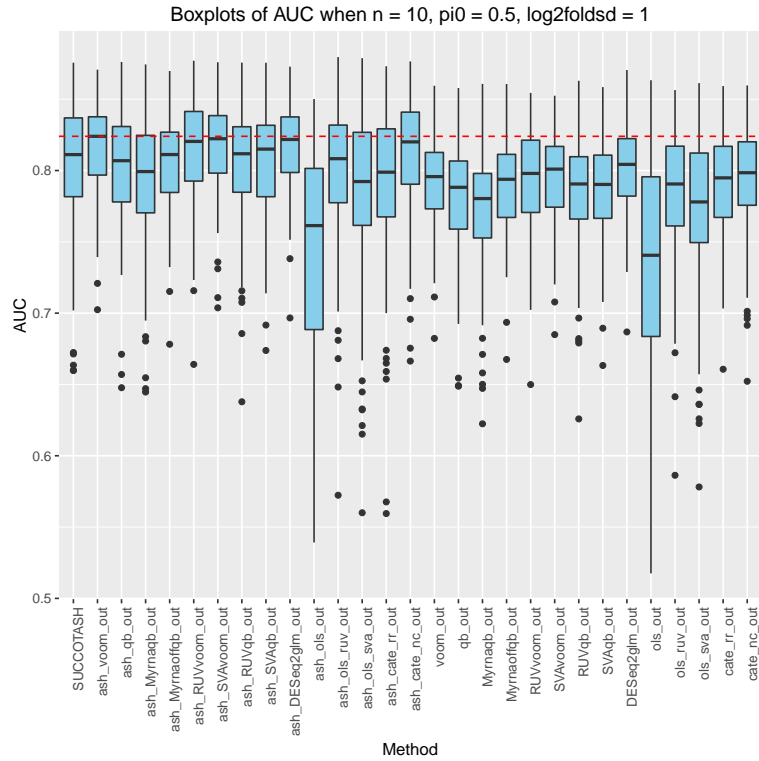
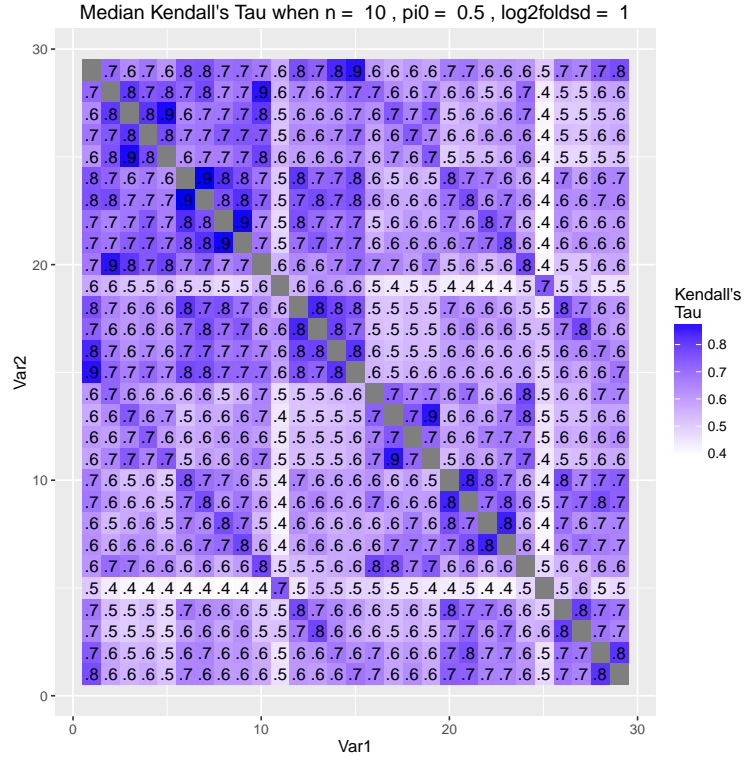
Median Kendall's Tau when  $n = 6$ ,  $\pi_0 = 0.9$ ,  $\log_2\text{foldsd} = 5$

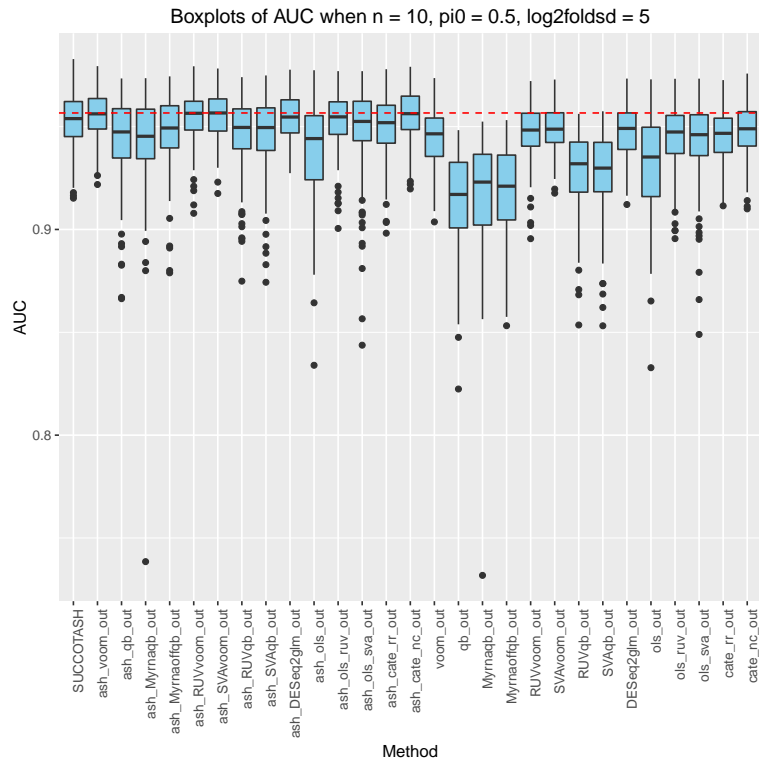
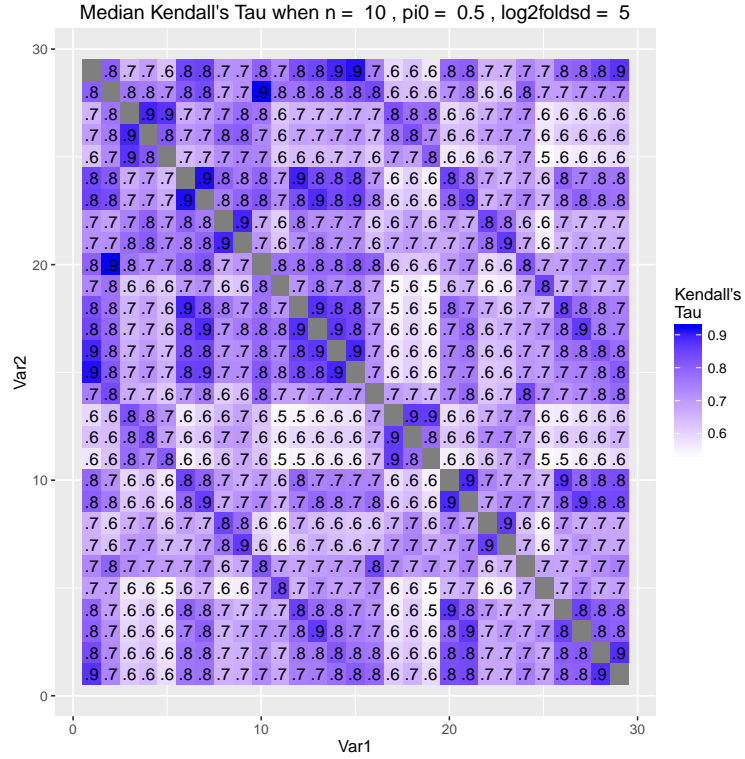


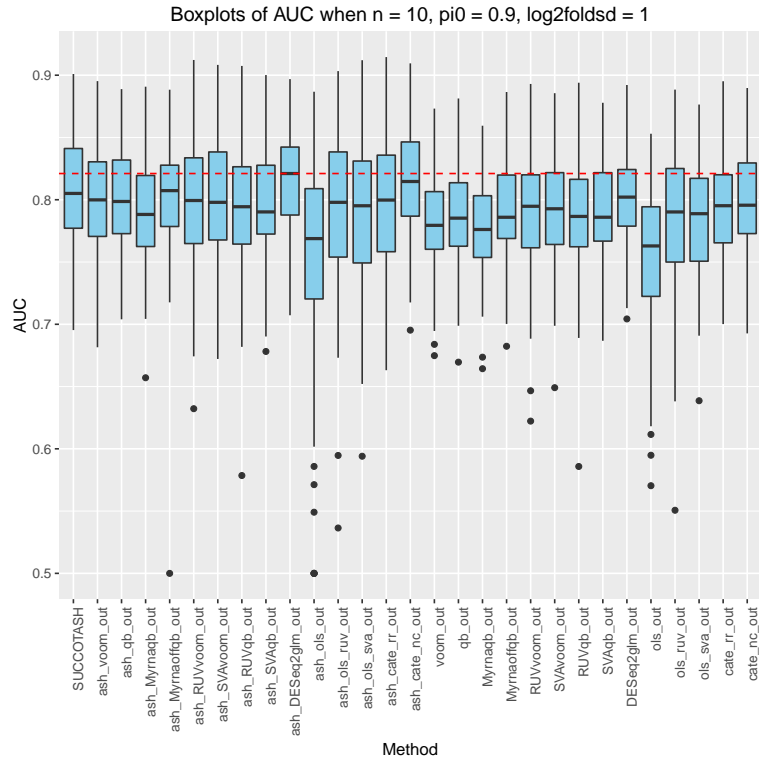
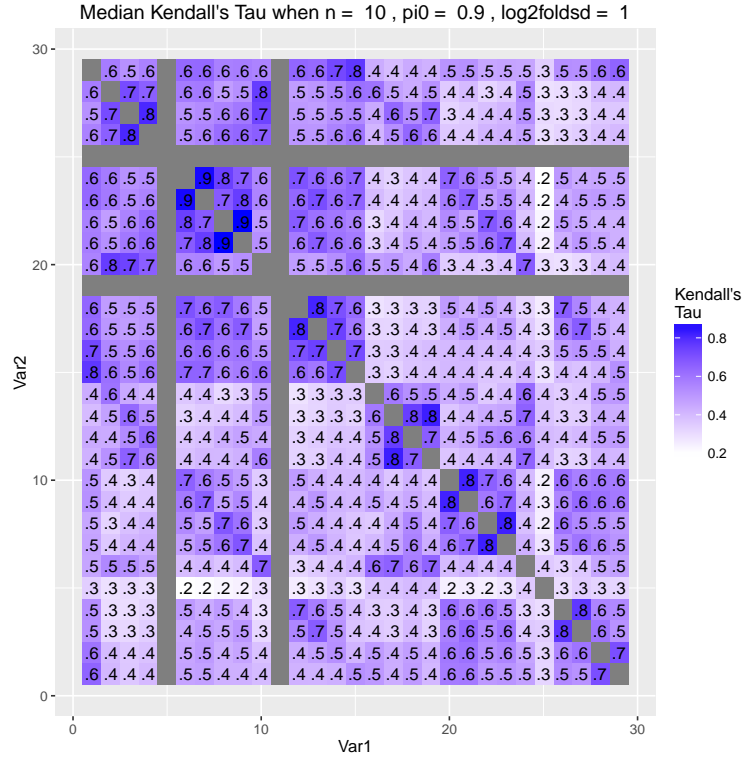
Boxplots of AUC when  $n = 6$ ,  $\pi_0 = 0.9$ ,  $\log_2\text{foldsd} = 5$

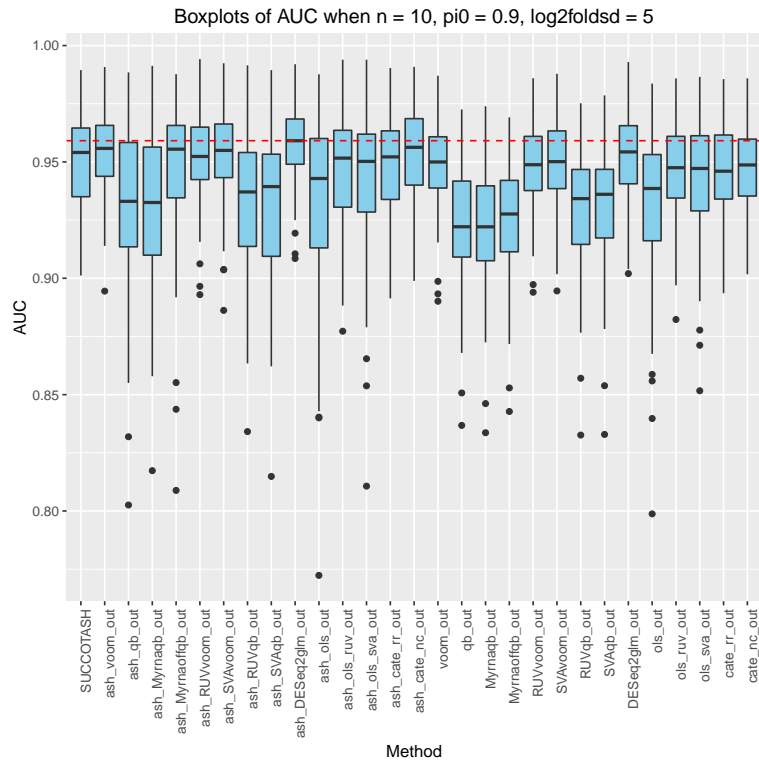
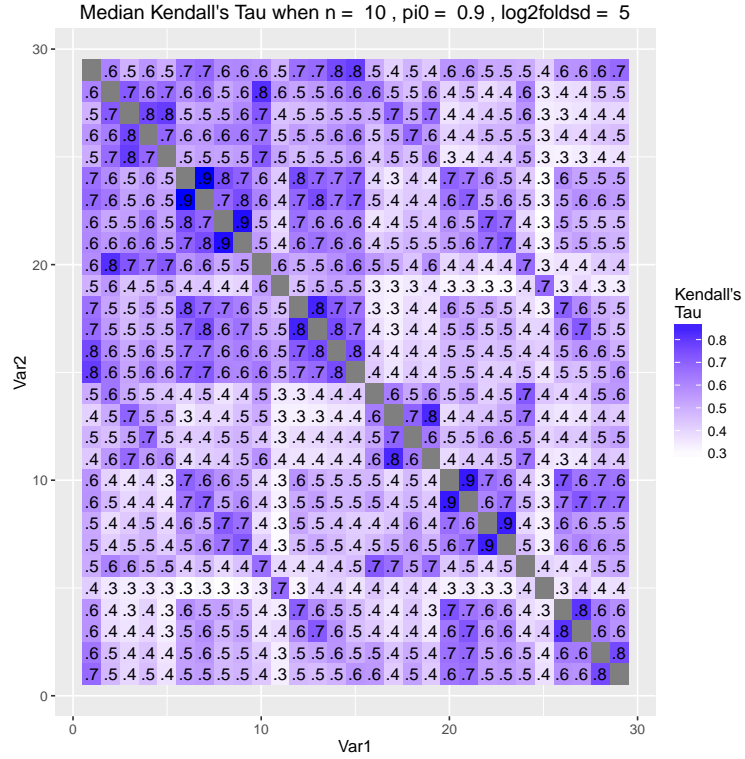




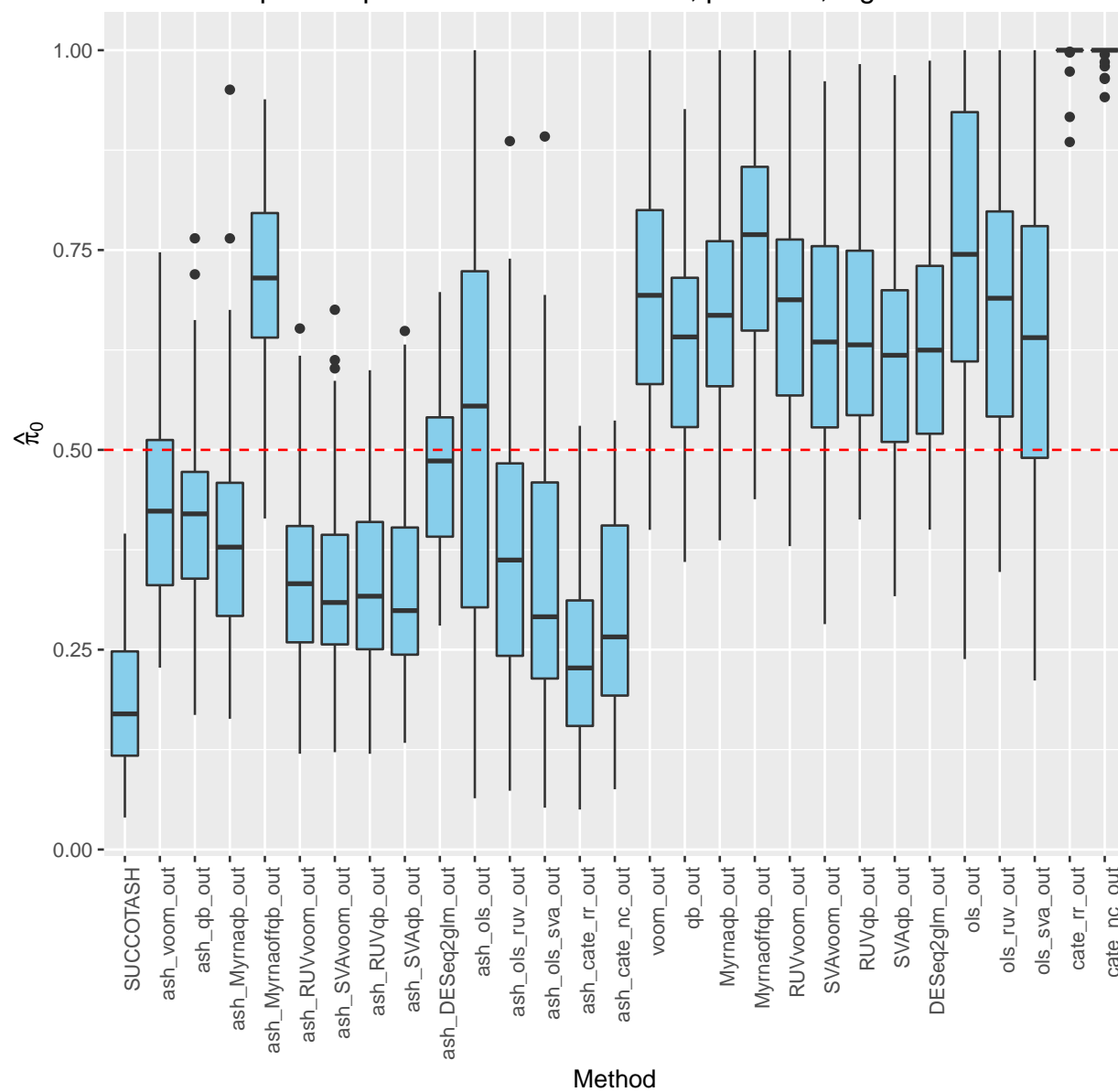




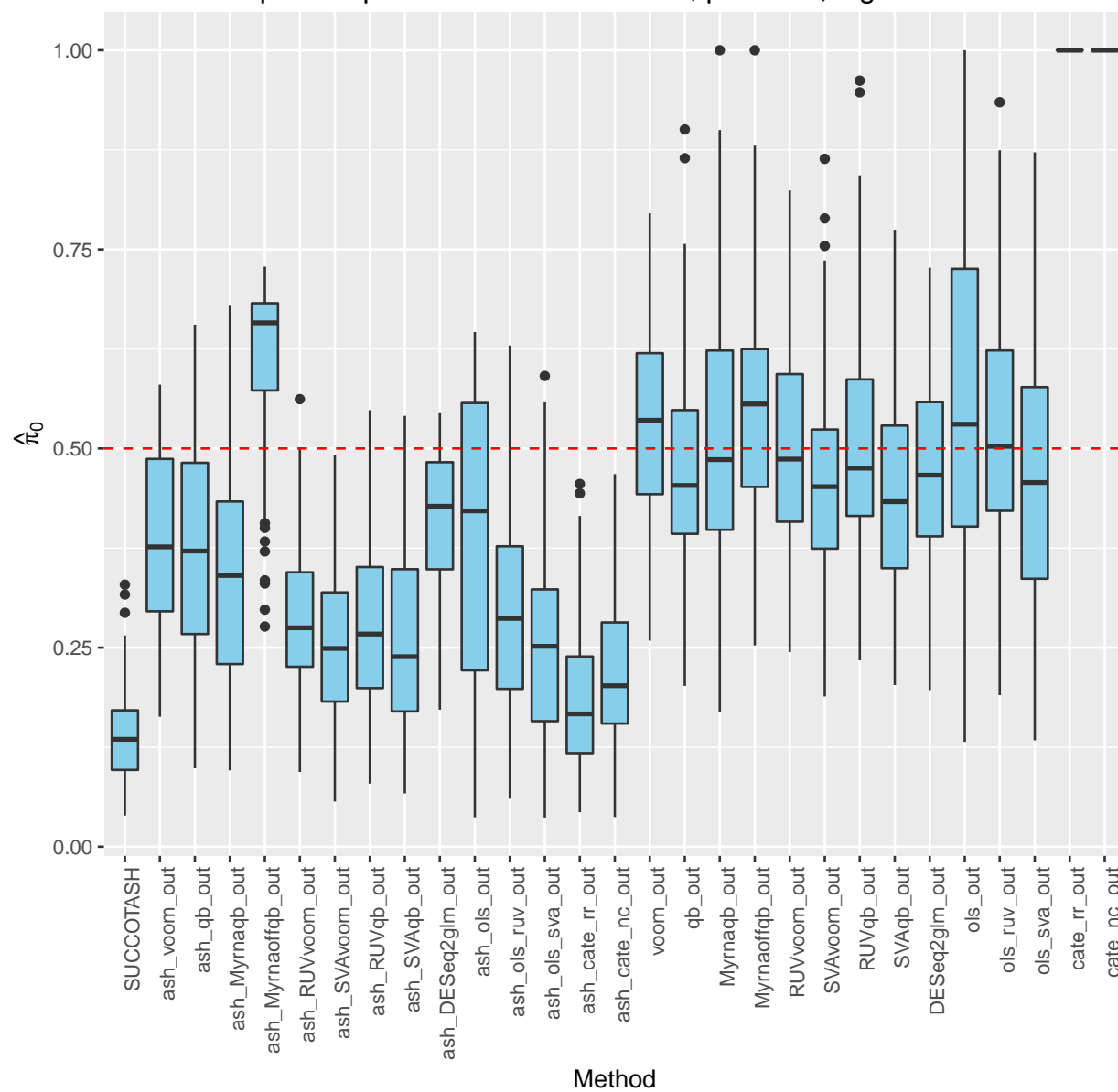




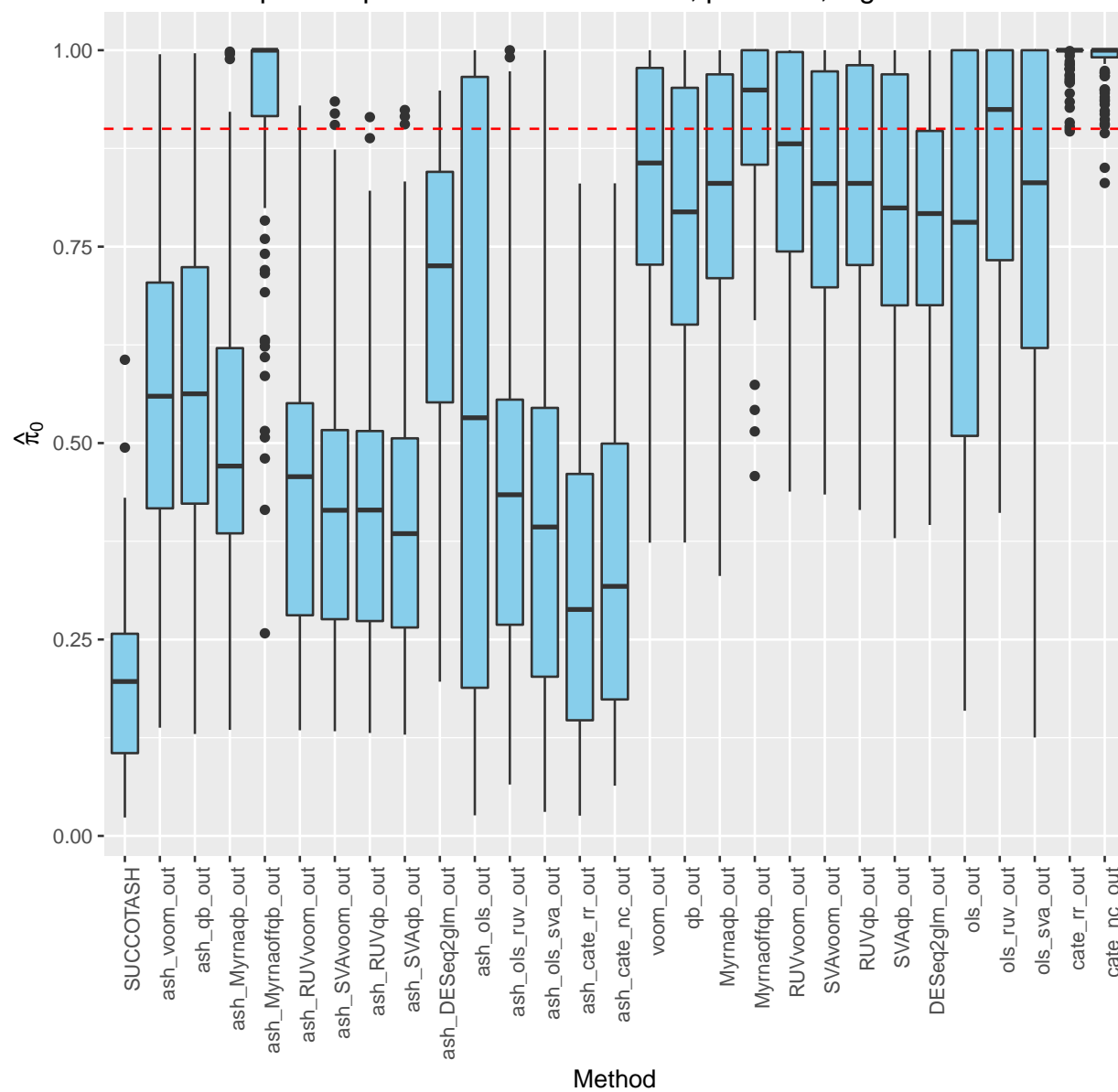
Boxplots of  $\pi_0$  estimates when  $n = 6$ ,  $\pi_0 = 0.5$ ,  $\log_2\text{foldsd} = 1$



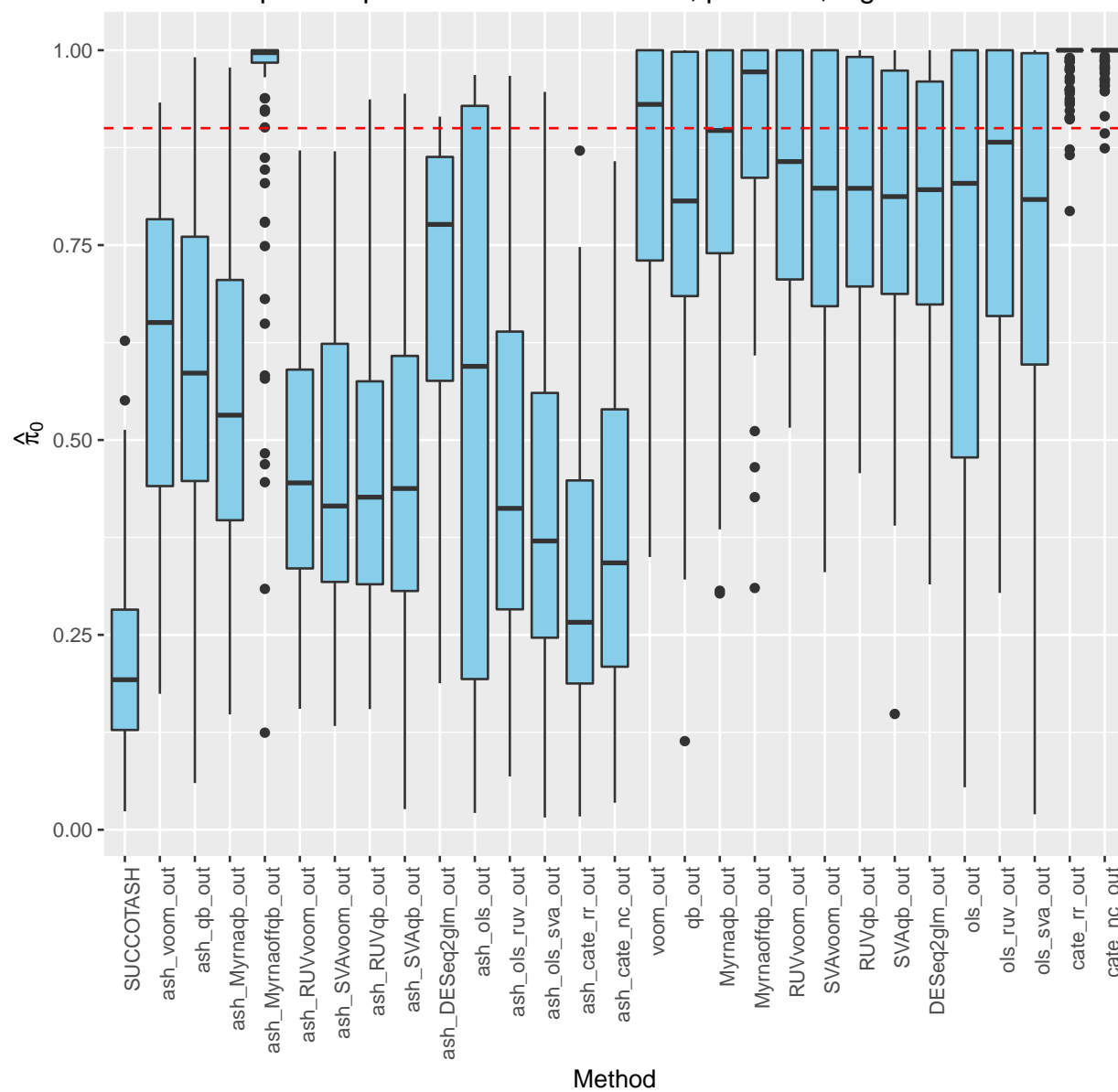
Boxplots of  $\pi_0$  estimates when  $n = 6$ ,  $\pi_0 = 0.5$ ,  $\log_2\text{foldsd} = 5$



Boxplots of  $\hat{\pi}_0$  estimates when  $n = 6$ ,  $\pi_0 = 0.9$ ,  $\log_2\text{foldsd} = 1$

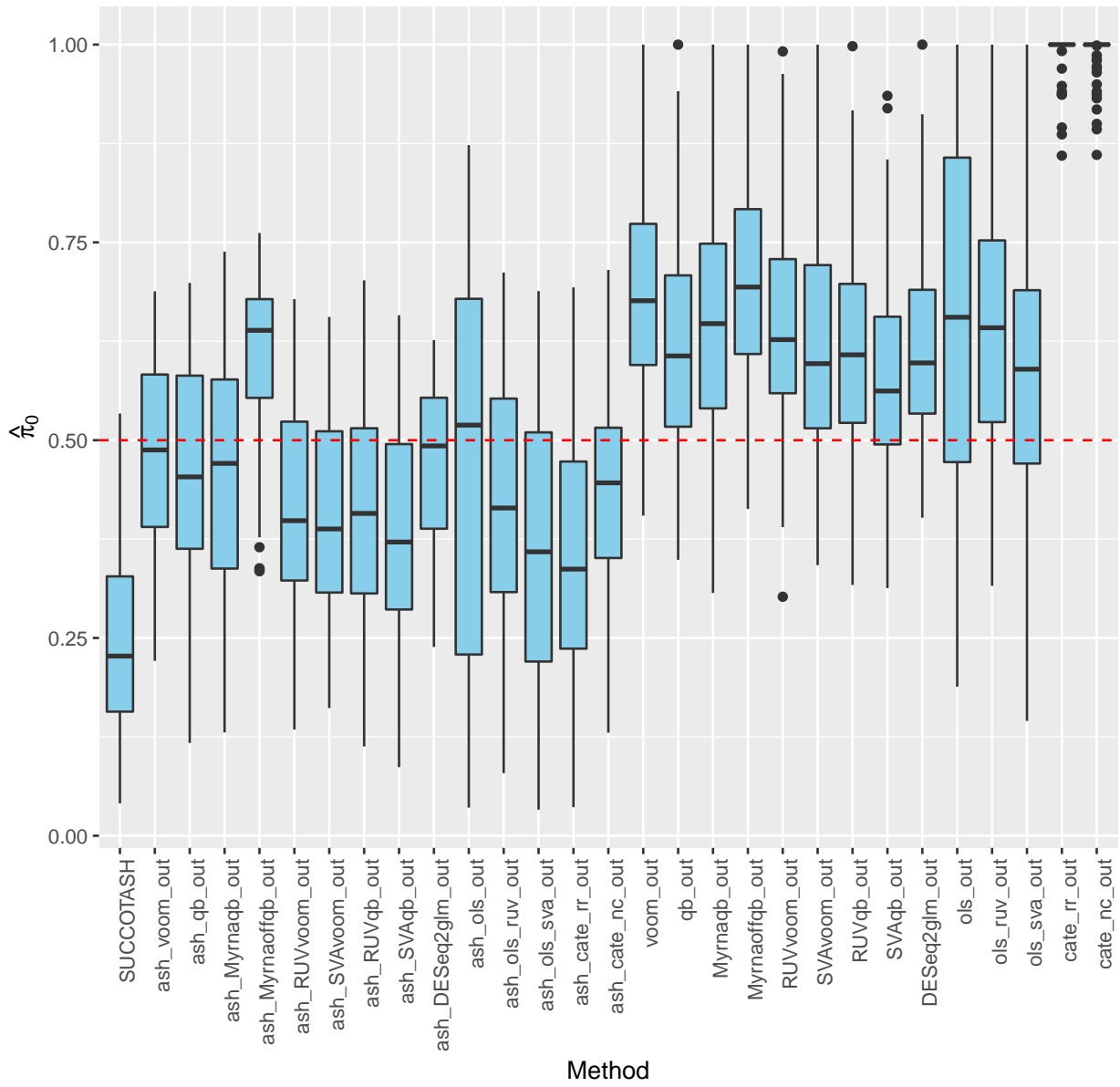


Boxplots of  $\hat{\pi}_0$  estimates when  $n = 6$ ,  $\pi_0 = 0.9$ ,  $\log_2\text{foldsd} = 5$

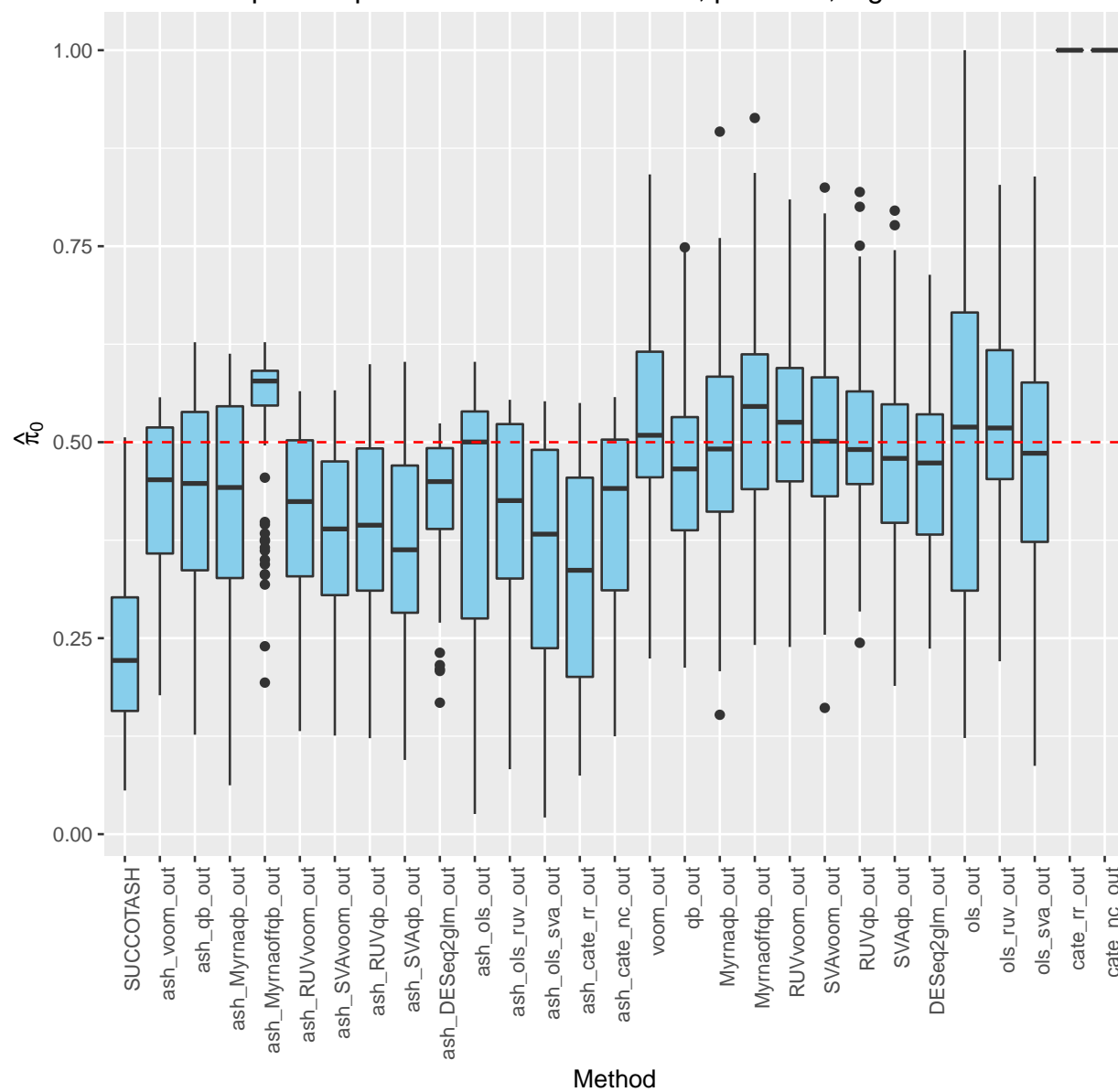




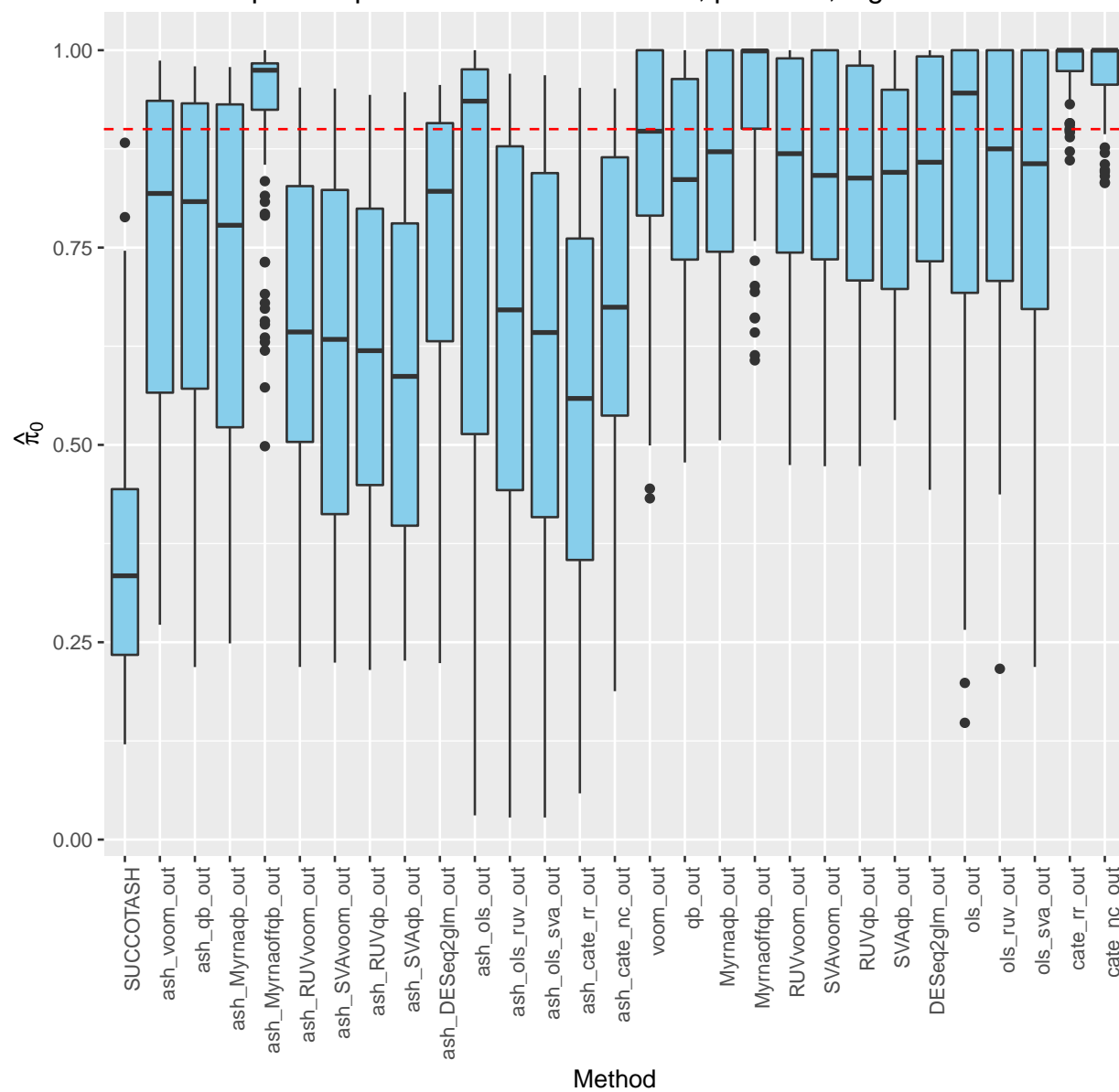
Boxplots of  $\pi_0$  estimates when  $n = 10$ ,  $\pi_0 = 0.5$ ,  $\log_2\text{foldsd} = 1$

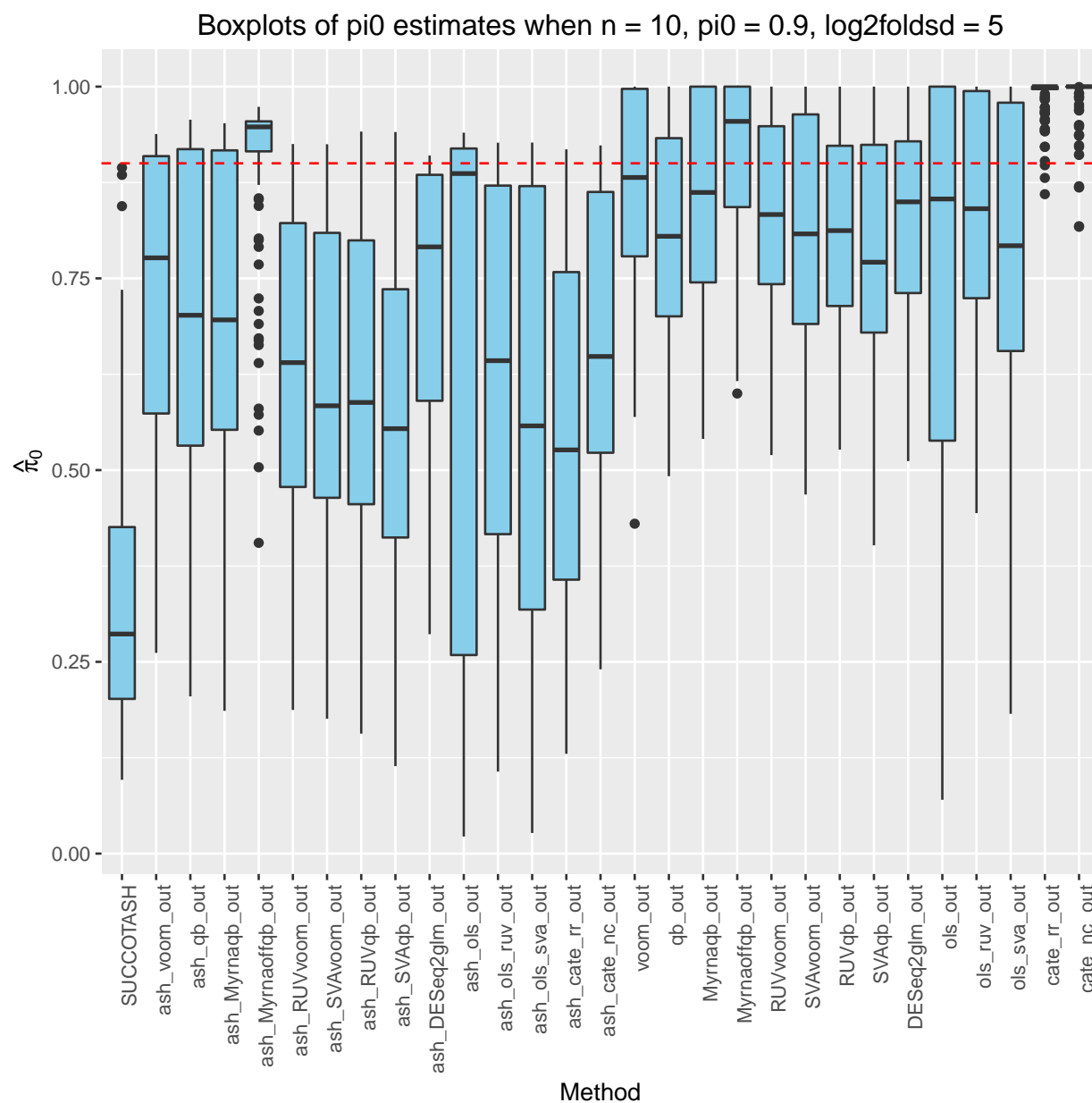


Boxplots of  $\pi_0$  estimates when  $n = 10$ ,  $\pi_0 = 0.5$ ,  $\log_2\text{foldsd} = 5$



Boxplots of  $\pi_0$  estimates when  $n = 10$ ,  $\pi_0 = 0.9$ ,  $\log_2\text{foldsd} = 1$





## References

- Jushan Bai, Kunpeng Li, et al. Statistical analysis of factor models of high dimension. *The Annals of Statistics*, 40(1):436–465, 2012.
- Andreas Buja and Nermin Eyuboglu. Remarks on parallel analysis. *Multivariate behavioral research*, 27(4):509–540, 1992.
- J Gagnon-Bartsch, L Jacob, and TP Speed. Removing unwanted variation from high dimensional

- data with negative controls. Technical report, Technical Report 820, Department of Statistics, University of California, Berkeley, 2013.
- Ben Langmead, Kasper D Hansen, Jeffrey T Leek, et al. Cloud-scale rna-sequencing differential expression analysis with myrna. *Genome Biol*, 11(8):R83, 2010.
- Charity W Law, Yunshun Chen, Wei Shi, and Gordon K Smyth. Voom: precision weights unlock linear model analysis tools for rna-seq read counts. *Genome Biol*, 15(2):R29, 2014.
- Jeffrey T Leek. svaseq: removing batch effects and other unwanted noise from sequencing data. *Nucleic acids research*, page gku864, 2014.
- Jeffrey T Leek and John D Storey. Capturing heterogeneity in gene expression studies by surrogate variable analysis. *PLoS Genet*, 3(9):1724–1735, 2007.
- Michael I Love, Wolfgang Huber, and Simon Anders. Moderated estimation of fold change and dispersion for rna-seq data with deseq2. *Genome Biol*, 15(12):550, 2014.
- Davide Risso, John Ngai, Terence P Speed, and Sandrine Dudoit. Normalization of rna-seq data using factor analysis of control genes or samples. *Nature biotechnology*, 32(9):896–902, 2014.
- Gordon K Smyth. Limma: linear models for microarray data. In *Bioinformatics and computational biology solutions using R and Bioconductor*, pages 397–420. Springer, 2005.
- John D Storey. A direct approach to false discovery rates. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64(3):479–498, 2002.
- Yunting Sun, Nancy R Zhang, Art B Owen, et al. Multiple hypothesis testing adjusted for latent variables, with an application to the agemap gene expression data. *The Annals of Applied Statistics*, 6(4):1664–1688, 2012.
- Jingshu Wang, Qingyuan Zhao, Trevor Hastie, and Art B Owen. Confounder adjustment in multiple hypotheses testing. *arXiv preprint arXiv:1508.04178*, 2015.