SUCCOTASH sims when α, the covariance, and the grid are known.

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Abstract

This simulation study looks at the model assumed in the second step of SUCCOTASH. I compare SUCCOTASH with the second step of LEAPP. No other confounder adjustment procedure is applicable for comparison when assuming this model. I assume that α, Σ, and τ are all known.

1 Model Description

$$Y_{p\times 1} = \beta_{p\times 1} + \alpha_{p\times k} Z_{k\times 1} + E_{p\times 1}, \tag{1}$$

such that

- α is known.
- $E \sim N_p(0, I_p)$, so we explore homoscedastic case.

2 Procedure

- $p = 100$,
- $k \in \{5, 10, 50\}$,
- $\beta_j \sim N(0, \tau_k^2)$ w.p. π_k,
- $\tau_k^2 = 0, 1, 100$ for $k = 0, 1, 2$ when we have a three mixture and $\tau_k = 0, 100$ for $k = 0, 1$ when we have a two mixture model,
- $\pi \in \{(0.5, 0.5), (0.9, 0.1), (1, 0, 0), (0.9, 0.1, 0), (0.9, 0, 0.1), (0.5, 0.5, 0), (0.5, 0, 0.5), (0.5, 0.25, 0.25)\}$
- $Z_j \overset{i.i.d.}{\sim} N(0, 1)$,
- $\alpha_{ij} \overset{i.i.d.}{\sim} N(0, 1)$,
- 400 iterations for each π by k combination, sampling a new Z and α at each iteration.

- I did not regularize the estimates of π for SUCCOTASH.
- At each iteration, I calculated the Sum of Squared Errors (SSE) for the posterior means under SUCCOTASH, and the estimates of β given by the second step of LEAPP.
- I also calculated the SSE when using just Y to estimate β (called OLS in Figures and Tables below).
- I also calculated $\hat{\pi}_0$ given by SUCCOTASH and LEAPP at each iteration.
- LEAPP uses an L_1 penalty, so I called its $\hat{\pi}_0$ to just be the proportion of elements of β it sets to 0.

- The only comparable procedure using Model (1) is the second step of LEAPP [Sun et al., 2012].
- CATE [Wang et al., 2015] is not applicable because its model for its second step is

$$Y_{p\times 1} \sim N_p(\beta_{p\times 1} + \alpha_{p\times k}\gamma_{k\times 1}, \alpha\alpha^T + I_p), \tag{2}$$

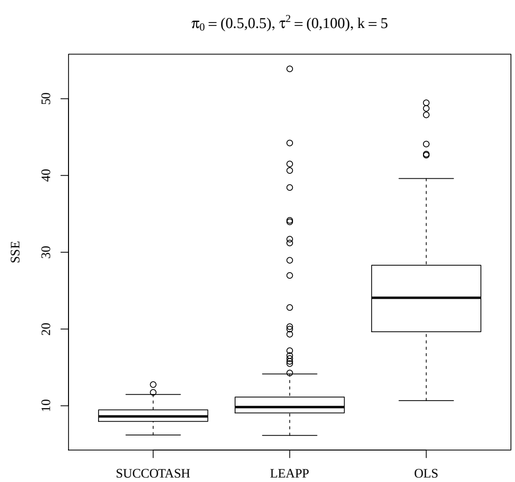
  where γ describes the linear relationship between the observed and unobserved variables.
- RUV [Gagnon-Bartsch et al., 2013] is not applicable because we don't assume we have any control genes.
- SVA [Leek and Storey, 2008] is not applicable because it doesn't use this two-step procedure.

3 Results

SUCCOTASH always beats LEAPP in terms of SSE (Table 1), especially when k is large. LEAPP seems to perform horribly whenever there are a lot of confounders.

When k is small ($k = 5$ or 10) and we don't include $\tau_1 = 1$, SUCCOTASH estimates π_0 fairly accurately (Table 3). This accuracy decreases when k is large ($k = 50$). But increasing p corrects for this (Table 4).

4 SSE Boxplots



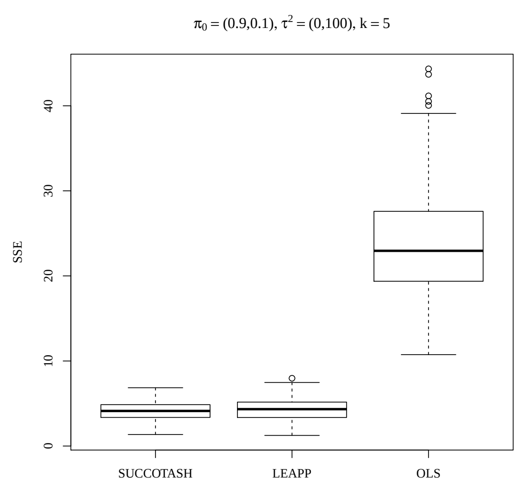


Table 1: Average Sum of Squared Errors for SUCCOTASH, LEAPP, and OLS at given k and π values.

k	π_0	π_1	π_2	SUCCOTASH	LEAPP	OLS
5	.5	NA	.5	8.7	10.9	24.4
5	.9	NA	.1	4.1	4.3	23.6
5	1	0	0	0.2	0.8	23.7
5	.9	.1	0	3.0	3.5	23.7
5	.9	0	.1	4.2	4.5	23.6
5	.5	.5	0	5.9	7.3	23.7
5	.5	0	.5	8.9	10.7	24.2
5	.5	.25	.25	7.6	8.4	23.7
10	.5	NA	.5	10.0	19.4	32.1
10	.9	NA	.1	4.3	4.6	32.5
10	1	0	0	0.2	1.3	32.4
10	.9	.1	0	3.0	3.7	32.7
10	.9	0	.1	4.3	4.7	31.9
10	.5	.5	0	6.0	7.4	32.7
10	.5	0	.5	10.1	18.2	32.5
10	.5	.25	.25	8.1	9.7	32.5
50	.5	NA	.5	54.2	99.5	70.8
50	.9	NA	.1	7.5	98.8	70.9
50	1	0	0	0.0	98.5	69.8
50	.9	.1	0	3.1	98.7	71.1
50	.9	0	.1	7.4	97.9	71.2
50	.5	.5	0	7.4	99.3	70.9
50	.5	0	.5	52.8	100.4	71.2
50	.5	.25	.25	23.2	99.9	71.1

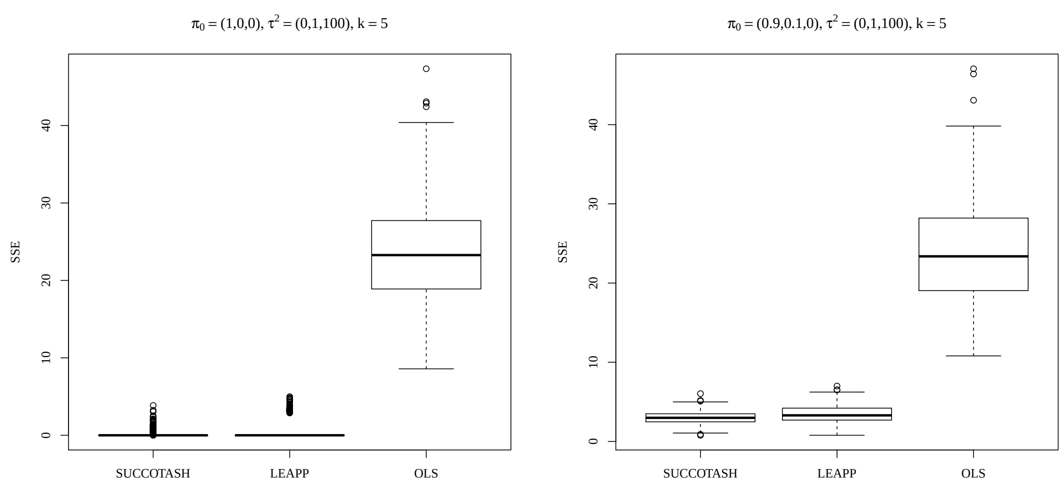


Table 2: Average Sum of Squared Errors for SUCCOTASH, LEAPP, and OLS at given k and π values. Here, $p = 500$.

k	π_0	π_1	π_2	SUCCOTASH	LEAPP	OLS
50	.5	NA	.5	22.1	134.4	159.0
50	.9	NA	.1	10.0	10.9	159.1

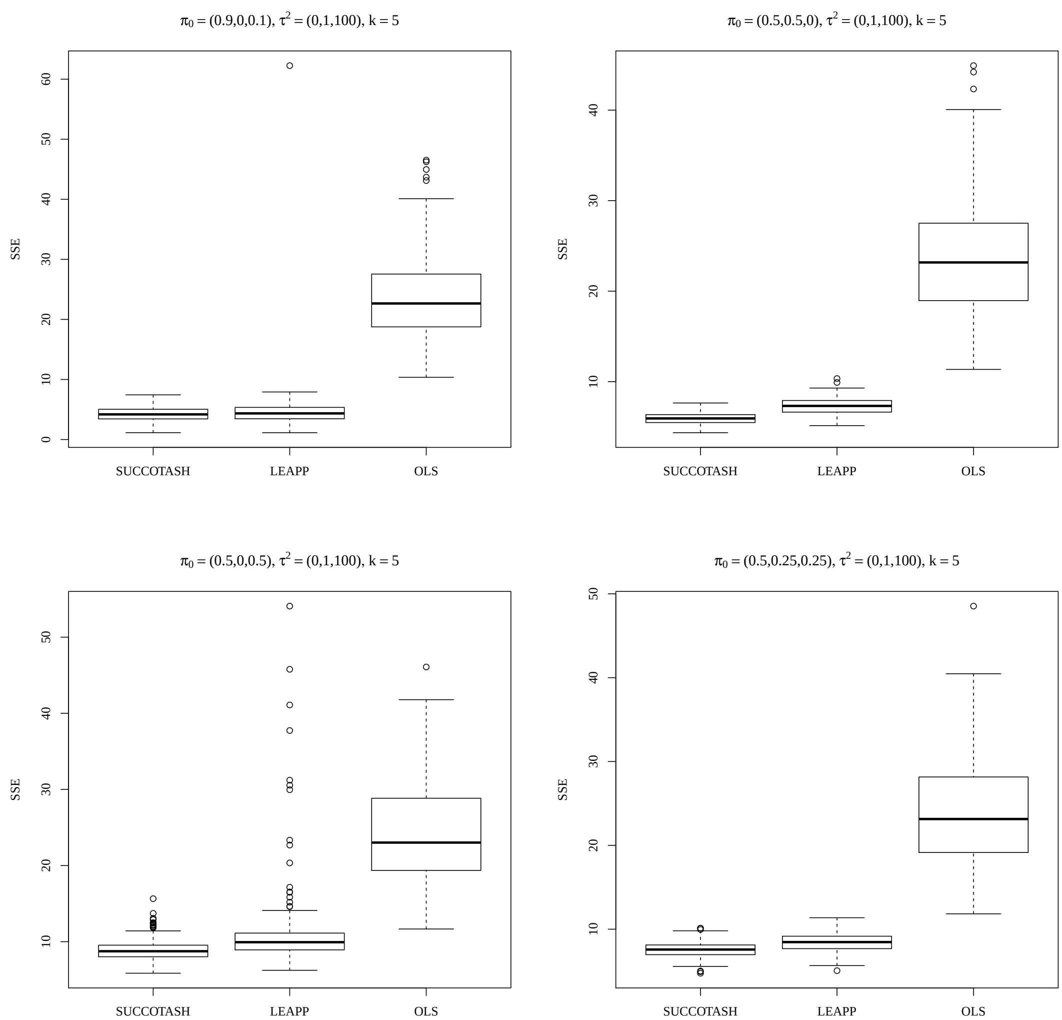


Table 3: Mean $\hat{\pi}_0$ for SUCCOTASH and LEAPP at given k and π values.

k	π_0	π_1	π_2	SUCCOTASH	LEAPP
5	.5	NA	.5	.51	.62
5	.9	NA	.1	.90	.92
5	1	0	0	.98	1.00
5	.9	.1	0	.92	.99
5	.9	0	.1	.86	.92
5	.5	.5	0	.57	.98
5	.5	0	.5	.48	.62
5	.5	.25	.25	.57	.80
10	.5	NA	.5	.52	.58
10	.9	NA	.1	.91	.92
10	1	0	0	.98	1.00
10	.9	.1	0	.94	.99
10	.9	0	.1	.88	.92
10	.5	.5	0	.65	.98
10	.5	0	.5	.50	.58
10	.5	.25	.25	.61	.79
50	.5	NA	.5	.66	.17
50	.9	NA	.1	.92	.20
50	1	0	0	1.00	.20
50	.9	.1	0	1.00	.21
50	.9	0	.1	.93	.20
50	.5	.5	0	1.00	.20
50	.5	0	.5	.68	.17
50	.5	.25	.25	.81	.19

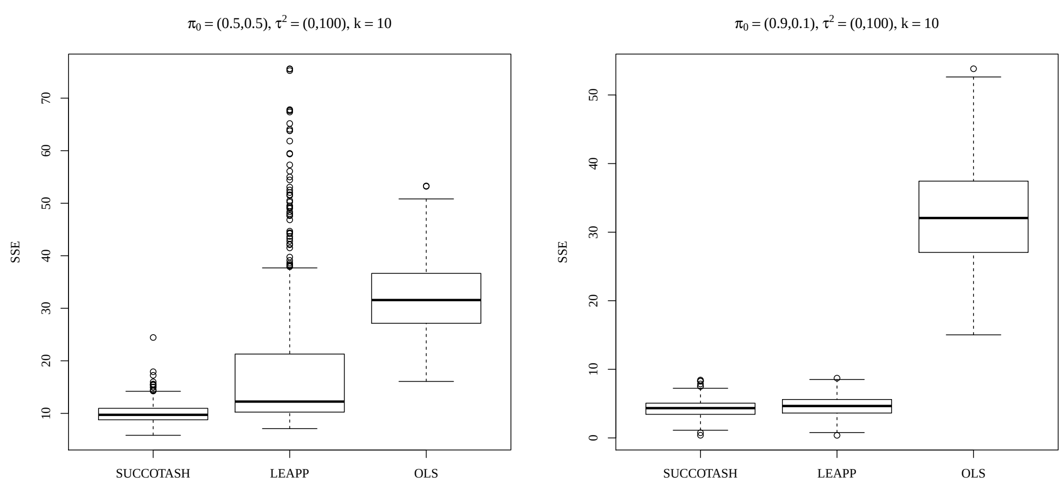
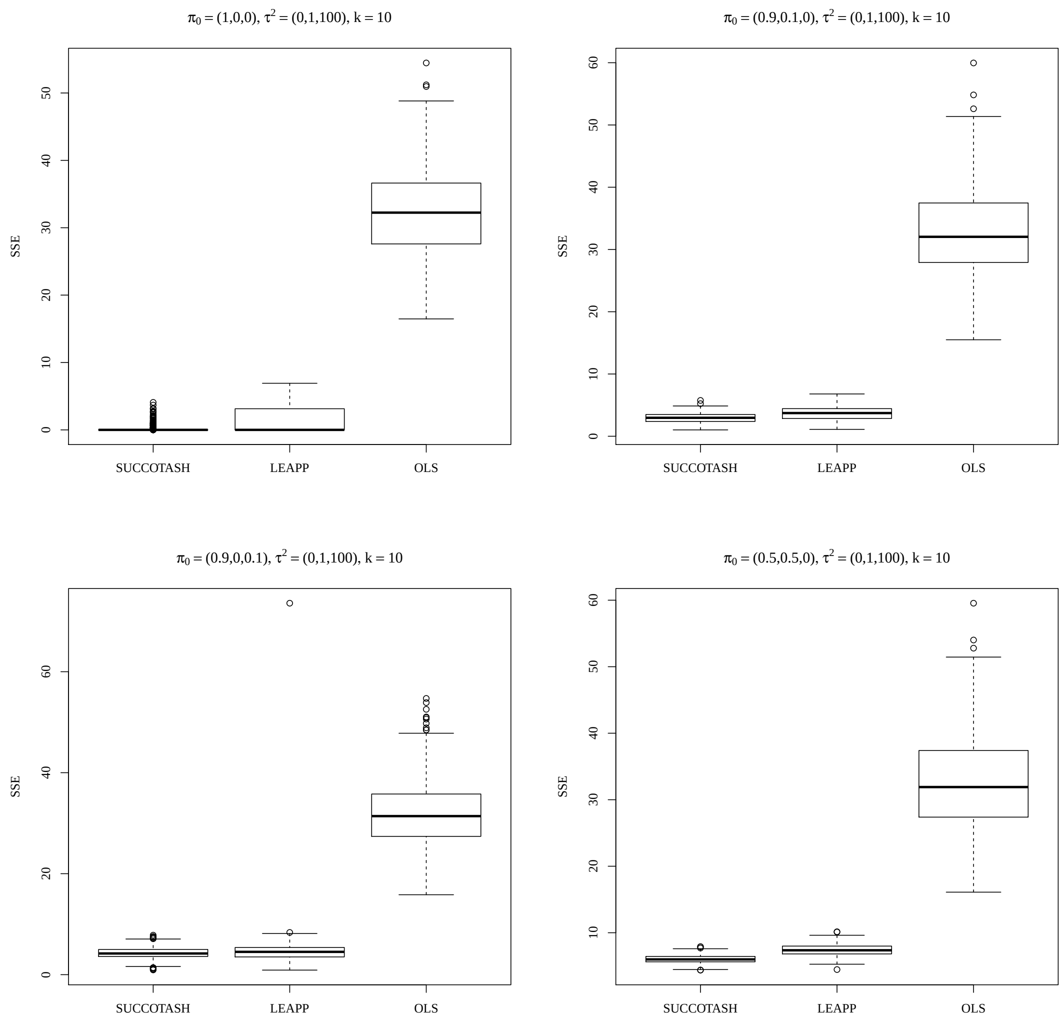
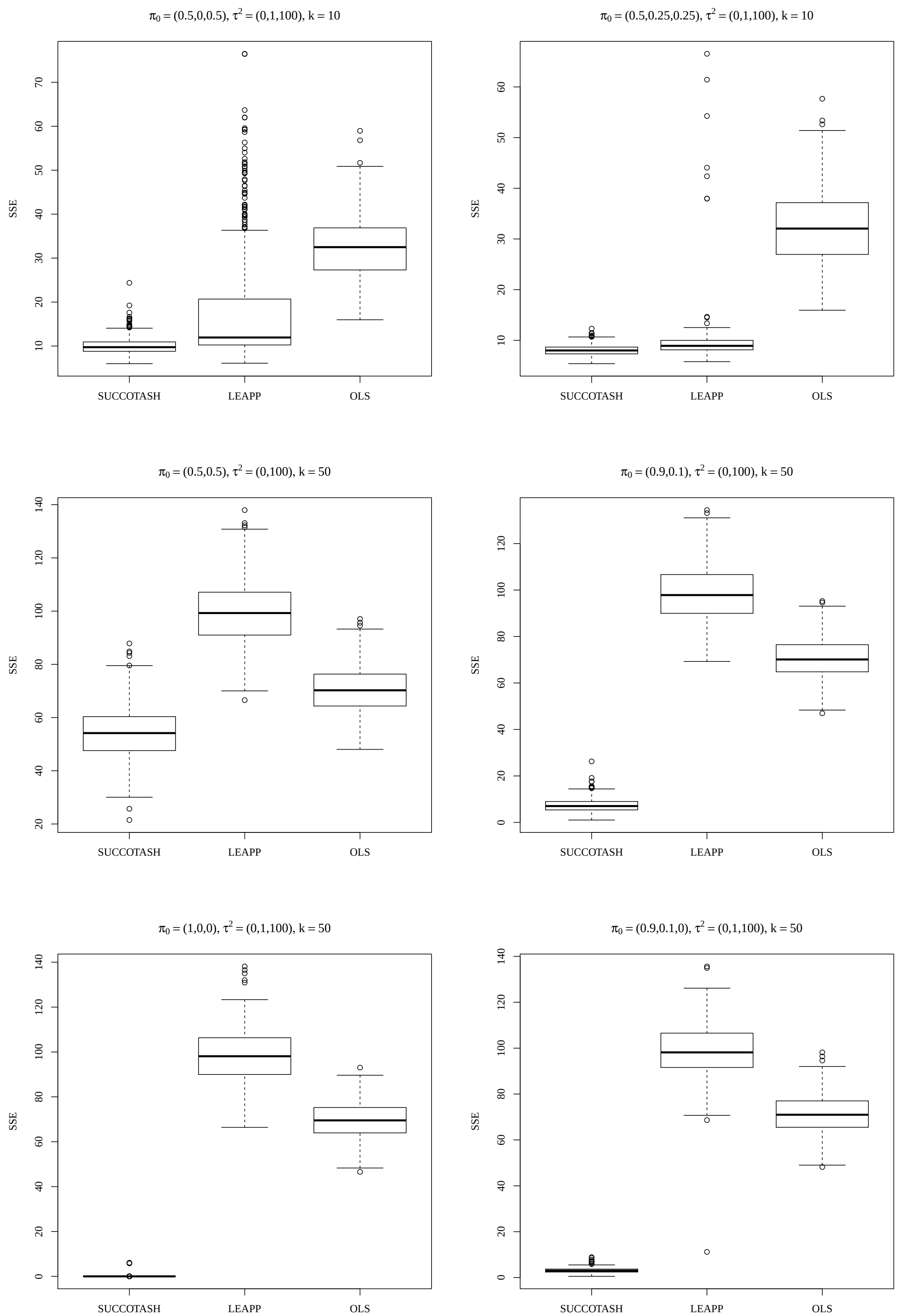


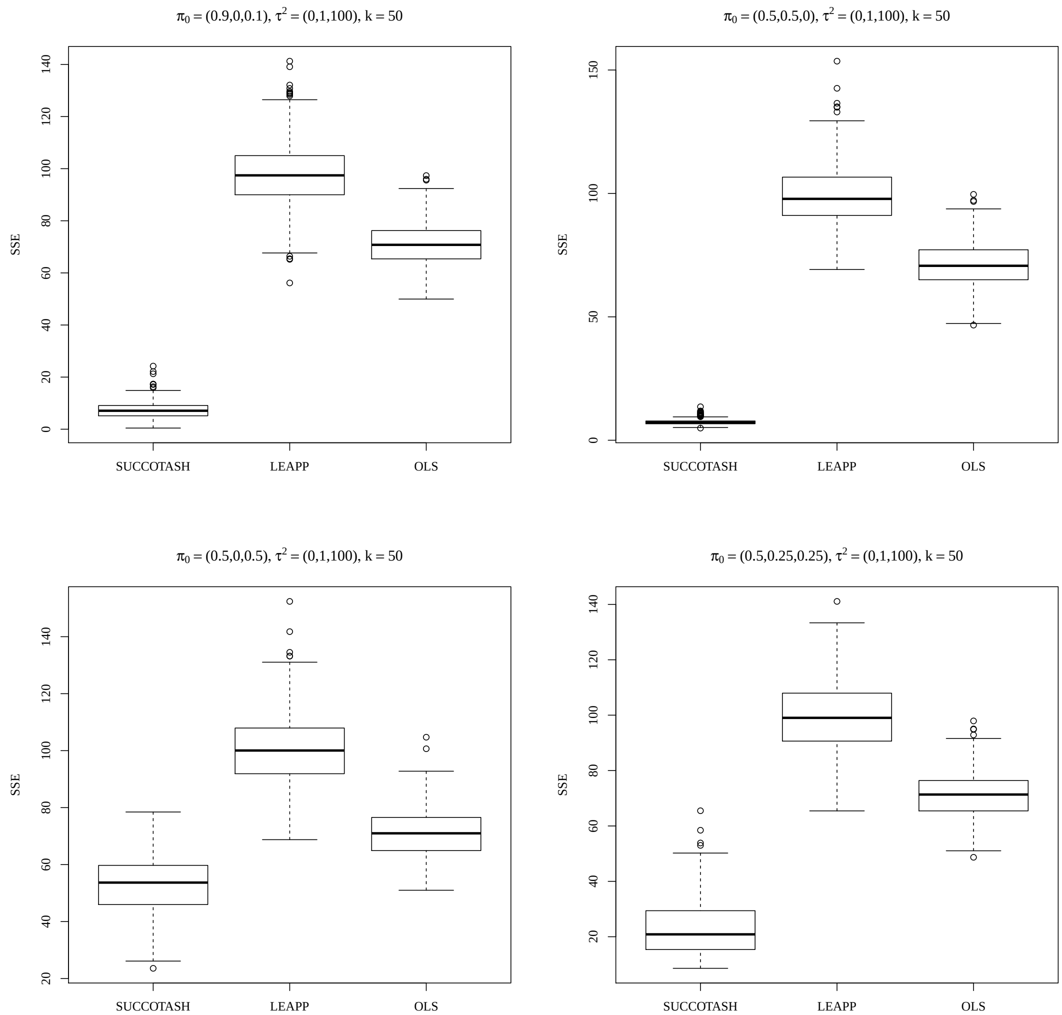
Table 4: Mean $\hat{\pi}_0$ for SUCCOTASH and LEAPP at given k and π values. Here, $p = 500$.

k	π_0	π_1	π_2	SUCCOTASH	LEAPP
50	.5	NA	.5	.52	.45
50	.9	NA	.1	.90	.93

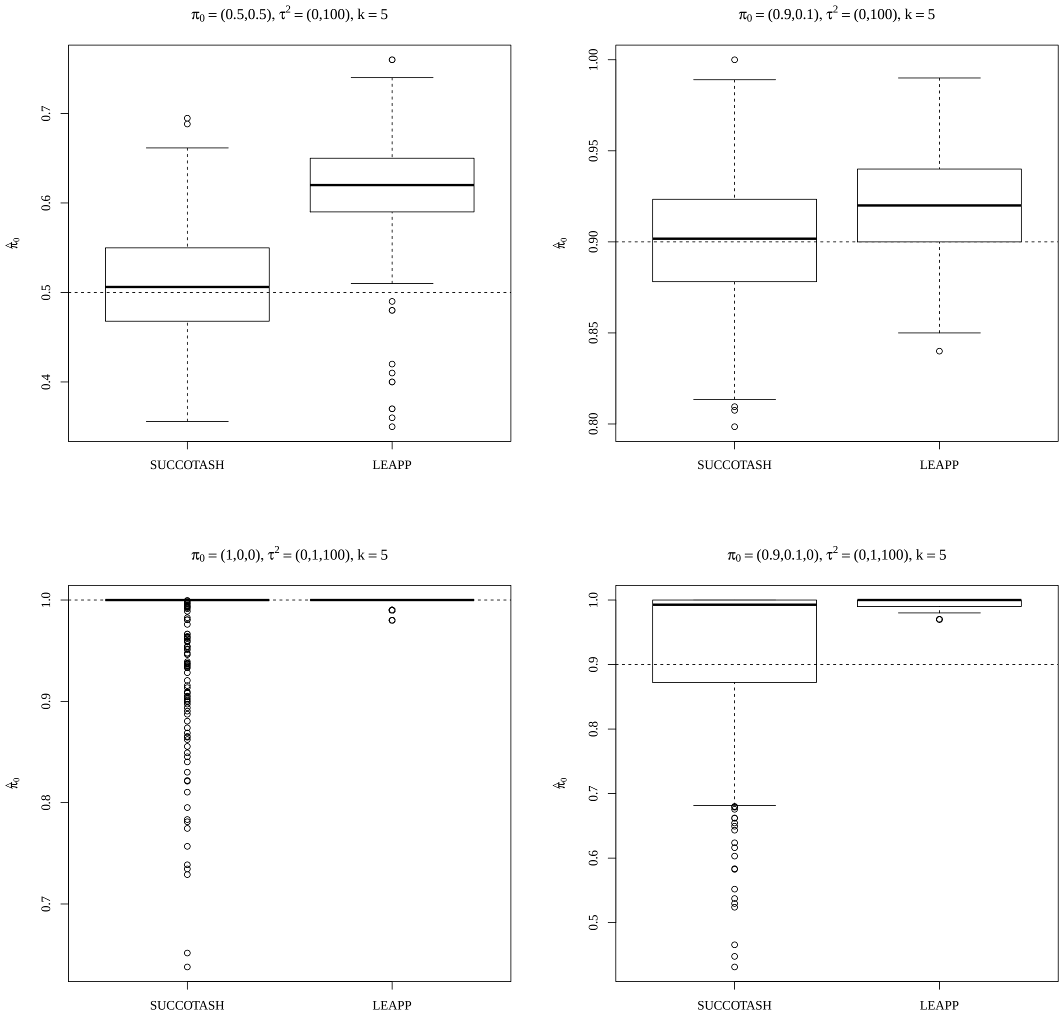


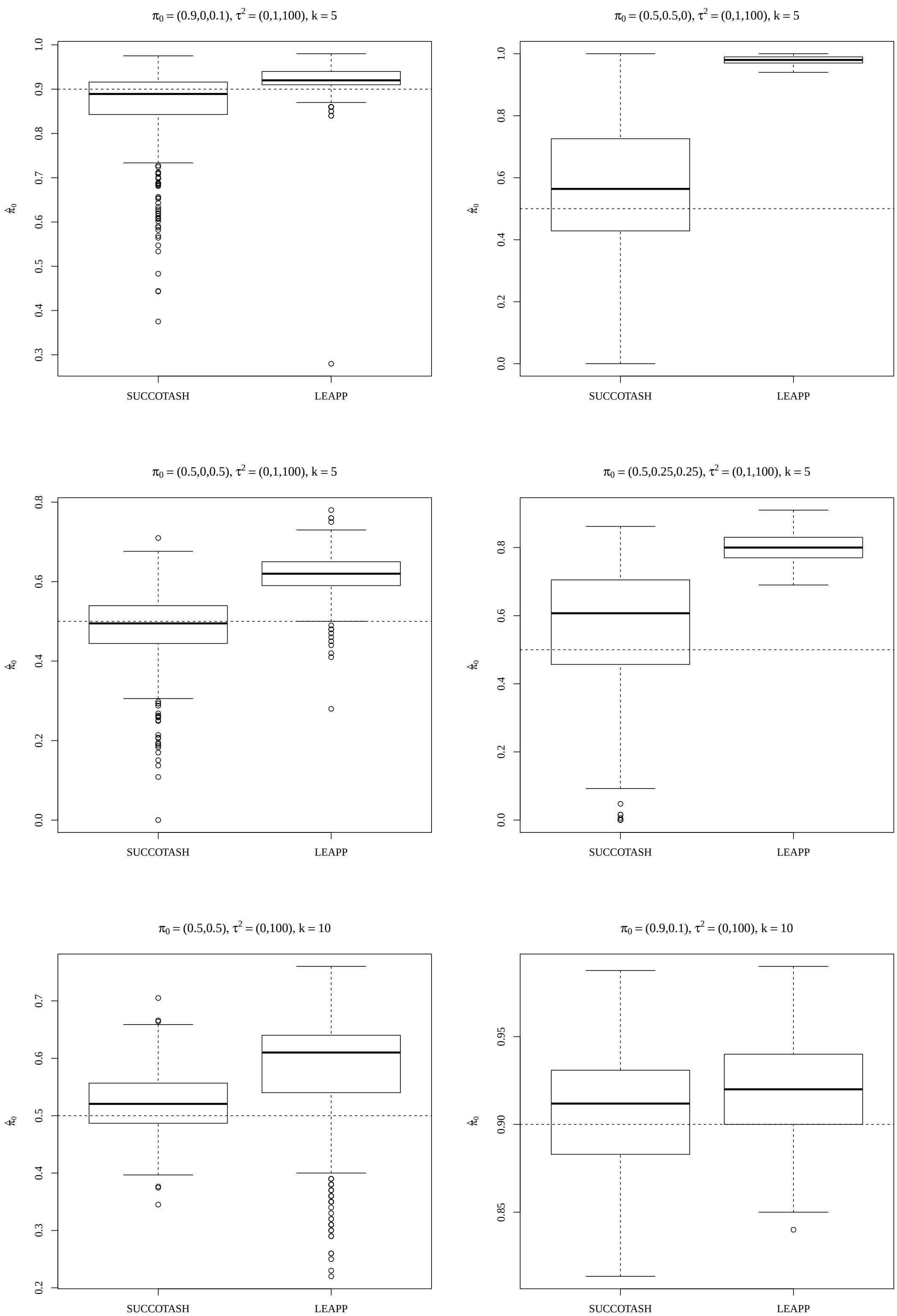


$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, k = 10
$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, k = 10
$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, k = 50
$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, k = 50
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, k = 50
$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, k = 50
SSE
SUCCOTASH
LEAPP
OLS

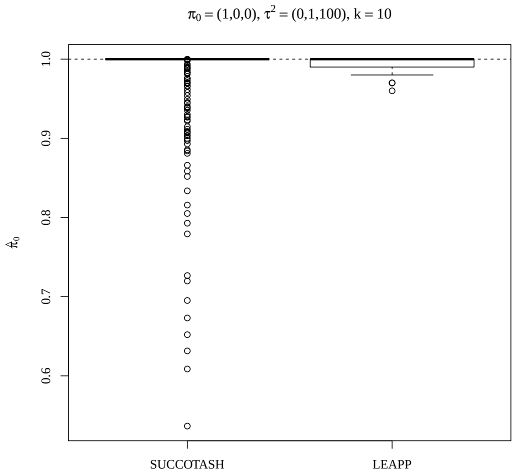

π0 = (0.9,0,0.1), τ2 = (0,1,100), k = 50
π0 = (0.5,0.5,0), τ2 = (0,1,100), k = 50
π0 = (0.5,0,0.5), τ2 = (0,1,100), k = 50
π0 = (0.5,0.25,0.25), τ2 = (0,1,100), k = 50
SSE
SUCCOTASH
LEAPP
OLS

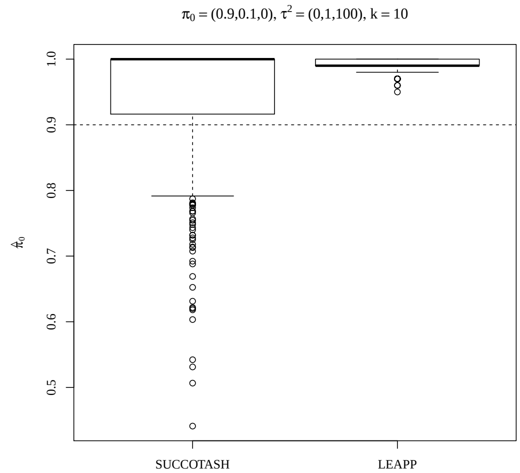
5 $\hat{\pi}_0$ Boxplots

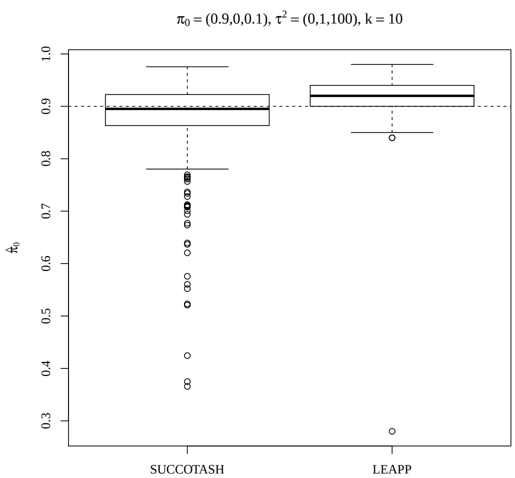


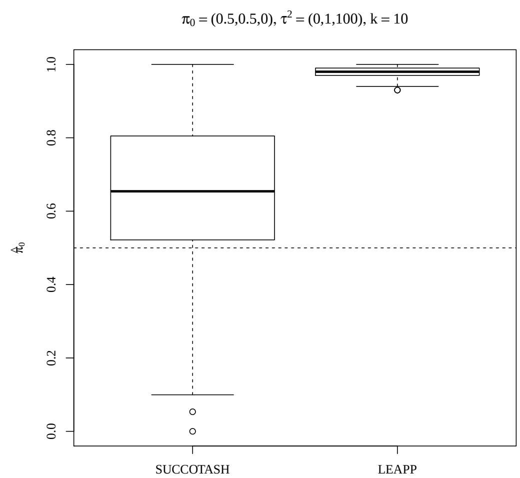


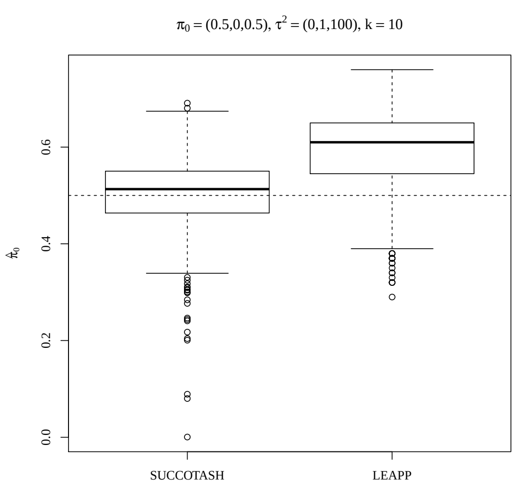
π0 = (0.9,0,0.1), τ2 = (0,1,100), k = 5
π0 = (0.5,0.5,0), τ2 = (0,1,100), k = 5
π0 = (0.5,0,0.5), τ2 = (0,1,100), k = 5
π0 = (0.5,0.25,0.25), τ2 = (0,1,100), k = 5
π0 = (0.5,0.5), τ2 = (0,100), k = 10
π0 = (0.9,0.1), τ2 = (0,100), k = 10
π̂0
SUCCOTASH
LEAPP

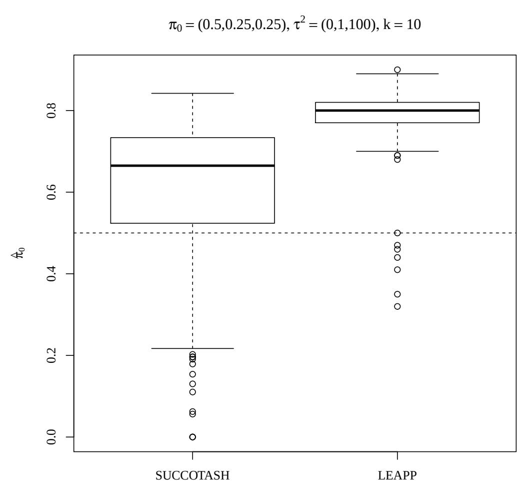

$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP

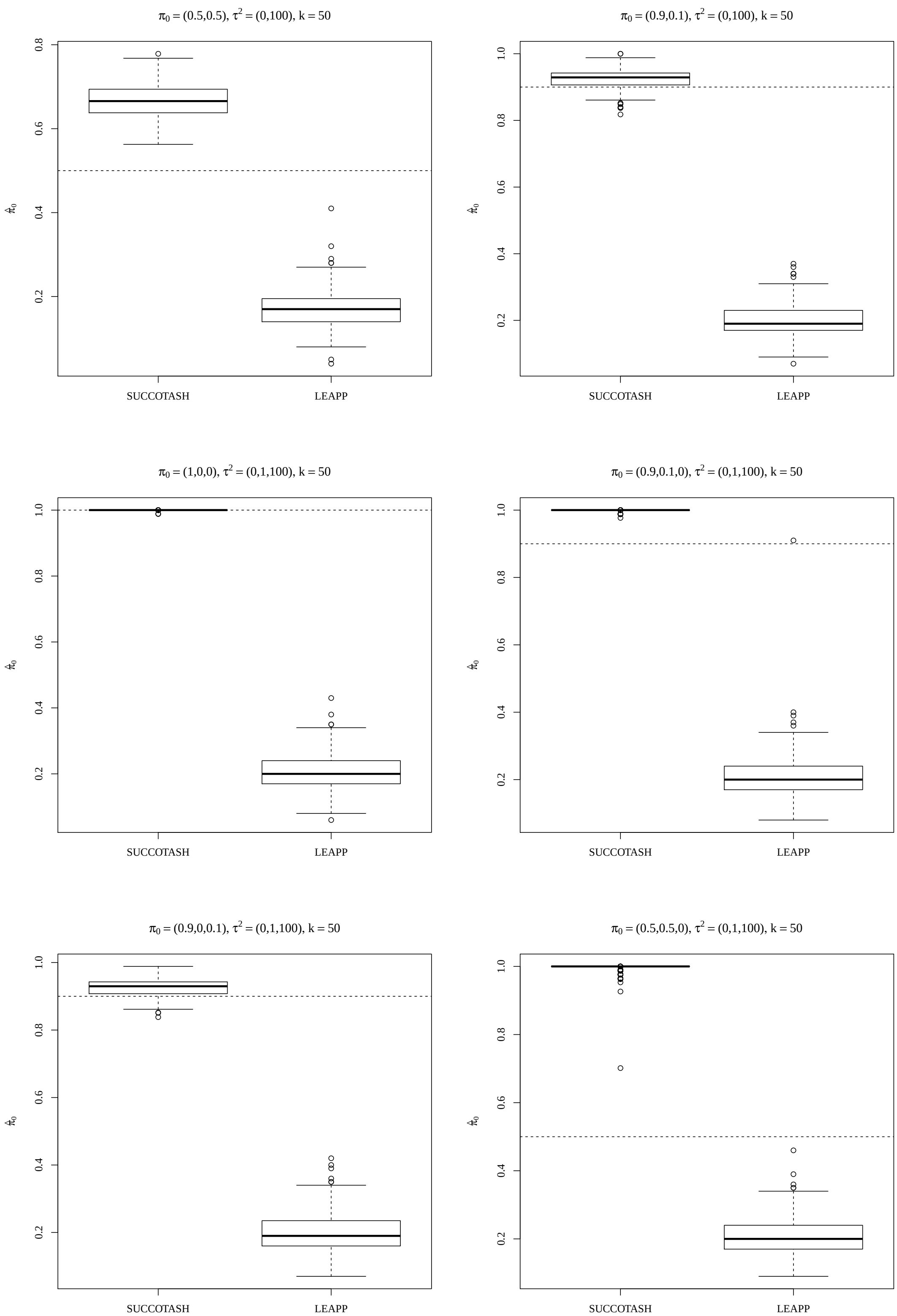

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP


$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
SUCCOTASH
LEAPP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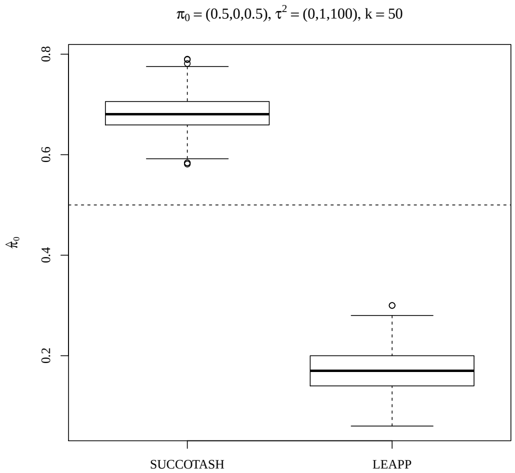
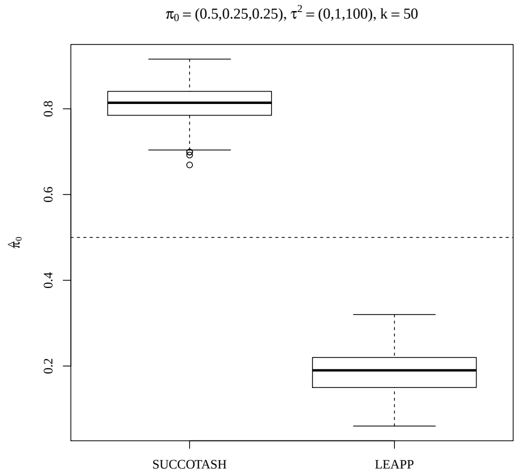
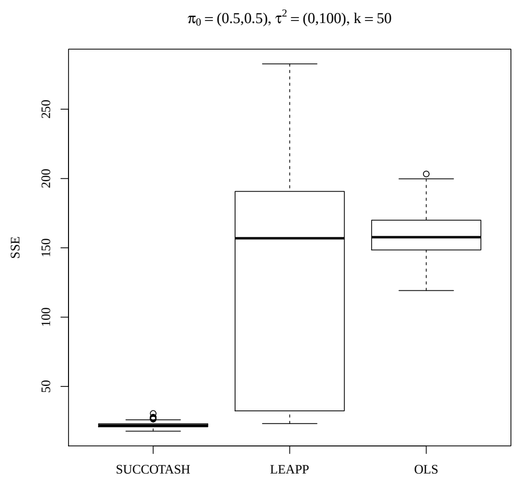
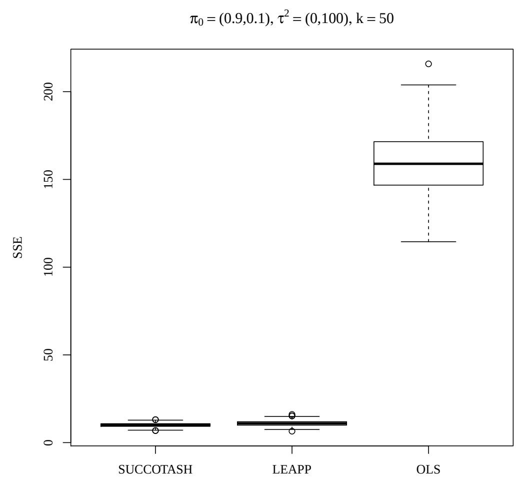
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 50$
$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\hat{\pi}_0$
SUCCOTASH
LEAPP

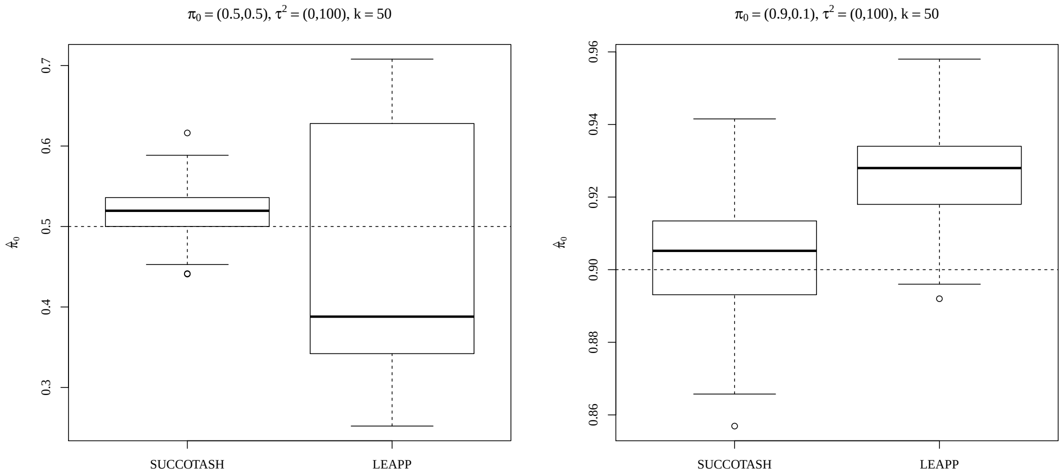




6 Boxplots when $p = 500$







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