SUCCOTASH when α and τ are known, but Σ is "estimated".

David Gerard

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Abstract

Here, I compare SUCCOTASH to LEAPP when the assumed Σ is a noisy version of the true Σ.

1 Model Description

$$Y_{p\times 1} = \beta_{p\times 1} + \alpha_{p\times k} Z_{k\times 1} + E_{p\times 1}, \tag{1}$$

such that

- α is known.
- $E \sim N_p(0, \Sigma)$, $\Sigma = \text{diag}\{\sigma_1^2, \dots, \sigma_p^2\}$.

2 Procedure

- $p = 100$,
- $k \in \{5, 10, 50\}$,
- $\beta_j \sim N(0, \tau_k^2)$ w.p. π_k,
- $\tau_k^2 = 0, 1, 100$ for $k = 0, 1, 2$ when we have a three mixture and $\tau_k = 0, 100$ for $k = 0, 1$ when we have a two mixture model,
- $\pi \in \{(0.5, 0.5), (0.9, 0.1), (1, 0, 0), (0.9, 0.1, 0), (0.9, 0, 0.1), (0.5, 0.5, 0), (0.5, 0, 0.5), (0.5, 0.25, 0.25)\}$
- $Z_j \overset{i.i.d.}{\sim} N(0, 1)$,
- $\alpha_{ij} \overset{i.i.d.}{\sim} N(0, 1)$,
- 400 iterations for each π by k combination, sampling a new Z and α at each iteration.
- The true Σ I used to generate the data is $\Sigma = I_p$.
- I drew $\hat{\Sigma} = \text{diag}\{\hat{\sigma}_1, \dots, \hat{\sigma}_p\}$ from $\hat{\sigma}_i \sim \sigma_i^2 \chi_1^2/1$. This roughly corresponds to having a sample size of 3 (if we are just estimating a slope and intercept). I used this $\hat{\Sigma}$ when estimating β and Z in (1). But again, $\sigma_i^2 = 1$ for all $i = 1, \dots, p$.

- I did not regularize the estimates of π for SUCCOTASH.
- At each iteration, I calculated the Sum of Squared Errors (SSE) for the posterior means under SUCCOTASH, and the estimates of β given by the second step of LEAPP.
- I also calculated the SSE when using just Y to estimate β (called OLS in Figures and Tables below).
- I also calculated $\hat{\pi}_0$ given by SUCCOTASH and LEAPP at each iteration.

- LEAPP uses an L_1 penalty, so I called its $\hat{\pi}_0$ to just be the proportion of elements of β it sets to 0.

3 Results

SUCCOTASH beats LEAPP in every scenario (Table 1). It also seems to estimate π_0 fairly comparably to the case when Σ is known (Table 2). When df=1, it sometimes performs better than when $\sigma_i^2 \sim \chi_5^2/5$, which is a litte weird. These scenarios are $k = 50 and \pi = (0.5, 0.5)$, and then $k = 5$ or 10 for $\pi = (0.5, 0.25, 0.25)$. The only scenario where it performs much worse (in terms of estimating π_0) is when $k = 50$ and $\pi = (0.9, 0.1)$.

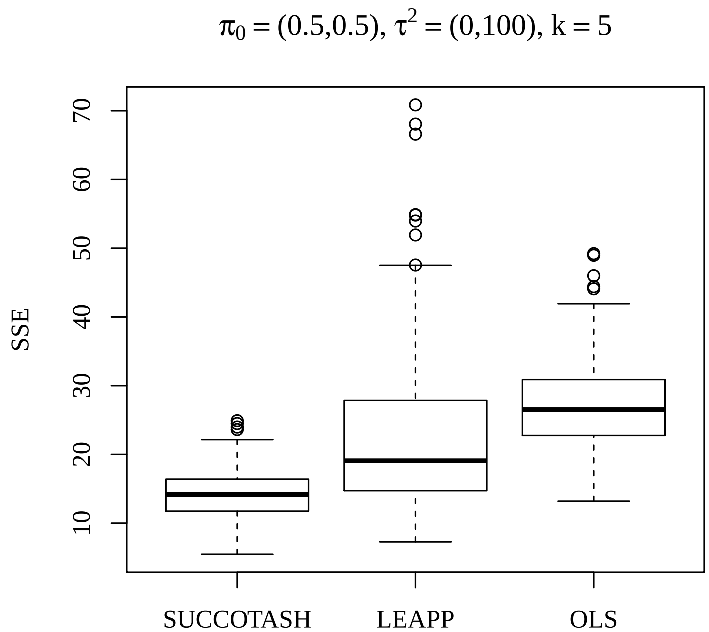
k	π_0	π_1	π_2	SUC	LEAPP	OLS
5	.5	-	.5	14.2	22.4	27.1
5	.9	-	.1	9.5	10.7	26.5
5	1	0	0	10.9	14.0	29.2
5	.9	.1	0	12.6	15.6	30.5
5	.9	0	.1	7.0	8.1	25.2
5	.5	.5	0	11.8	16.0	29.1
5	.5	0	.5	16.5	22.6	28.4
5	.5	.25	.25	13.2	18.6	27.1
10	.5	-	.5	13.4	32.1	34.1
10	.9	-	.1	8.4	11.7	33.4
10	1	0	0	4.8	7.9	34.4
10	.9	.1	0	8.2	11.0	35.0
10	.9	0	.1	9.1	11.2	33.9
10	.5	.5	0	5.8	8.9	33.0
10	.5	0	.5	14.0	33.3	34.6
10	.5	.25	.25	12.1	19.1	34.3
50	.5	-	.5	56.1	101.7	72.5
50	.9	-	.1	20.5	100.8	72.3
50	1	0	0	4.8	100.1	71.6
50	.9	.1	0	5.8	99.2	71.9
50	.9	0	.1	15.2	101.5	73.3
50	.5	.5	0	13.6	101.3	72.8
50	.5	0	.5	54.7	99.7	71.9
50	.5	.25	.25	32.7	100.2	72.7

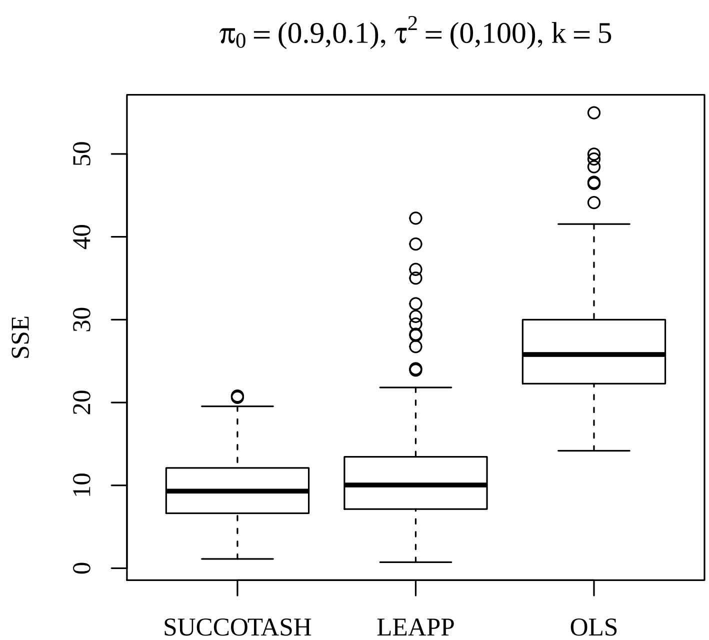
Table 1: Average Sum of Squared Errors for SUCCOTASH, LEAPP, and OLS at given k and π values.

k	π_0	π_1	π_2	SUC df=1	SUC df=5	SUC Sig Known	LEAPP
5	.5	-	.5	.49	.50	.51	.48
5	.9	-	.1	.88	.89	.90	.88
5	1	0	0	.98	1.00	.98	.98
5	.9	.1	0	.93	.95	.92	.95
5	.9	0	.1	.89	.88	.86	.89
5	.5	.5	0	.58	.60	.57	.81
5	.5	0	.5	.49	.49	.48	.52
5	.5	.25	.25	.53	.64	.57	.60
10	.5	-	.5	.49	.51	.52	.43
10	.9	-	.1	.88	.89	.91	.84
10	1	0	0	.99	1.00	.98	.98
10	.9	.1	0	.91	.97	.94	.94
10	.9	0	.1	.88	.89	.88	.87
10	.5	.5	0	.63	.63	.65	.87
10	.5	0	.5	.48	.51	.50	.43
10	.5	.25	.25	.59	.67	.61	.65
50	.5	-	.5	.55	.65	.52	.20
50	.9	-	.1	.77	.92	.90	.26
50	1	0	0	.95	1.00	1.00	.24
50	.9	.1	0	.97	1.00	1.00	.18
50	.9	0	.1	.87	.92	.93	.23
50	.5	.5	0	.88	.99	1.00	.24
50	.5	0	.5	.63	.67	.68	.16
50	.5	.25	.25	.74	.79	.81	.24

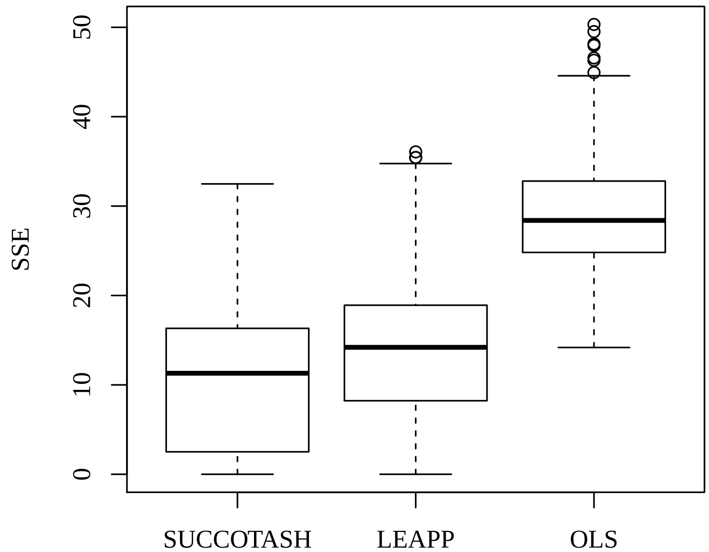
Table 2: Mean $\hat{\pi}_0$ for SUCCOTASH and LEAPP at given k and π values. "df $= k$" means σ_i^2 was drawn from a χ_k^2/k distribution. Also included are mean π_0 estimates when Σ is known.

4 SSE Plots

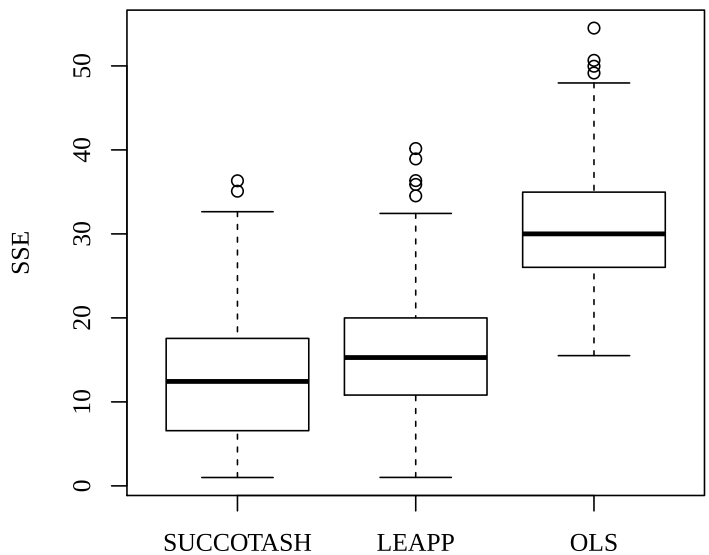

$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 5$
SSE
70
60
50
40
30
20
10
SUCCOTASH
LEAPP
OLS


$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 5$
SSE
50
40
30
20
10
0
SUCCOTASH
LEAPP
OLS

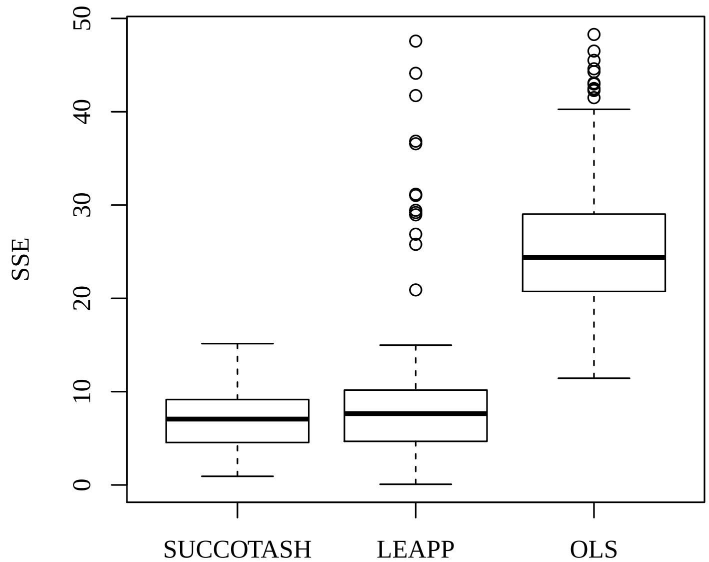
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 5$



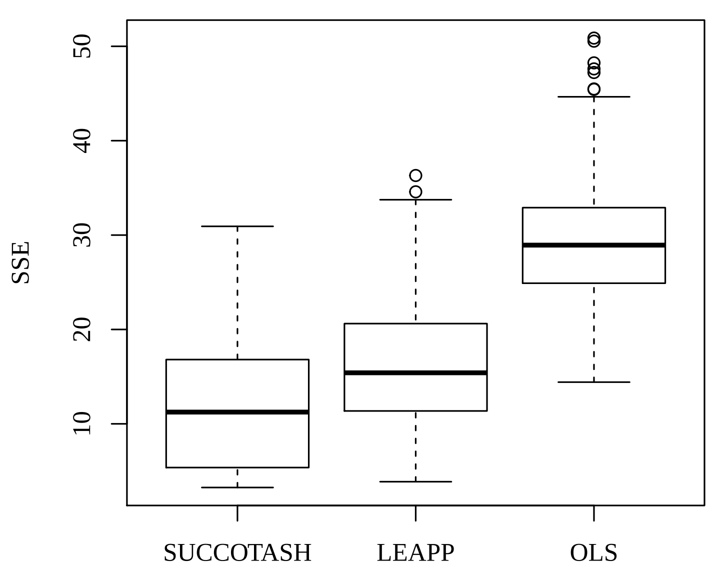
$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 5$



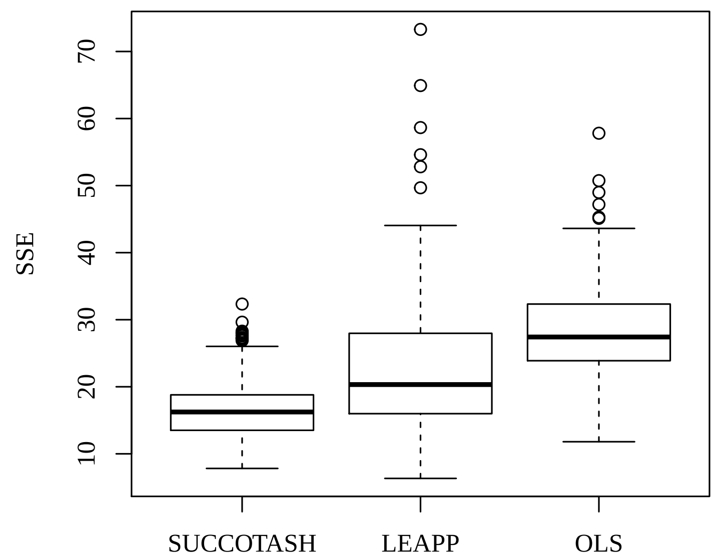
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 5$



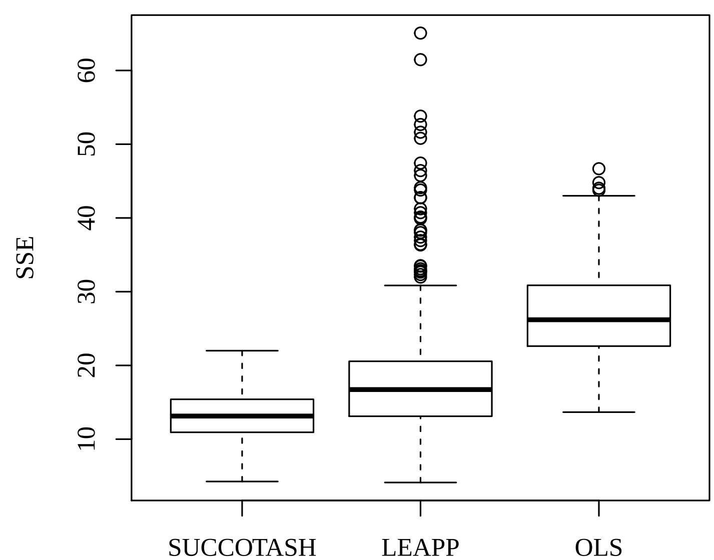
$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 5$

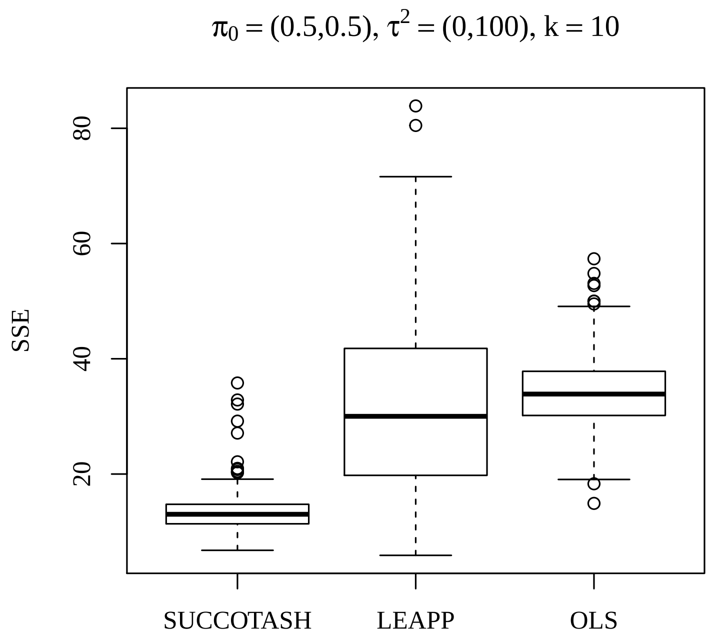


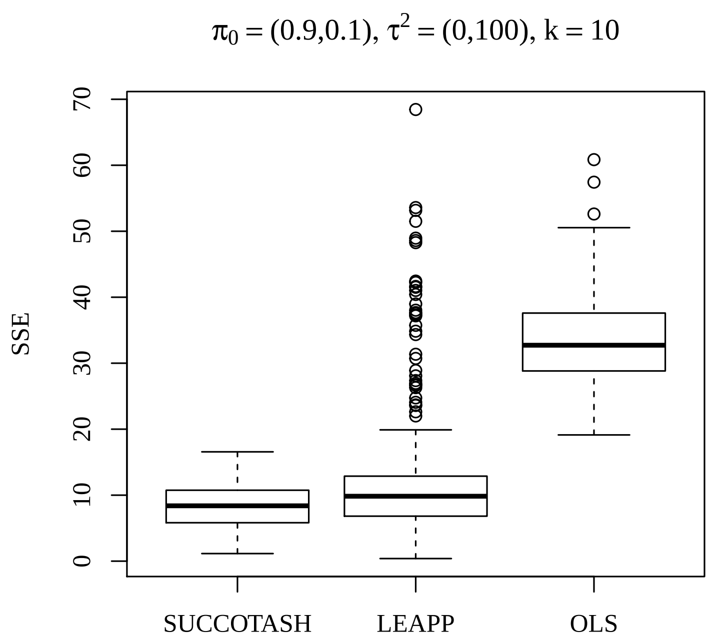
$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 5$

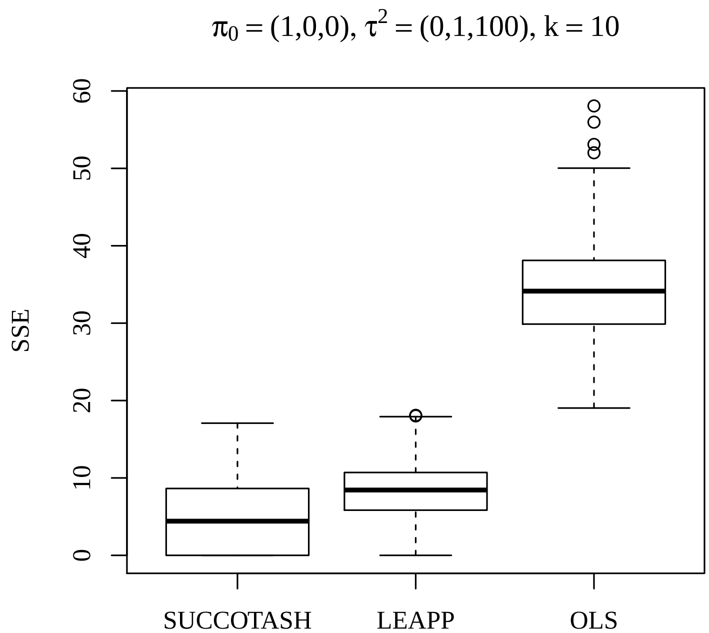


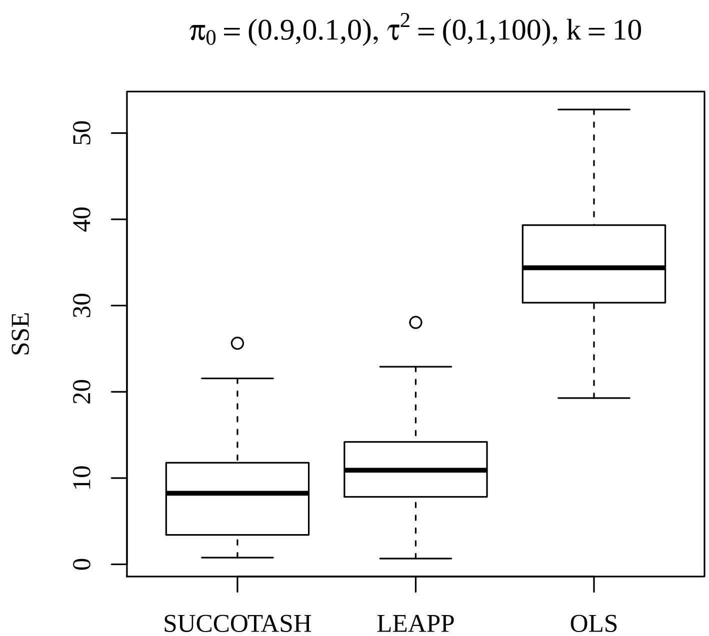
$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, $k = 5$

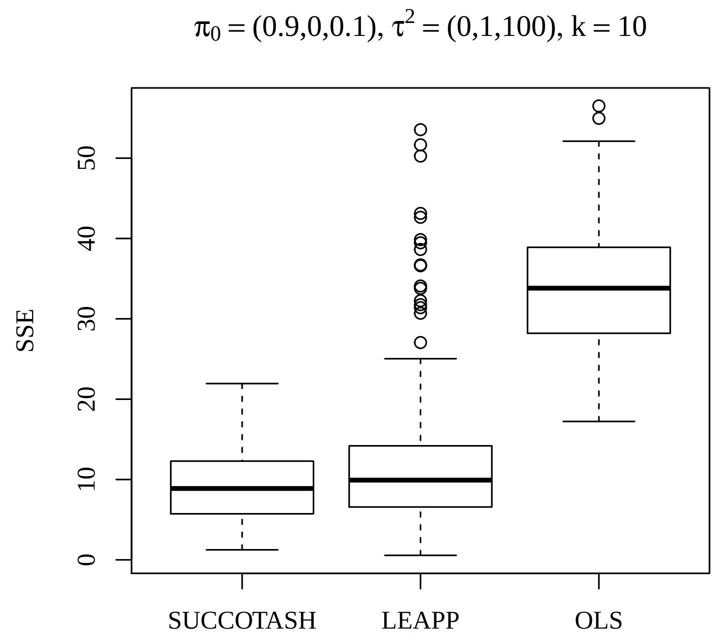



$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 10$
SSE
20
40
60
80
SUCCOTASH
LEAPP
OLS

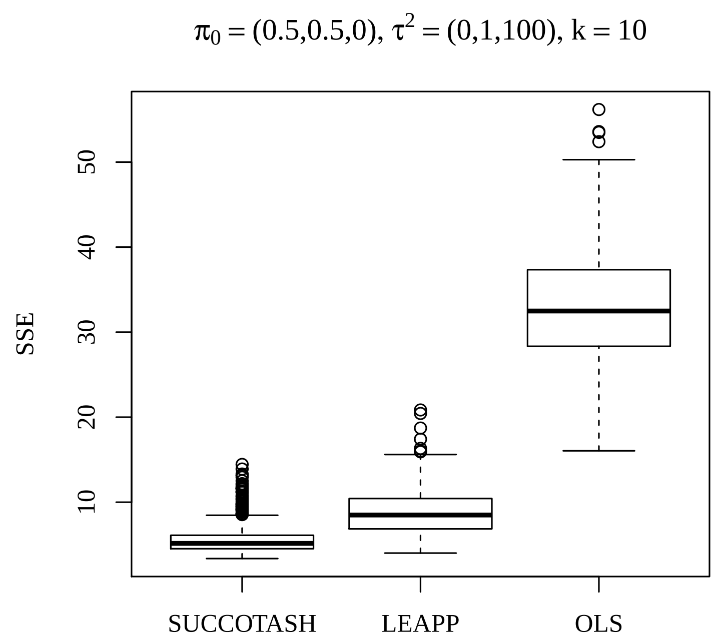

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 10$
SSE
0
10
20
30
40
50
60
70
SUCCOTASH
LEAPP
OLS


$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
0
10
20
30
40
50
60
SUCCOTASH
LEAPP
OLS

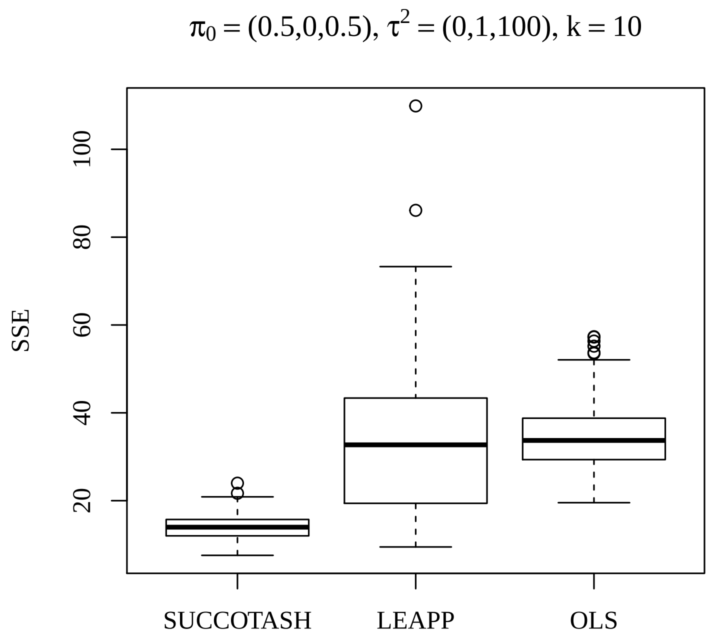

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
0
10
20
30
40
50
SUCCOTASH
LEAPP
OLS

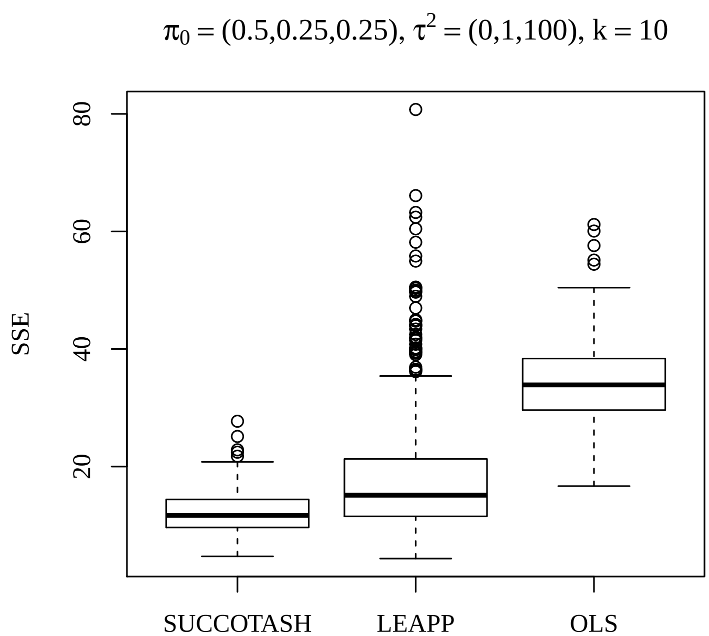


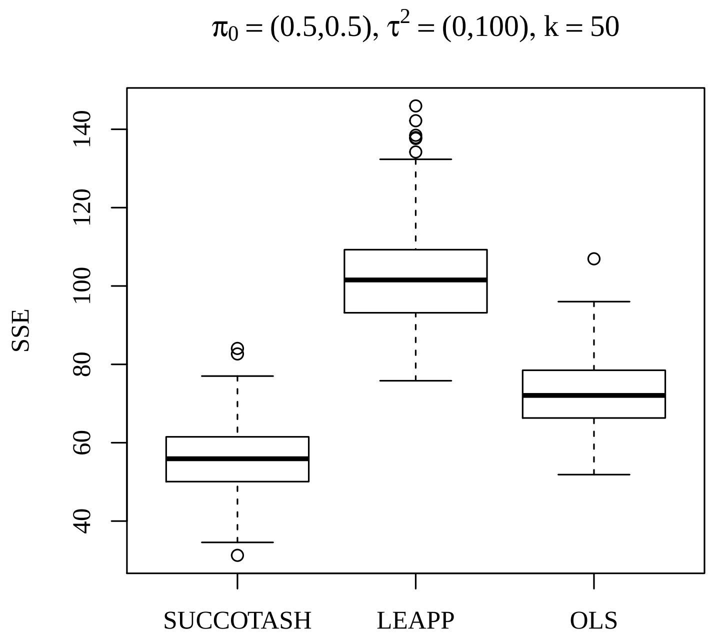
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
0
10
20
30
40
50
SUCCOTASH
LEAPP
OLS

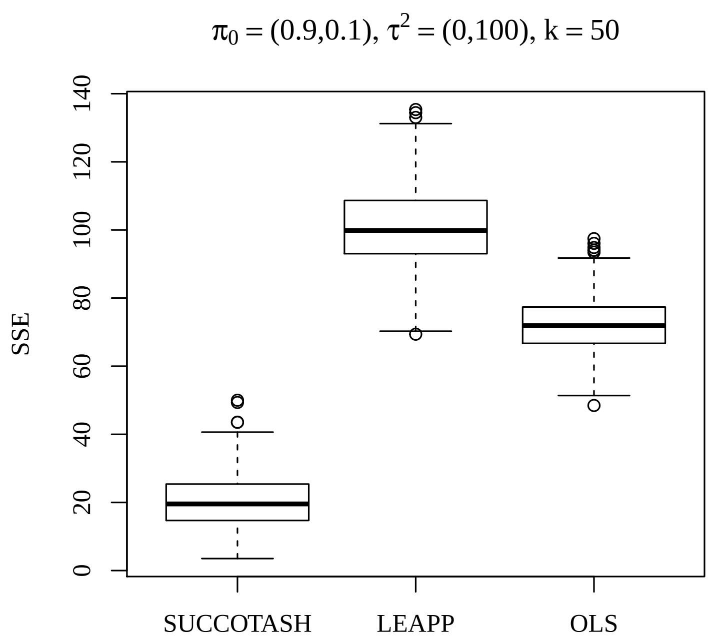


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
10
20
30
40
50
SUCCOTASH
LEAPP
OLS

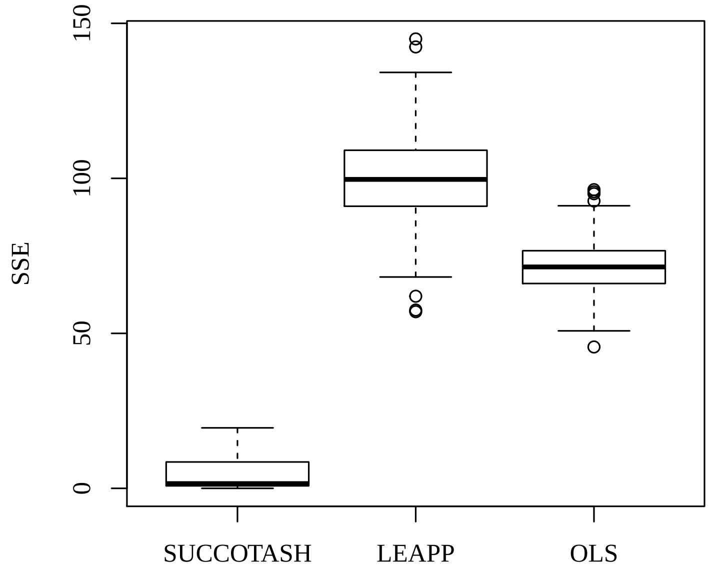

$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
20
40
60
80
100
SUCCOTASH
LEAPP
OLS


$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, $k = 10$
SSE
20
40
60
80
SUCCOTASH
LEAPP
OLS

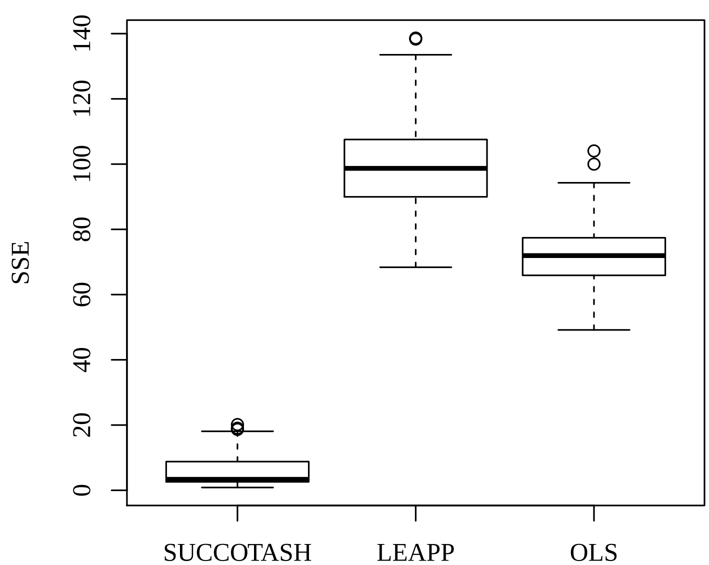

$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 50$
SSE
140
120
100
80
60
40
SUCCOTASH
LEAPP
OLS

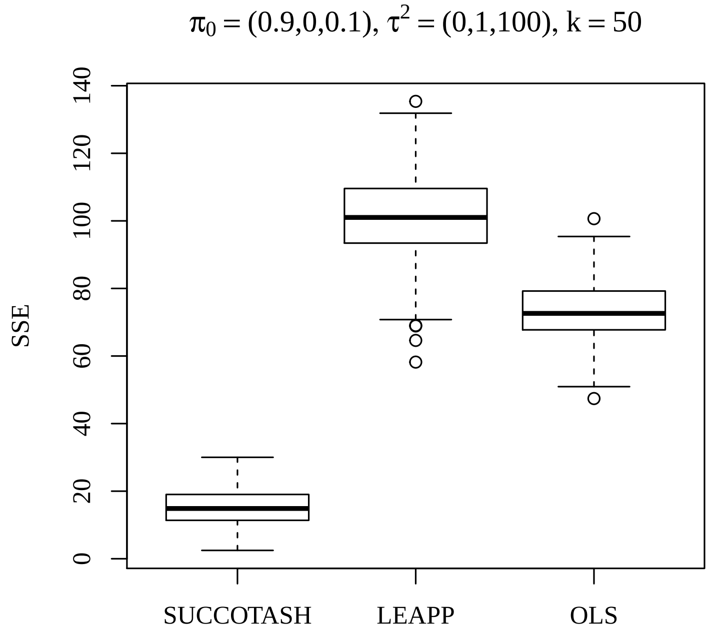

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 50$
SSE
140
120
100
80
60
40
20
0
SUCCOTASH
LEAPP
OLS

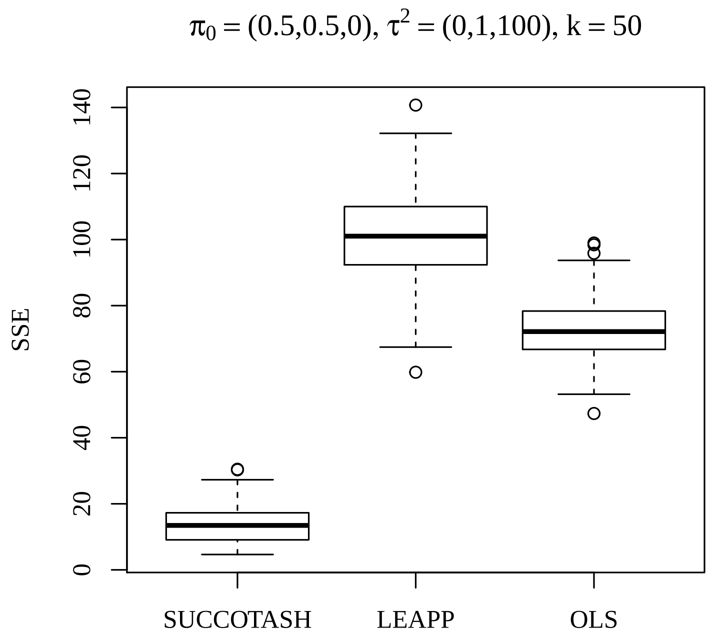
$\pi_0 = (1,0,0),\ \tau^2 = (0,1,100),\ k = 50$



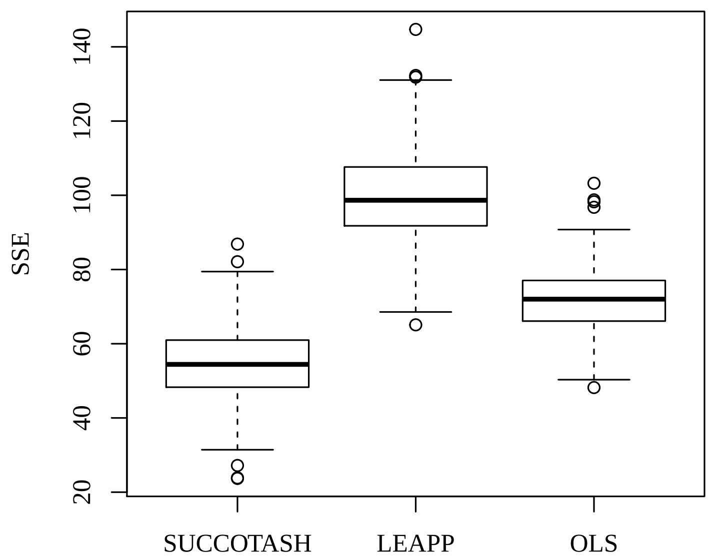
$\pi_0 = (0.9,0.1,0),\ \tau^2 = (0,1,100),\ k = 50$



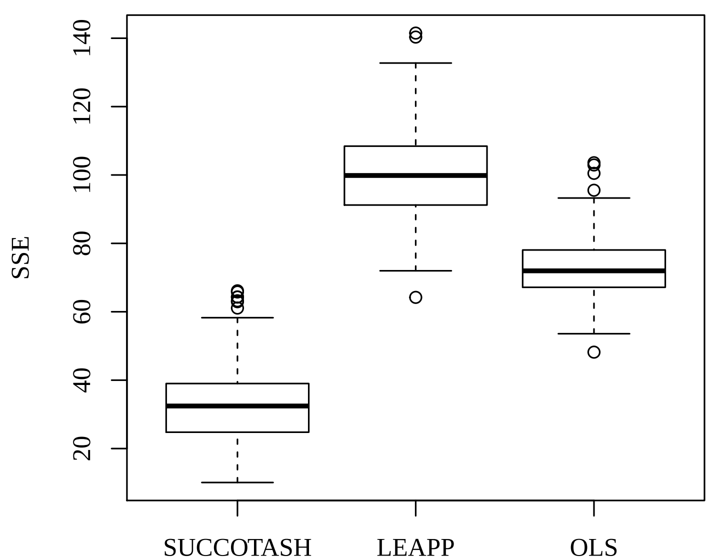

$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 50$
SSE
0
20
40
60
80
100
120
140
SUCCOTASH
LEAPP
OLS


$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 50$
SSE
0
20
40
60
80
100
120
140
SUCCOTASH
LEAPP
OLS

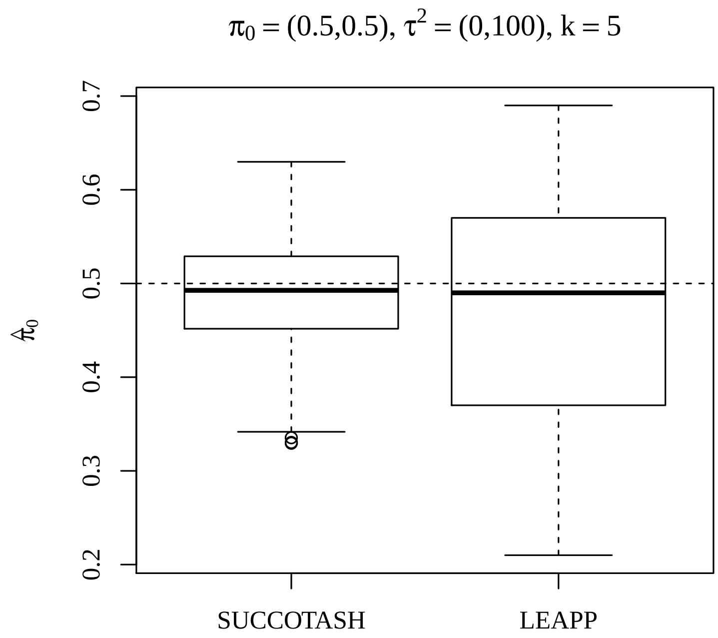
$\pi_0 = (0.5,0,0.5),\ \tau^2 = (0,1,100),\ k = 50$

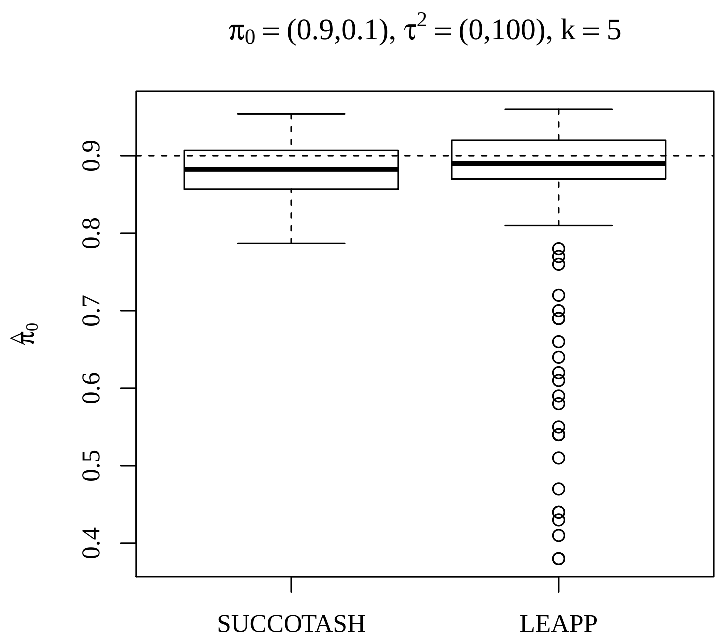


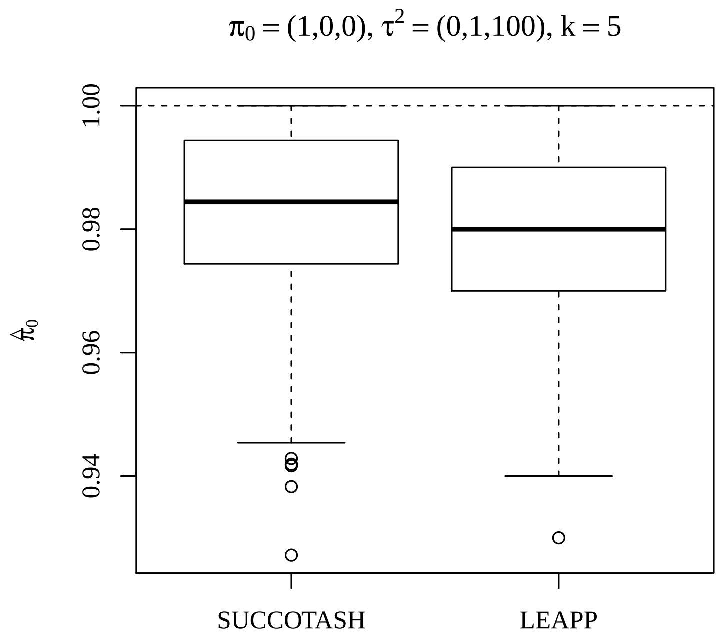
$\pi_0 = (0.5,0.25,0.25),\ \tau^2 = (0,1,100),\ k = 50$



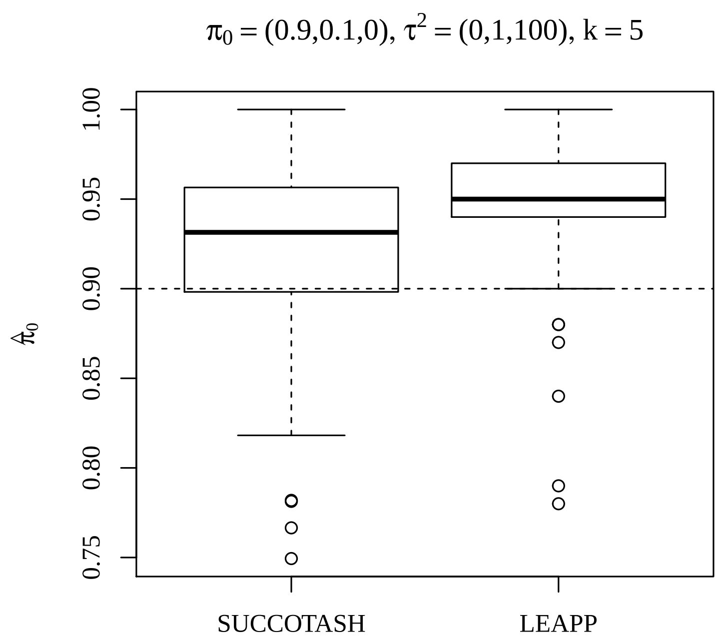
5 $\hat{\pi}_0$ Plots



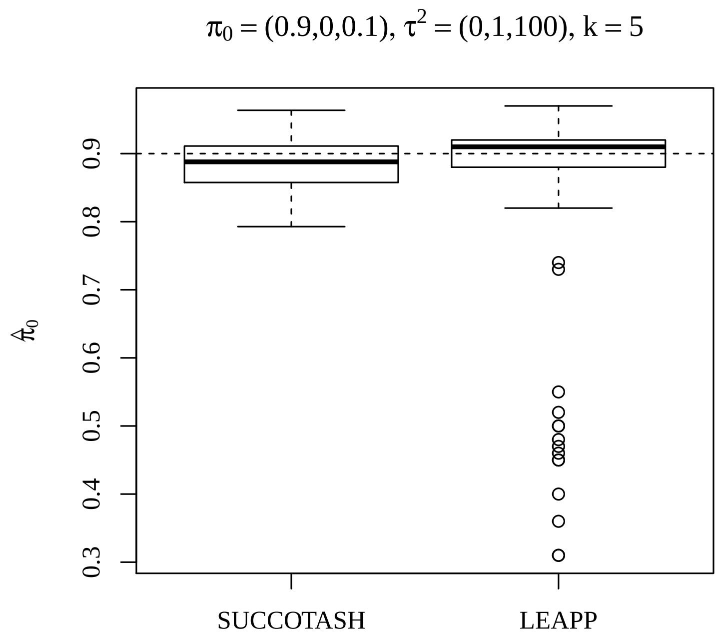




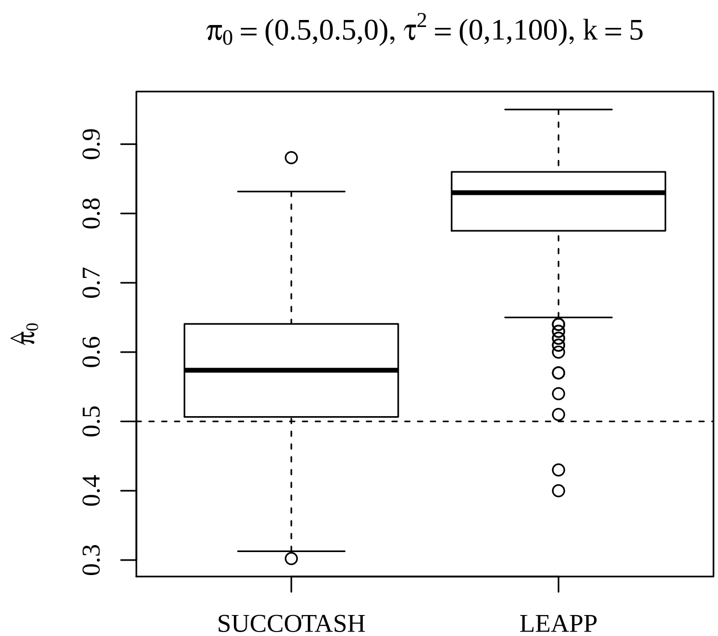
$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
1.00
0.98
0.96
0.94
SUCCOTASH
LEAPP



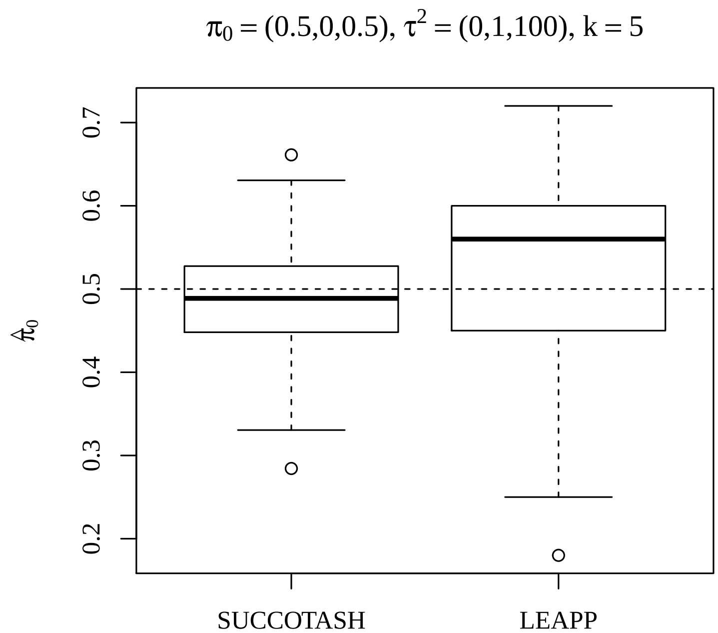
$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
1.00
0.95
0.90
0.85
0.80
0.75
SUCCOTASH
LEAPP

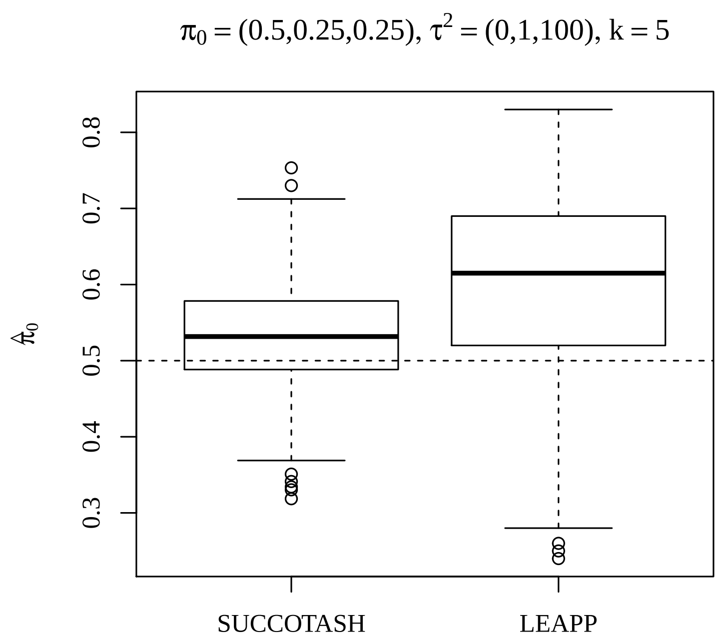


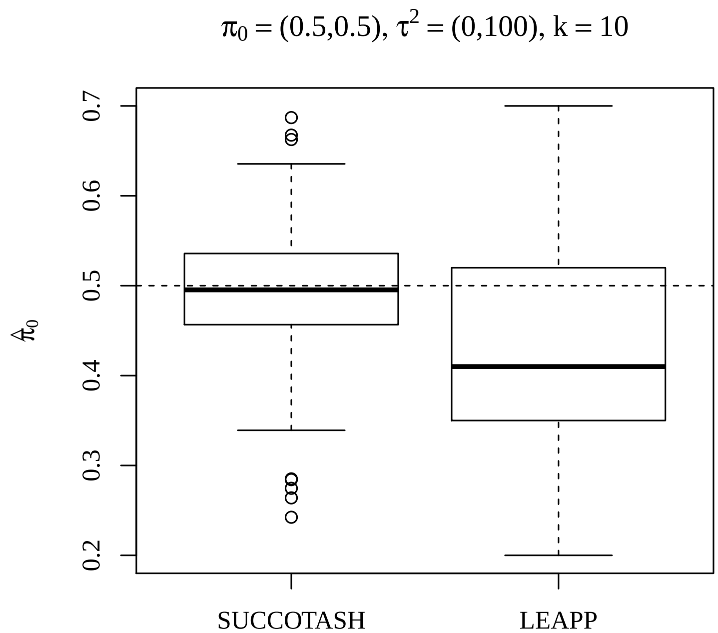
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
0.3
0.4
0.5
0.6
0.7
0.8
0.9
SUCCOTASH
LEAPP

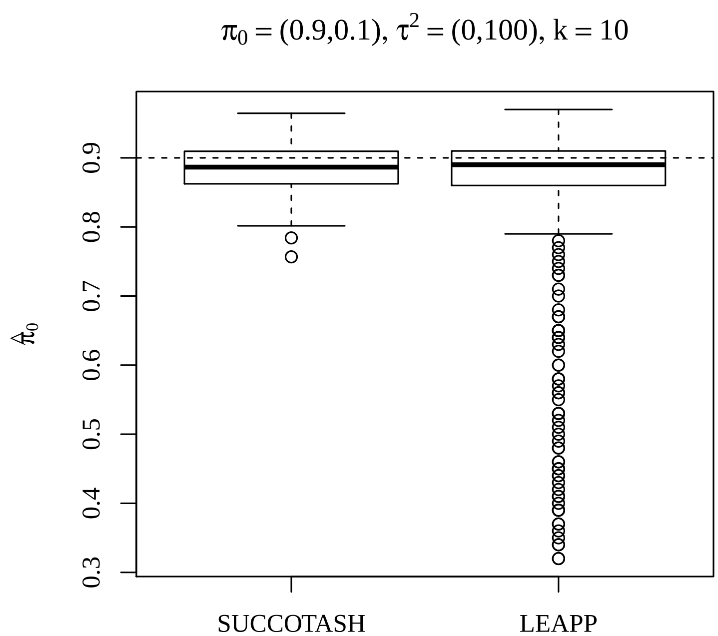


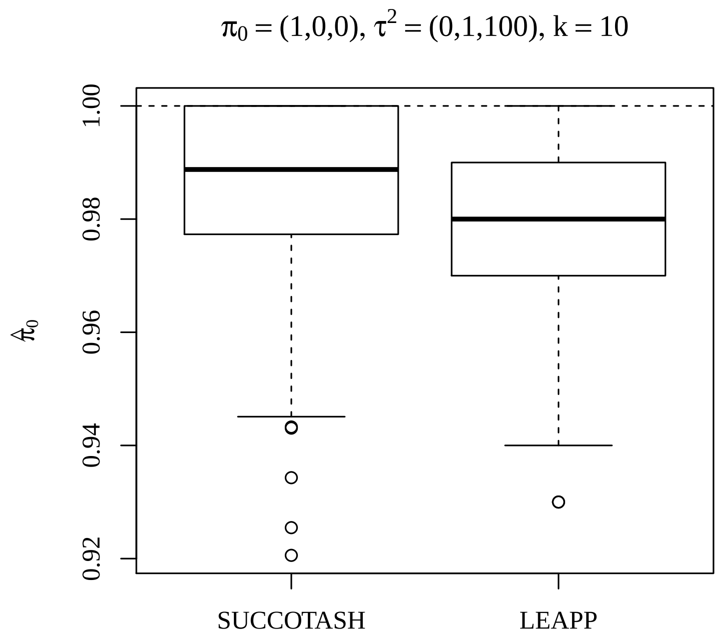
$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
0.3
0.4
0.5
0.6
0.7
0.8
0.9
SUCCOTASH
LEAPP

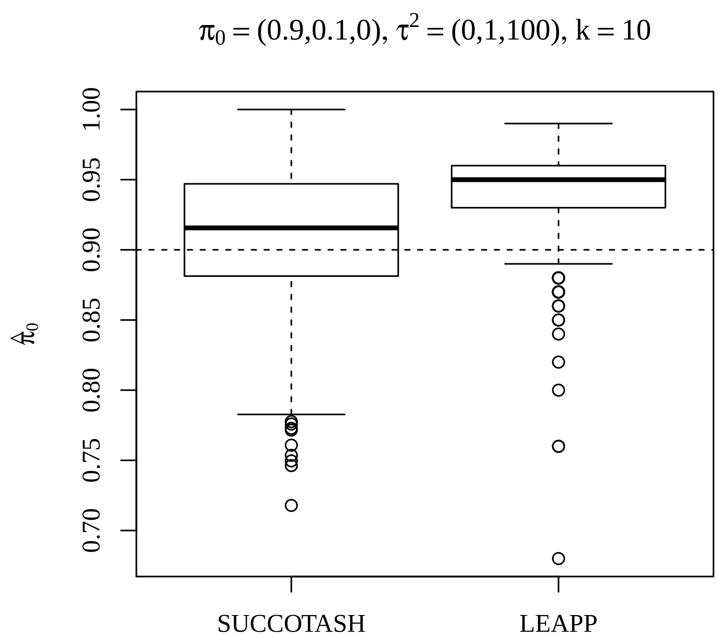

$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
0.2
0.3
0.4
0.5
0.6
0.7
SUCCOTASH
LEAPP

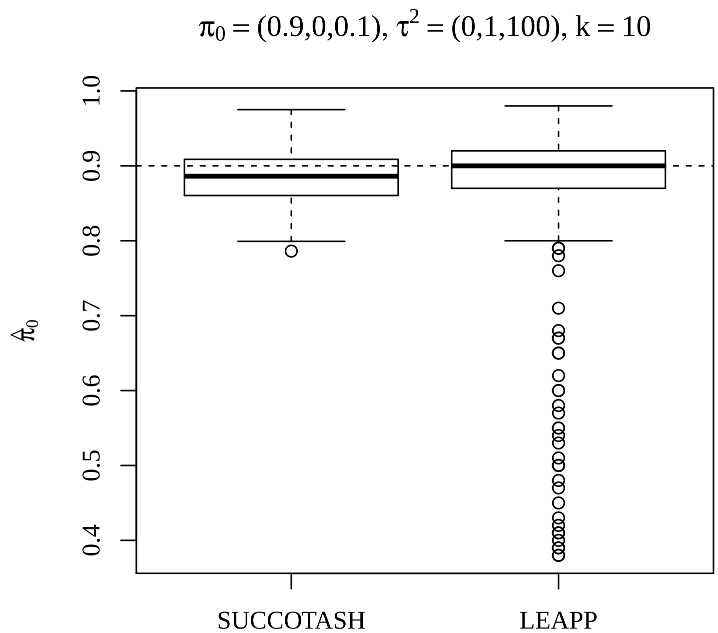

$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, $k = 5$
$\hat{\pi}_0$
0.3
0.4
0.5
0.6
0.7
0.8
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, $k = 10$
$\hat{\pi}_0$
0.2
0.3
0.4
0.5
0.6
0.7
SUCCOTASH
LEAPP

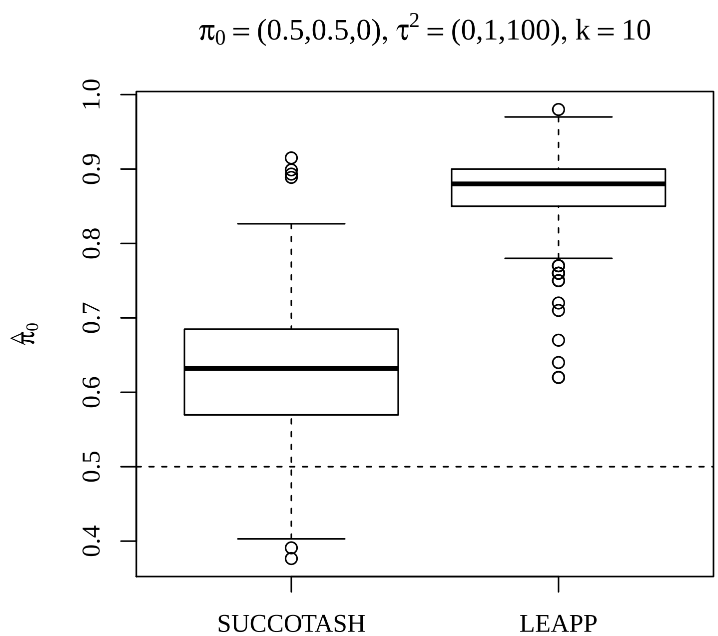

$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, $k = 10$
$\hat{\pi}_0$
0.3
0.4
0.5
0.6
0.7
0.8
0.9
SUCCOTASH
LEAPP


$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, k = 10
1.00
0.98
0.96
0.94
0.92
$\hat{\pi}_0$
SUCCOTASH
LEAPP

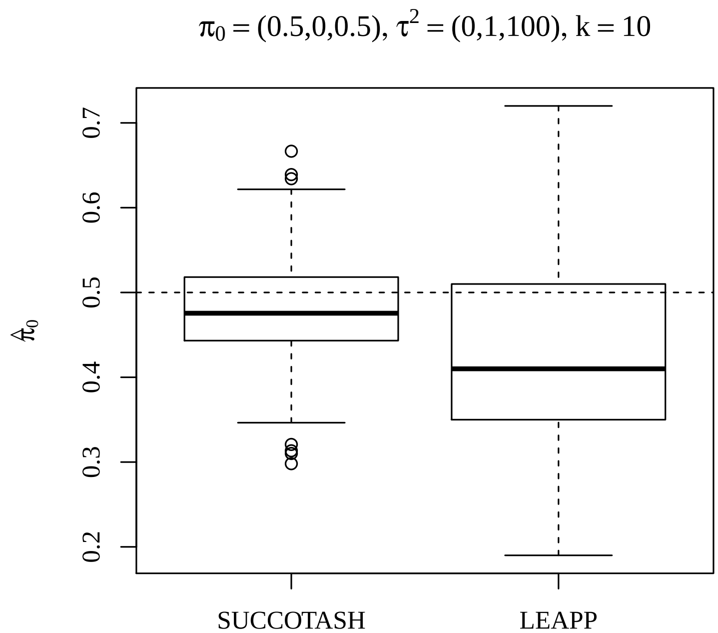

$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, k = 10
1.00
0.95
0.90
0.85
0.80
0.75
0.70
$\hat{\pi}_0$
SUCCOTASH
LEAPP

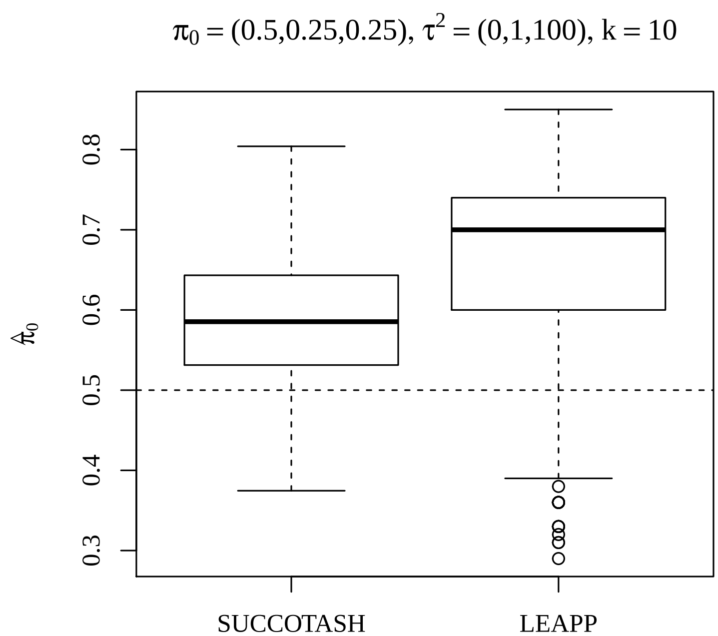


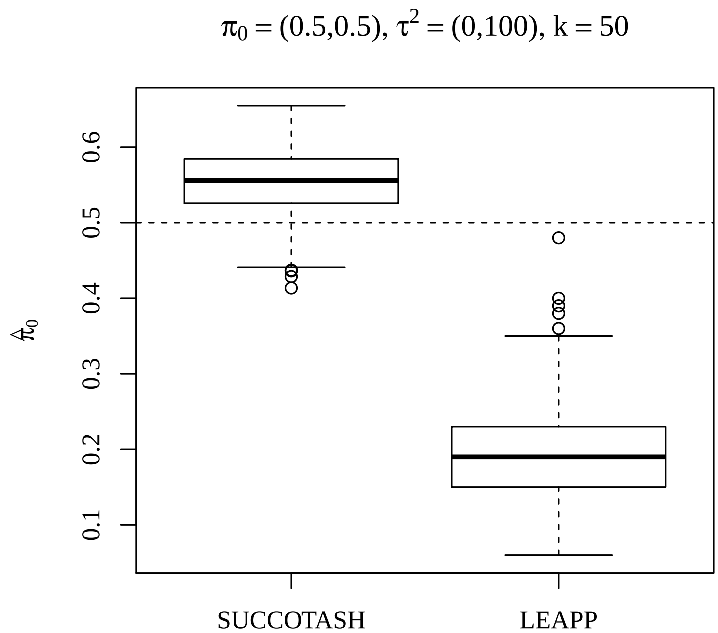
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
SUCCOTASH
LEAPP



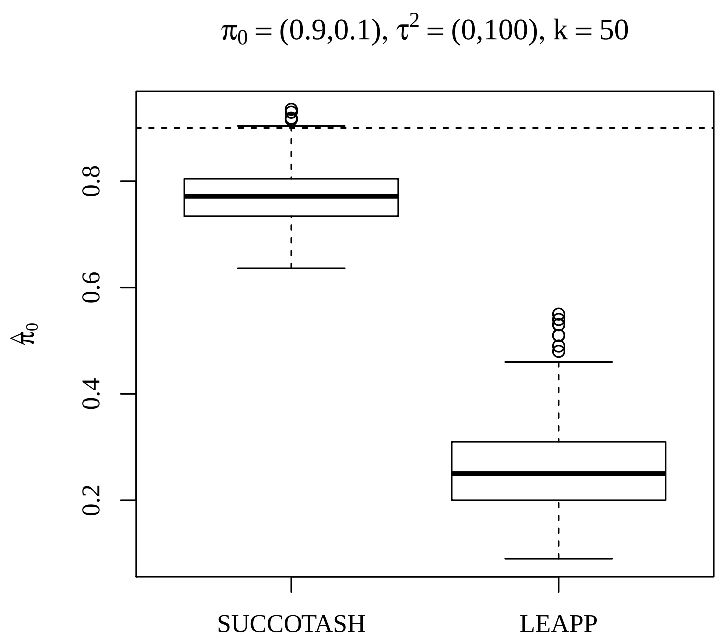
$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, k = 10
$\hat{\pi}_0$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
SUCCOTASH
LEAPP


$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
0.2
0.3
0.4
0.5
0.6
0.7
SUCCOTASH
LEAPP

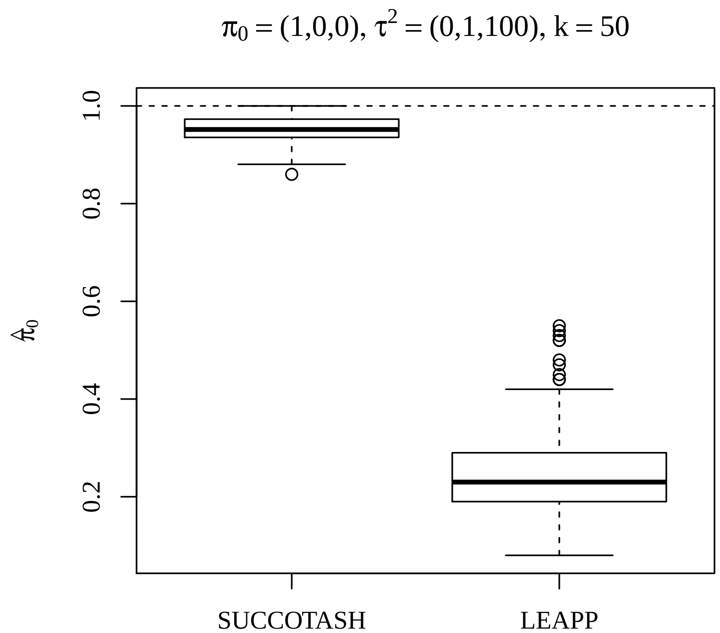

$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, $k = 10$
$\hat{\pi}_0$
0.3
0.4
0.5
0.6
0.7
0.8
SUCCOTASH
LEAPP



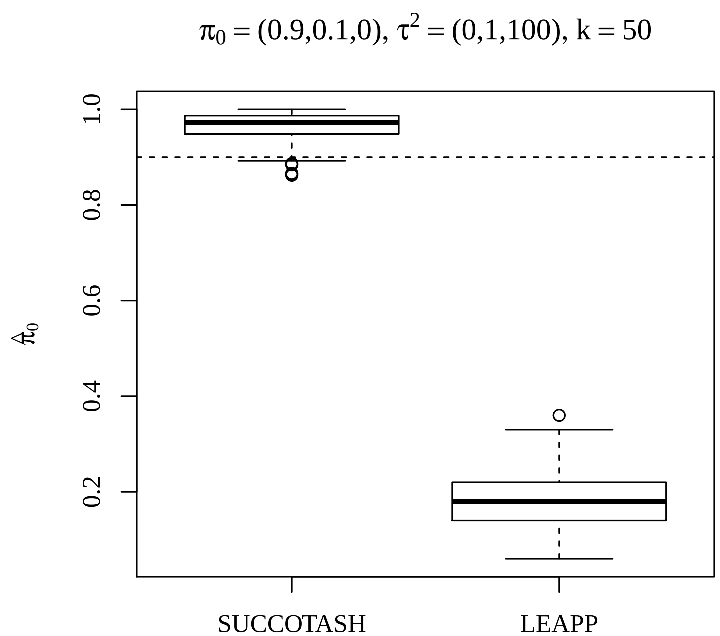
$\pi_0 = (0.5,0.5)$, $\tau^2 = (0,100)$, k = 50
$\hat{\pi}_0$
0.1
0.2
0.3
0.4
0.5
0.6
SUCCOTASH
LEAPP



$\pi_0 = (0.9,0.1)$, $\tau^2 = (0,100)$, k = 50
$\hat{\pi}_0$
0.2
0.4
0.6
0.8
SUCCOTASH
LEAPP

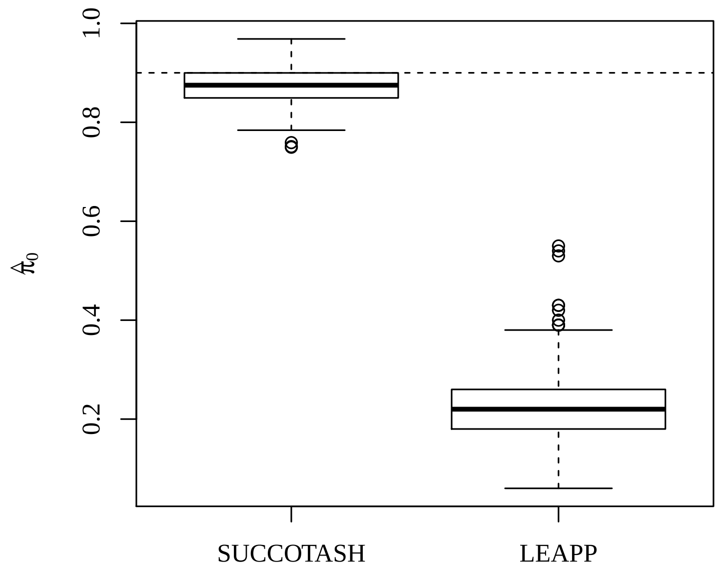


$\pi_0 = (1,0,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\hat{\pi}_0$
0.2
0.4
0.6
0.8
1.0
SUCCOTASH
LEAPP

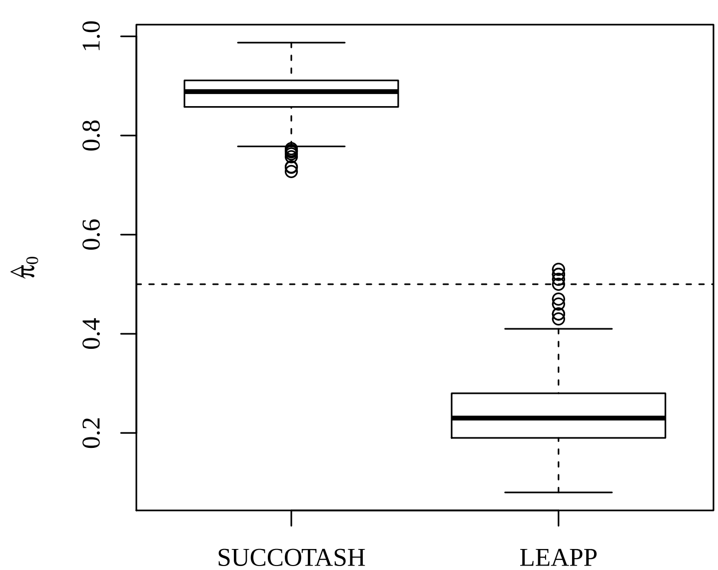


$\pi_0 = (0.9,0.1,0)$, $\tau^2 = (0,1,100)$, $k = 50$
$\hat{\pi}_0$
0.2
0.4
0.6
0.8
1.0
SUCCOTASH
LEAPP

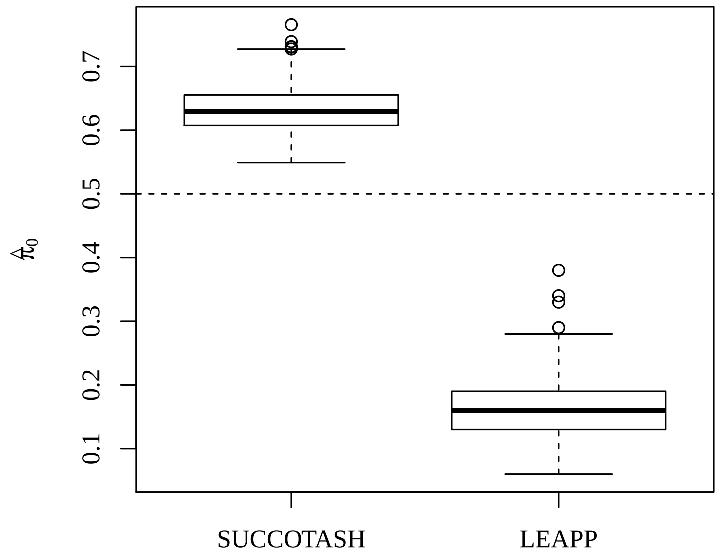
$\pi_0 = (0.9,0,0.1)$, $\tau^2 = (0,1,100)$, $k = 50$



$\pi_0 = (0.5,0.5,0)$, $\tau^2 = (0,1,100)$, $k = 50$



$\pi_0 = (0.5,0,0.5)$, $\tau^2 = (0,1,100)$, $k = 50$



$\pi_0 = (0.5,0.25,0.25)$, $\tau^2 = (0,1,100)$, $k = 50$