

# Different Alternative Types

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## Abstract

This the same setup as here except I use bicrossvalidation (using the package `cate`) to estimate the number of hidden confounders rather than `sva:num.sv`. The results are basically the same, with SUCCOTASH and RUVASH doing very well compared to other methods.

## Simulation Setup

I ran through 200 repetitions of generating data from GTEX muscle data under the following parameter conditions:

- $n \in \{10, 20, 40\}$ ,
- $p = 1000$ .
- $\pi_0 \in \{0.5, 0.9\}$ ,
- The alternative distribution being either spiky, near-normal, flattop, skew, big-normal, or bimodal, where these are the same alternatives defined in Stephens (2016) and the following table. New alternatives are generated every iteration.

Scenario	Alternative Distribution
Spiky	$0.4N(0, 0.25^2) + 0.2N(0, 0.5^2) + 0.2N(0, 1^2), 0.2N(0, 2^2)$
Near Normal	$2/3N(0, 1^2) + 1/3N(0, 2^2)$
Flattop	$(1/7)N(-1.5, .5^2) + N(-1, .5^2) + N(-.5, .5^2) + N(0, .5^2) + N(0.5, .5^2) + N(1.0, .5^2) + N(1.5, .5^2)$
Skew	$(1/4)N(-2, 2^2) + (1/4)N(-1, 1.5^2) + (1/3)N(0, 1^2) + (1/6)N(1, 1^2)$
Big-normal	$N(0, 4^2)$
Bimodal	$0.5N(-2, 1^2) + 0.5N(2, 1^2)$

I extracted the most expressed  $p$  genes from the GTEX muscle data and  $n$  samples are chosen at random. Half of these samples are randomly given the “treatment” label 1, the other half given the “control” label 0. Of the  $p$  genes,  $\pi_0 p$  were chosen to be non-null. Signal was added by a Poisson-thinning approach, where the log-2 fold change was sampled from one of five the alternative models above. That is

$$A_1, \dots, A_{p/2} \sim f \tag{1}$$

$$B_i = 2^{A_i} \text{ for } i = 1, \dots, p/2, \tag{2}$$

where  $f$  is from the table above. If  $A_i > 0$  then we replace  $Y_{[1:(n/2), i]}$  with  $\text{Binom}(Y_{[j, i]}, 1/B_i)$  for  $j = 1, \dots, n/2$ . If  $A_i < 0$  then we replace  $Y_{[(n/2+1):n, i]}$  with  $\text{Binom}(Y_{[j, i]}, B_i)$  for  $j = n/2 + 1, \dots, n$ .

I now describe the justification for this. Suppose that

$$Y_{ij} \sim \text{Poisson}(\lambda_j). \tag{3}$$

Let  $x_i$  be the indicator of treatment vs control for individual  $i$ . Let  $\Omega$  be the set of non-null genes. Let  $Z$  be the new dataset derived via the steps above. That is

$$Z_{ij}|Y_{ij} = \begin{cases} \text{Binom}(Y_{ij}, 2^{A_j x_i}) & \text{if } A_j < 0 \text{ and } j \in \Omega \\ \text{Binom}(Y_{ij}, 2^{-A_j(1-x_i)}) & \text{if } A_j > 0 \text{ and } j \in \Omega \\ Y_{ij} & \text{if } j \notin \Omega. \end{cases} \quad (4)$$

Then

$$Z_{ij}|A_j, A_j < 0, j \in \Omega \sim \text{Poisson}(2^{A_j x_i} \lambda_j) \quad (5)$$

$$Z_{ij}|A_j, A_j > 0, j \in \Omega \sim \text{Poisson}(2^{-A_j(1-x_i)} \lambda_j), \quad (6)$$

and

$$E[\log_2(Z_{ij}) - \log_2(Z_{kj})|A_j, A_j < 0, j \in \Omega] \approx A_j x_i - A_j x_k, \text{ and} \quad (7)$$

$$E[\log_2(Z_{ij}) - \log_2(Z_{kj})|A_j, A_j > 0, j \in \Omega] \approx -A_j(1 - x_i) + A_j(1 - x_k). \quad (8)$$

if individual  $i$  is in the treatment group and individual  $k$  is in the control group, then this just equals  $A_j$ . I treat the  $A_j$ 's as the true coefficient values when calculating the MSE below.

## Methods

I first normalized the counts by  $\log_2(COUNTS + 1)$ . The number of hidden confounders was estimated using `cate::est.confounder.num` in R.

The confounder adjustment methods I look at in this write-up are:

- OLS + qvalue.
- OLS + ASH
- SUCCOTASH using normal mixtures and heteroscedastic PCA as the factor-analysis method.
- The robust regression version of CATE using PCA as the factor analysis method + qvalue.
- Ridge version of LEAPP + qvalue.
- SVA + qvalue.
- RUVASH
- Negative control version of CATE using PCA as the factor analysis method + qvalue.
- RUV2 + qvalue.
- RUV4 + qvalue.
- RUV4 + ASH (no variance inflation).

## Results

Note that in the plots below,  $n$  refers to the size of each group, not the total size.

## Estimates of $\pi_0$

- SUCCOTASH has slightly anti-conservative estimates of  $\pi_0$  in the Flat-top and bimodal Scenarios. It does well for every other scenario for larger  $n$ . RUVASH also does very well.
- Without the variance inflation, RUV4ThenASH does very poorly.
- No method using  $q$ -value ever performed as well as succotash. Indeed, none exhibited this “conservative with high-probability” behavior that is desirable.

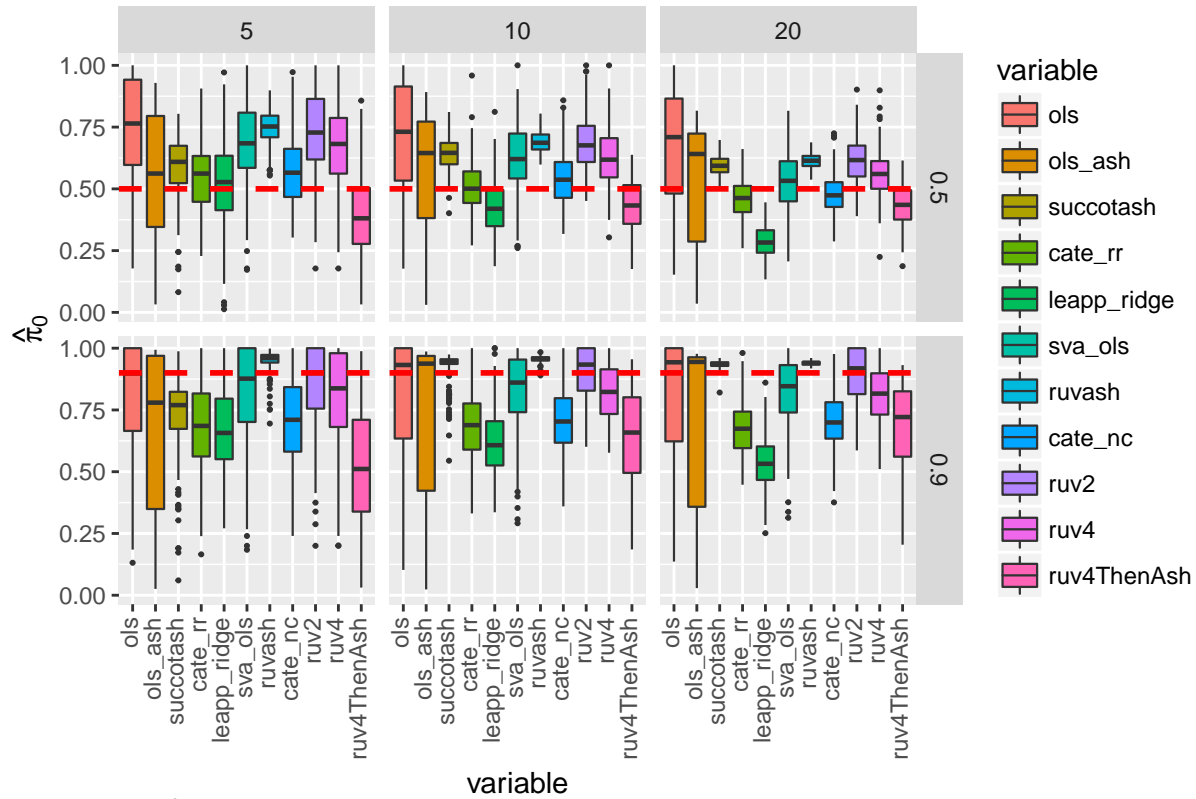
## AUC performance.

The ASH-like methods always have higher AUC, even in the bimodal and big-normal scenarios where LEAPP estimated  $\pi_0$  more accurately.

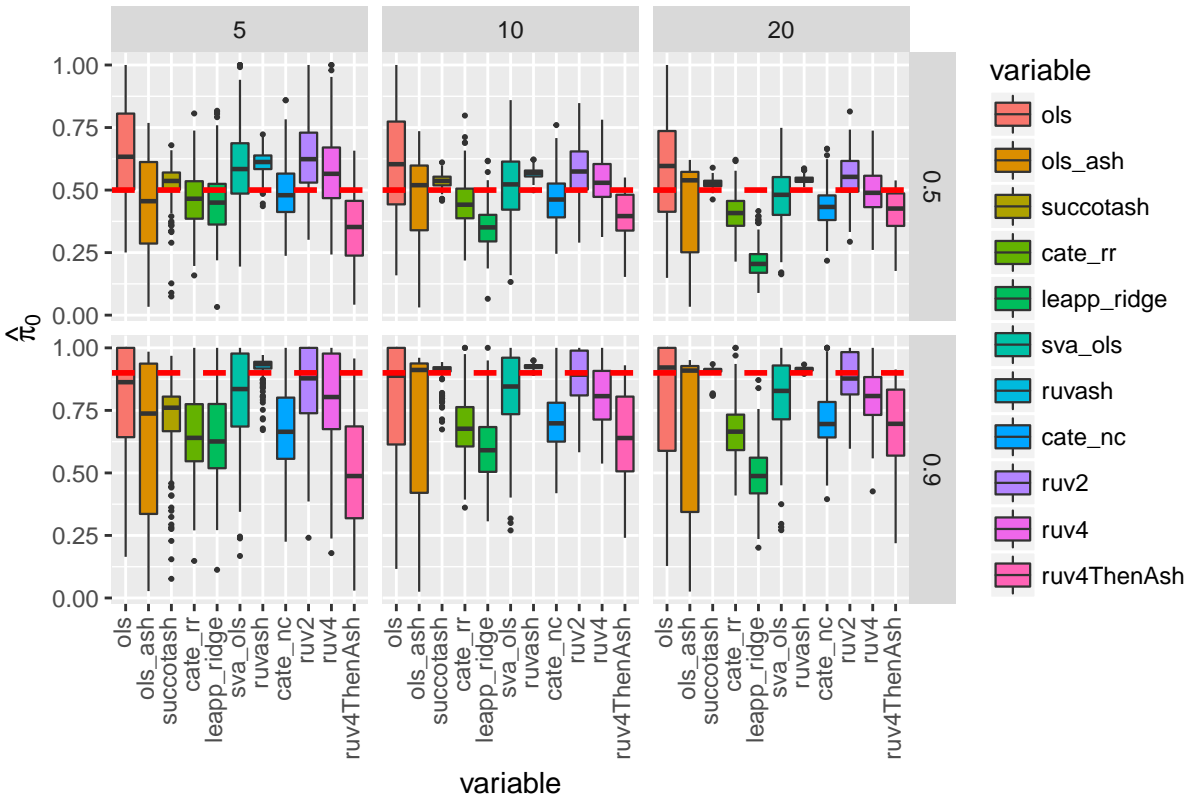
## MSE

The ASH-like methods have superior performance in term of AUC compared to other methods. LEAPP has terrible MSE performance in all cases except the all-null setting. Even then, SUCCOTASH has better performance.

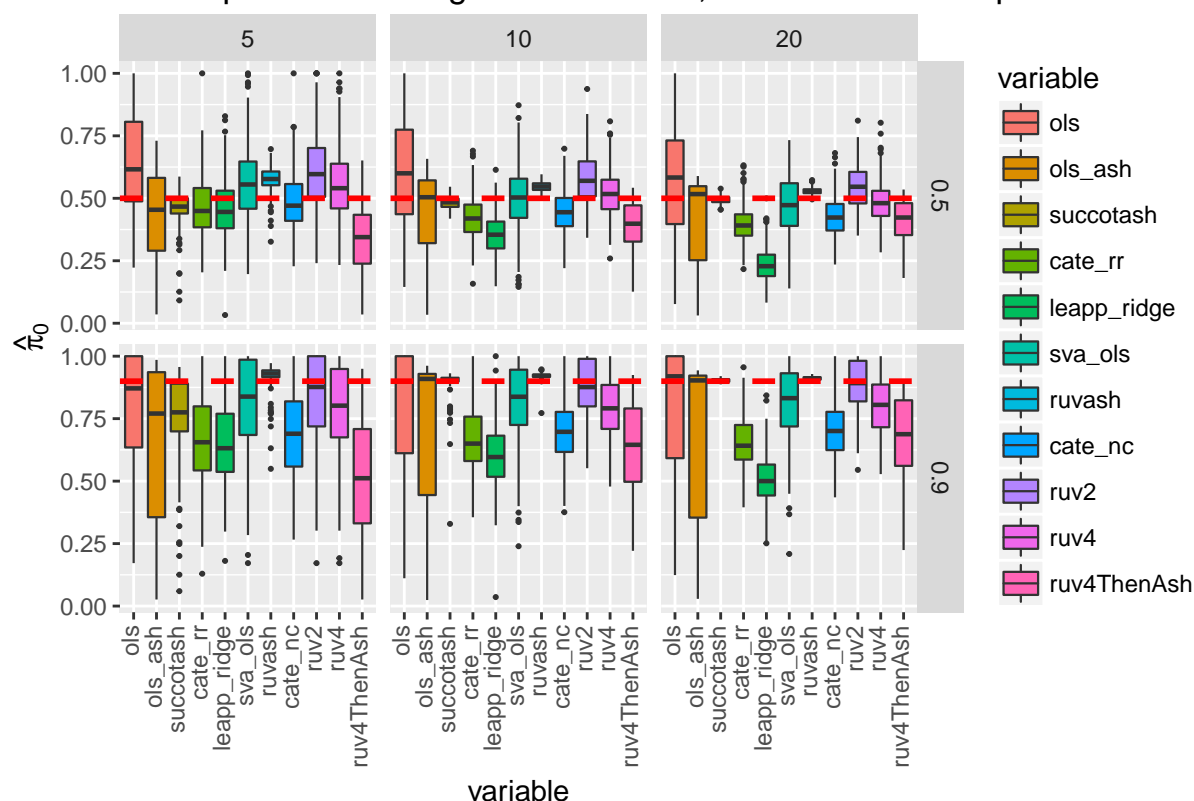
### Estimates of $\pi_0$ When Using Muscle Tissue, Alternative = spiky



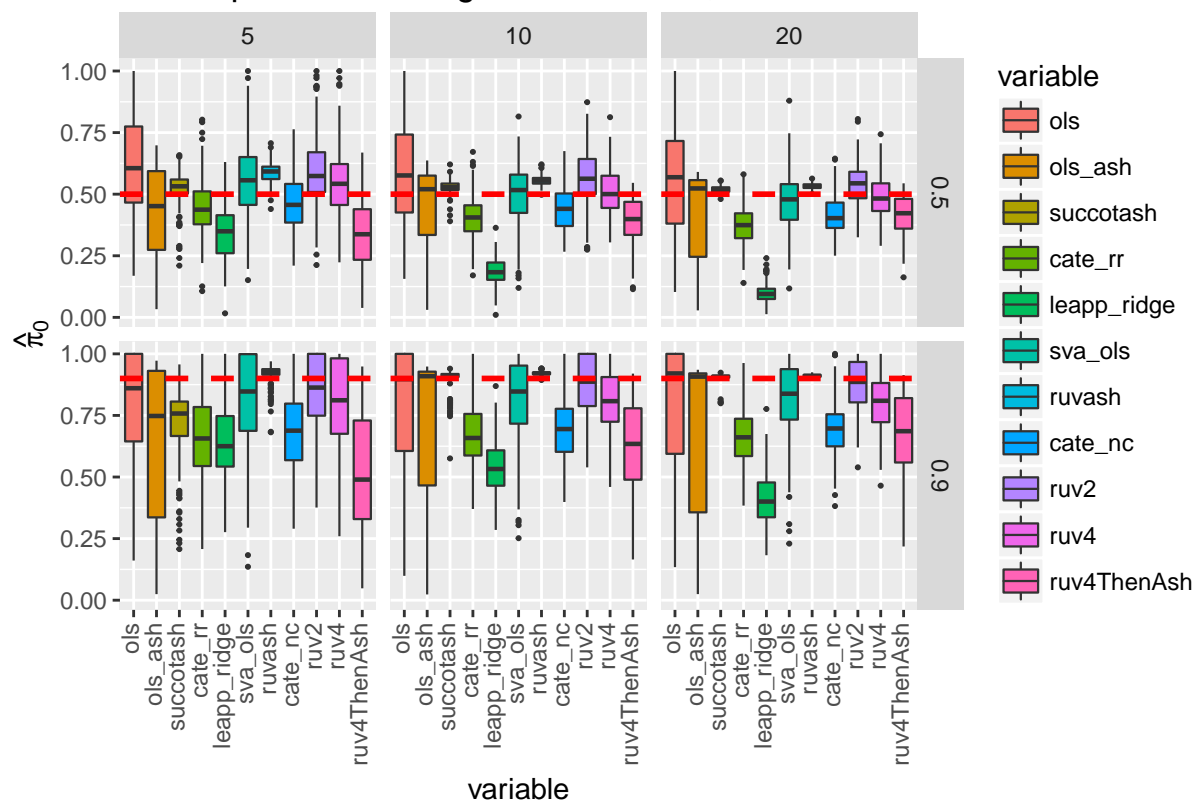
### Estimates of $\pi_0$ When Using Muscle Tissue, Alternative = near\_normal



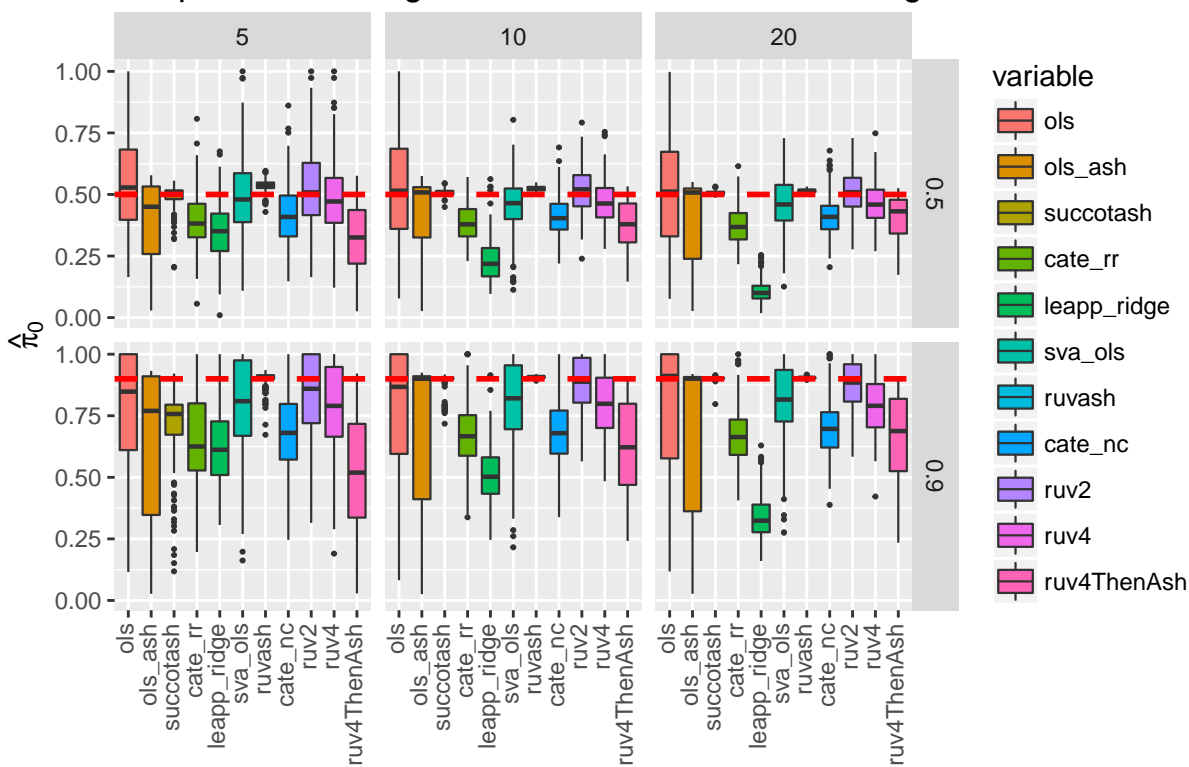
Estimates of  $\pi_0$  When Using Muscle Tissue, Alternative = flattop



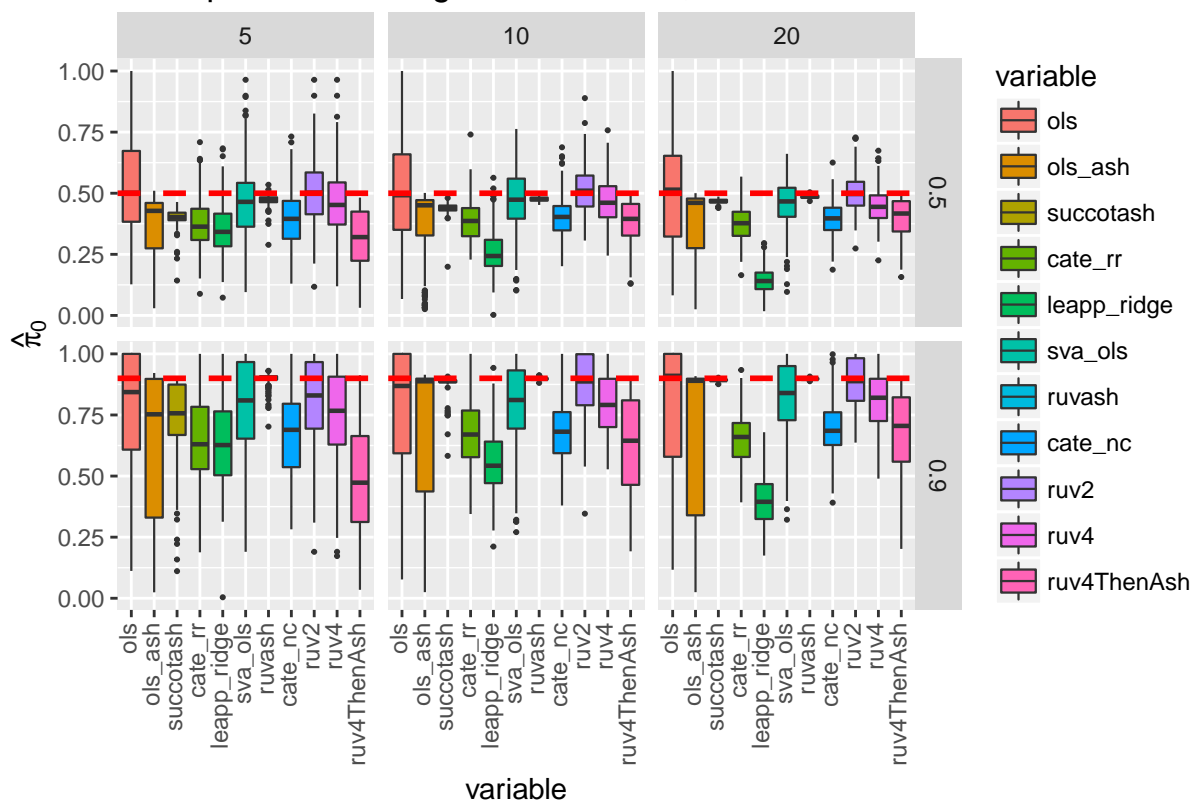
Estimates of  $\pi_0$  When Using Muscle Tissue, Alternative = skew



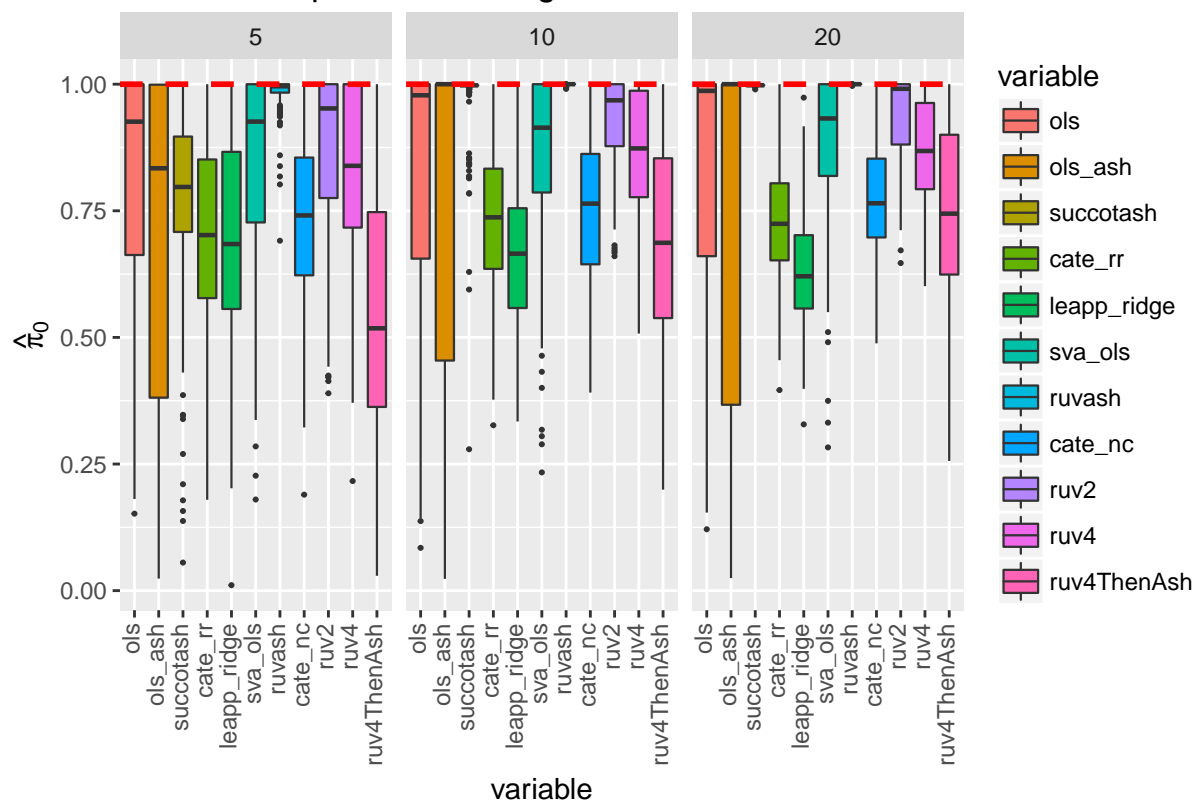
Estimates of  $\pi_0$  When Using Muscle Tissue, Alternative = big\_normal



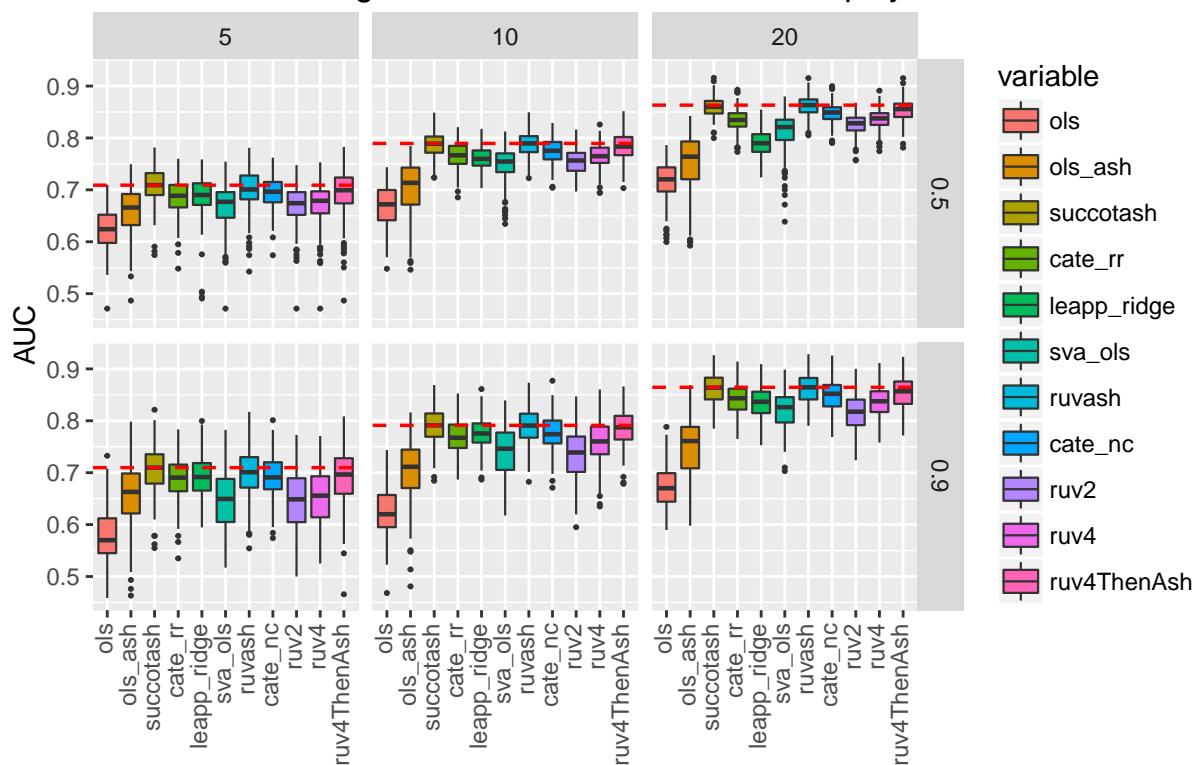
Estimates of  $\pi_0$  When Using Muscle Tissue, Alternative = bimodal



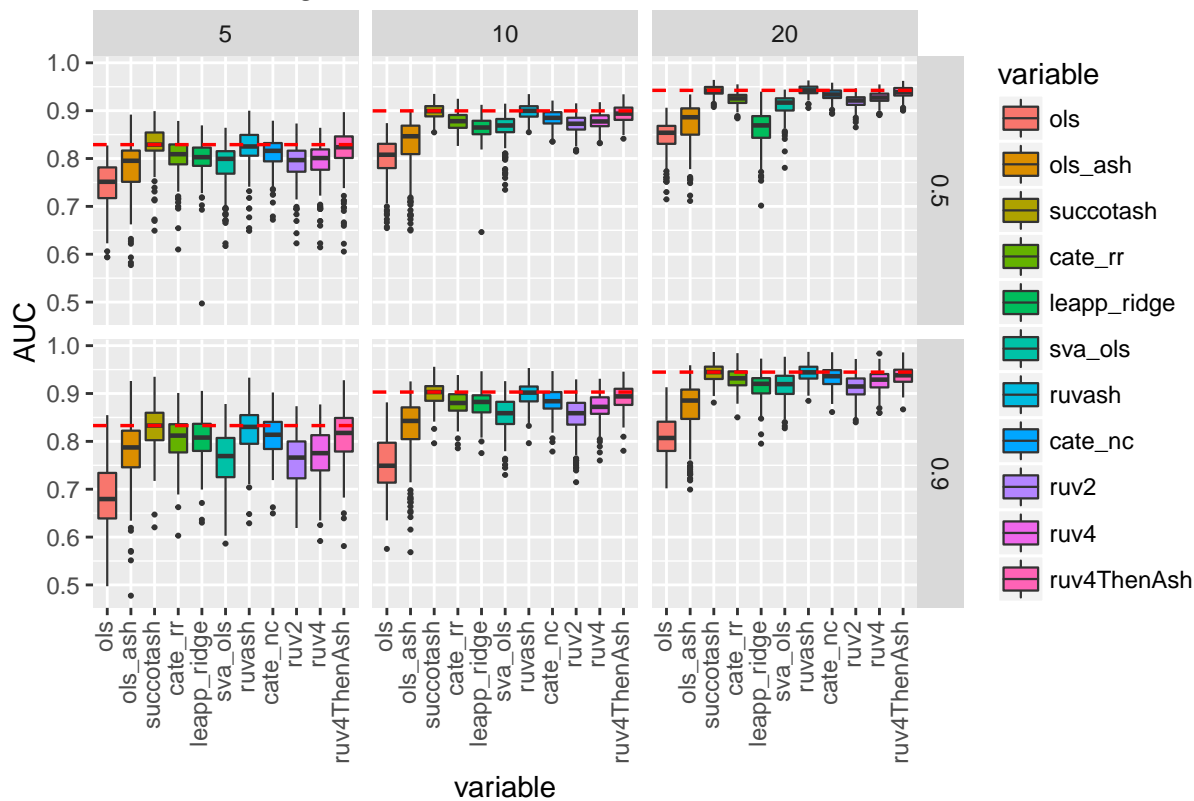
Estimates of  $\pi_0$  When Using Muscle Tissue and All Null



## AUC When Using Muscle Tissue, Alternative = spiky

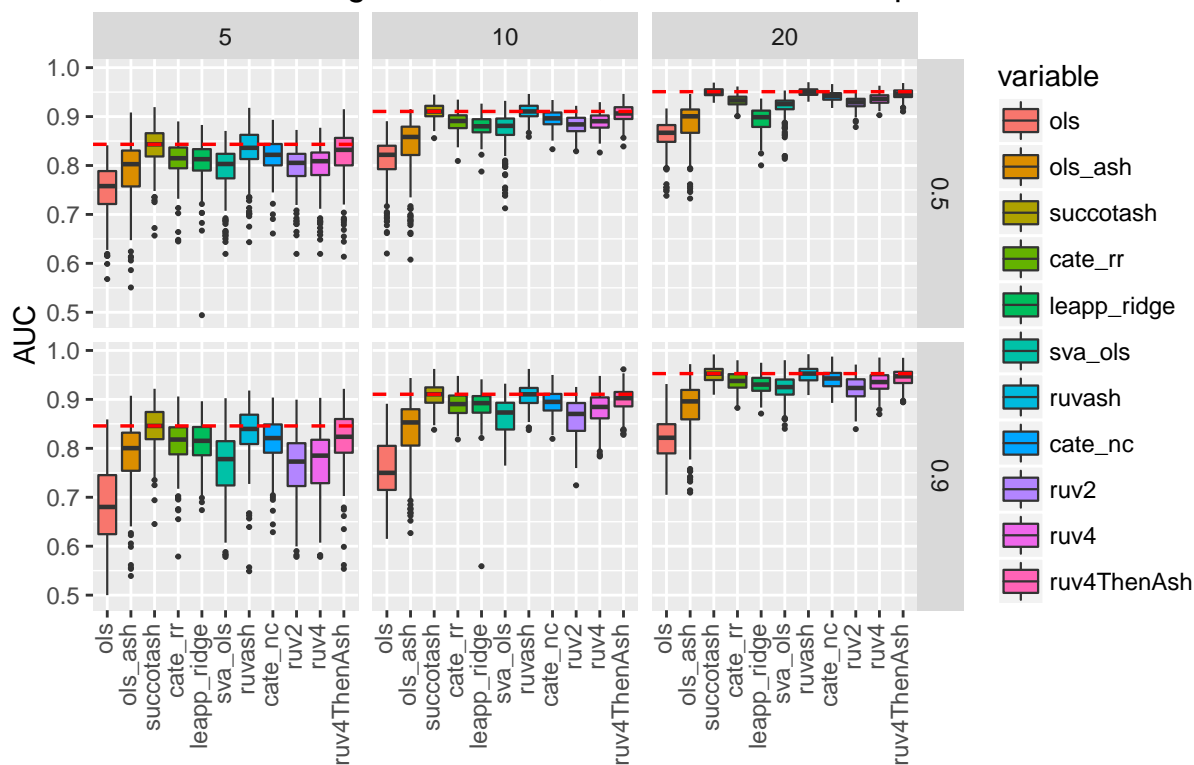


## AUC When Using Muscle Tissue, Alternative = near\_normal

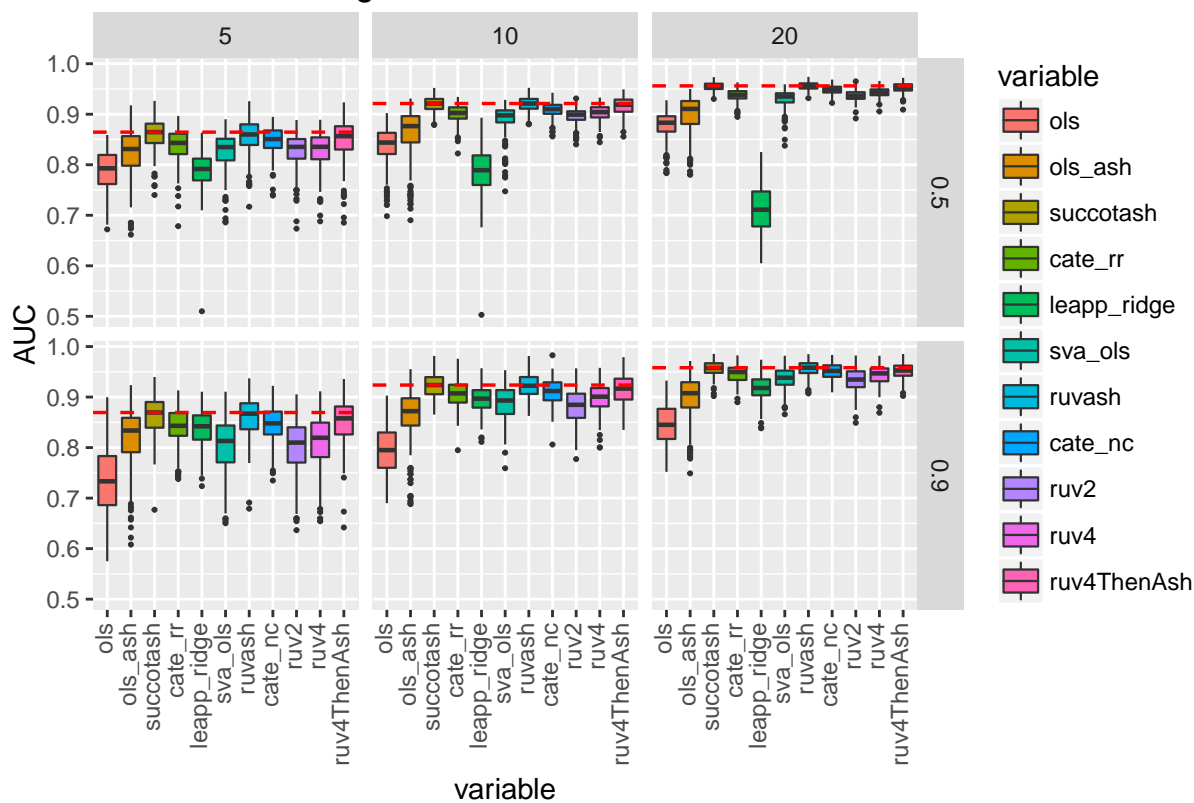




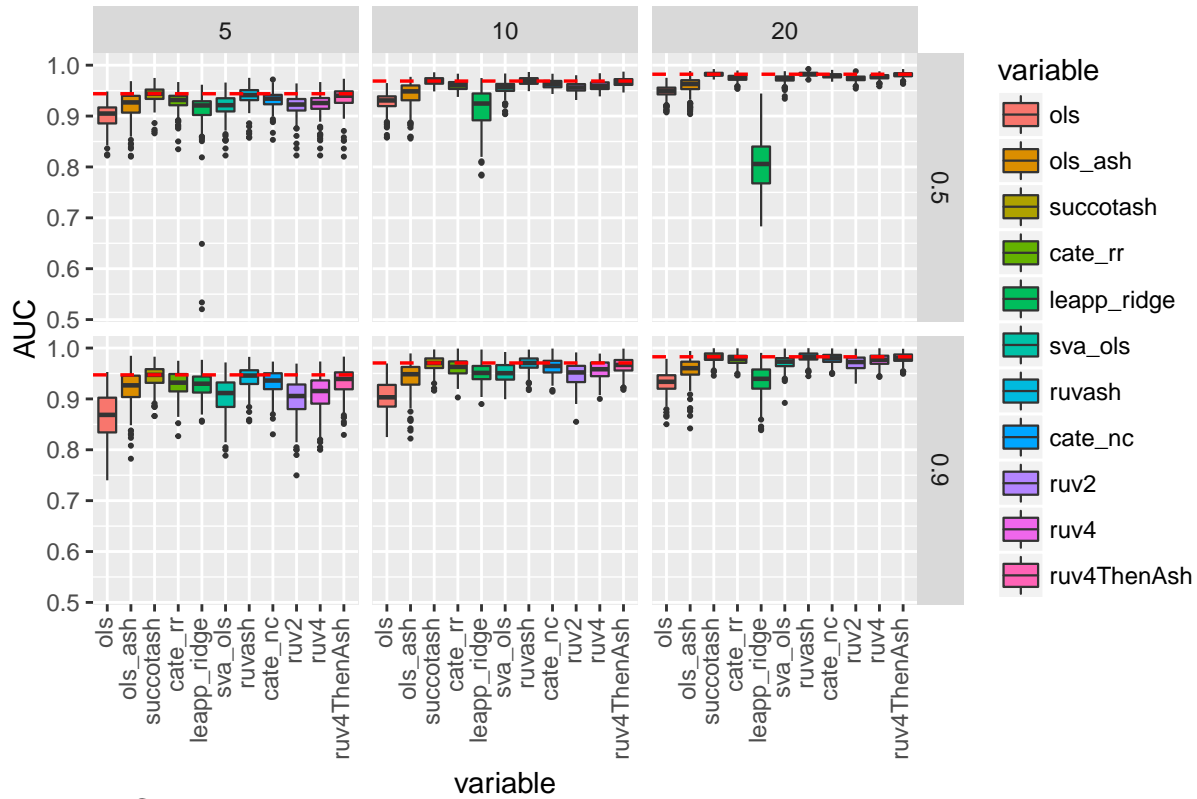
AUC When Using Muscle Tissue, Alternative = flattop



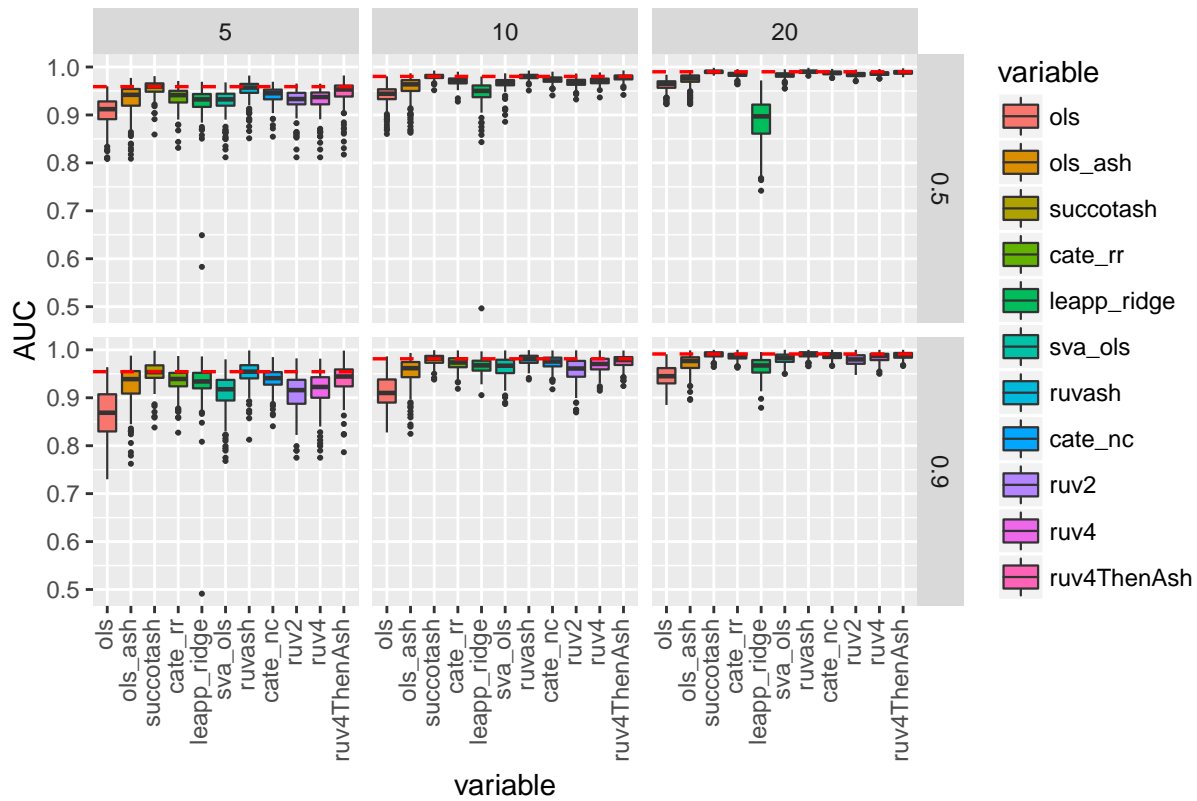
AUC When Using Muscle Tissue, Alternative = skew



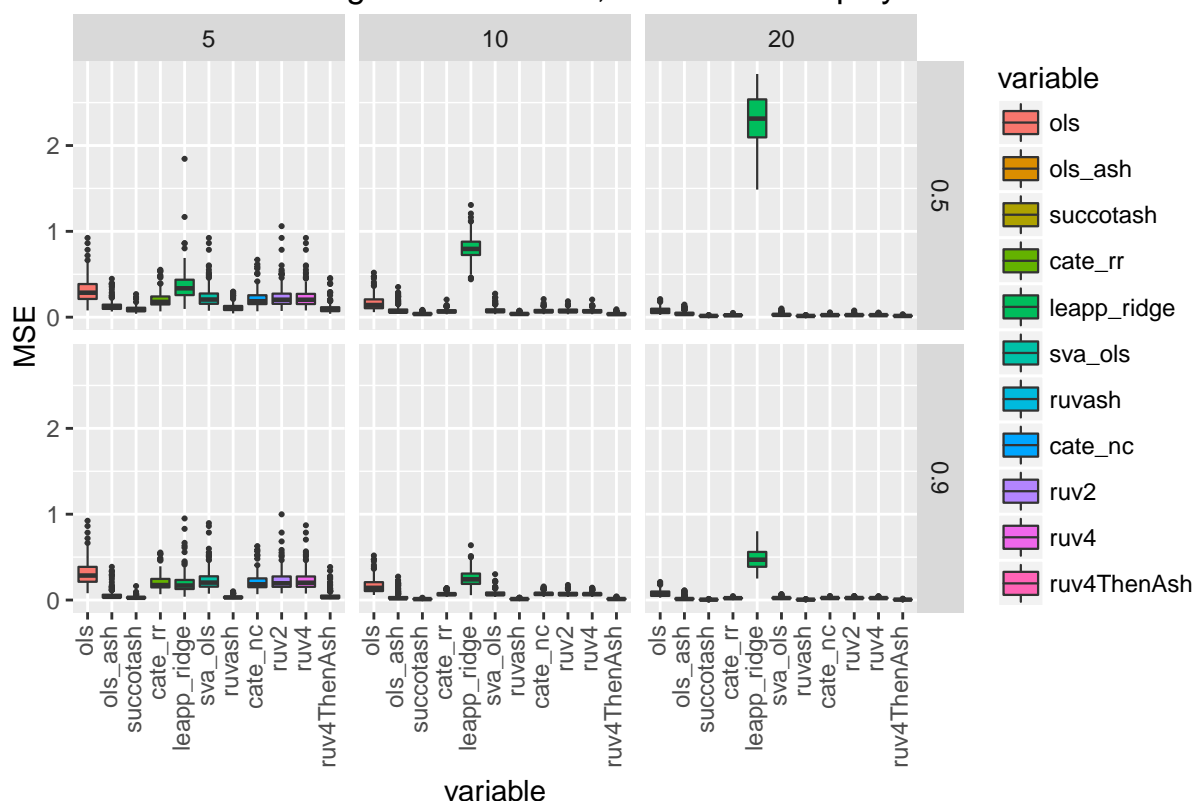
AUC When Using Muscle Tissue, Alternative = big\_normal



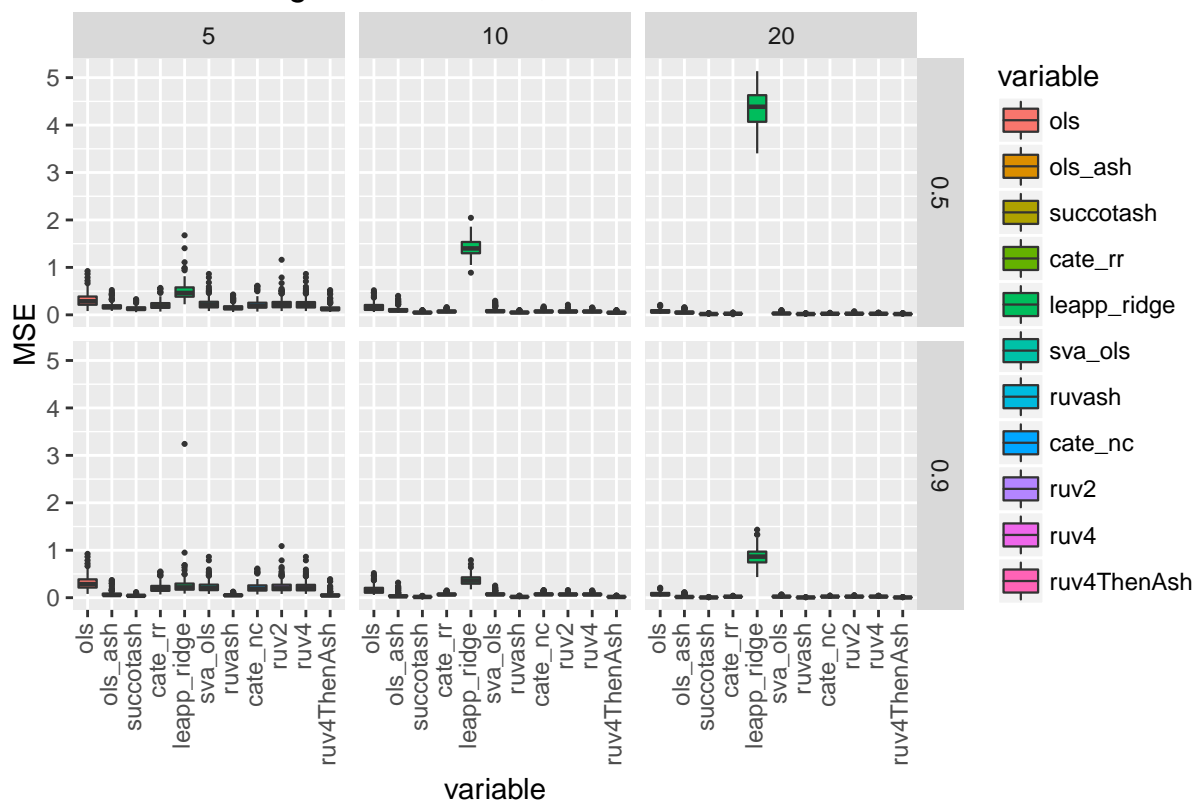
AUC When Using Muscle Tissue, Alternative = bimodal



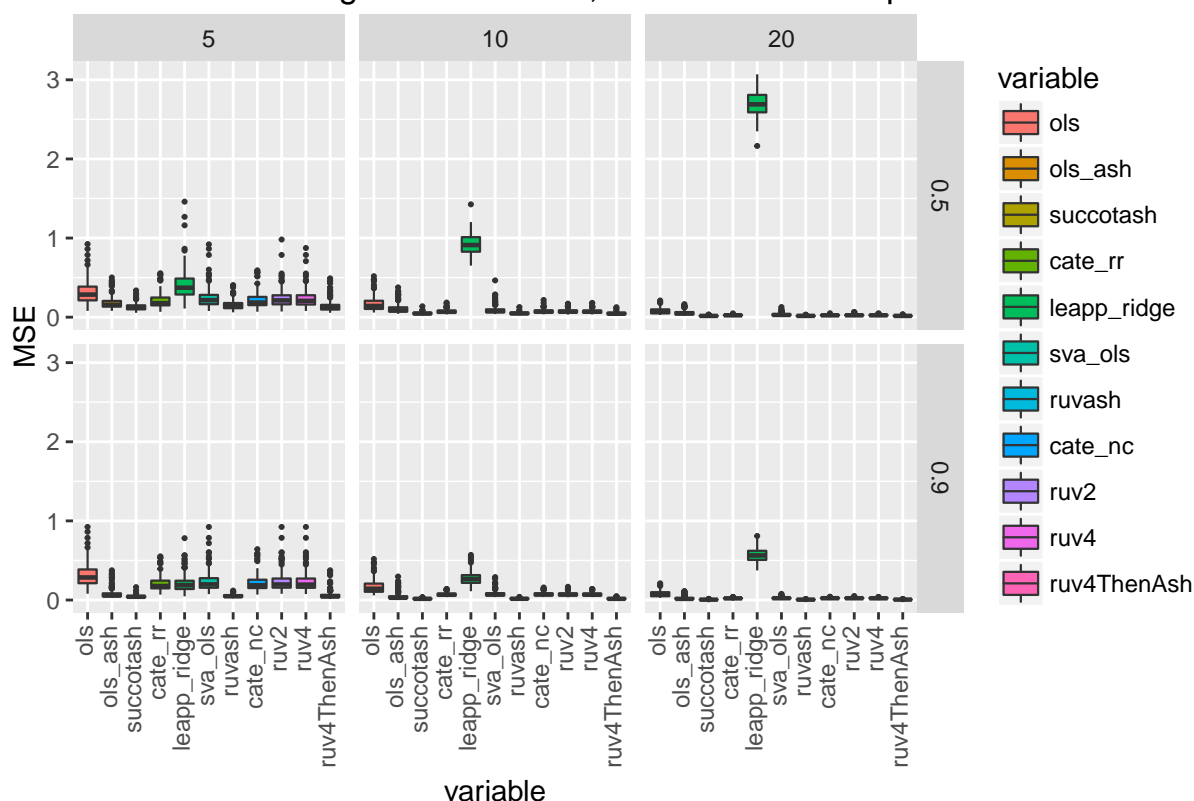
MSE When Using Muscle Tissue, Alternative = spiky



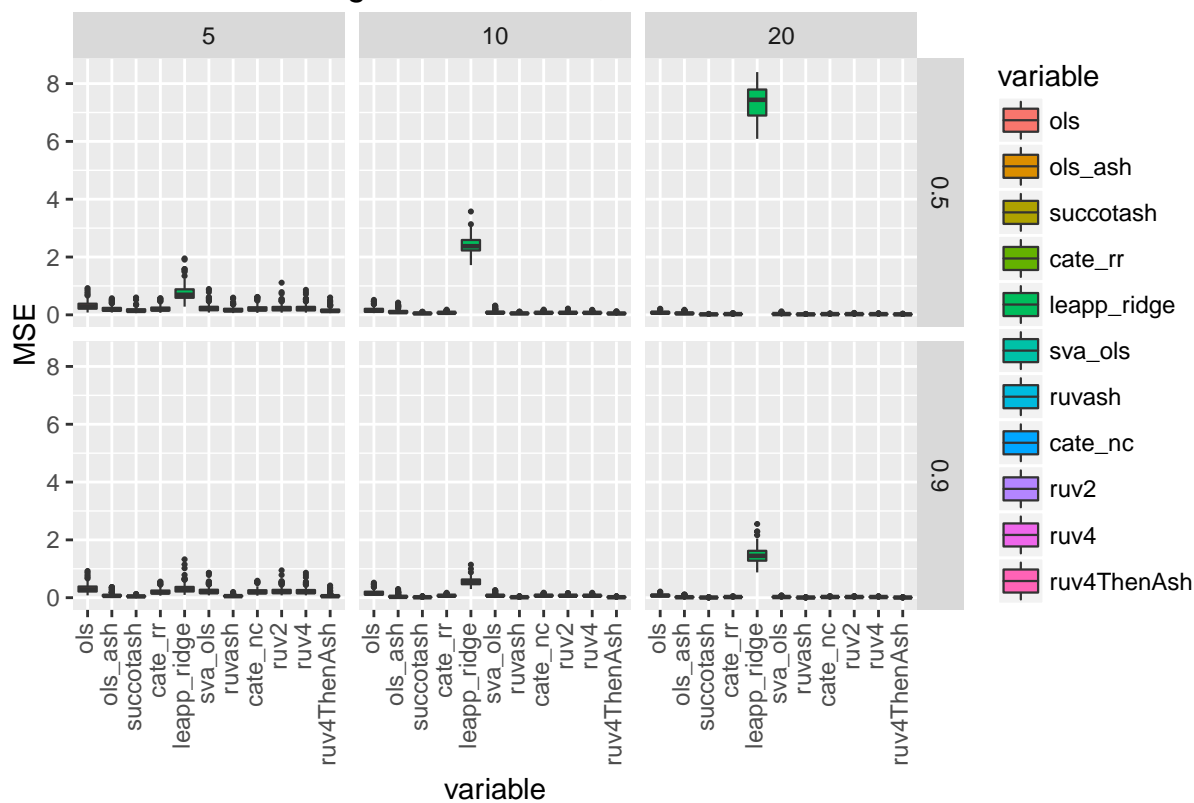
MSE When Using Muscle Tissue, Alternative = near\_normal



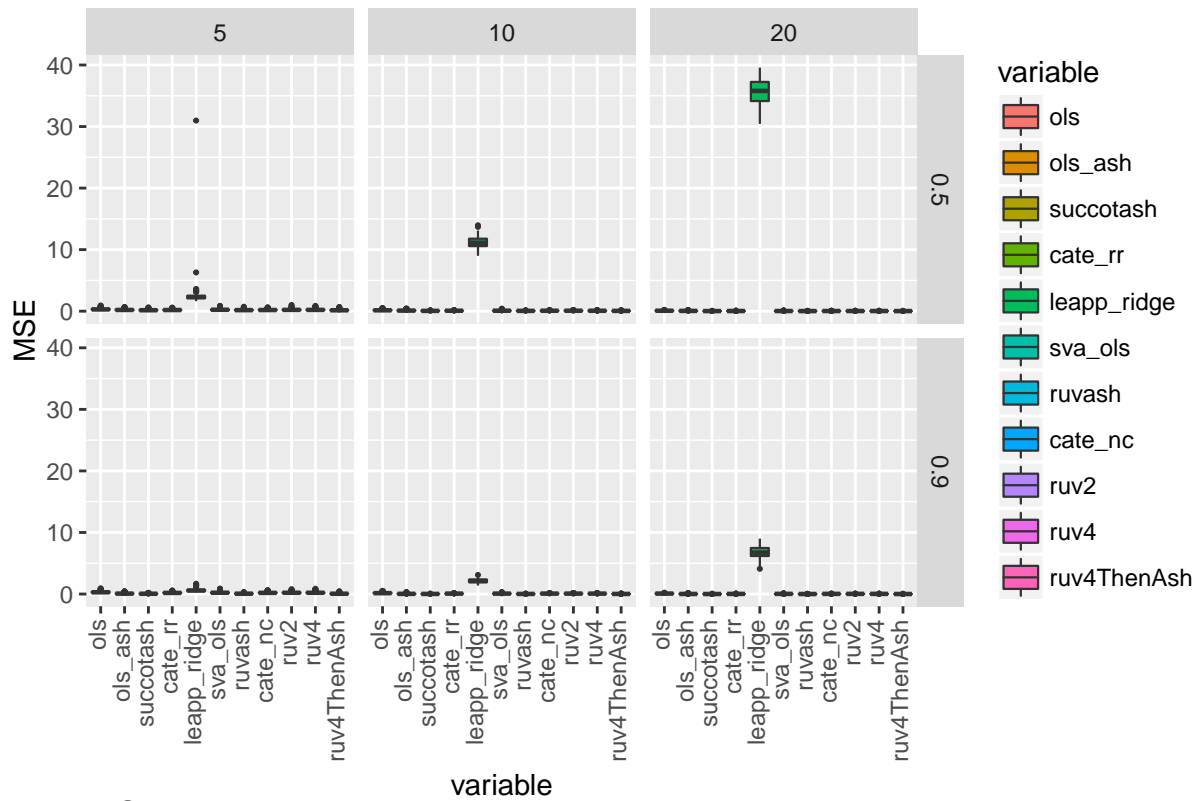
MSE When Using Muscle Tissue, Alternative = flattop



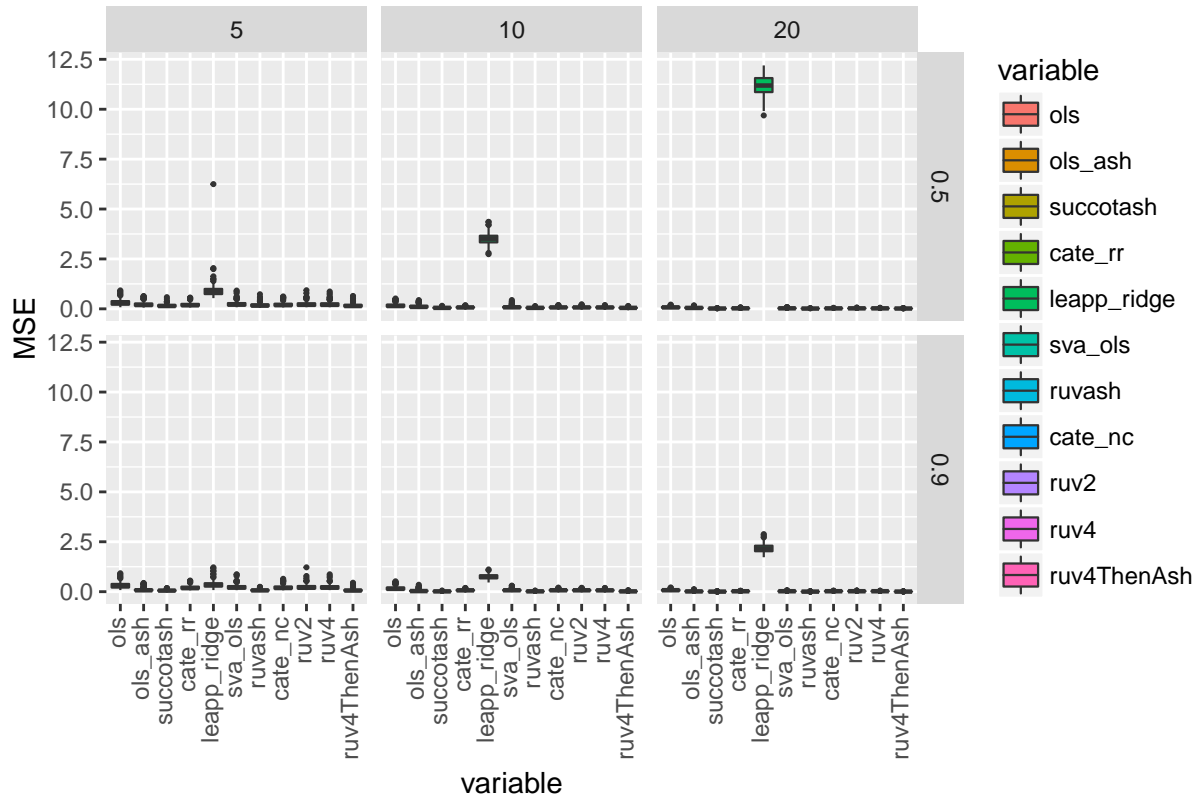
MSE When Using Muscle Tissue, Alternative = skew



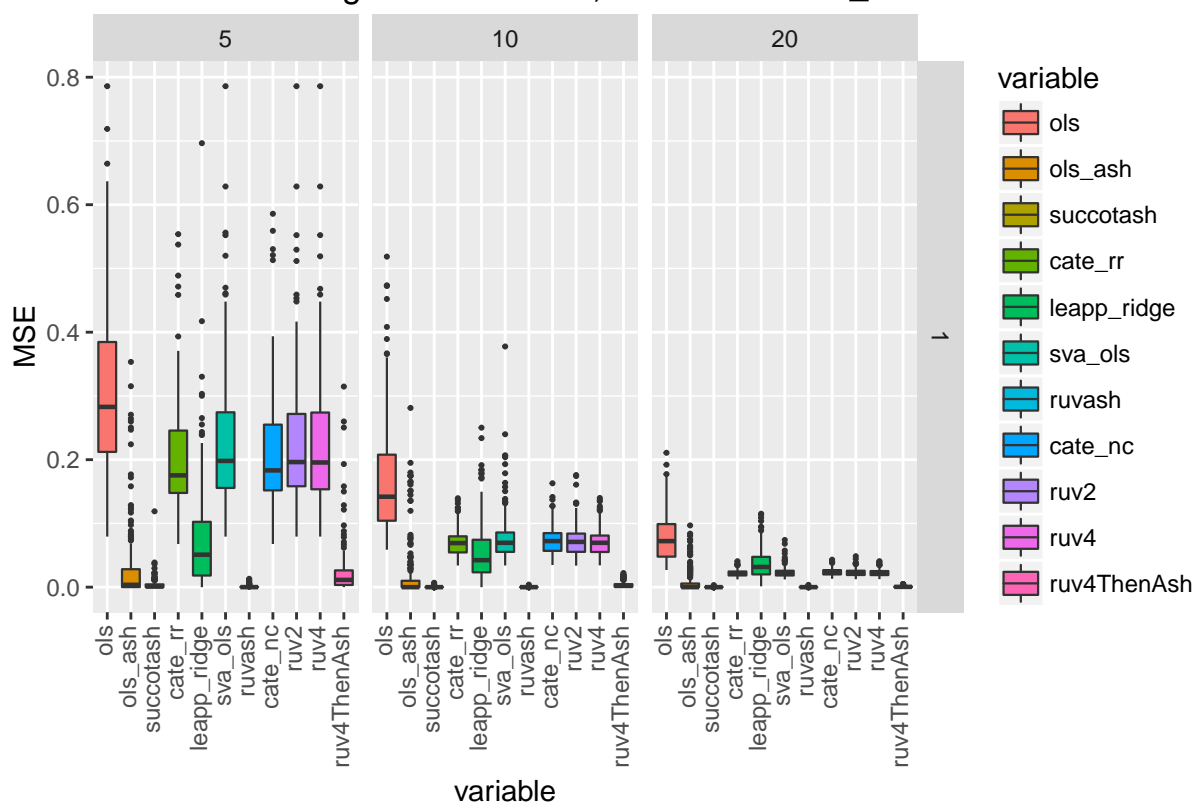
MSE When Using Muscle Tissue, Alternative = big\_normal



MSE When Using Muscle Tissue, Alternative = bimodal



## MSE When Using Muscle Tissue, Alternative = all\_null



sessionInfo()

```
## R version 3.3.0 (2016-05-03)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 14.04.4 LTS
##
## locale:
##  [1] LC_CTYPE=en_US.UTF-8      LC_NUMERIC=C
##  [3] LC_TIME=en_US.UTF-8      LC_COLLATE=en_US.UTF-8
##  [5] LC_MONETARY=en_US.UTF-8  LC_MESSAGES=en_US.UTF-8
##  [7] LC_PAPER=en_US.UTF-8     LC_NAME=C
##  [9] LC_ADDRESS=C             LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats      graphics  grDevices  utils      datasets  methods   base
##
## other attached packages:
## [1] ggplot2_2.1.0
##
## loaded via a namespace (and not attached):
##  [1] Rcpp_0.12.5      compiler_3.3.0    formatR_1.3
##  [4] plyr_1.8.3       tools_3.3.0       digest_0.6.9
##  [7] RSQLite_1.0.0    annotate_1.48.0   evaluate_0.9
## [10] gtable_0.2.0     cate_1.0.4        nlme_3.1-127
## [13] lattice_0.20-33  mgcv_1.8-12       Matrix_1.2-6
## [16] DBI_0.4          yaml_2.1.13       parallel_3.3.0
```

```
## [19] genefilter_1.52.0    stringr_1.0.0      knitr_1.12.28
## [22] IRanges_2.4.6       S4Vectors_0.8.6    stats4_3.3.0
## [25] grid_3.3.0          Biobase_2.30.0     ruv_0.9.6
## [28] AnnotationDbi_1.32.3 survival_2.39-2     XML_3.98-1.3
## [31] rmarkdown_0.9.6     leapp_1.2          sva_3.18.0
## [34] corpcor_1.6.8       magrittr_1.5       splines_3.3.0
## [37] scales_0.4.0        htmltools_0.3.5    MASS_7.3-45
## [40] BiocGenerics_0.16.1 svd_0.3.3-2        colorspace_1.2-6
## [43] xtable_1.8-2        esaBcv_1.2.1       stringi_1.0-1
## [46] munsell_0.4.3
```

Stephens, Matthew. 2016. “False Discovery Rates: A New Deal.” *BioRxiv*. Cold Spring Harbor Labs Journals, 038216.