(Kerr) Naked Singularities Lurking in the Shadows

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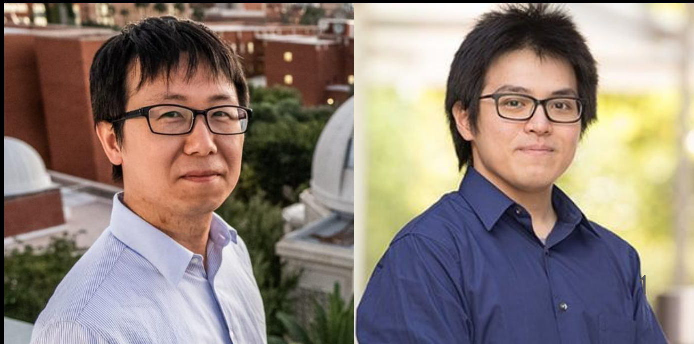
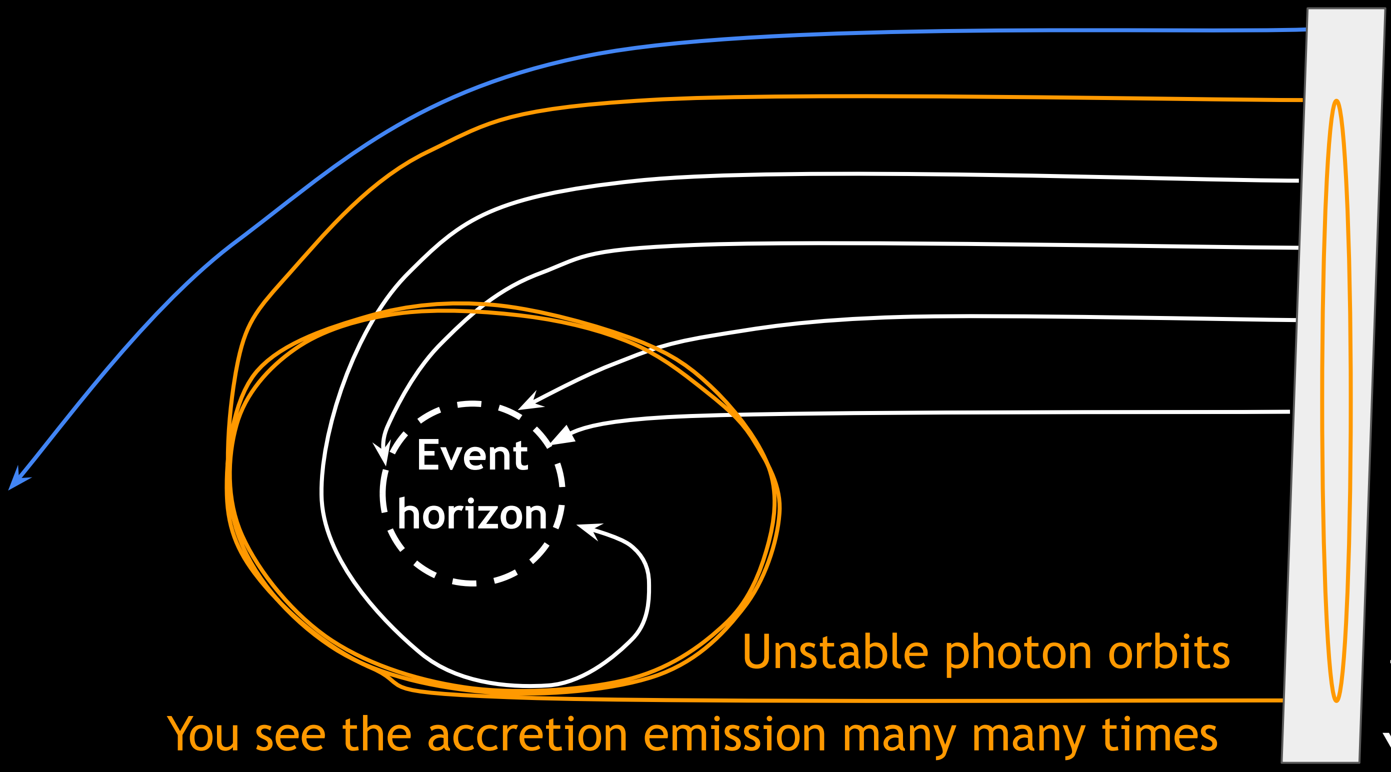
Nguyen et al. 2023
ApJ 954, 78

THE UNIVERSITY OF ARIZONA
COLLEGE OF SCIENCE
STEWARD OBSERVATORY

Black Hole PIRE



BLACK HOLE SHADOW

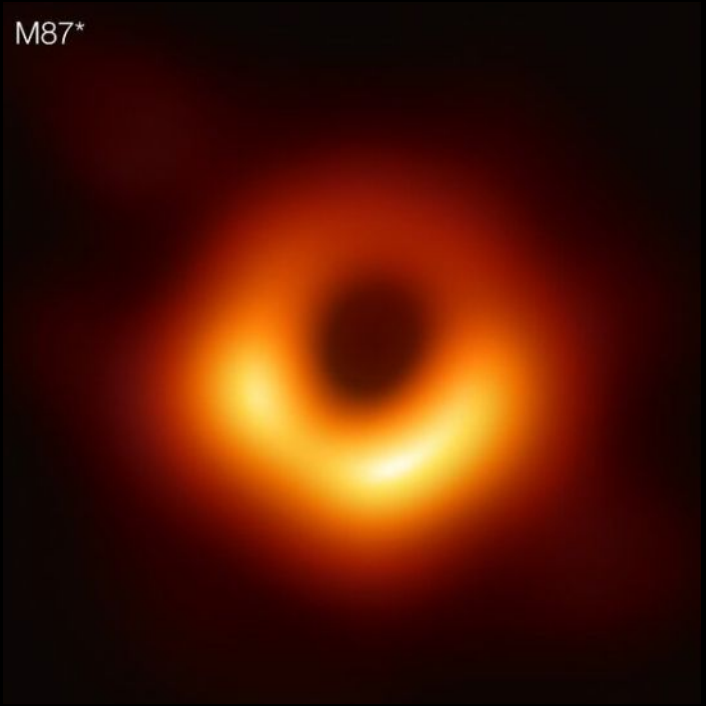


Disclaimer: there are subtleties in the shadow definition and what this image shows (Gralla+ 2019)

Black Hole "Shadow" - projection of unstable photon orbits

MOTIVATION

- EHT images of M87* and Sgr A*
- Shadows are signatures of Kerr (rotating) black holes in GR (Falcke et al. 2000, Johannsen & Psaltis 2010)
  - Projection of unstable photon orbits
  - Test of gravity
  - Independent from accretion physics
- Future space-based EHT will provide an even higher sensitive test of gravity!

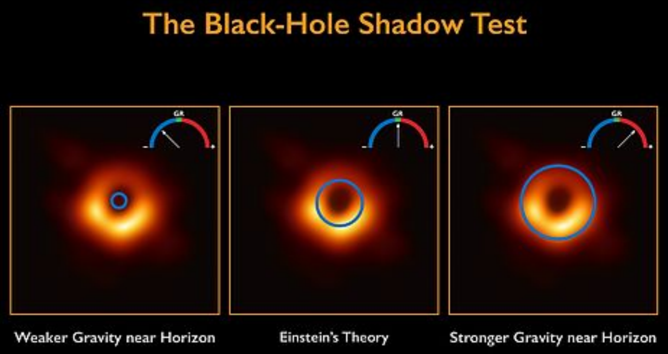


BLACK HOLE MIMICKERS

Shadows are signatures of **Kerr black holes** in GR

BUT shadows can also be mimicked by **other objects or gravity theories**

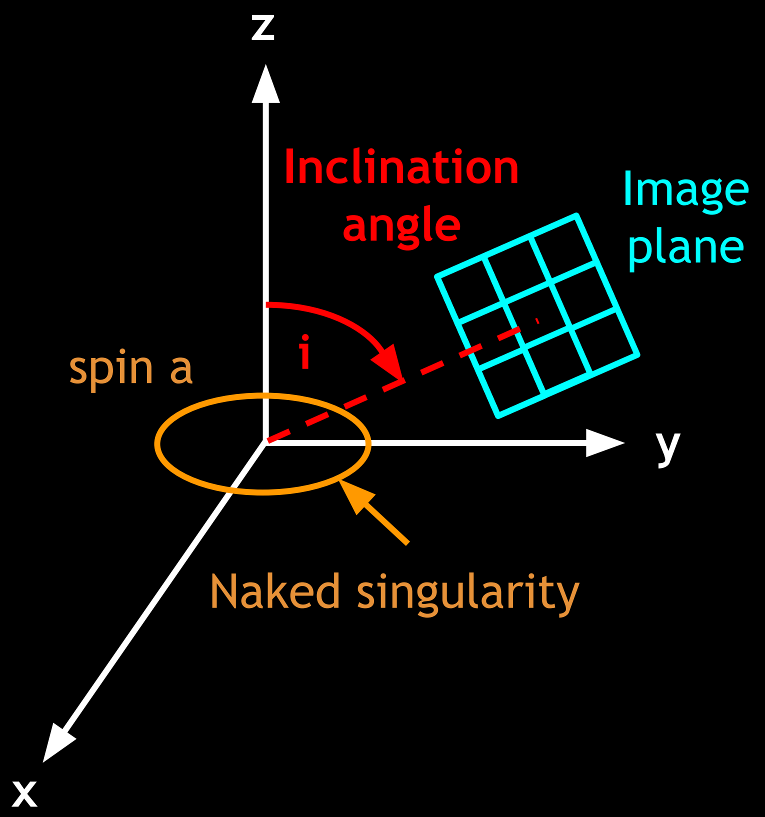
- ➢ Naked singularities (Hioki & Maeda 2009, Shaikh+ 2018)
- ➢ Boson stars (Vincent et al. 2016)
- ➢ No-horizon spacetimes (Kumar & Ghosh 2021)
- ➢ Deviations from Kerr spacetime (Vigeland et al. 2011, Johannsen & Psaltis 2011, Medeiros+ 2020)
- ➢ Constraints on these alternative models from images (M87* VI 2019, Sgr A* VI 2022)



Psaltis+ 2020

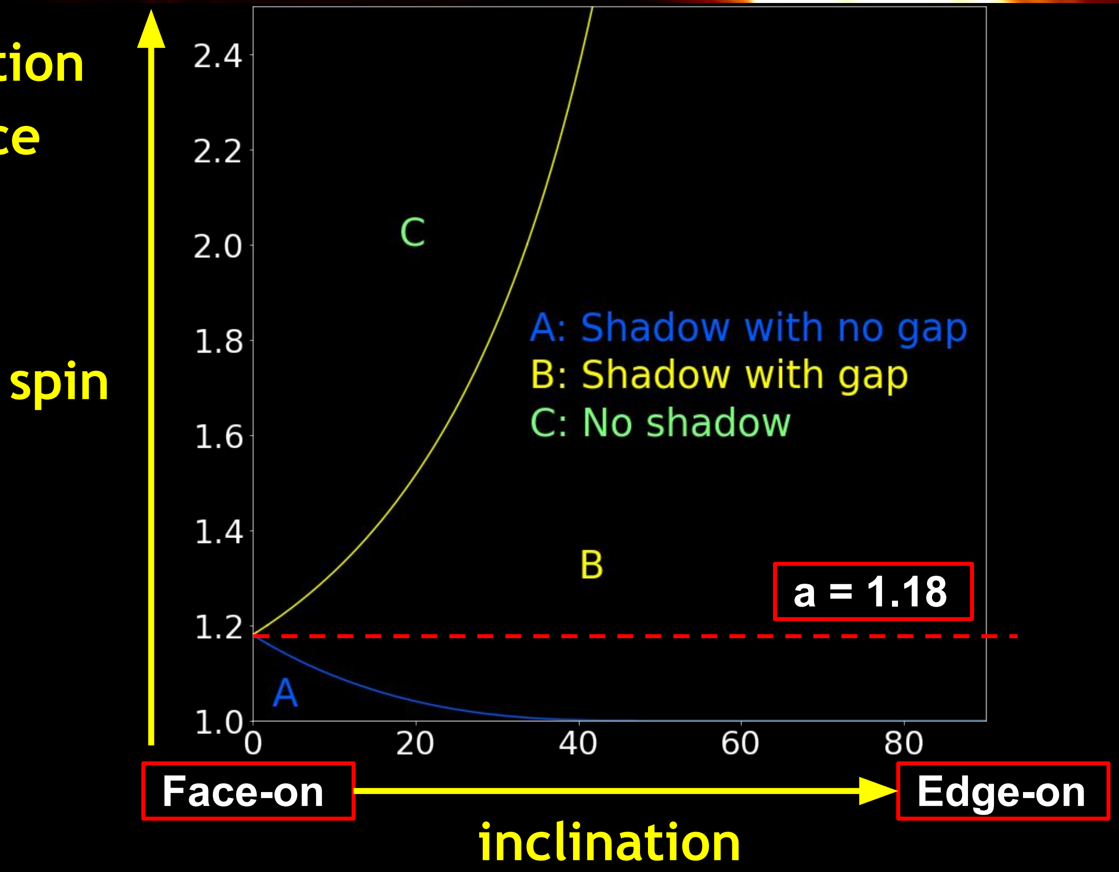
KERR NAKED SINGULARITY (KNS)

- **Super-spinning "black holes", spin |a| > 1**
  - Event horizon vanishes
- **Theoretical motivation**
  - Cosmic censorship (Penrose 1969)
  - String theory (Gimon & Horava 2009)
  - KNS is unstable in GR → test deviation from GR (Cardoso+ 2008, Dotti+ 2008)
  - M87* shadow is consistent with KNS with quantum gravity effects (Bambi+ 2018)

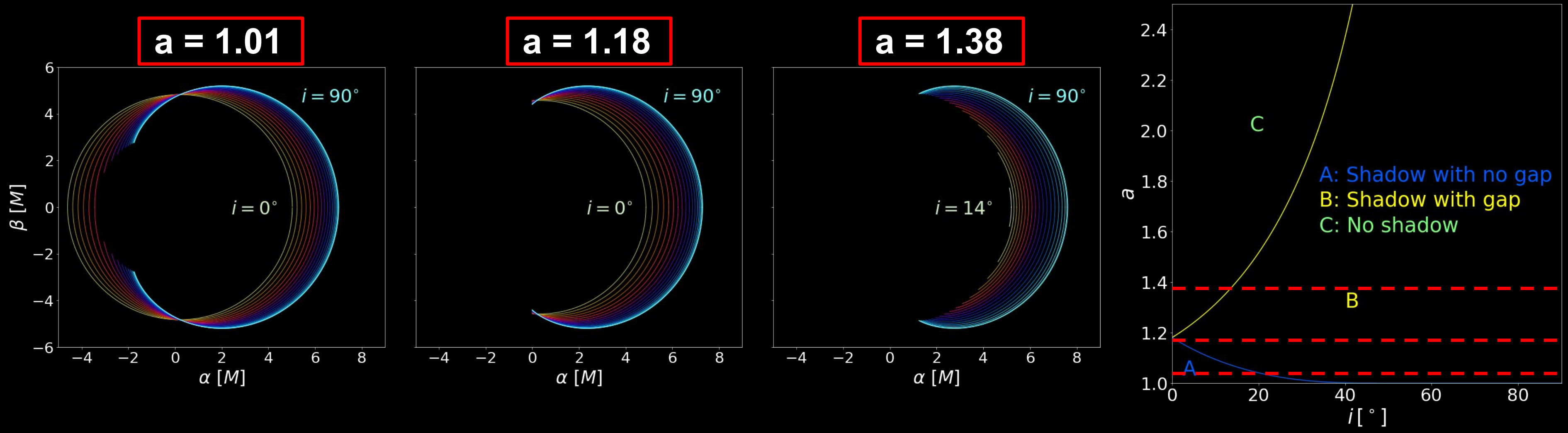


KNS SHADOW

Spin-inclination phase space



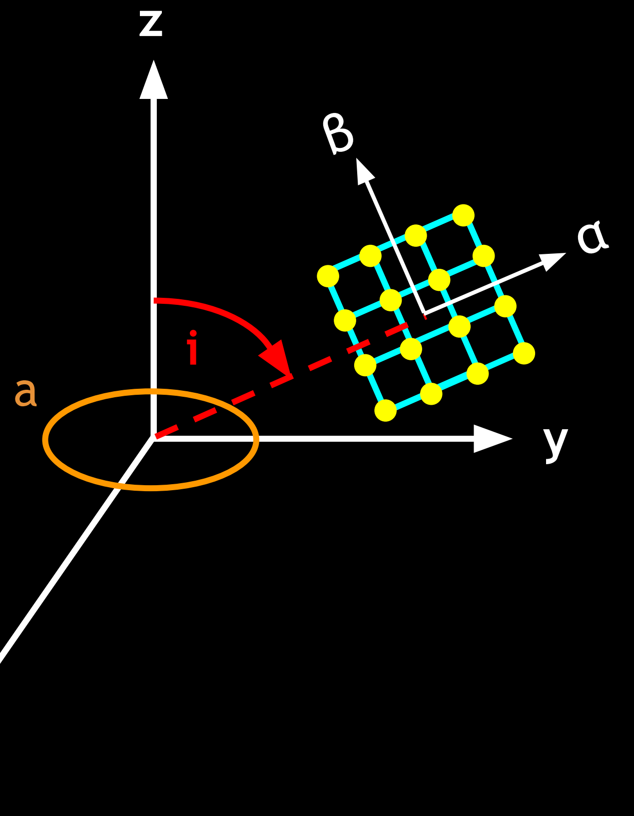
Fixed spin, varied inclination



NUMERICAL RAY TRACING

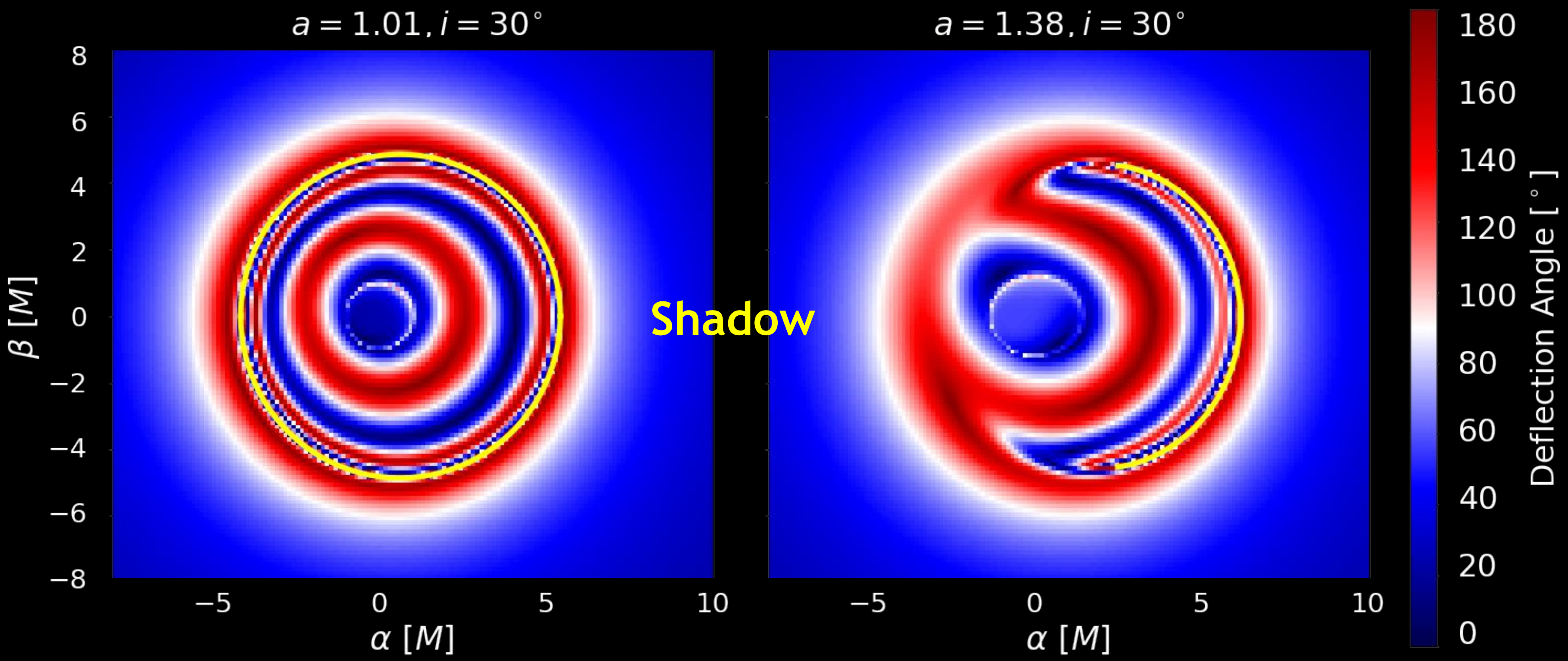
- Image plane with a 128 x 160 grid of photons
- Integrate light rays backward in time
- Differential geometry package **fadge**:

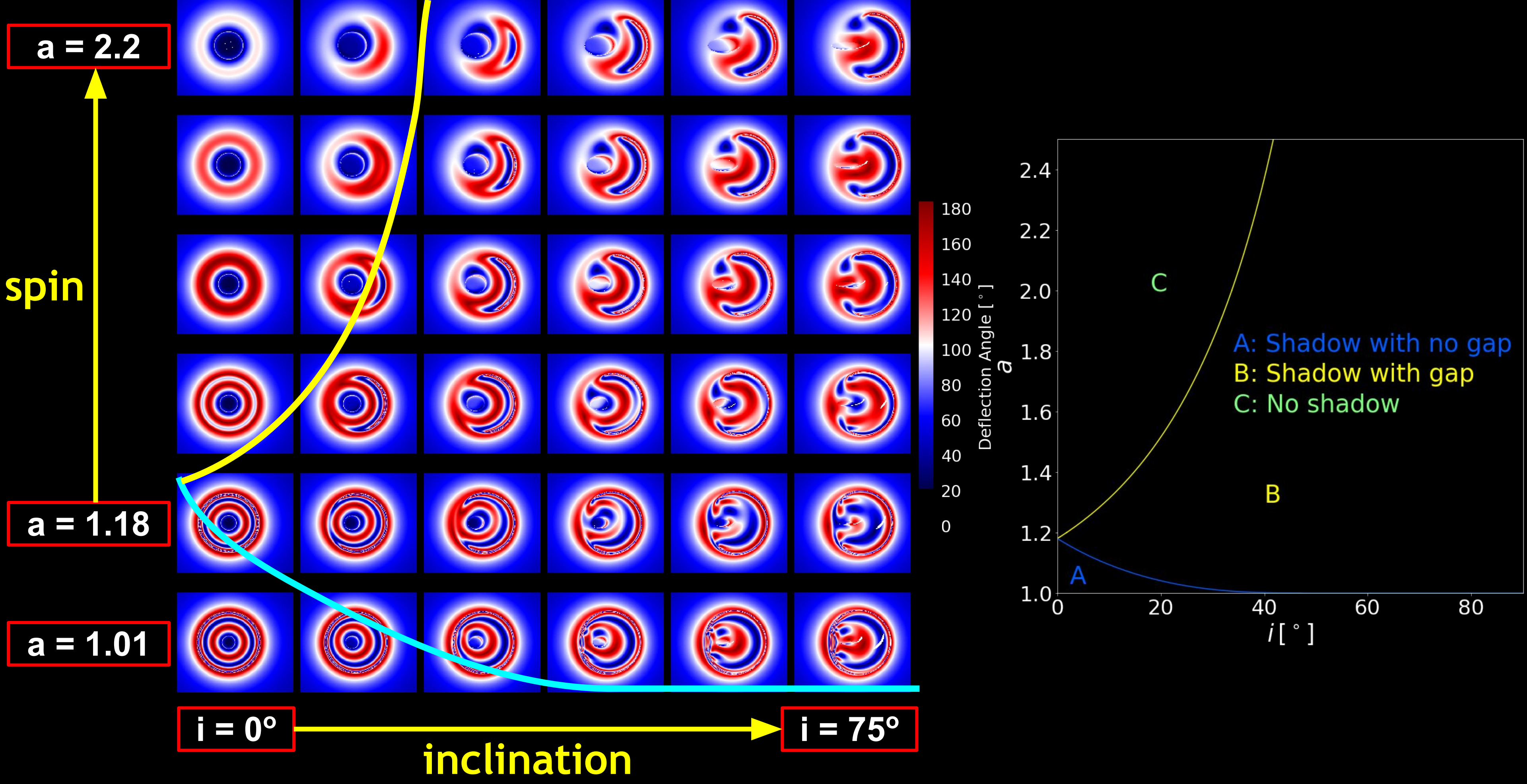
  https://github.com/adxsrc/fadge (CK Chan)



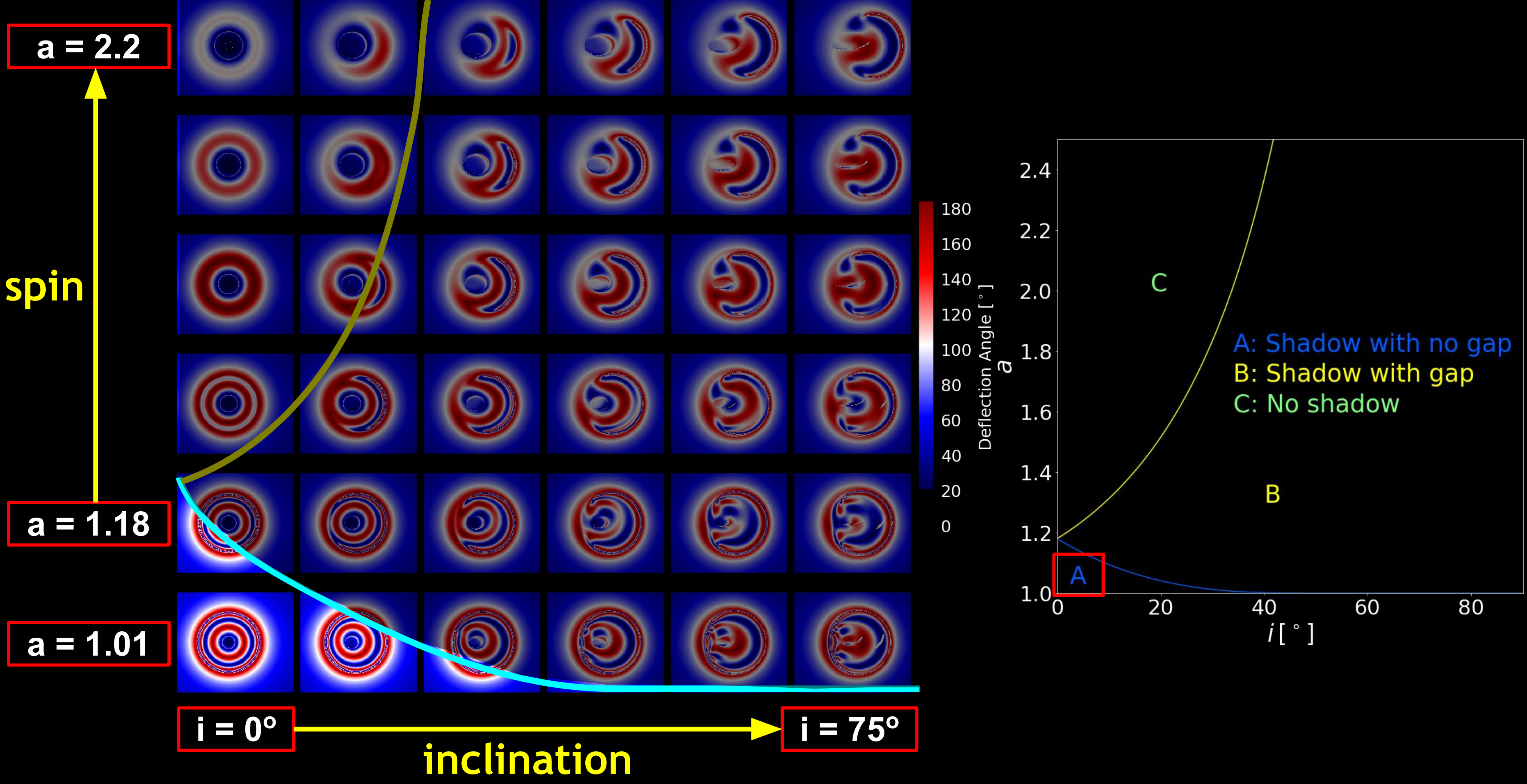
NUMERICAL RESULTS

Deflection angle plots





a = 2.2
spin
a = 1.18
a = 1.01
i = 0°
i = 75°
inclination
180
160
140
120
100
80
60
40
20
0
Deflection Angle [°]
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
a
0
20
40
60
80
i [°]
C
B
A
A: Shadow with no gap
B: Shadow with gap
C: No shadow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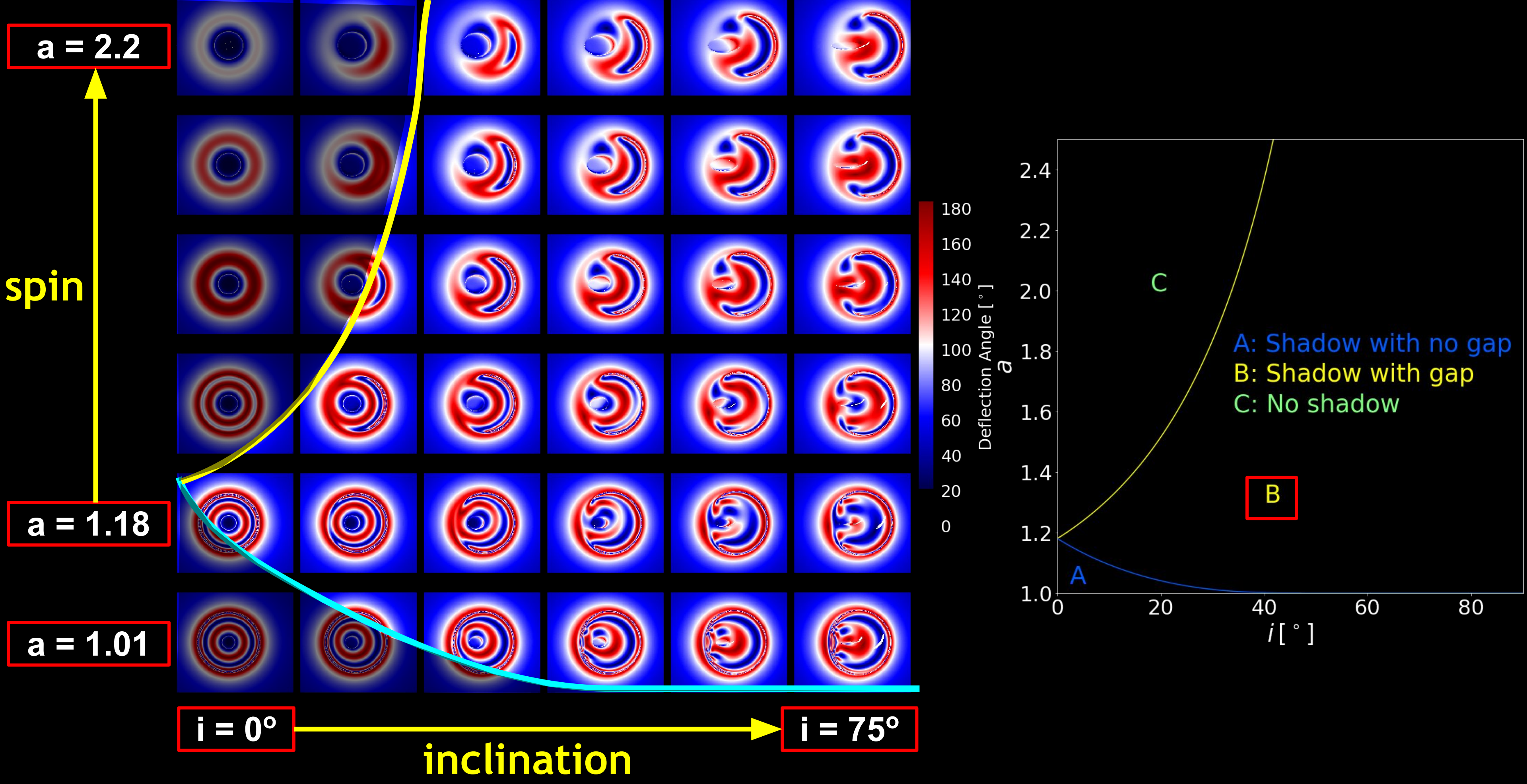
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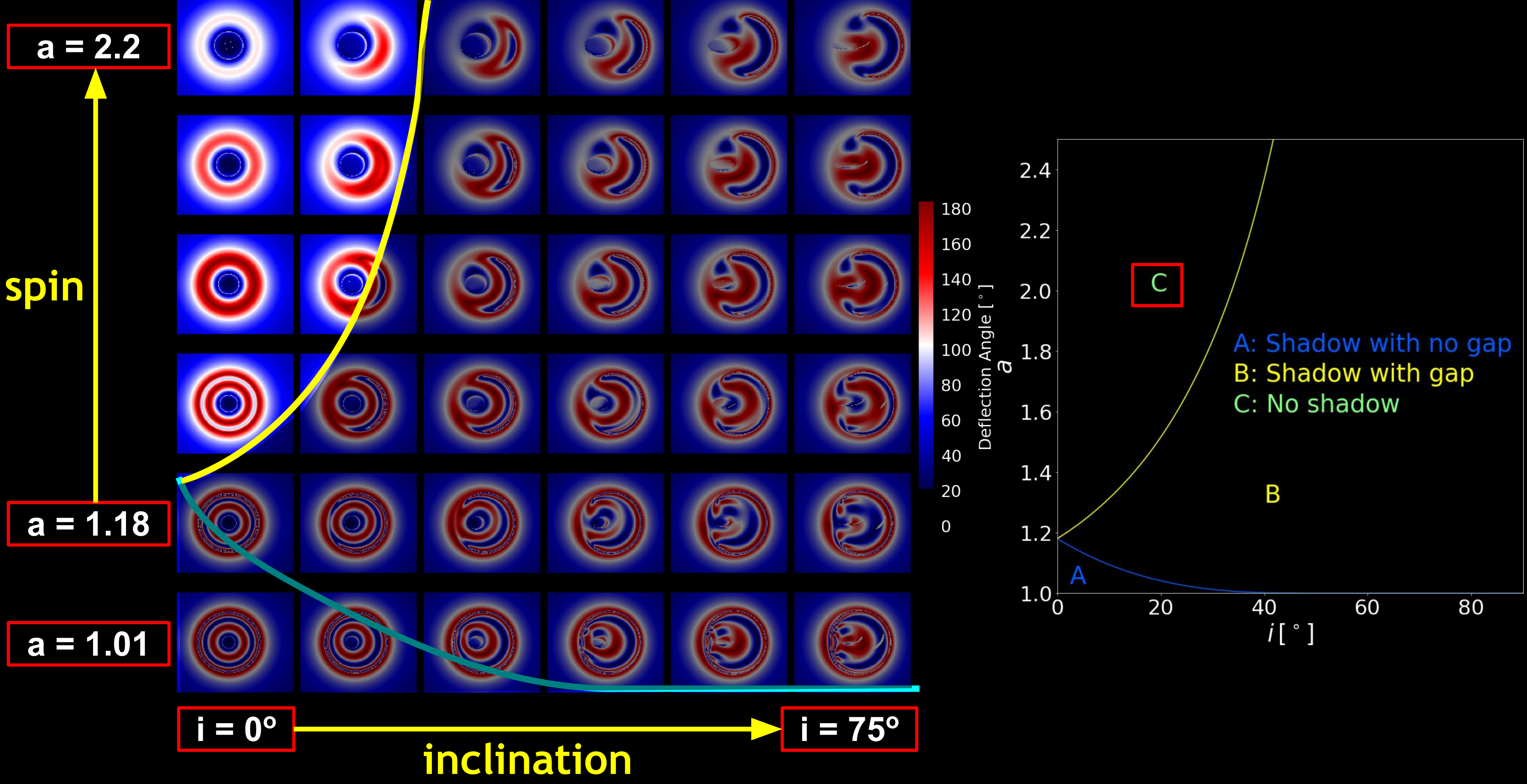
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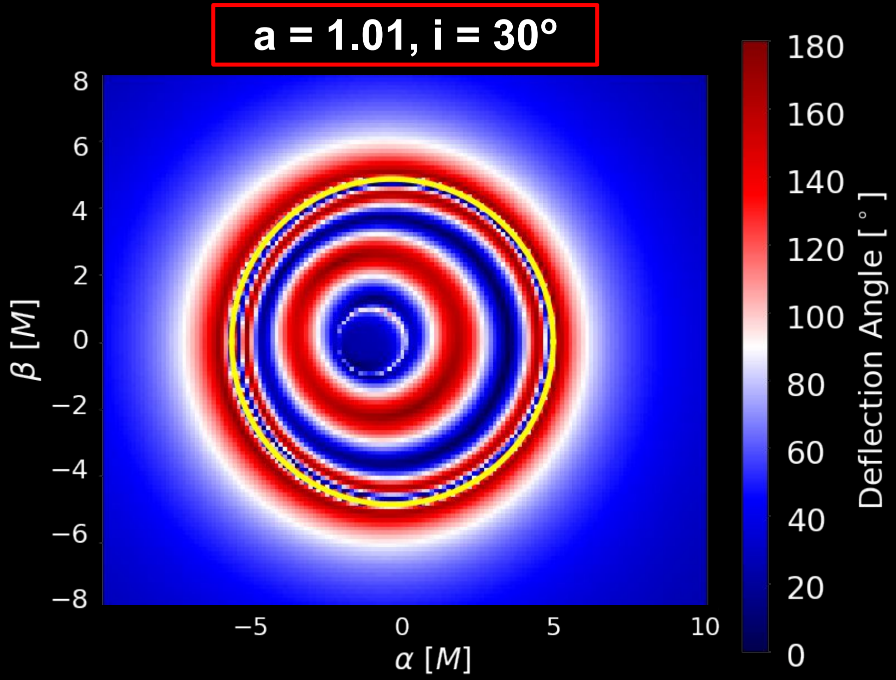
1.8
1.6
1.4
1.2
1.0
a
C
A: Shadow with no gap
B: Shadow with gap
C: No shadow
B
A
0
20
40
60
80
i [°]

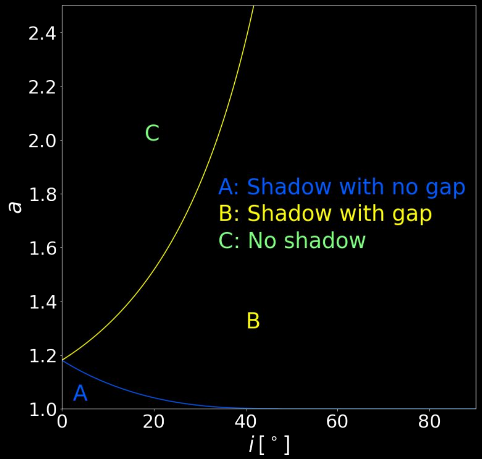
IMPLICATIONS

- ➢ Misconceptions that KNS shadow can't be closed (Kumar & Ghosh 2020)
- ➢ Test cosmic censorship from horizon-scale imaging
- ➢ Springboard to study perturbations of KNS and other naked singularity types



CONCLUSION

- KNS shadows can be closed, open, or vanishing
- Analytical results are reproduced by numerical ray tracing simulations
- Implications in testing gravitational physics through horizon-scale imaging



ApJ Paper

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Black Hole PIRE (U. Arizona)

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U. Arizona Department of Physics

THANK YOU FOR LISTENING!

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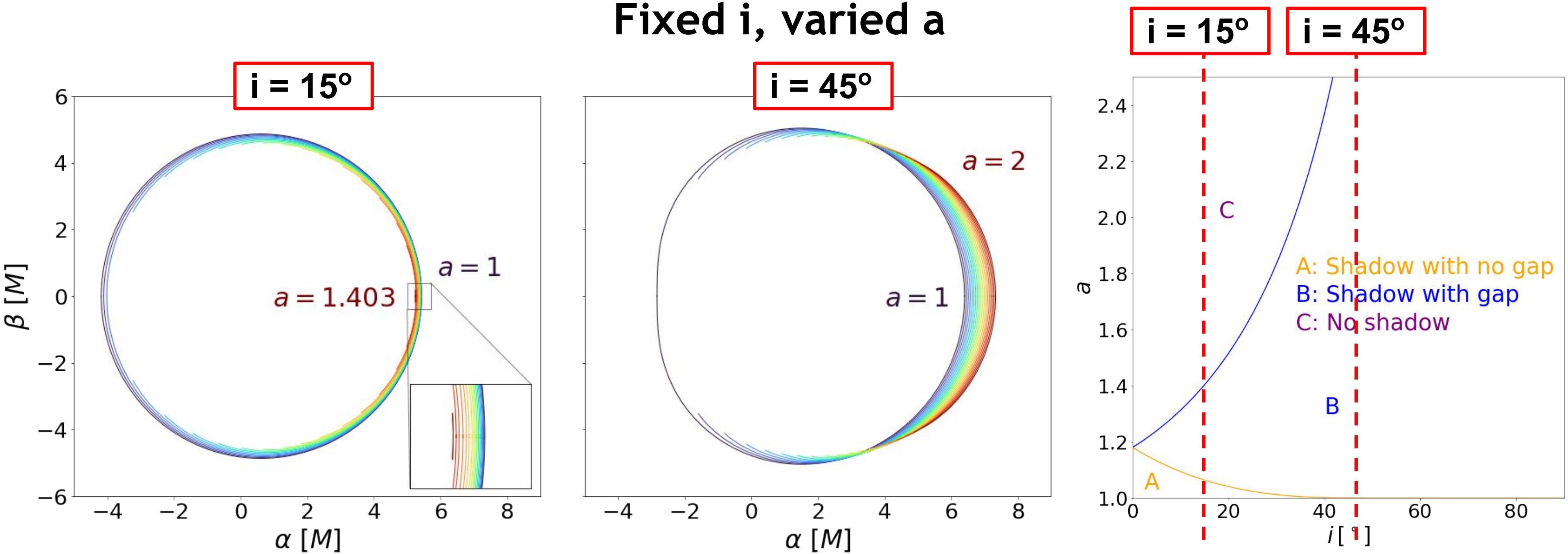
Fixed i, varied a



APPENDIX

Kerr metric

$$ds^2 = \left(\frac{2Mr}{\Sigma} - 1\right) dt^2 - \frac{4Mar}{\Sigma} \sin^2\theta \, dt \, d\phi + \frac{\Sigma}{\Delta} + \Sigma \, d\theta^2 + \frac{\beta}{\Sigma} \sin^2\theta$$

$$\Sigma = r^2 + a^2 \cos\theta,$$
$$\Delta = r^2 + a^2 - 2Mr,$$
$$\beta = (r^2 + a^2)^2 - \Delta \, a^2 \sin^2\theta.$$

APPENDIX

Conserved: E = energy, L_z = angular momentum, C = Carter's Constant

Parameters: $\Phi = L_z/E$, $Q = C/E$

Kerr metric

Hamilton-Jacobi equation (Carter 1968)

Radial effective potential $R(r,\Phi,Q)$

$$R(r) = r^4 + r^2(a^2 - \Phi^2 - Q)$$
$$+ 2Mr[(a-\Phi)^2 + Q] - a^2 Q$$

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Kerr metric

Hamilton-Jacobi equation (Carter 1968)

Radial effective potential $R(r,\Phi,Q)$

Unstable SPOs: $R(r) = dR/dr = 0$ (Teo 2003)

$\Phi(r)$, $Q(r)$

$$\Phi = -\frac{r^3 - 3r^2 + a^2 r + a^2}{a(r-1)}$$

$$Q = -\frac{r^3(r^3 - 6r^2 + 9r - 4a^2)}{a^2(r-1)^2}$$

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Hamilton-Jacobi equation (Carter 1968)

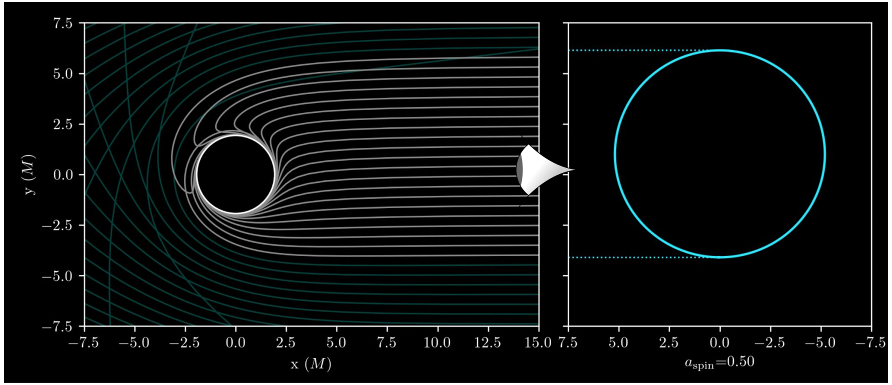
Radial effective potential $R(r,\Phi,Q)$

Unstable SPOs: $R(r) = dR/dr = 0$ (Teo 2003)

$\Phi(r)$, $Q(r)$

Projection (Bardeen et al. 1972)

$\alpha[\Phi(r), Q(r)]$, $\beta[\Phi(r), Q(r)]$



Credit: CK Chan (Arizona)

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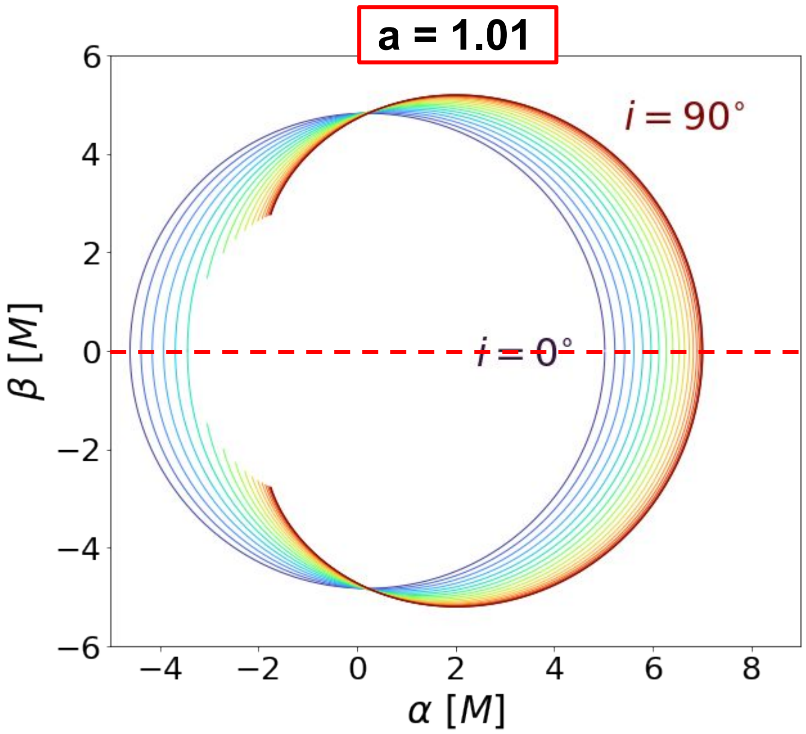
Projection (Bardeen et al. 1972)

$\alpha[\Phi(r), Q(r)]$, $\beta[\Phi(r), Q(r)]$

Shadow α-axis symmetry

$\beta(r) = 0 \rightarrow$ solve for r_p (- r_{ms})



APPENDIX

Conserved: E = energy, L_z = angular momentum, C = Carter's Constant

Parameters: $\Phi = L_z/E$, $Q = C/E$

Marginally stable radius: $r_{ms} = 1 + (a^2-1)^{1/3}$

Kerr metric

Hamilton-Jacobi equation (Carter 1968)

Radial effective potential $R(r,\Phi,Q)$

Unstable SPOs: $R(r) = dR/dr = 0$ (Teo 2003)

$\Phi(r)$, $Q(r)$

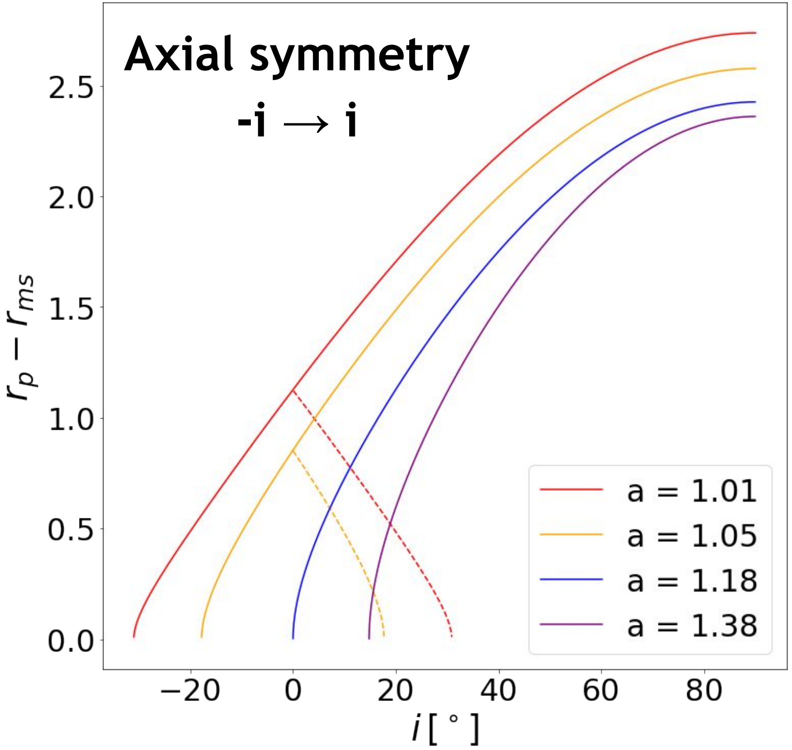
Projection (Bardeen et al. 1972)

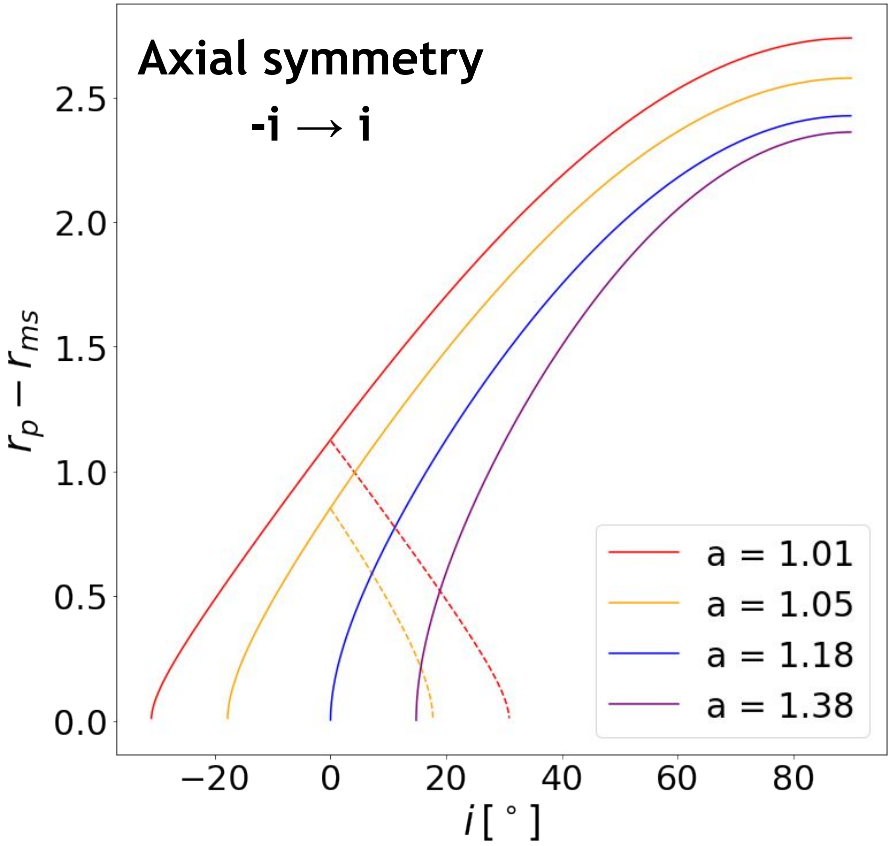
$\alpha[\Phi(r), Q(r)]$, $\beta[\Phi(r), Q(r)]$

Shadow α-axis symmetry

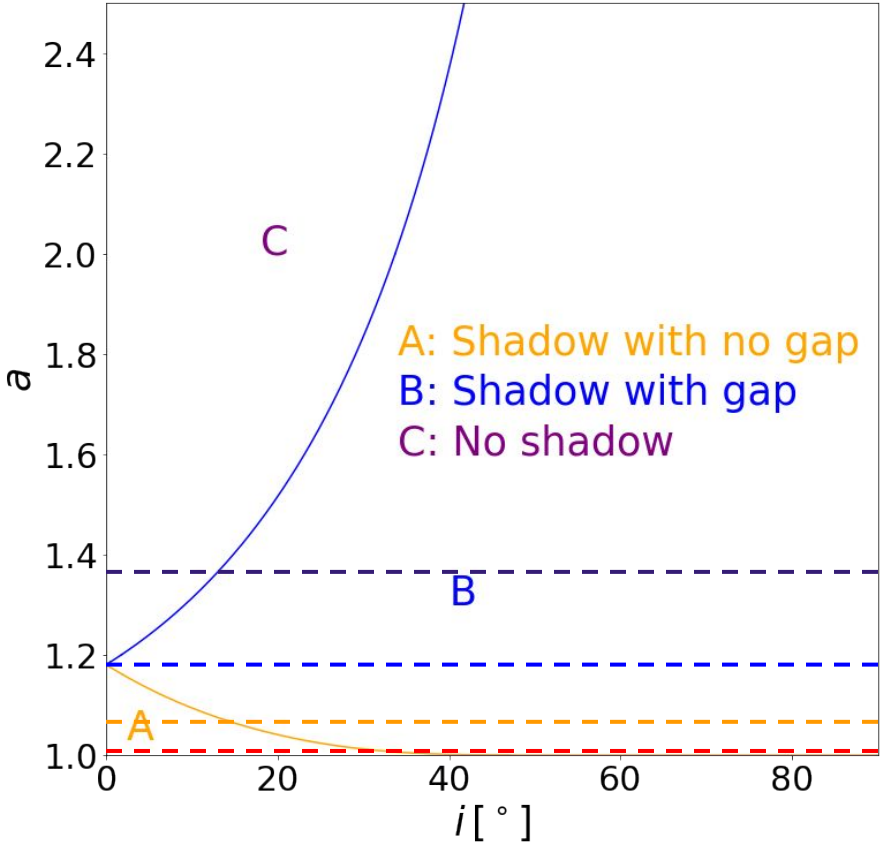
$\beta(r) = 0 \rightarrow$ solve for r_p ($- r_{ms}$)

(More on null geodesics/SPOs: Chandrasekhar 1983)





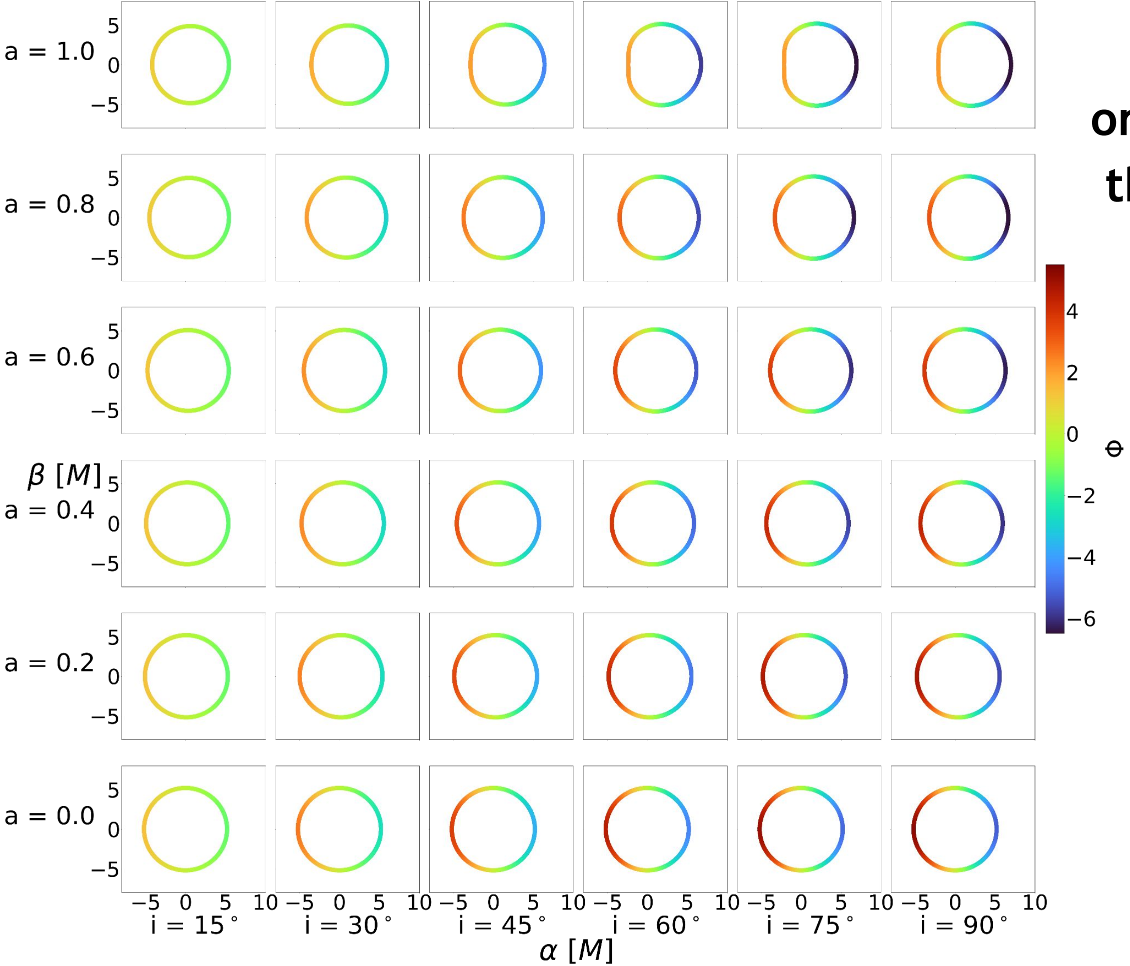
Axial symmetry
-i → i
$r_p - r_{ms}$
$i\,[\,^\circ\,]$
a = 1.01
a = 1.05
a = 1.18
a = 1.38



C
A: Shadow with no gap
B: Shadow with gap
C: No shadow
B
A
a
$i\,[\,^\circ\,]$

APPENDIX

Φ of different orbits projected to the image plane's shadow



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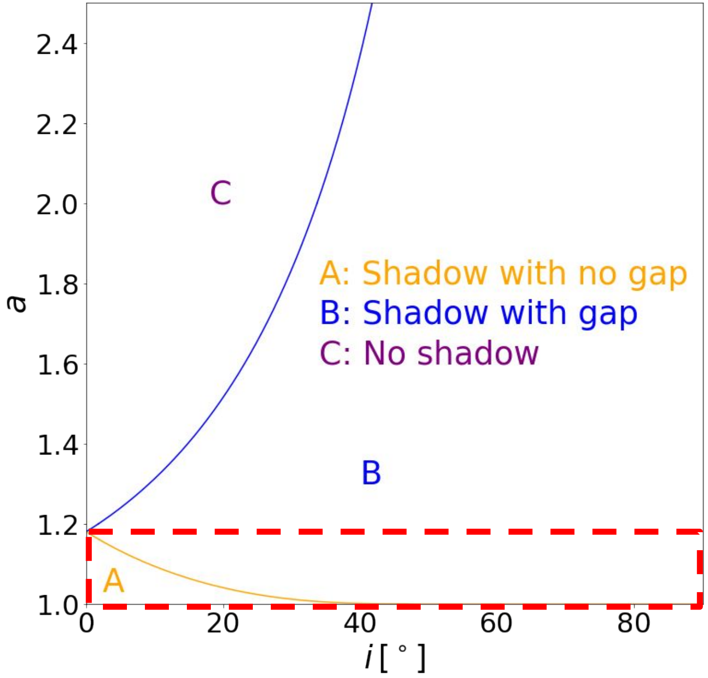
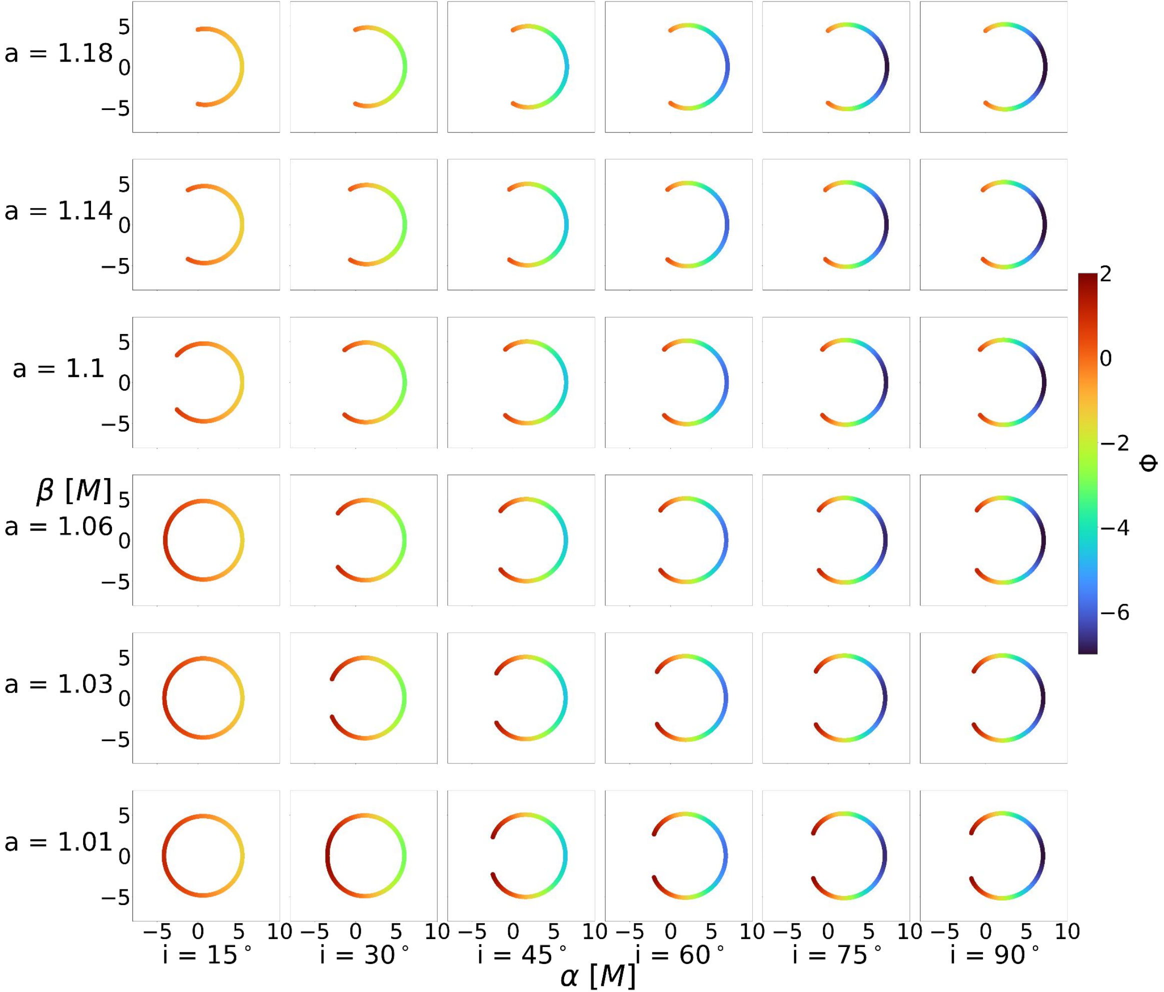
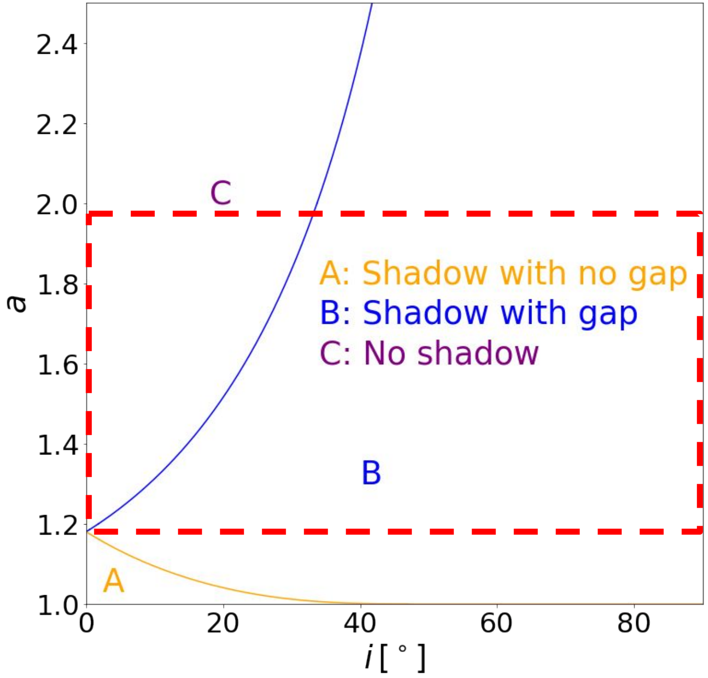


C
A: Shadow with no gap
B: Shadow with gap
C: No shadow
B
A
a
i [°]
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
20
40
60
80

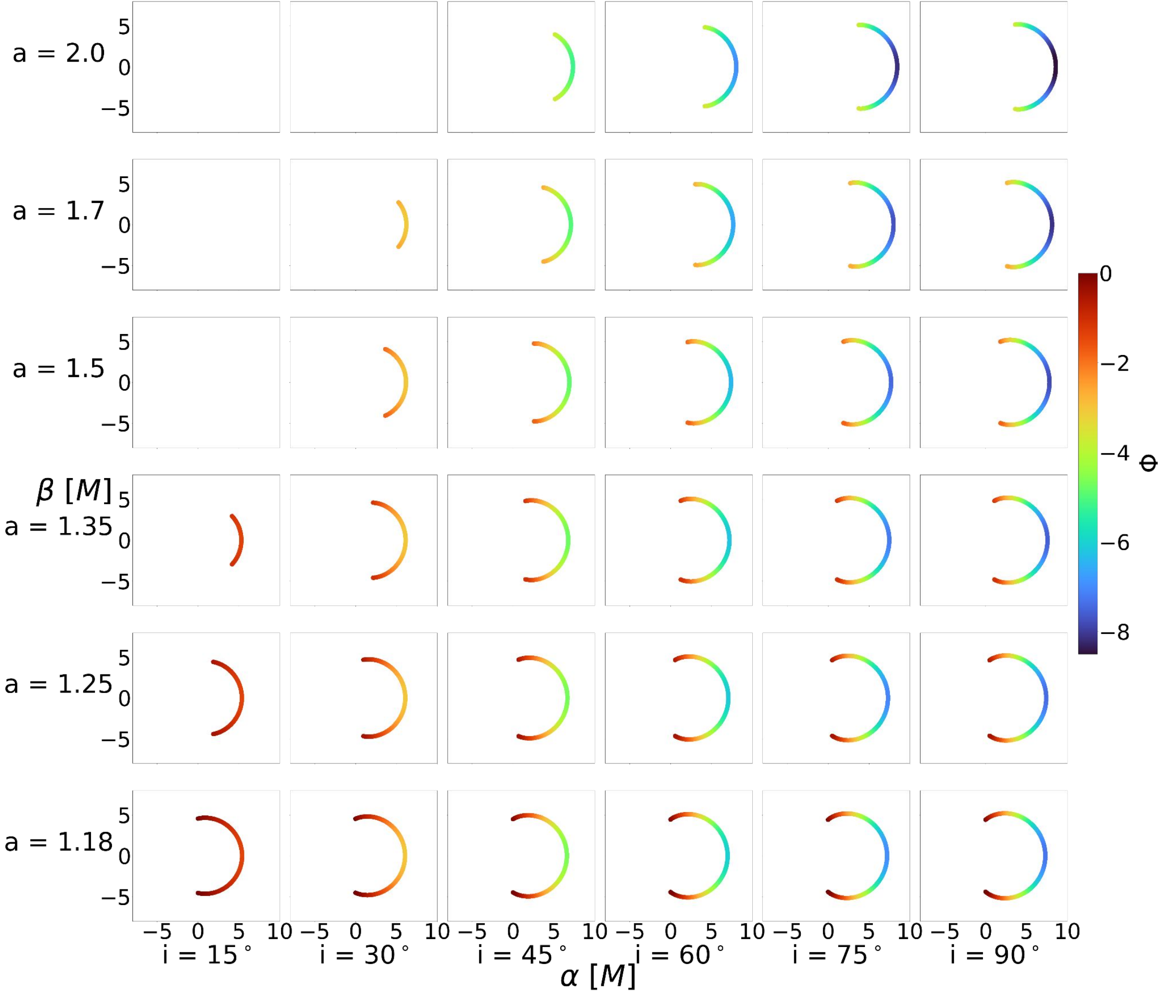


a = 1.18
a = 1.14
a = 1.1
a = 1.06
a = 1.03
a = 1.01
β [M]
α [M]
i = 15 °
i = 30 °
i = 45 °
i = 60 °
i = 75 °
i = 90 °
Φ
2
0
−2
−4
−6

APPENDIX



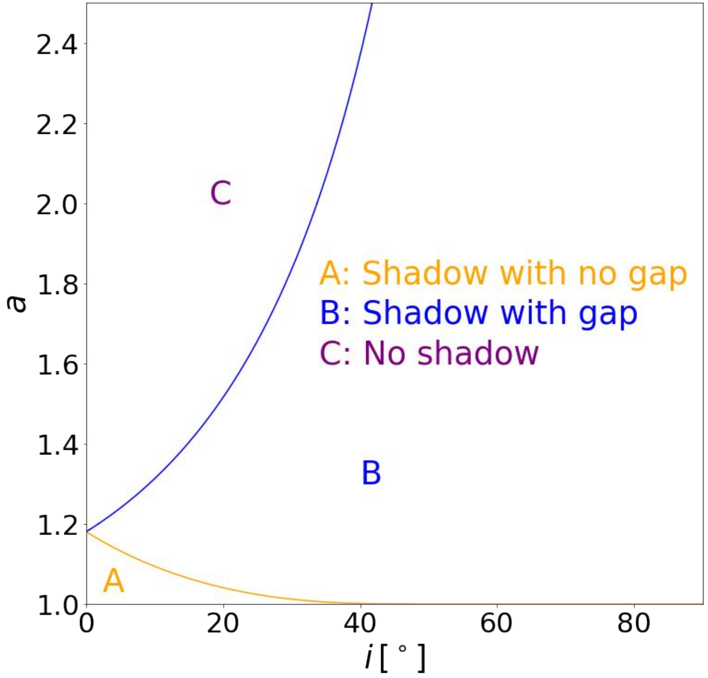
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
a
0
20
40
60
80
$i\,[^\circ]$
C
A: Shadow with no gap
B: Shadow with gap
C: No shadow
B
A

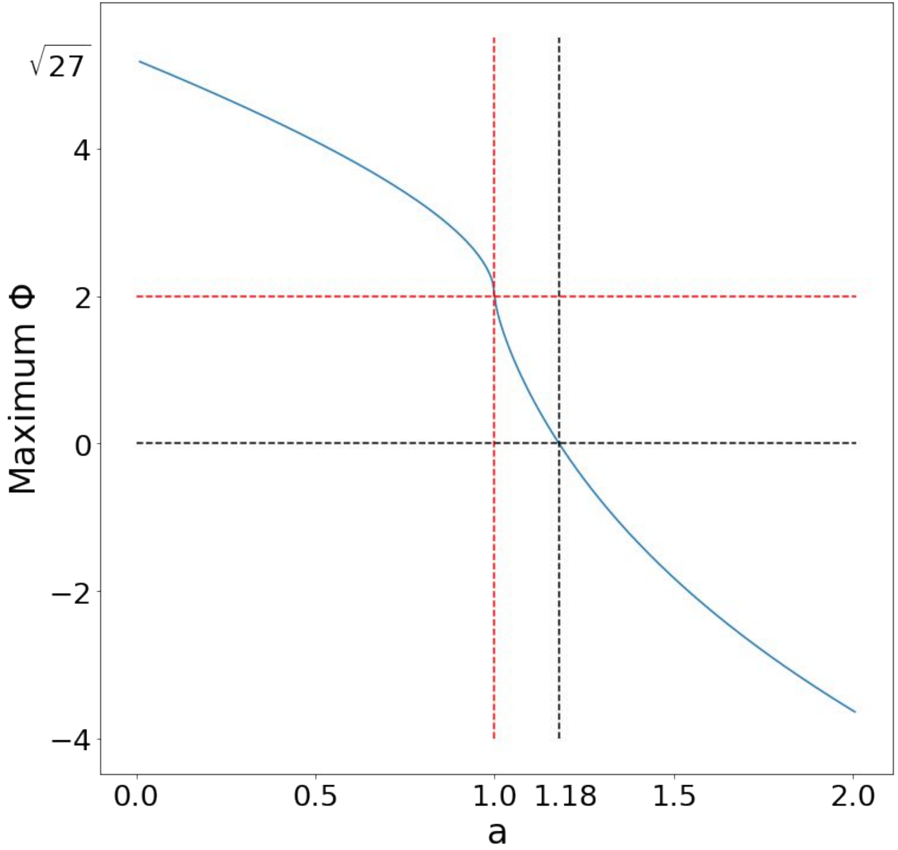


a = 2.0
a = 1.7
a = 1.5
a = 1.35
a = 1.25
a = 1.18
$\beta\ [M]$
5
0
−5
−5 0 5 10
i = 15°
i = 30°
i = 45°
i = 60°
i = 75°
i = 90°
$\alpha\ [M]$
0
−2
−4
−6
−8
Φ

APPENDIX

Maximum Φ to sustain spherical orbits





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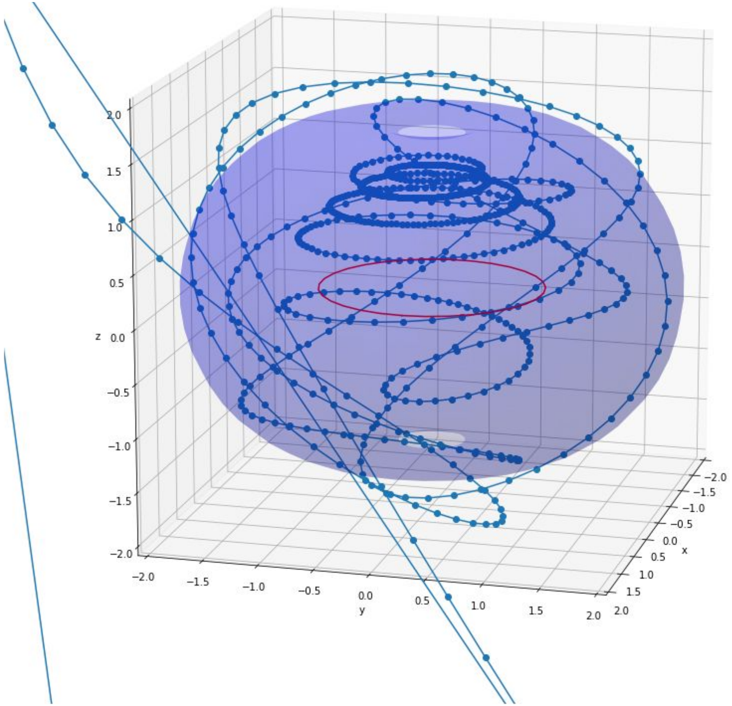
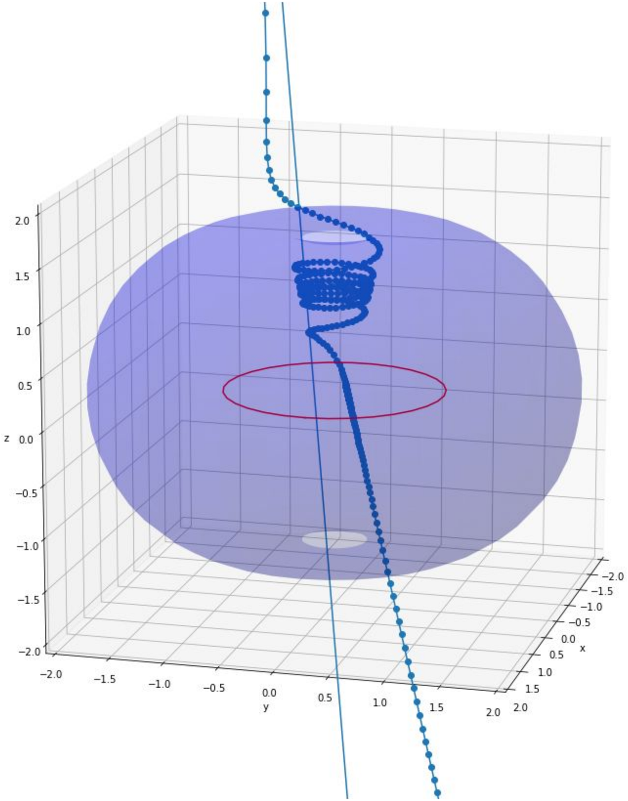


Close to unstable spherical photon orbit



Asymmetry between ingoing and outgoing geodesics