# A family of qualitative theories for continuous spatio-temporal change as a spatio-temporalisation of $\mathcal{ALC}(D)$ —first results[1]

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**Abstract.** We present a family of qualitative theories for continuous motion of spatial scenes, and for continuous spatio-temporal change in general, consisting of a subfamily of the $\mathcal{ALC}(D)$ family of description logics (DLs) with a concrete domain, with Allen-like temporal relations as roles, and RCC8-like spatial relations as predicates. Examples illustrating the importance and usefulness of the theories will be provided, as well as a discussion characterising continuous spatio-temporal change.

## 1 Introduction

Qualitative Spatial and Temporal Reasoning (QSTR), and more generally Qualitative Reasoning (QR), differs from quantitative reasoning by its particularity of remaining at a description level as high as possible. In other words, QSTR sticks at the idea of "making only as many distinctions as necessary" [7], idea borrowed to naïve physics [12]. The core motivation behind this is that, whenever the number of distinctions that need to be made is finite, the reasoning issue can get rid of the calculation details of quantitative models, and be transformed into a simple matter of symbol manipulation; in the particular case of constraint-based spatial and temporal reasoning, this generally means a finite relation algebra (finite RA), with tables recording the results of applying the different operations to the different atoms, and the reasoning issue reduced to a matter of table look-ups: the best illustration to this is certainly Allen's algebra of time intervals [1].

Considered separately, Qualitative Spatial Reasoning (QSR) and Qualitative Temporal Reasoning (QTR) have an important place in Artificial Intelligence (AI), especially their constraint-based subarea, symbolised by works such as the $\mathcal{RCC}8$ calculus [20, 8] and the time interval algebra [1]. However, an important goal for research in AI is to come with well-founded languages for the representation of motion, and it is fairly clear that the way such languages combine time and space is crucial for their "strength". There is, of course, a reasonably important literature on the subject [10, 12, 19]: the notion of continuous change and contiuous motion, closely related to the notion of conceptual neighbourhood [9] (see also [17]), is crucial for these works.

Consider, for instance, Muller's mereo-topological theory[2] of motion [19], which is mainly a spatio-temporal extension of Asher and Vieu's topological theory [3], which in turn derives from Clarke's calculus [6], and differs from the $\mathcal{RCC}8$ calculus [20] in that it differentiates between open and closed regions. The primitive entities of Muller's theory [19] are spatio-temporal regions, which "can be interpreted as the trajectories of physical objects and events [19]". The relations of the theory are spatio-temporal and temporal relations on the spatio-temporal regions. A crucial difference with our work is that we consider time and space as "unrelated", but "working" together, through the "description logics" bridge, to achieve a common goal: the representation of motion.

We provide a family of qualitative theories for continuous motion of spatial scenes, consisting of a subfamily of the $\mathcal{ALC}(D)$ family of description logics with a concrete domain [4] (see also [2, 18]), with Allen-like temporal relations [1] as roles, and RCC8-like spatial relations [20, 8] as predicates of the concrete domain. The use of temporal relations as roles is done in a way similar to the use of Allen's relations [1] in Halpern and Shoham's modal logic of time intervals [11]. Examples illustrating the importance and usefulness of the theories will be provided, as well as a discussion characterising continuous spatio-temporal change.

## 2 A quick overview of the temporal and spatial RAs to be used in the theories of motion

The family of qualitative theories for continuous motion is a family (x,y)-*DL* of description logics with a concrete domain [4]: $x$ stands for a temporal Relation Algebra (RA) whose relations will be used as roles; and $y$ stands for a spatial RA, whose relations will be used as predicates of the concrete domain. We focus in this work on $x$ being either the temporal interval RA, $\mathcal{LIA}$, in [1], or its cyclic time counterpart, $\mathcal{CIA}$, in [5]; and on $y$ being either the $\mathcal{RCC}8$ binary calculus in [20] (see also [8]), or the $\mathcal{CYC}_t$ ternary calculus of 2-dimensional orientations in [14].

**The RA $\mathcal{LIA}$.** Allen's RA [1] is well-known. Its importance for this work is primordial, since it handles relations on temporal intervals, instead of relations on temporal points such as the RA in [24]: as such, it captures much better the idea of continuous spatial change [10]. Briefly, the algebra is qualitative and contains 13 atoms, which allow to differentiate between the 13 possible configurations of two intervals on the time line. The atoms are $<$ (before), *m* (meets), *o* (overlaps), *s* (starts), *d* (during), *f* (finishes); their respective con-

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[2] See, for instance, [23] for a survey of mereo-topology.

verses $>$ (after), *mi* (met-by), *oi* (overlapped-by), *si* (started-by), *di* (contains), *fi* (finished-by); and *eq* (equals), which is its proper converse.

**The RA $\mathcal{CIA}$.** The cyclic counterpart of Allen's linear time RA [1] is Balbiani and Osmani's RA of cyclic time intervals, recently known in the literature [5]. We assume cyclic time to be anticlockwise. Because time is cyclic (modelled, for example, by the circle centred at $O$ and of unit radius), the number of possible configurations of two intervals is 16. The corresponding 16 atoms of the RA are *c-dc* (disconnected), *c-m* (meets), *c-o* (overlaps), *c-fi* (finished-by), *c-di* (contains), *c-s* (starts), *c-eq* (equals), *c-si* (started-by), *c-d* (during), *c-f* (finishes), *c-oi* (overlapped-by), *c-moi* (meets and overlapped-by), *c-ooi* (overlaps and overlapped-by), *c-mi* (met-by), *c-mmi* (meets and met-by), *c-omi* (overlaps and met-by) —see the illustration of Figure 1(Left), where *X-* and *X+*, for $X \in \{I, J\}$, stand, respectively, for the left and right endpoints of interval $X$.

**The RA $\mathcal{RCC}$8.** Topology is certainly the aspect the most developed so far by the QSR Community. This is illustrated by the well-known RCC theory [20], from which derives the RCC-8 calculus [20, 8]. The RCC theory, on the other hand, stems from Clarke's "calculus of individuals" [6], based on a binary "connected with" relation —sharing of a point of the arguments. Clarke's work, in turn, was developed from classical mereology [15, 16] and Whitehead's "extensionally connected with" relation [25]. The RCC-8 calculus [20, 8] consists of a set of eight Jointly Exhaustive and Pairwise Disjoint (JEPD) atoms, *DC* (DisConnected), *EC* (Externally Connected), *TPP* (Tangential Proper Part), *PO* (Partial Overlap), *EQ* (EQual), *NTPP* (Non Tangential Proper Part), and the converses, *TPPi* and *NTPPi*, of *TPP* and *NTPP*, respectively. The reader is referred to Figure 1(Right) for an illustration of the atoms.

**The RA $\mathcal{CYC}_t$.** The set 2D$\mathcal{O}$ of 2-d orientations is defined in the usual way, and is isomorphic to the set of directed lines incident with a fixed point, say $O$. Let $h$ be the natural isomorphism, associating with each orientation $x$ the directed line (incident with $O$) of orientation $x$. The angle $\langle x, y\rangle$ between two orientations $x$ and $y$ is the anticlockwise angle $\langle h(x), h(y)\rangle$. Isli and Cohn [14] have defined a binary RA of 2D orientations, $\mathcal{CYC}_b$, that contains four atoms: $e$ (equal), $l$ (left), $o$ (opposite) and $r$ (right). For all $x, y \in$ 2D$\mathcal{O}$: $e(y, x) \Leftrightarrow \langle x, y\rangle = 0$; $l(y, x) \Leftrightarrow \langle x, y\rangle \in (0, \pi)$; $o(y, x) \Leftrightarrow \langle x, y\rangle = \pi$; $r(y, x) \Leftrightarrow \langle x, y\rangle \in (\pi, 2\pi)$. Based on $\mathcal{CYC}_b$, Isli and Cohn [14] have built a ternary RA, $\mathcal{CYC}_t$, for cyclic ordering of 2D orientations: $\mathcal{CYC}_t$ has 24 atoms, thus $2^{24}$ relations. The atoms of $\mathcal{CYC}_t$ are written as $b_1b_2b_3$, where $b_1, b_2, b_3$ are atoms of $\mathcal{CYC}_b$, and such an atom is interpreted as follows: $(\forall x, y, z \in \text{2D}\mathcal{O})(b_1b_2b_3(x, y, z) \Leftrightarrow b_1(y, x) \wedge b_2(z, y) \wedge b_3(z, x))$. The reader is referred to [14] for more details.

## 3 The (x,y)-*DL* description logics, with $x \in \{\mathcal{LIA}, \mathcal{CIA}\}$ and $y \in \{\mathcal{RCC}\mathbf{8}, \mathcal{CYC}_t\}$

Let $x \in \{\mathcal{LIA}, \mathcal{CIA}\}$ and $y \in \{\mathcal{RCC}8, \mathcal{CYC}_t\}$. The set of role names of (x,y)-*DL* is the set x-*at* of $x$ atoms, which we also refer to as $N_R^x$. The set of predicate names of (x,y)-*DL* is the set y-*at* of $y$ atoms, which we also refer to as $N_P^y$. It is known from the literature that the set $\{\{r\}|r \in$ x-*at*$\}$ of $x$ atomic relations is decidable, for all $x \in \{\mathcal{LIA}, \mathcal{CIA}\}$; as well as the set $\{\{r\}|r \in$ y-*at*$\}$ of $y$ atomic relations, for all $y \in \{\mathcal{RCC}8, \mathcal{CYC}_t\}$:
1. van Beek [22] has shown that the pointisable part of Allen's $\mathcal{LIA}$ [1], of which the set of atomic relations is a subset, is tractable. A problem expressed in the subset can be checked for consistency with Allen's constraint propagation algorithm [1].
2. The $\mathcal{CIA}$ atomic relations have been shown to form a decidable subset of $\mathcal{CIA}$ by Isli [13]. A problem expressed in the subset can be checked for consistency by first translating it into Isli and Cohn's RA $\mathcal{CYC}_t$ of 2D orientations [14], and then using the (complete) solution search algorithm in [14].
3. The $\mathcal{RCC}8$ atomic relations have been shown to form a tractable subset of $\mathcal{RCC}8$ by Renz and Nebel [21]. A problem expressed in the subset can be checked for consistency using Allen's constraint propagation algorithm [1].
4. Isli and Cohn [14] have provided a propagation algorithm achieving 4-consistency for CSPs expressed in their RA $\mathcal{CYC}_t$, and shown that the propagation is complete for the subset of atomic relations.

Let $x$ be either of the temporal RAs in $\{\mathcal{LIA}, \mathcal{CIA}\}$ and $y$ either of the spatial RAs in $\{\mathcal{RCC}8, \mathcal{CDA}\}$. We denote by $R^x$ the $x$ universal relation, which we also refer to as the (x,y)-*DL* top role: $R^x = \bigcup_{R \in N_R^x} R$. The set $N_R^x$ of (x,y)-*DL* role names constitutes the set of (x,y)-*DL* atomic roles. (Possibly complex) (x,y)-*DL* roles are obtained by considering the closure of $N_R^x$ under the set-theoretic operations of complement with respect to $R^x$, union and intersection. Formally, if the complement $R^x \setminus R$ of $R$ with respect to $R^x$ is represented as $\overline{R}$, then we have the following:

**Definition 1 ((x,y)-*DL* roles)** *The set of (x,y)-*DL *roles is the smallest set such that: (1) every role name $R \in N_R^x$ is an (x,y)-*DL *role; and (2) if $R_1$ and $R_2$ are (x,y)-*DL *roles, then so are: $\overline{R_1}$, $R_1 \cup R_2$ and $R_1 \cap R_2$.*

Because the $x$ atoms, which constitute the (x,y)-*DL* role names (atomic roles), are JEPD, the set of (x,y)-*DL* roles reduces to the set $2^{\text{x-}at}$ of all $x$ relations: each (x,y)-*DL* role $R$ can be transformed into a role of the form $R_1 \cup \ldots \cup R_n$, where the $R_i$'s are $x$ atoms. We can therefore assume, without loss of generality, that each (x,y)-*DL* role is written as a union, or as a set, of role names: i.e., either as $R_1 \cup \ldots \cup R_n$ or as $\{R_1 \ldots, R_n\}$, where the $R_i$'s are role names.

We denote by $P^y$ the $y$ universal relation, which we also refer to as the (x,y)-*DL* top predicate: $P^y = \bigcup_{P \in N_P^y} P$. The set $N_P^y$ of (x,y)-*DL* predicate names constitutes the set of (x,y)-*DL* atomic predicates. (Possibly complex) (x,y)-*DL* predicates are all $y$ relations. They are defined formally as follows.

**Definition 2 ((x,y)-*DL* predicates)** *The set of (x,y)-*DL *predicates is the smallest set such that: (1) every predicate name $P \in N_P^y$ is an (x,y)-*DL *predicate; and (2) if $P_1$ and $P_2$ are (x,y)-*DL *predicates, then so are: $\overline{P_1}$, $P_1 \cup P_2$, and $P_1 \cap P_2$.*

Similarly to an (x,y)-*DL* role, and without loss of generality, each (x,y)-*DL* predicate is written as a union, or as a set, of predicate names; i.e., either as $P_1 \cup \ldots \cup P_n$ or as $\{P_1 \ldots, P_n\}$, where the $P_i$'s are predicate names.

**Definition 3 ((x,y)-*DL* concepts)** *Let $x \in \{\mathcal{LIA}, \mathcal{CIA}\}$ and $y \in \{\mathcal{RCC}8, \mathcal{CYC}_t\}$. Let $N_C$, $N_{aF}$ and $N_{cF}$ be mutually disjoint and countably infinite sets of concept names, abstract features, and concrete features, respectively. A path $u$ is a composition $f_1 \ldots f_n g$ of $n \geq 0$ abstract features $f_1, \ldots, f_n$ and one concrete feature $g$. The set of (x,y)-*DL *concepts is the smallest set such that:*

1. *$\top$ and $\bot$ are (x,y)-*DL *concepts*
2. *every (x,y)-*DL *concept name is an (x,y)-*DL *concept*
3. *if $C$ and $D$ are (x,y)-*DL *concepts, $u_1$, $u_2$ and $u_3$ are paths, $P$ is an (x,y)-*DL *predicate (a $y$ relation), then the following expressions are also (x,y)-*DL *concepts: (a) $\neg C$, $C \sqcap D$, $C \sqcup D$, $\exists R.C$, $\forall R.C$, $g_1\uparrow$; (b) $\exists(u_1)(u_2).P$, if $y$ binary; and (c) $\exists(u_1)(u_2)(u_3).P$, if $y$ ternary.*

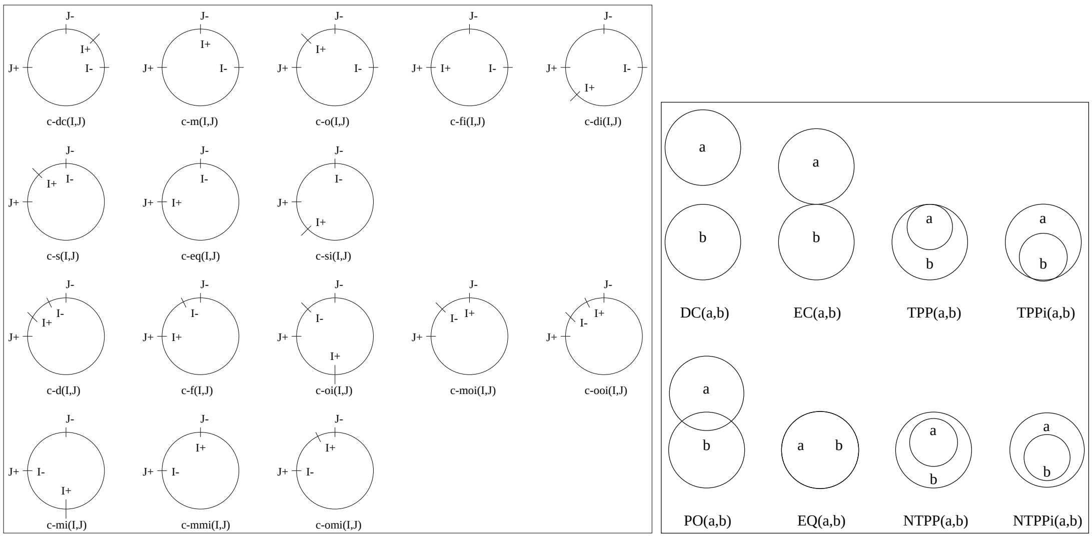


**Figure 1.** (Left) Illustration of the 16 $\mathcal{CIA}$ atomic relations. (Right) Illustration of the RCC-8 atoms.

The sublanguage of (x,y)-*DL* obtained by restricting feature chains to length one (in other words, by considering only feature chains consisting of concrete features), will be referred to as (x,y)-$DL^1$.

An ((x,y)-*DL* terminological) axiom is an expression of the form $A \doteq C$, $A$ being a concept name and $C$ a concept. A TBox is a finite set of axioms, with the condition that no concept name appears more than once as the left hand side of an axiom.

Let $T$ be a TBox. $T$ contains two kinds of concept names: concept names appearing as the left hand side of an axiom of $T$ are defined concepts; the others are primitive concepts. A defined concept $A$ "*directly uses*" a defined concept $B$ iff $B$ appears in the right hand side of the axiom defining $A$. If "*uses*" is the transitive closure of "*directly uses*" then $T$ contains a cycle iff there is a defined concept $A$ that "*uses*" itself. $T$ is cyclic if it contains a cycle; it is acyclic otherwise.

We denote atomic concepts by the letter $A$, possibly complex concepts by the letters $C$, $D$, $E$, roles by the letter $R$, concrete features by the letters $g$ and $h$, (possibly complex) predicates by the letter $P$.

**Definition 4 (concrete domain)** *A concrete domain $\mathcal{D}$ consists of a pair $(\Delta_\mathcal{D}, \Phi_\mathcal{D})$: $\Delta_\mathcal{D}$ is the set of objects of $\mathcal{D}$ (called the domain), and $\Phi_\mathcal{D}$ is the set of predicates of $\mathcal{D}$. Each predicate $P \in \Phi_\mathcal{D}$ is associated with an arity $n$: $P \subseteq (\Delta_\mathcal{D})^n$.*

The description logic (x,y)-*DL* can be viewed as equipped with the concrete domain

1. $\mathcal{D} = (\mathcal{TS}, 2^{\mathcal{RCC}8\text{-}at})$ if $x = \mathcal{RCC}8$, where $\mathcal{TS}$ is the set of regions of a topological space.
2. $\mathcal{D} = (2\mathrm{D}\mathcal{O}, 2^{\mathcal{CYC}_t\text{-}at})$ if $x = \mathcal{CYC}_t$, where 2D$\mathcal{O}$, as we have already seen, is the set of 2-dimensional orientations.

**Definition 5 (admissibility)** *A concrete domain $\mathcal{D}$ is admissible if: (1) the set of its predicates is closed under negation and contains a predicate for $\Delta_\mathcal{D}$; and (2) the satisfiability problem for finite conjunctions of predicates is decidable.*

Admissibility of the concrete domains associated with the (x,y)-*DL* description logics derive from decidability (and tractability), for each of the spatial RAs $y$, of the consistency of a finite conjunction of constraints over $y$'s atomic relations, which we have already discussed.

(x,y)-*DL* is equipped with a Tarski-style semantics. It is important to note that, because of the interpretation of the roles as Allen-like temporal relations [1], the abstract objects (i.e., the elements of the abstract domain -see Definition below) of an (x,y)-*DL* interpretation are temporal intervals:

**Definition 6** *An interpretation $\mathcal{I}$ consists of a pair $(\Delta_\mathcal{I}, .^\mathcal{I})$, where $\Delta_\mathcal{I}$ is a set called the (abstract) domain and $.^\mathcal{I}$ is an interpretation function mapping each concept name $C$ to a subset $C^\mathcal{I}$ of $\Delta_\mathcal{I}$, each role name $R$ to a subset $R^\mathcal{I}$ of $\Delta_\mathcal{I} \times \Delta_\mathcal{I}$, each concrete feature $g$ to a partial function $g^\mathcal{I}$:*

1. from $\Delta_\mathcal{I}$ to the set of regions of a topological space, if $y = \mathcal{RCC}8$; and
2. from $\Delta_\mathcal{I}$ to the set 2D$\mathcal{O}$ of 2-dimensional orientations, if $y = \mathcal{CYC}_t$.

*The interpretation function is extended to arbitrary concept terms as follows:*

$$
\begin{array}{rcl}
(\top)^\mathcal{I} & := & \Delta_\mathcal{I} \\
(\bot)^\mathcal{I} & := & \emptyset \\
(\neg C)^\mathcal{I} & := & \Delta_\mathcal{I} \setminus C^\mathcal{I} \\
(C \sqcap D)^\mathcal{I} & := & C^\mathcal{I} \cap D^\mathcal{I} \\
(C \sqcup D)^\mathcal{I} & := & C^\mathcal{I} \cup D^\mathcal{I} \\
(\exists R.C)^\mathcal{I} & := & \{a \in \Delta_\mathcal{I} | \exists b \in \Delta_\mathcal{I} : (a,b) \in R^\mathcal{I} \wedge b \in C^\mathcal{I}\} \\
(\forall R.C)^\mathcal{I} & := & \{a \in \Delta_\mathcal{I} | \forall b : (a,b) \in R^\mathcal{I} \rightarrow b \in C^\mathcal{I}\} \\
(\exists u_1, u_2.P)^\mathcal{I} & := & \{a \in \Delta_\mathcal{I} | \exists reg_1, reg_2 \in \mathcal{TS} : u_1^\mathcal{I}(a) = reg_1, \\
 & & u_2^\mathcal{I}(a) = reg_2, P(reg_1, reg_2)\} \\
(\exists (u_1)(u_2)(u_3).P)^\mathcal{I} & := & \{a \in \Delta_\mathcal{I} | \exists o_1, o_2, o_3 \in 2\mathrm{D}\mathcal{O} : u_1^\mathcal{I}(a) = o_1, \\
 & & u_2^\mathcal{I}(a) = o_2, u_3^\mathcal{I}(a) = o_3, P(o_1, o_2, o_3)\} \\
(g\uparrow)^\mathcal{I} & := & \{a \in \Delta_\mathcal{I} | g^\mathcal{I}(a) \text{ is undefined}\}
\end{array}
$$

*An interpretation $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ iff it satisfies $A^\mathcal{I} \doteq C^\mathcal{I}$ for all terminological axioms $A \doteq C$ in $\mathcal{T}$. $\mathcal{I}$ is a model of a concept $C$ w.r.t. a TBox $\mathcal{T}$ iff $\mathcal{I}$ is a model of $\mathcal{T}$ and $C^\mathcal{I} \neq \emptyset$. A concept $C$ is satisfiable w.r.t. a TBox $\mathcal{T}$ iff there exists a model $\mathcal{I}$ of $\mathcal{T}$ that is also* *model of $C$. A concept $C$ subsumes a concept $D$ w.r.t. $\mathcal{T}$ (written $D \sqsubseteq_{\mathcal{T}} C$) iff $D^{\mathcal{I}} \subseteq C^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{T}$.*

## 4 Illustrating examples

We now provide two illustrating examples. A motion is a particular case of spatio-temporal event. We denote by atomic spatio-temporal event an event during which the associated spatial situation remains the same. When, for instance, spatial knowledge is represented with $\mathcal{RCC}8$ [20, 8], a motion of an $n$-object spatial scene is atomic just in case the relation on any pair of the objects remains continuously the same during the entire motion. We make the following assumptions:

1. temporal intervals are durative;
2. atomic spatio-temporal events (thus atomic motions) take place during intervals (continuity); and
3. if an atomic spatio-temporal event takes place during an interval, then it takes place during all subintervals (homogeneity).

**Example 1 (illustration of ($\mathcal{CIA}$,$\mathcal{CYC}_t$)-$DL^1$)** *We consider an environment with four landmarks, L1, L2, L3 and L4, as depicted in Figure 2(Left). The lines through the different pairs of landmarks partition the plane into a tessellation of two-, one- and zero-dimensional convex regions. Nine of these regions are numbered R1, ..., R9 in Figure 2(Left). A robot R has to navigate all the way through from some point Pi in Region R1 to some point Pf in Rgion R9, traversing in between Regions R2, ..., R8, in that order. We denote by $M$ the entire motion the navigation to be planned should consist of. For all $i = 1 \ldots 9$, we denote by $M_i$ the restriction of $M$ to region $R_i$ (i.e., $M_i$ is the part of motion $M$ taking place in Region $R_i$). With each submotion $M_i, i = 1 \ldots 9$, we associate an interval $I_i$ and a concept $C_i$:*

1. *$I_i$ is the interval during which $M_i$ takes place; and*
2. *$C_i$ is a concept describing the (constant) configuration of the spatial scene during all interval $I_i$, and providing as well:*
   (a) *the knowledge on which atomic submotion should take place next; and*
   (b) *how the interval $I_i$ relates to the interval during which the next atomic submotion should take place.*

*Since the atomic submotions follow immediately one another, the "meets" interval relation is crucial for the example. We make use of four concrete features $g_1, \ldots, g_4$, which "perceive" at each time instant the orientations $o_1$, $o_2$, $o_3$ and $o_4$ of the directed lines joining the robot to Landmarks L1, L2, L3 and L4, respectively (Figure 2(Left)). The panorama of the robot at a specific time point consists in the conjunction of $\mathcal{CYC}_t$ constraints associating with each triple of the four orientations the $\mathcal{CYC}_t$ relation it satisfies. Within the same region, the panorama is constant. The navigation of the robot can thus be seen as a chronological evolution of the changing panorama. The TBox with the following axioms provides a plan describing a path the robot has to follow to reach the goal.*

$$
\begin{aligned}
C_1 &\doteq \exists(g_1)(g_2)(g_3).rrr \sqcap \exists(g_1)(g_2)(g_4).rrr \sqcap \exists(g_1)(g_3)(g_4).rrr \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).rrr \sqcap \exists m.C_2 \\
C_2 &\doteq \exists(g_1)(g_2)(g_3).rrr \sqcap \exists(g_1)(g_2)(g_4).rro \sqcap \exists(g_1)(g_3)(g_4).rro \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).rrr \sqcap \exists m.C_3 \\
C_3 &\doteq \exists(g_1)(g_2)(g_3).rrr \sqcap \exists(g_1)(g_2)(g_4).rrl \sqcap \exists(g_1)(g_3)(g_4).rrl \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).rrr \sqcap \exists m.C_4 \\
C_4 &\doteq \exists(g_1)(g_2)(g_3).rro \sqcap \exists(g_1)(g_2)(g_4).rol \sqcap \exists(g_1)(g_3)(g_4).orl \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).rro \sqcap \exists m.C_5 \\
C_5 &\doteq \exists(g_1)(g_2)(g_3).rrl \sqcap \exists(g_1)(g_2)(g_4).rll \sqcap \exists(g_1)(g_3)(g_4).lrl \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).rrl \sqcap \exists m.C_6 \\
C_6 &\doteq \exists(g_1)(g_2)(g_3).rol \sqcap \exists(g_1)(g_2)(g_4).rll \sqcap \exists(g_1)(g_3)(g_4).lrl \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).orl \sqcap \exists m.C_7 \\
C_7 &\doteq \exists(g_1)(g_2)(g_3).rll \sqcap \exists(g_1)(g_2)(g_4).rll \sqcap \exists(g_1)(g_3)(g_4).lrl \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).lrl \sqcap \exists m.C_8 \\
C_8 &\doteq \exists(g_1)(g_2)(g_3).rll \sqcap \exists(g_1)(g_2)(g_4).rll \sqcap \exists(g_1)(g_3)(g_4).lel \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).lel \sqcap \exists m.C_9 \\
C_9 &\doteq \exists(g_1)(g_2)(g_3).rll \sqcap \exists(g_1)(g_2)(g_4).rll \sqcap \exists(g_1)(g_3)(g_4).lll \sqcap \\ &\quad \exists(g_2)(g_3)(g_4).lll
\end{aligned}
$$

*Concept $C_4$, for instance, provides the information that the orientations $o_1, \ldots, o_4$ should satisfy the constraints that the $\mathcal{CYC}_t$ relation on the triple $(o_1, o_2, o_3)$ is* rro*, the one on the triple $(o_1, o_2, o_4)$ is* rol*, the one on the triple $(o_1, o_3, o_4)$ is* orl*, and the one on the triple $(o_2, o_3, o_4)$ is* rro *—which is a description of the panorama of the robot while in Region $R_4$. $C_4$ also tells about the concept describing the submotion to take place next, and relates temporally the interval during which the current submotion takes place to the interval during which the next submotion will take place ($\exists$m.$C_5$).*

**Example 2 (illustration of ($\mathcal{CIA}$, $\mathcal{RCC}$8)-$DL^1$)** *Consider the moving spatial scene depicted in Figure 2(Right), consisting of two subscenes: a subscene 1, composed of three objects o1, o2 and o3; and a subscene 2, also composed of three objects, q1, q2 and q3:*

1. *For subscene 1, four snapshots of four atomic submotions, each holding invariably during an interval, are presented, and labelled A, B, C and D; the arrows show the transitions from the current atomic submotion to the next. The motion is cyclic. It starts with the configurtion A, with o1 touching o2 and tangential proper part of o3, and o2 tangential proper part of o3. The scene's configuration then moves to configuration B, involving the change of the $\mathcal{RCC}$8 relation on the pair (o2,03) from* TPP *to its conceptual neighbour* NTPP*. The next atomic submotion is given by configuration C, involving the object o1 to move completely inside o3, becoming thus* NTPP *to it. The atomic submotion immediately following C is D, presenting the same spatial configuration as the atomic submotion B. But from D, the motion moves, not to atomic submotion C, as from B, but back to the very first atomic submotion, A, and repeats the steps in a non terminating loop.*
2. *For subscene 2, two snapshots of two atomic submotions, each holding invariably during an interval, are presented and labelled E and F; the arrows show the transitions from the current atomic submotion to the next. The motion is cyclic. It starts with the configurtion E, with q1 touching q2 and tangential proper part of q3, and q2 tangential proper part of q3. The scene's configuration then moves to configuration F, involving the change of the $\mathcal{RCC}$8 relation on the pair (q2,q3) from* TPP *to its conceptual neighbour* NTPP*. From F, the motion moves back to E, and repeats the steps in a non terminating loop.*
3. *The label m on the arrows within each of the two subscenes, says that the interval during which takes place a submotion meets the interval during which takes place the immediately following submotion.*
4. *The two subscenes start their motion simultaneously, but the switch from submotion A to submotion B, in subscene 1, happens before the switch from submotion E to submotion F, in subscene 2: i.e., the interval during which submotion A takes place starts the interval during which submotion E takes place, indicated by the label s on the dashed arrow from A to E.*

*We make use of the concrete features $g_1$, $g_2$ and $g_3$ to refer to the actual regions corresponding to objects o1, o2 and o3 in Subscene 1, and of the concrete features $h_1$, $h_2$ and $h_3$ to refer to the actual*

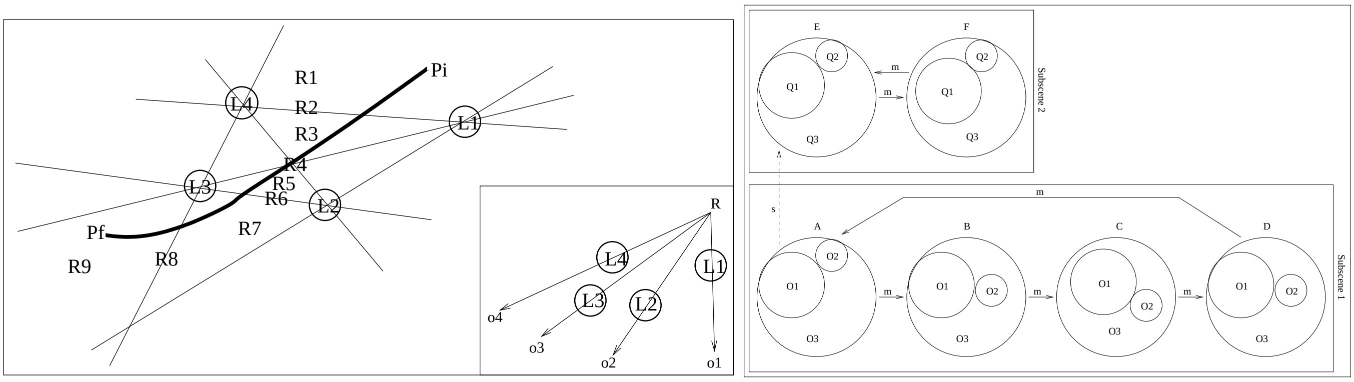


**Figure 2.** (Left) Illustration of ($\mathcal{LIA}$,$\mathcal{CYC}_t$)-$DL^1$. (Right) Illustration of ($\mathcal{CIA}$, $\mathcal{RCC}8$)-$DL^1$.

*regions corresponding to objects q1, q2 and q3 in Subscene 2. The (cyclic) TBox composed of the following axioms represents the described moving spatial scene:*

$$C_A \doteq \exists g_1, g_2.EC \sqcap \exists g_1, g_3.TPP \sqcap \exists g_2, g_3.TPP \sqcap \exists m.C_B \sqcap \exists s.C_E$$
$$C_B \doteq \exists g_1, g_2.EC \sqcap \exists g_1, g_3.TPP \sqcap \exists g_2, g_3.NTPP \sqcap \exists m.C_C$$
$$C_C \doteq \exists g_1, g_2.EC \sqcap \exists g_1, g_3.NTPP \sqcap \exists g_2, g_3.NTPP \sqcap \exists m.C_D$$
$$C_D \doteq \exists g_1, g_2.EC \sqcap \exists g_1, g_3.TPP \sqcap \exists g_2, g_3.NTPP \sqcap \exists m.C_A$$
$$C_E \doteq \exists h_1, h_2.EC \sqcap \exists h_1, h_3.TPP \sqcap \exists h_2, h_3.TPP \sqcap \exists m.C_F$$
$$C_F \doteq \exists h_1, h_2.EC \sqcap \exists h_1, h_3.NTPP \sqcap \exists h_2, h_3.TPP \sqcap \exists m.C_E$$

*The concept $C_A$, for instance, says that o1 and o2 are related by the* EC *relation ($\exists g_1, g_2$.EC); that o1 and o3 are related by the* TPP *relation ($\exists g_1, g_3$.TPP); that o2 and o3 are also related by the* TPP *relation ($\exists g_2, g_3$.TPP); that submotion $A$ immediately precedes submotion $B$, represented by the concept $C_B$ ($\exists$m.$C_B$); and, finally, that the interval during which takes place submotion $A$ starts the one during which takes place submotion $E$, represeted by the concept $C_E$ ($\exists$s.$C_E$).*

## 5 Discussion: characterisation of continuous spatio-temporal change

We have now reached a point where a discussion is needed on the relation between spatial change (over time), continuity [10], and conceptual neighbourhoods [9] (see also [17]). The theory of conceptual neighbourhoods is well-known in qualitative spatial and temporal reasoning (QSTR). QSTR constraint-based languages consist mainly of RAs. In the spatial case, for instance, the atoms of such an RA, in finite number, are built by defining an appropriate partition of the spatial domain at hand on which the RA is supposed to represent knowledge, as constraints on n-tuples of objects, where n is the arity of the relations. We say appropriate partition, in the sense that the partition has to fulfil some requirements, such as cognitive adequacy criteria, so that the obtained RA reflects, for instance, the common-sense reasoning, or the reasoning required by the task the RA is meant to be used for, as much as possible. The regions of the partition are generally continuous, and each groups together elements of the universe which do not need to be distinguished, because, for instance, the task at hand does not need, or Humans do not make, such a distinction. Given two atoms $r_1$ and $r_2$ of such an RA, $r_2$ is said to be a conceptual neighbour of $r_1$, if the union of the corresponding regions in the partition is continuous, so that one can move from one to the other without traversing a third region of the partition. The conceptual neighbouhood of $r_1$ is nothing else than the set of all its conceptual neighbours, including $r_1$ itself. The conceptual neighbourhood of a general relation, which is a set of atoms, is the union of the conceptual neighbourhoods of its atoms. The meets relation in Allen's RA [1], for instance, has two conceptual neighbours other than itself, which are before ($<$) and overlaps ($o$); the $\mathcal{RCC}8$ relation *TPP* has *PO*, *EQ* and *NTPP* as conceptual neighbours. The conceptual neighbouhoods of the atoms, $e$, $l$, $o$ and $r$, of the $\mathcal{CYC}_b$ binary RA of 2-dimensional orientations in [14] are, respectively, $\{e, l, r\}$, $\{e, l, o\}$, $\{l, o, r\}$ and $\{e, o, r\}$. Concerning the ternary RA $\mathcal{CYC}_t$ in [14], an atom $b'_1b'_2b'_3$ is a conceptual neighbour of an atom $b_1b_2b_3$, where the $b_i$'s and the $b'_i$'s are $\mathcal{CYC}_b$ atoms, if and only if the $\mathcal{CYC}_b$ atoms $b_i$ and $b'_i$, $i = 1 \ldots 3$, are such that $b'_i$ is a conceptual neighbour of $b_i$. In the two examples above, it is the case that the relation on any triple (Example 1) or pair (Example 2) of the involved objects (the orientations $o_1$, $o_2$, $o_3$ and $o_4$ in Example 1; the objects o1, o2 and o3 in Example 2, Subscene 1; and the objects q1, q2 and q3 in Example 2, Subscene 2), when moving from the current atomic submotion to the next, either remains the same, or changes to a relation that is a conceptual neighbour. Since a spatio-temporal event is a special case of a temporal event, in the sense that it has additionally a spatial situation associated with it, the well-known continuity property that atomic temporal events hold during (durative) intervals can be assumed for spatio-temporal events, and we have been assuming it in the previous two examples: atomic spatio-temporal events (thus atomic submotions in the examples) hold during intervals. The homogeneity property also applies to atomic spatio-temporal events: if an atomic spatio-temporal event takes place during an interval, then it takes place during all subintervals. In Example 1, the concept $C_2$ corresponds to the crossing of the 1-dimensional region R1, and the concept $C_4$ to the crossing of the 0-dimensional region R4; but even so the time required for the robot's motion to achieve the crossing is considered as an interval. See, again, [10] for more details on this rather philosophical question: this is related to the question of whether an interval includes its endpoints, or whether a region includes its boundary, which emerges, for instance, when considering the defintion of relations such as Allen's relations or the $\mathcal{RCC}8$ relations, and generally solved by having recourse to topology, so that, for instance, two intervals meet if and only if their topological closures have a one-point intersection. To summarise these properties of spatio-temporal events, "*atomic spatio-temporal events hold during intervals (continuity)*" and "*if an atomic spatio-temporal event holds during an interval then it holds during all subintervals (homogeneity)*", whatever snapshot of a moving spatial scene we consider, the snapshot gives the spatial configuaration of the scene that remains unchanged during an interval including the time point at which the snapshot has been taken.

Without loss of generality, we restrict the remainder of the discussion to one member of our (x,y)-*DL* family of theories of continuous spatio-temporal change, which is $(\mathcal{LIA}, \mathcal{RCC}8)$-*DL*. We denote by $M$ a motion of a spatial scene $\mathcal{S}$ composed on $n$ objects, $O_1, \ldots, O_n$.[3] For all $i, j \in \{1, \ldots, n\}$, $i < j$, we denote by $\mathcal{S}_{ij}$ the subscene of $\mathcal{S}$ composed of objects $O_i$ and $O_j$; by $M_{ij}$ the restriction of motion $M$ to subscene $\mathcal{S}_{ij}$; by $M_{ij}^1, \ldots, M_{ij}^{n_{ij}}$ the $n_{ij} \geq 1$ atomic submotions of $M_{ij}$; by $I_{ij}^k$, $k \in \{1, \ldots, n_{ij}\}$, the interval during which atomic submotion $M_{ij}^k$ takes place; and by $r_{ij}^k$, $k \in \{1, \ldots, n_{ij}\}$, the $\mathcal{RCC}8$ relation of the pair $(O_i, O_j)$ during Submotion $M_{ij}^k$ of Subscene $\mathcal{S}_{ij}$. Submotions $M_{ij}^k$ and $M_{ij}^{k+1}$, $k \in \{1, \ldots, n_{ij} - 1\}$, of $M_{ij}$ are such that $M_{ij}^{k+1}$ immediately follows $M_{ij}^k$; in other words, the intervals during which they hold are related by the *meets* relation: $m(I_{ij}^k, I_{ij}^{k+1})$.

**Definition 7 (maximal atomic submotion)** *The atomic submotion $M_{ij}^k$, $k \in \{1, \ldots, n_{ij}\}$, of subscene $\mathcal{S}_{ij}$ is said to be maximal if $M_{ij}$ has no atomic submotion that stricly subsumes $M_{ij}^k$.*

**Definition 8 (continuous motion of a 2-object subscene)** *The motion of Subscene $\mathcal{S}_{ij}$ is said to be continuous if it can be decomposed into $n_{ij}$ maximal atomic submotions $M_{ij}^1, \ldots, M_{ij}^{n_{ij}}$ such that $r_{ij}^{k+1}$ is a conceptual neighbourhood of $r_{ij}^k$, for all $k = 1, \ldots, n_{ij} - 1$.*

**Definition 9 (continuous motion of scene $\mathcal{S}$)** *The motion of Scene $\mathcal{S}$ is said to be continuous if its restriction to any of its 2-object subscenes is continuous.*

## 6 Conclusion

We have provided first results of our work on a highly challenging topic: qualitative representation of continuous spatio-temporal change (in particular, of continuous motion of spatial scenes). We used only two RAs for $x$ and two RAs for $y$ in our definition of the family of (x,y)-*DL* description logics. The result generalises, in an obvious way, to all temporal RAs $x$ and spatial RAs $y$ for which the atoms are Jointly Exhaustive and Pairwise Disjoint (JEPD), and such that the atomic relations form a decidable subclass.

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