

Claim: For all lists $l1$ and $l2$, $\text{rev_append } l1 \ l2 = \text{rev_append}' \ l1 \ l2$.

Proof: By induction on $l1$.

- **Base case:** $l1 = []$.

In this case, we have:

$$\begin{aligned} \text{rev_append } l1 \ l2 &= \text{rev_append } [] \ l2 && \text{by assumption} \\ &= l2 && \text{by definition of } \text{rev_append} \\ &= \text{append } [] \ l2 && \text{by definition of } \text{append} \\ &= \text{append } (\text{rev } []) \ l2 && \text{by definition of } \text{rev} \\ &= \text{append } (\text{rev } l1) \ l2 && \text{by assumption} \\ &= \text{rev_append}' \ l1 \ l2. && \text{by definition of } \text{rev_append}' \end{aligned}$$

- **Step:** $l1 = h::t$ for some element h and list t .

Inductive hypothesis: For all lists $l2$, $\text{rev_append } t \ l2 = \text{rev_append}' \ t \ l2$.

In this case, we have:

$$\begin{aligned} \text{rev_append } l1 \ l2 &= \text{rev_append } (h::t) \ l2 && \text{by assumption} \\ &= \text{rev_append } t \ (h::l2) && \text{by definition of } \text{rev_append} \\ &= \text{rev_append}' \ t \ (h::l2) && \text{by the inductive hypothesis} \\ &= \text{append } (\text{rev } t) \ (h::l2) && \text{by definition of } \text{rev_append}' \\ (\star) &= \text{append } (\text{rev } t) \ (h::(\text{append } [] \ l2)) && \text{by definition of } \text{append} \\ &= \text{append } (\text{rev } t) \ (\text{append } [h] \ l2) && \text{by definition of } \text{append} \\ &= \text{append } (\text{append } (\text{rev } t) \ [h]) \ l2 && \text{by associativity of } \text{append} \\ &= \text{append } (\text{rev } (h::t)) \ l2 && \text{by definition of } \text{rev} \\ &= \text{rev_append}' \ (h::t) \ l2 && \text{by definition of } \text{rev_append}' \\ &= \text{rev_append}' \ l1 \ l2. && \text{by assumption} \end{aligned}$$

Note: skipping the step marked (\star) will not lose any marks.