Homework 5 - Probability

Dwaipayan Chanda

1) Each question gets asked to a new student

$$\frac{{}_{15}P_8}{15^8}$$

${}_{15}P_8$ ← ways to arrange it so that a new student answers each time

15^8 ← equal chance of each question going to a student

$= \boxed{0.101}$

2)

$\underline{5} \cdot \underline{4} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5}$

odd digits: ${}_5C_2 \cdot 2$

7 digits left

even: ${}_5C_1$

so 4200 digits meet the criteria.

so Probability of getting it once $= \frac{4200}{10^5} = 0.042$

To get it exactly 5/8 times:

$$(0.042)^5(1-0.042)^3 \, {}_8C_5 = \boxed{6.43 \times 10^{-6}}$$

3) A: $\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) {}_3C_2 + \left(\frac{1}{2}\right)^3 {}_3C_3 = 1/2$

↑ getting >=4

B: $\frac{6}{6^3} = \frac{1}{36}$

$P(A \cap B)$: all 3 dice have same value, above 4.

$$P(A \cap B) = \frac{3}{6^3} = \frac{1}{72}$$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \underline{\frac{1}{72}} = P(A \cap B)$,

So $\boxed{\text{YES}}$, A and B are <u>independent</u>.

4) Probability of getting a flush:

$$\frac{{}_4C_1 \cdot {}_{13}C_5}{{}_{52}C_5} = \underline{0.00198}$$

so expended # of hands until he gets one is $1/p$, or $1/0.00198 \approx \boxed{504.85 \text{ hands}}$

5) S → superstar plays

W → they win

L → they lose

N → NOT

<u>we know:</u>

$P(W|S) = 0.7$

$P(W|NS) = 0.5$

Find $P(S|W4) = \frac{P(W4|S)\,P(S)}{P(W4)}$

$P(S)$ is $\underline{0.75}$ (probability superstar plays next 5 games)

$$P(S|W4) = \frac{P(W4|S)\,P(S)}{P(W4|S)\,P(S) + P(W4|NS)\,P(NS)}$$

$$= \frac{(0.7)^4(0.3)^1 \, {}_5C_4 \cdot 0.75}{(0.7)^4(0.3)^1 \, {}_5C_4 \cdot 0.75 + (0.5)^4(0.5)^1 \cdot {}_5C_4 \cdot 0.25}$$

$= \boxed{0.874}$ is the probability the superstar played.