Homework 5 - Counting

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1) unusual

↓ NON-u letters

all possibilities →

- 1 u: $_4C_4 = 1$
- 2 u's: $_4C_3 = 4$
- 3 u's: $_4C_2 = 6$

So # of unique 5-letter subsets is $1+4+6 =$ **11 subsets**

Strings that can be made:

$$\underbrace{1 \cdot \frac{5!}{1!}}_{1u} + \underbrace{4 \cdot \frac{5!}{2!}}_{2u} + \underbrace{6 \cdot \frac{5!}{3!}}_{3u} = 5! + 2 \cdot 5! + 5!$$

$=$ **480 strings**

2) choose 2 values: $_{13}C_2$

choose 2 suits for each: $_4C_2 \cdot {}_4C_2$

some other card: $_{44}C_1$

so: $_{13}C_2 \cdot {}_4C_2 \cdot {}_4C_2 \cdot {}_{44}C_1$

$= \frac{13 \cdot 12}{2} \cdot 6 \cdot 6 \cdot 44$

$=$ **123552 hands**

3) 7 "bins" (couples)

16 songs to distribute

Two cases

- the flight couple gets a song
- they don't

They get a song:

Divide 15 songs amongst the rest, so: $\frac{(5+15)!}{5!\,15!} = 15504$

(6 bins left, so $(6-1)!$; 15 songs)

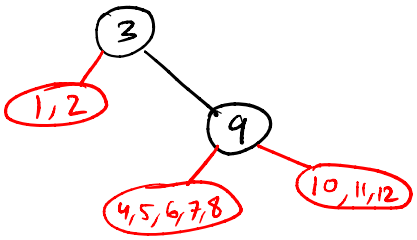
+

They don't:

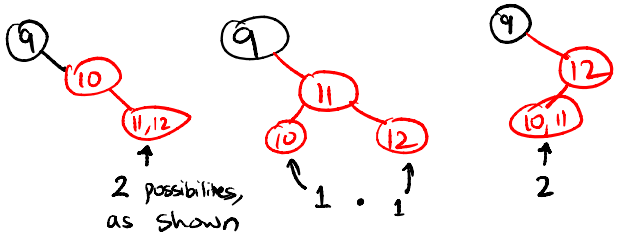
Divide 16 songs amongst the rest, so: $\frac{(5+16)!}{5!\,16!} = 20349$

so, in total, there are $\boxed{35,853}$ ways

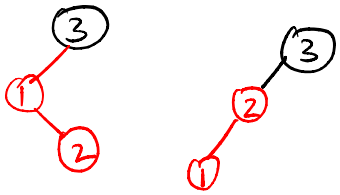
4)



For 10, 11, 12 :



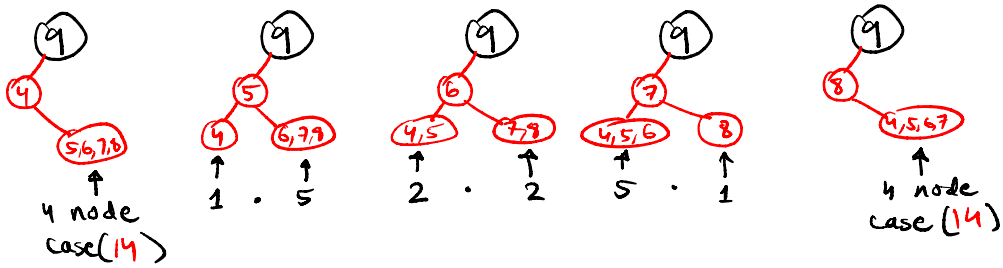
For 1, 2 :



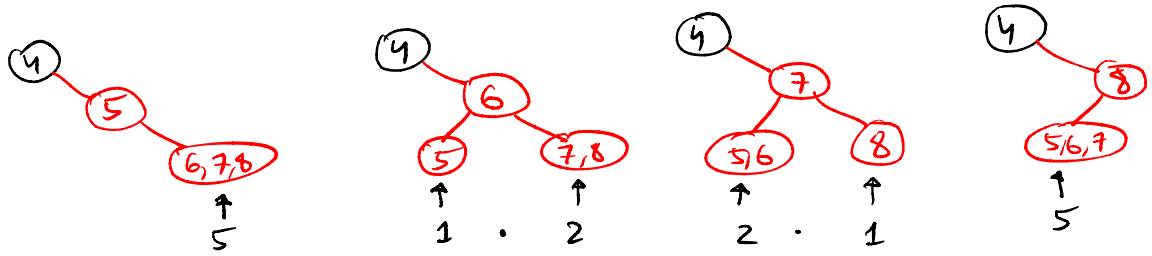
are the only ways, true for all 2 node subtrees.

So, in total, $2 + (1 \cdot 1) + 2$ cases $= 5$ cases for 3 nodes.

For 4, 5, 6, 7, 8 :



For 4 nodes (5,6,7,8, for example):



so, for 4 nodes, there are $5 + (1 \cdot 2) + (2 \cdot 1) + 5$ cases

$= 14$ cases

so, for the five node case, there are $14 + (1 \cdot 5) + (2 \cdot 2) + (5 \cdot 1) + 14$ cases

$= 42$ cases

For the overall BST, therefore, there are:

1,2 cases · 10, 11, 12 cases · 4, 5, 6, 7, 8 cases

$= 2 \cdot 5 \cdot 42$

$= \boxed{420 \text{ possibilities}}$

5) 4 identical nurses, 1 may / may not be at break

10 friends

at least 1 person will be served / available nurse.

Case 1: 4 nurses

(4) < {1, 1, 1, 7}, {1, 1, 2, 6}, {1, 1, 3, 5}, {1, 1, 4, 4}

{1, 2, 2, 5}, {1, 2, 3, 4}, {1, 3, 3, 3}, {2, 2, 2, 4}, {2, 2, 3, 3}

Case 2: 3 nurses (are on break)

⑧ < {1,1,8}, {1,2,7}, {1,3,6}, {1,4,5},
{2,2,6}, {2,3,5}, {2,4,4}
{3,3,4}

so, in total, 8+9 = 17 cases.