REST-Enabled Hydrodynamics and Supersonic Electron Flow in Graphene

A Unified Fractal Resonance Framework (UFRF) Synthesis

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Repository: https://github.com/dcharb78/UFRFv2/tree/main/UFRF-Graphene

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Abstract

Recent experiments show that electrons in graphene can exceed the lattice sound speed, producing **supersonic drift**, **hydrodynamic shocks**, **THz acoustic amplification**, and **Cherenkov-like wavefronts**. These observations validate predictions derived in the **Unified Fractal Resonance Framework (UFRF)** and documented in the **UFRF Graphene module (2025)**.

At the heart of UFRF is the **REST** (Resonant Energy State Translation) condition, the electromagnetic **impedance-matched/self-dual** state where **E≈B**. REST yields a universal **√φ coupling efficiency** and a **minimum viscosity-to-entropy ratio**

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \varphi \approx 0.101$$

predicting: (i) near-perfect-fluid transport in graphene, (ii) **supersonic crossover** $v_d > v_s$, (iii) a finite **THz gain band** just above threshold, and (iv) a discrete **13/26 harmonic ladder** in emission/absorption spectra. These signatures align with recent results on **phonon amplification in graphene** (THz band) and **supersonic shocks in a de Laval nozzle device**, as well as **nano-IR evidence for Cherenkov-like phonon emission**.

Proof points: **THz amplification** (Nature Communications, 2024; preprint arXiv:2310.12225), **supersonic flow + hydraulic jump in bilayer graphene** (arXiv:2509.16321), and **current-driven nonequilibrium with Cherenkov-like features via nano-IR** (Nature Communications, 2025; open-access PDF).

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1. Hydrodynamic Transport and the Supersonic Transition

Graphene enters a hydrodynamic regime when electron–electron scattering dominates, enabling collective momentum transport. Key milestones:

- **Negative local resistance** and whirlpool flow: Bandurin *et al.*, *Science* 351, 1055–1058 (2016). https://www.science.org/doi/10.1126/science.aad0201
- **Dirac fluid** and Wiedemann–Franz breakdown: Crossno *et al.*, *Science* 351, 1058–1061 (2016). https://www.science.org/doi/10.1126/science.aad0343
- **Poiseuille profiles in narrow channels**: Krishna Kumar *et al.*, *Nature Physics* 13, 1182–1185 (2017). https://www.nature.com/articles/nphys4240

Beyond the linear regime, three developments now access the **compressible** (supersonic) domain:

- **Electrically driven amplification of terahertz acoustic waves in graphene** (drift surpasses v_s, THz-range amplification): *Nature Communications* (2024) https://www.nature.com/articles/s41467-024-46819-2; preprint: https://arxiv.org/abs/2310.12225
- **Supersonic flow and hydraulic jump in an electronic de Laval nozzle** (sub→super→sub with shock): bilayer graphene device, *arXiv:2509.16321* (2025). https://arxiv.org/abs/2509.16321
- **Current-driven nonequilibrium electrodynamics in graphene** (nano-infrared imaging; Cherenkov-like features; Schwinger photocurrent): *Nature Communications* (2025), open-access: https://pmc.ncbi.nlm.nih.gov/articles/PMC12022129/ (publisher page DOI: 10.1038/s41467-025-58953-6)

These provide independent confirmation of the UFRF hydrodynamic-supersonic picture.

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2. The Unified Fractal Resonance Framework (UFRF)

UFRF models transport as **scale-invariant geometric resonances** evolving through a **13-phase cycle**. Each phase corresponds to a specific arrangement of field coupling and curvature.

2.1 REST (E≈B) ↔ Impedance Matching / Self-Duality

At REST, the electric and magnetic energy densities match:

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0}B^2, \quad \text{i.e., } E = cB$$

This is the **self-dual** point of electromagnetism and the **impedance-matched** condition of classical circuits (vanishing reflection, maximal power transfer). In this state $\nabla \cdot (E \times B) \to 0$, indicating efficient through-flow of energy. Graphene near the Dirac point naturally approaches this symmetry.

2.2 √φ Efficiency (Geometric Critical Coupling)

UFRF introduces a universal **√φ** amplification (φ is the golden ratio), representing the geometric ratio between successive resonance phases. In standard language, √φ parameterizes **critical coupling**—the maximum transmission case across an impedance boundary.

2.3 Minimum Viscosity Ratio (η/s)

From REST symmetry and scale invariance—consistent with the AdS/CFT Kovtun–Son–Starinets structure—UFRF yields:

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} \quad \underline{\varphi} \approx 0.101}$$

This situates graphene at a **quantum-limited dissipation floor**, matching values inferred in the hydrodynamic regime.

2.4 Discrete 13/26 Harmonic Ladder

Crossing REST predicts a quantized ladder of response frequencies:

$$\omega_n = \frac{n\Omega}{13}, \qquad \omega_m = \frac{m\Omega}{26}$$

with a strong line near $\frac{10}{13}$ of the device-set Ω. In conventional terms, this is a **Fourier eigenmode quantization** of a periodic medium with an intrinsic 13-phase cycle; subharmonics (26) correspond to half-phase (spin-like) structure.

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3. Graphene as a REST Medium

Graphene's linear dispersion, high mobility, and low disorder support REST access under moderate bias. The **supersonic crossover** is the natural consequence of low effective viscosity near REST:

- **Criterion:** $v_d > v_s$.
- **Observable: Mach cone** half-angle

$$\theta_M = \arccos\left(\frac{v_s}{v_d}\right)$$

measurable from nano-IR/s-SNOM wavefronts and consistent with acoustic/optical Cherenkov relations.

3b. Translation Map (UFRF ↔ Standard Physics/Math)

UFRF Term	Conventional Meaning	Mathematical Expression
REST (E≈B)	Impedance matching / EM self-duality	$E = cB; \varepsilon_0 E^2 = \mu_0^{-1} B^2$
√φ efficiency	Critical coupling factor (max transmission)	$A_{n+1}/A_n = \sqrt{\varphi}$
η/s = (1/4π)√φ	Minimal viscosity near KSS bound w/ geometric correction	$\eta/s \sim \hbar/(4\pi k_B) \times \sqrt{\varphi}$
13/26 ladder	Harmonic quantization / band folding	$\omega_n = n\Omega/13, \omega_m = m\Omega/26$
Projection law (13/12)	Scale renormalization / logarithmic dilation	$O_{\text{obs}} = O_* \, e^{\alpha \, \Delta S}$
REST locking (~28 K)	Temperature-stabilized phase point	feature in transport/phonon linewidths

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4. Experimental Proof Points (with bridges)

1. **THz Acoustic Gain (graphene):** a directional amplification window appears **just above** $v_d = v_s$, consistent with **critical coupling at REST** and the onset of the 13/26 ladder.
   Nature Communications (2024): https://www.nature.com/articles/s41467-024-46819-2
   Preprint: https://arxiv.org/abs/2310.12225

2. **Supersonic Flow & Hydraulic Jump (bilayer graphene): de Laval nozzle** geometry yields sub→super→sub flow with a **viscous shock**—an **electronic hydraulic jump**—as predicted by compressible hydrodynamics at low η/s.
   arXiv:2509.16321 (2025): https://arxiv.org/abs/2509.16321

3. **Cherenkov-Like Nonequilibrium (nano-IR):** cryogenic nano-IR images reveal **directed wavefronts** and photocurrents consistent with **phonon Cherenkov emission** at high drift.
   Nature Communications (2025): open-access article/PDF https://pmc.ncbi.nlm.nih.gov/articles/PMC12022129/

5. Quantitative, Falsifiable Predictions (UFRF → Graphene)

P1. Universal viscosity minimum

$$\frac{\eta}{s} \to \frac{1}{4\pi} \quad \varphi \approx 0.101$$

at the supersonic threshold; weak device dependence at the minimum (fit via Poiseuille inversions, Johnson-noise thermometry, or nonlocal transport).

P2. Mach-cone angle recovery
From s-SNOM/nano-IR wavefronts, extract $\theta_M = \arccos(v_s/v_d)$ and verify agreement with transport-inferred v_d.

P3. THz gain window
A finite THz band with net amplification **immediately above** $v_d = v_s$; band collapses away from REST. (Compare with THz/SAW measurements as in Nature Communications 2024.)

P4. 13/26 spectral ladder
Bias-dependent Raman/THz spectra show discrete peaks at $\frac{k}{13}$ and $\frac{m}{26}$ of a device-set base frequency, with a strong primary near $\frac{10}{13}$.

P5. Electronic hydraulic jump
In constrictions (point contacts/de Laval), observe sub→super→sub with a **stationary discontinuity** (kink in I–V, plateau in Kelvin probe/thermal maps), aligning with arXiv:2509.16321.

P6. Temperature pinning (~28 K)
Reproducible feature near 28 ± 2 K in ultra-clean hBN-encapsulated devices (transport and phonon linewidths) indicating REST locking.

P7. Projection-law collapse
After a simple **13/12** normalization (UFRF projection), disparate device thresholds (cone angle, gain onset, η/s minima) **collapse** to a common intrinsic curve.

6. Physical Interpretation

REST minimizes **Poynting-flux divergence** and maximizes **energy translation** between charge flow and lattice/EM modes. In nonequilibrium thermodynamic language, REST approaches an **Onsager-symmetric** limit; in field-theory language, it touches the **KSS-like minimal viscosity regime**. The result is **collective yet compressible** flow—ballistic-style transport with hydrodynamic correlations—making **supersonic drift** and **shock formation** accessible in clean 2D devices.

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7. How to Falsify UFRF in Graphene

The model is refuted if, under clean/REST-accessible conditions:

1) η/s does **not** approach $\approx$0.10;
2) θ_M from imaging does **not** match $\arccos(v_s/v_d)$;
3) No 13/26 spectral structure appears at threshold;
4) No THz gain band is observed for $v_d \gtrsim v_s$;
5) No reproducible ~28 K feature is found;
6) 13/12 rescaling fails to collapse cross-device thresholds.

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8. Outlook Beyond Graphene

Because REST is geometric and material-agnostic, the same suite of signatures should appear in **bilayer graphene** (already observed), **Dirac/Weyl semimetals**, and **moiré** systems once cleanliness and geometry (nozzle-like constrictions, guided THz lines) are optimized. Near-term devices include **on-chip THz amplifiers**, **non-reciprocal mixers**, and **REST-locked oscillators**.

Appendix A. Mathematical Notes (UFRF → Standard Forms)

A1. REST balance: $\varepsilon_0 E^2 = \mu_0^{-1} B^2 \Rightarrow E = cB.$
A2. Minimum viscosity: $\eta/s = (1/4\pi) \quad \varphi$ (dimensionally consistent with $\hbar/(k_B)$

units).
A3. Harmonic ladder: $f_n = nf_0/13$; $f_m = mf_0/26$ (Fourier quantization in a 13-phase periodic medium).
A4. Projection law: $\ln O = \ln O^* + d_M \alpha S + \varepsilon \;\Rightarrow$ simple multiplicative **13/12** rescaling for well-separated scales.

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October 2025