A Generalized Vehicle Routing Problem Optimization based on Gavish and Graves Formulation (1978)

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Outline

Introduction

- The GVR problem that was modeled is the Ganish and Graves (1976) formulation.
- The goal of the GVRP is to minimize total traversal costs across the set of arcs A by servicing all cluster demands using exactly one node within it.

$$\min \sum_{(i,j)\in A} c_{ij}x_{ij}$$

where:

- A is the set of all arcs in the directed Graph $G(V, A)$.
- c_{ij} is the cost of the arc (i, j)
- $x_{ij} \in \{0, 1\}$ is the arc (i, j). If the arc is used it is 1.

Problem Formulation

- Modeled as an MILP with the arc variable being a integer and the flow variable by implication is integer although it is modeled as continuous.
- Constraints:
  - Cluster Visit Rule
  - Vehicle Usage Rule
  - Flow Conservation Rule
  - Commodity Flow Balance
  - Capacity Constraint
- Objective: **Minimize travel cost.**

GVRP Mathematical Formulation

Objective Function:

$$\text{Minimize} \quad \sum_{(i,j)\in A} c_{ij} x_{ij}$$

Subject to:

- **Cluster Visit Rule:**

$$x(\delta^+(C_k)) = 1, \quad x(\delta^-(C_k)) = 1 \quad \forall k \in M \setminus \{0\}$$

- **Vehicle & Flow Conservation:**

$$x(\delta^+(C_0)) = K, \quad x(\delta^-(C_0)) = K$$
$$x(\delta^+(i)) = x(\delta^-(i)) \quad \forall i \in V$$

- **Commodity Flow Balance:**

$$f(\delta^+(i)) - f(\delta^-(i)) = \frac{1}{2} q_{\alpha(i)} (x(\delta^-(i)) + x(\delta^+(i)))$$

- **Capacity & Domain Constraints:**

$$q_{\alpha(i)} \leq f_{ij} \leq (Q - q_{\alpha(i)}) x_{ij} \quad \forall (i,j) \in A$$
$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in A.$$

Using Gurobi as a Solver

- **1. Instance Generation (`gvrppgenerator`)**
  - We create randomized problem instances using our custom generator.
  - **Inputs:** Grid size ($N \times N$),Total number of Vertices (V),Number of Clusters, Vehicles (K), Capacity (Q), and the number of problems.
  - **Output:** A .txt file containing all the information needed about the custom problem.
- **2. Model Construction & Execution**
  - The generated data is loaded into **Gurobi**, and the SCF constraints (Equations 1-9) are initialized.
  - The standard `model.optimize()` function is called, triggering Gurobi's internal **Branch-and-Cut** algorithm to find the best solution.
- **3. Solution Extraction**
  - We extract the active arcs ($x_{ij} = 1$) to reconstruct the optimal route.

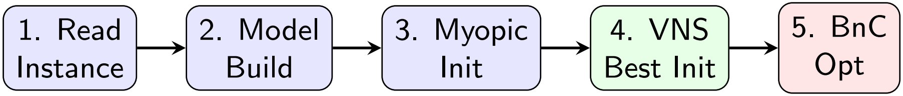
Branch-and-Bound Method

The Proposed methodology will be compared to the PyOmo model.

- Utilizes **Best-First Search** technique.
- The nodes are inserted into a min-heap based on the relaxed objective solution.
- The most fractional variable of the node is explored first (Most-Fractional technique) $\rightarrow min_{i \in Ov}(value_i - 0.5)$
- A Cutting Plane method is implemented inside this algorithm.

Heuristics

- Heuristics are implemented to speed up the branching algorithm
- First, a Myopic Heuristic to extract an initial cost
- Then a metaheuristic VNS to improve the initial
- VNS's cost becomes the upper bound of the BnC



Variable Neighborhood Heuristic

- Parameters k_{max} and max_iterations are set.
- Finds routes through the process:
  - Shaking then local search
  - Increases the intensity of shaking (if no improvement)
  - The heuristic moves to the improved solution S" only if feasibility criteria are checked.

$$(S, k) \leftarrow \begin{cases} (S'', 1) & \text{if } C(S'') < C(S) - \epsilon \\ (S, k+1) & \text{otherwise} \end{cases} \tag{1}$$

Capacity Cut

- The cut implemented in this algorithm is the *Rounded Capacity Inequality*
- The cut enforces that at least $2r(S)$ arcs must cross the subset's boundary (accounting for both entry and exit).

$$r(S) = \left\lceil \frac{\sum_{k \in S} q_k}{Q} \right\rceil \leq \frac{1}{2} \sum_{(i,j) \in \delta(S)} x_{ij} \qquad (2)$$

where:

- S is a subset
- $D(S)$ its total demand
- and $r(S)$ min vehicles required

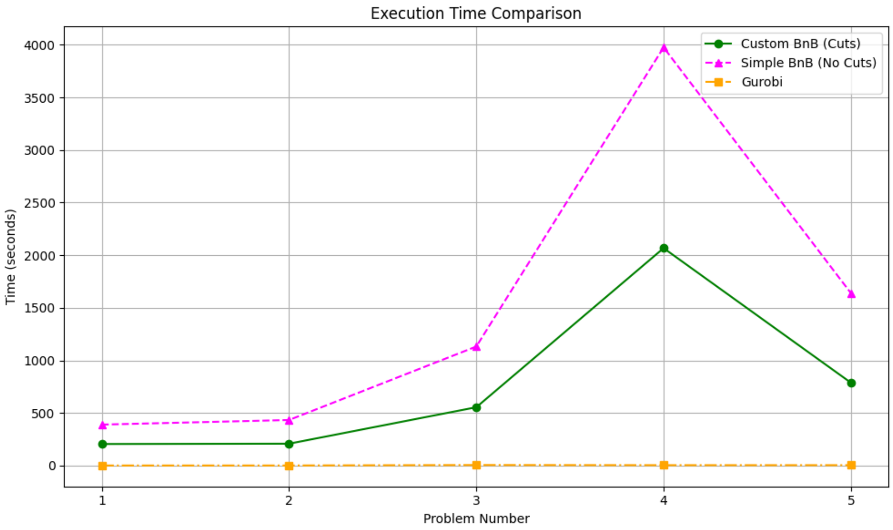
Performance Metrics

- Key Metric 1: **Computation Time**.
- Key Metric 2: **Nodes Explored**
- Comparison between:
    - ILP Model (Gurobi)
    - Custom Branch and Cut Method
    - Simple Branch and Bound

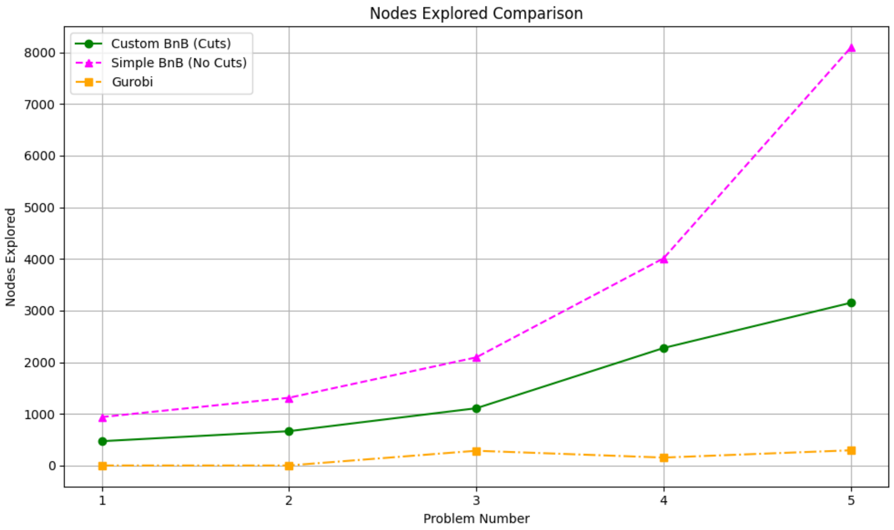
Experimental Setup

- Implemented using **Python's GurobiPy API**.
- 5 problems with grid dimensions: $\{10, 12, 14, 15, 35, 45\}$ generated.
- All Problems are solved by the BnC (final), BnB (simple) and Gurobi.

Execution Time



Nodes Explored



Key Observations

- The execution time increases with the size of the problem (exponential growth) but no distinct correlation with problem's order.
- The state-of-the-art has the best performance.
- Simple BnB explores twice the number of nodes of BnC. This is attributed to the Cut method
- BnB provides competitive performance for small-to-medium instances.

Conclusion

- The Problem's order is not the only difficulty metric. A computational analysis that accounts more factors for problem's difficulty growth.
- The proposed Branch and Cut can be further optimized from the heuristic aspect.
- More Cutting Plane techniques can be added to further improve the nodes explored number.

Thank You!