PCA

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1 Biến đổi lại công thức từ trên lớp

$$\mathbf{x} = \begin{pmatrix} ...x_1^T... \\ ...x_2^T... \\ ...x_3^T... \\ ... \\ ...x_N^T... \end{pmatrix} \in R^{N.D}$$

Ma trận hệ số $\mathbf{B} = \begin{pmatrix} b_1 & b_2 & ... & b_M \\ b_1 & b_2 & ... & b_M \\ ... & ... & ... & ... \\ b_1 & b_2 & ... & b_M \end{pmatrix} \in R^{D.M}$

$$\mathbf{z} = \begin{pmatrix} ...z_1^T... \\ ...z_2^T... \\ ...z_3^T... \\ ... \\ ...z_N^T... \end{pmatrix} = \begin{pmatrix} ...x_1^T... \\ ...x_2^T... \\ ...x_3^T... \\ ... \\ ...x_N^T... \end{pmatrix} . \begin{pmatrix} b_1 & b_2 & ... & b_M \\ b_1 & b_2 & ... & b_M \\ ... & ... & ... & ... \\ b_1 & b_2 & ... & b_M \end{pmatrix} = \begin{pmatrix} x_1^T b_1 & x_1^T b_2 & ... & x_1^T b_M \\ x_2^T b_1 & x_2^T b_2 & ... & x_2^T b_M \\ ... & ... & ... & ... \\ x_N^T b_1 & x_N^T b_2 & ... & x_N^T b_M \end{pmatrix}$$

$\Rightarrow maxvariance(\mathrm{x}_1^T b_1, x_2^T b_1, ..., x_N^T b_1)$

Asumption mean = 0

$$\begin{aligned} \mu_z &= \frac{x_1^T b_1 + x_2^T b_1 + ... + x_N^T b_1}{N} \\ &= \frac{b_1^T \sum_{n=1}^N x_i}{N} \\ &= b_1^T . \mu_x \end{aligned}$$

$$
\begin{aligned}
Var_z &= \frac{\sum_{n=1}^{N}(x_i^T b_1 - \mu_z)^2}{N} \\
&= \frac{\sum_{n=1}^{N}(x_i^T b_1)^2}{N} \\
&= \frac{\sum_{n=1}^{N} b_1^T x_i x_i^T b_1}{N} \\
&= b_1^T \left(\frac{\sum_{n=1}^{N} x_i x_i^T}{N}\right) b_1 \\
&= b_1^T S b_1
\end{aligned}
$$

$\Rightarrow Maximize \mathrm{b}_1^T S b_1$

$L(b_1, \lambda) = b_1^T S b_1 - \lambda(b_1^T b_1 - 1)$

$\frac{\delta L}{\delta b_1} = 0$

$\Leftrightarrow 2\mathrm{Sb}_1 - 2\lambda b_1 = 0$

$\Leftrightarrow \mathrm{Sb}_1 = \lambda b_1$

$\frac{\delta L}{\delta \lambda} = 0$

$\Leftrightarrow \mathrm{b}_1^T b_1 = 1$

$$
\begin{aligned}
Var_z &= b_1^T S b_1 \\
&= b_1^T \lambda b_1 \\
&= \lambda b_1^T b_1 \\
&= \lambda
\end{aligned}
$$