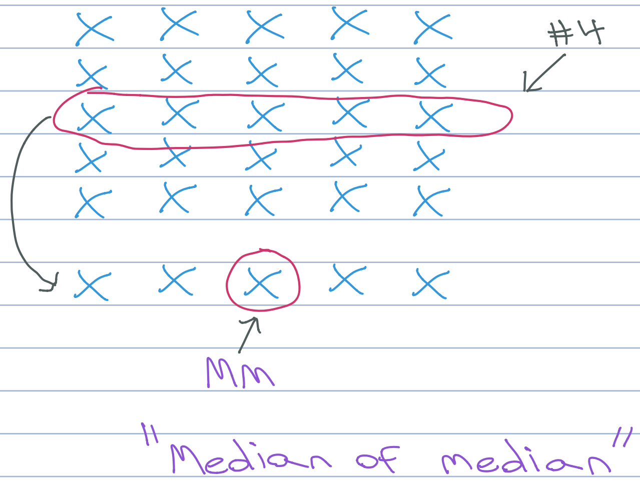
Linear time selection to select the ith smallest from n numbers.

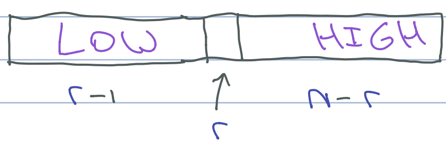
1. Cut the array into n/5 groups, so each group has 5 numbers.
2. Sort each group
3. Each group has a median so n/5 medians.
4. Recursively run linear time selection on the n/5 medians, selecting the n/10 smallest.



Note: $3 \cdot \frac{n}{10}$ numbers $\leq$ MM

Half of the items are below the median, so $\frac{n}{2}$ items. Split into 5 groups is $\frac{n}{10}$. 3 groups are less than or equal to MM, so $3 \cdot \frac{n}{10}$ items are less than or equal to the MM.

5. Swap MM with the first element of the original array.
6. Run partition on the input array.



7. ret match i {

   r=i ⇒ A[i]

   i<r ⇒ linear time selection on LOW

   i>r ⇒ linear time selection on HIGH

}

Linear time selection Proof

Step 1. Write a formula

$T_\omega(n) = T_\omega\left(\frac{7n}{10}\right) + T_\omega\left(\frac{n}{5}\right) + O(n)$

Notes Step #4 Step #6

Step 2. Guess.

$T_\omega(n) = O(n) \leq c \cdot n$

Need $c > 0$,

$\forall n \; T_\omega(n) \leq c \cdot O(n)$

Step 3. Check

$T_\omega(n) = T_\omega\left(\frac{7n}{10}\right) + T_\omega\left(\frac{n}{5}\right) + O(n)$

$= c \cdot \left(\frac{7n}{10}\right) + c \cdot \left(\frac{n}{5}\right) + a \cdot n$

$= c \cdot n \left(\frac{7}{10} + \frac{1}{5}\right) + a \cdot n$

$= \frac{9}{10} c \cdot n + a \cdot n$

$\leq c \cdot n$ when $c >> a$.

Note: So, if we recursively run linear time selection on LOW:

$|LOW| \geq 3 \cdot \frac{n}{10}$

Using symmetry,

$|HIGH| \geq 3 \cdot \frac{n}{10}$

which makes $|LOW| + |HIGH| \approx n$ roughly

So, our upperbounds will be:

$|LOW| \leq 7 \cdot \frac{n}{10}$

$|HIGH| \leq 7 \cdot \frac{n}{10}$

Therefore,

$\max\{|LOW|, |HIGH|\} \leq 7 \cdot \frac{n}{10}$ Upperbounded by