Symbolic Represataion

Say I need to send a list of integers

$1, 2, 3, 4, \ldots, 100$

May just send $1 \leq x \leq 100$ which is the symbolic rep. of the list. However we need to create a symbolic representation of a graph.

Can't store very large graphs in memory.

Boolean Formula

Over 4 variables $(x_1, x_2; y_1, y_2)$

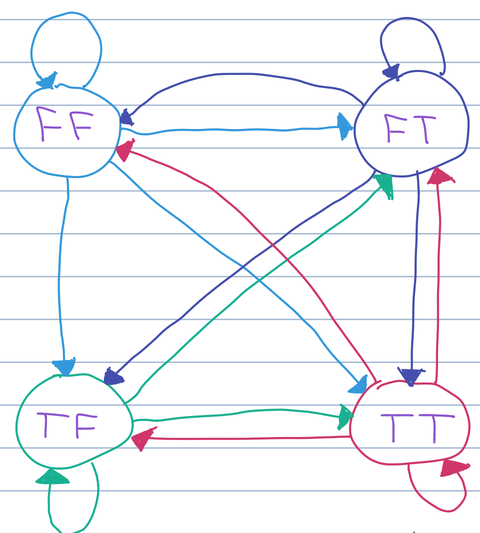
$(x_1 \wedge \bar{y}_1) \longrightarrow (x_2 \vee y_2)$

The bar is negation

x_1	x_2	y_1	y_2	$(x_1 \wedge \bar{y}_1) \longrightarrow (x_2 \vee y_2)$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

from | to

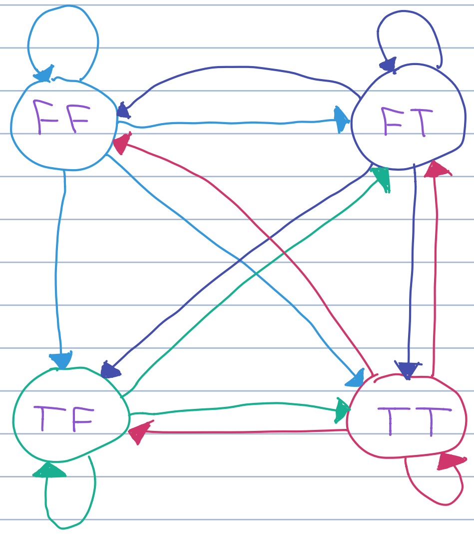
So, there are 4 nodes that are represented here:



Note: If an edge is missing, then the formula resolves to false.

Summary: A boolean formula may produce a graph with 2^k nodes using its truth table.

Suppose we have some very large graph



We have 15 edges, they are:

$E_{FFFF} \vee \ldots \vee E_{TTTT}$

For each edge, a boolean formula is used to represent it:

$E_{FFFT} = \bar{x}_1 \wedge \bar{x}_2 \wedge \bar{y}_1 \wedge y_2$

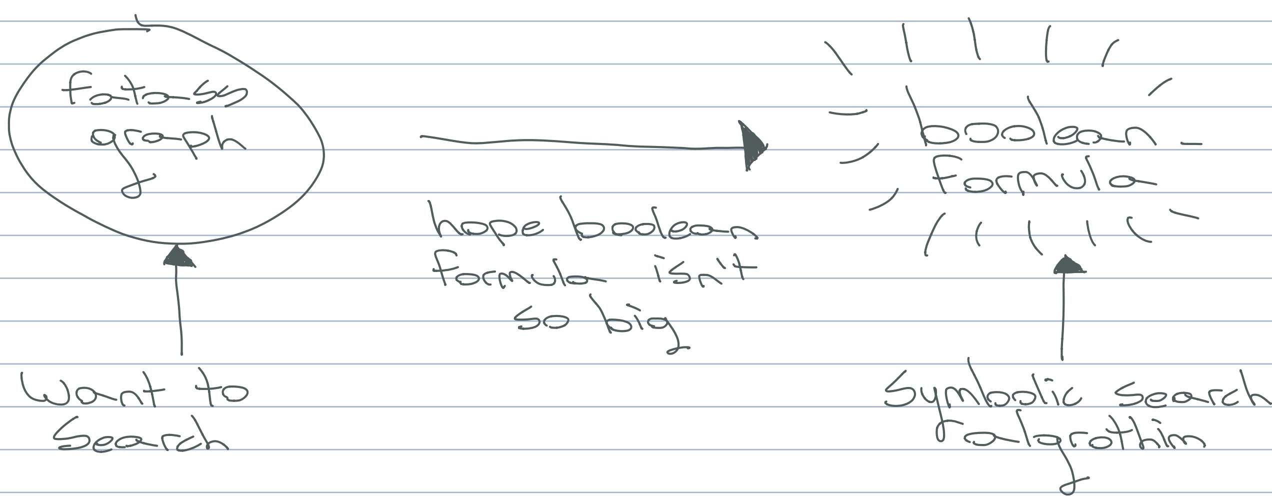
In this way, all 15 edges may become a boolean formula over four variables (by anding them all). Once it is simplified, we end up with an equivalent formula.

So,

$$E_{FFFF} \vee \ldots \vee E_{TTTT} \equiv (x_1 \wedge \bar{y}_1) \longrightarrow (x_2 \vee y_2)$$

A graph with 2^k nodes may be symbolicly represented as a boolean formula with 2^k variables.

Big Picture for Project



Symbolic Search Example

Let G, a graph be a boolean formula:

$R(x_1, \dots x_k; y_1, y_k)$

over $2k$ boolean variables.

Note:

Boolean formulas are closed under boolean operations

$\wedge, \vee, \neg, \rightarrow, \dots;$ as well as qualifiers $\exists$ and $\forall$.

Define a new boolean formula s.t.

$(R \circ R)(x_1, \dots, x_k; y_1, \dots y_k)$

Compose

which is defined as

$\exists z_1, \dots, z_k (R(x_1, \dots, x_k; z_1, \dots z_k) \wedge R(z_1, \dots z_k; y_1, \dots y_k))$

So we must interpt the meaning of $R \circ R$ on G

On graph G, a node $= k$ bits. Edge $=$ relatively on k bits.

$$(x_1, \ldots, x_k \longrightarrow y_1, \ldots, y_k) \equiv R(x_1, \ldots, x_k; y_1, \ldots y_k)$$

$$(x_1, \ldots, x_k \longrightarrow z_1, \ldots, z_k) \equiv R(x_1, \ldots, x_k; z_1, \ldots z_k)$$

$$(z_1, \ldots, z_k \longrightarrow y_1, \ldots, y_k) \equiv R(z_1, \ldots z_k; y_1, \ldots y_k)$$

$$(x_1, \ldots, x_k \rightarrow z_1, \ldots, z_k \rightarrow y_1, \ldots, y_k) \equiv R(x_1, \ldots, x_k; z_1, \ldots z_k) \wedge R(z_1, \ldots z_k; y_1, \ldots y_k)$$

$$(R \circ R)(x_1, \ldots, x_k; y_1, \ldots y_k) \equiv$$

$$\exists z_1, \ldots, z_k \left(R(x_1, \ldots, x_k; z_1, \ldots z_k) \wedge R(z_1, \ldots z_k; y_1, \ldots y_k) \right)$$