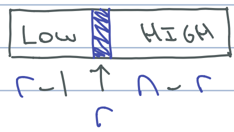
Quick Select

The ith smallest elements from n numbers.

1. Run partition on the array



2. if $i = r$ ret $A[r]$ $\frac{1}{n}$ proability

   if $i < r$
   Recursively run Quick-Select on LOW $\frac{r-1}{n}$ proability

   if $i > r$
   Recursively run Quick-Select on HIGH $\frac{n-r}{n}$ proability

Note: Worst-Case is $O(n^2)$ when the array is already sorted.

Quick Select Average Case

Step 1.

Write a formula.

From the proability of hitting each step

$$T_{AVG}(n) = \frac{1}{n} \sum_{r=1}^{n} \left\{ \frac{n-r}{n} T_{AVG}(n-r) + \frac{r-1}{n} T_{AVG}(r-1) + \frac{1}{n} O(1) + O(n) \right\}$$

Note:

i, A, LOW, HIGH, and r are all random.

Need to pick 1 to be random and the rest fixed.

So, 1 i will be random with the rest fixed. Then 2 r will be random.

Step 2.

Guess $T_{AVG}(n)$ is $O(n) \leq c \cdot n$ for some $c > 0$.

Induction Hypothesis.

$T_{AVG}(i) \leq c \cdot i$

Step 3.

Prove

$$T_{AVG}(n) = \frac{1}{n} \sum_{r=1}^{n} \left\{ \frac{n-r}{n} T_{AVG}(n-r) + \frac{r-1}{n} T_{AVG}(r-1) + \frac{1}{n} O(1) + O(n) \right\}$$

$$\leq \frac{1}{n} \sum_{r=1}^{n} \left\{ \frac{n-r}{n} \cdot c \cdot (n-r) + \frac{r-1}{n} \cdot c \cdot (r-1) + \frac{1}{n} \cdot a + a \cdot n \right\}$$

too small

$$= \frac{1}{n} \sum_{r=1}^{n} \frac{(n-r)^2}{n} \cdot c + \frac{1}{n} \sum_{r=1}^{n} \frac{(r-1)^2}{n} \cdot c + \frac{a}{n} + a \cdot n$$

$$= \frac{c}{n^2} \sum_{r=1}^{n} (n-r)^2 + \frac{c}{n^2} \sum_{r=1}^{n} (r-1)^2 + a \cdot n$$

$$= (n-1)^2 + (n-2)^2 \ldots$$

$$= \frac{2c}{n^2} \sum_{r=1}^{n} (r-1)^2 + a \cdot n$$

$$= \frac{2c}{n^2} \int_0^n x^2 \, dx + a \cdot n$$

$$= \frac{2c}{n^2} \cdot \frac{1}{3} n^3 + a \cdot n$$

$$= \frac{2}{3} \cdot c \cdot n + a \cdot n$$

$$\leq c \cdot n \quad \text{when } c \gg a.$$