Stanford CS224n HW A2

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Written Questions

(a) Since y is a one-hot encoded vector, it has zeros everywhere, and 1 where $w = o$. Thus, the only non-zero term in the sum is $-y_o \log(\hat{y_o})$, which is the RHS.

(b)

$$\begin{aligned}
\boldsymbol{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U}) &= -\log P(O = o, C = c) \\
&= -\log\left(\frac{\exp\left(\mathbf{u}_o^\top \mathbf{v}_c\right)}{\sum_{w\in\text{Vocab}} \exp\left(\mathbf{u}_w^\top \mathbf{v}_c\right)}\right) \\
&= \log\left(\sum_{w\in\text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)\right) - \mathbf{u}_o^\top \mathbf{v}_c
\end{aligned}$$

We have $\frac{\partial}{\partial \mathbf{v}_c} \mathbf{u}_o^\top \mathbf{v}_c = \mathbf{u}_o$, where we take the transpose of $\mathbf{u}_o^\top$ to keep the same shape as $\mathbf{v}_c$. The derivative of the left side is as follows:

$$\begin{aligned}
\frac{\partial}{\partial \mathbf{v}_c} \log\left(\sum_{w\in\text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)\right) &= \frac{1}{\sum_{w\in\text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \sum_{x\in\text{Vocab}} \frac{\partial}{\partial \mathbf{v}_c} \exp(\mathbf{u}_x^\top \mathbf{v}_c) \\
&= \frac{\sum_{x\in\text{Vocab}} \exp(\mathbf{u}_x^\top \mathbf{v}_c)\mathbf{u}_x}{\sum_{w\in\text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \\
&= \sum_{x\in\text{Vocab}} P(O = x, C = c)\mathbf{u}_x
\end{aligned}$$

Therefore,

$$\frac{\partial \boldsymbol{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = \left(\sum_{x\in\text{Vocab}} \hat{\mathbf{y}}_{\mathbf{x}} \mathbf{u}_{\mathbf{x}}\right) - \mathbf{u}_o$$

This can be interpreted as a difference of (expected - actual).

Let $\theta = U^\top \mathbf{v}_c$, and let the prediction function be $\hat{y} = softmax(\theta)$

$$\frac{\partial J}{\partial \theta} = (\hat{y} - y)^\top$$

$$\begin{aligned}
\frac{\partial J}{\partial \mathbf{v}_c} &= \frac{\partial J}{\partial \theta} \frac{\partial \theta}{\partial \mathbf{v}_c} \\
&= (\hat{y} - y)^\top \frac{\partial U^\top \mathbf{v}_c}{\partial \mathbf{v}_c} \\
&= U(\hat{y} - y)^\top
\end{aligned}$$

(c) Again, let $\theta = U^\top \mathbf{v}_c$, and let the prediction function be $\hat{y} = softmax(\theta)$

$$\begin{aligned}
\frac{\partial J}{\partial U} &= \frac{\partial J}{\partial \theta} \frac{\partial \theta}{\partial U} \\
&= (\hat{y} - y) \frac{\partial U^\top \mathbf{v}_c}{\partial U} \\
&= (\hat{y} - y)\mathbf{v}_c
\end{aligned}$$

Therefore, $\frac{\partial J}{\partial \mathbf{u}_w}$ is the wth row of $\frac{\partial J}{\partial U}$.

(d)

$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$$

$$\frac{d\sigma(x)}{dx} = \frac{0 - (-e^{-x})}{(1 + e^{-x})^2} = \frac{e^{-x}}{1 + e^{-x}} \sigma(x) = \sigma(x)(1 - \sigma(x))$$

(e) Let $f(x) = -\log(\sigma(x))$. Then we have:

$$f'(x) = \frac{\partial}{\partial x}[-\log(\sigma(x))] = -\frac{1}{\sigma(x)} \sigma(x)(1 - \sigma(x)) = \sigma(x) - 1$$

Let $a = \mathbf{u}_o^\top \mathbf{v}_c$ and b $= -\mathbf{u}_k^\top \mathbf{v}_c$. Then,

$$J = f(a) + \sum_{k=1}^{K} f(b)$$

Thus, $\frac{\partial J}{\partial a} = f'(a)$ and $\frac{\partial J}{\partial b} = \sum_{k=1}^{K} f'(b)$. Additionally, we have $\frac{\partial a}{\partial \mathbf{v}_c} = \mathbf{u}_o$ and $\frac{\partial b}{\partial \mathbf{v}_c} = -\mathbf{u}_k$. We can now calculate

$$\frac{\partial J}{\partial \mathbf{v}_c} = \frac{\partial J}{\partial a}\frac{\partial a}{\partial \mathbf{v}_c} + \frac{\partial J}{\partial b}\frac{\partial b}{\partial \mathbf{v}_c} = [\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1]\mathbf{u}_o + \sum_{k=1}^{K}[1 - \sigma(-\mathbf{u}_k^\top \mathbf{v}_c)]\mathbf{u}_k$$

Finally, $\frac{\partial a}{\partial \mathbf{u}_o} = \mathbf{v}_c$ and $\frac{\partial b}{\partial \mathbf{u}_k} = -\mathbf{v}_c$, so

$$\frac{\partial J}{\partial \mathbf{u}_o} = \frac{\partial J}{\partial a}\frac{\partial a}{\partial \mathbf{u}_o} = [\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1]\mathbf{v}_c$$

$$\frac{\partial J}{\partial \mathbf{u}_k} = \frac{\partial J}{\partial b}\frac{\partial b}{\partial \mathbf{u}_k} = [1 - \sigma(\mathbf{u}_k^\top \mathbf{v}_c)]\mathbf{v}_c, \forall k \in [1, K]$$

Negative sampling loss is much more efficient that naive softmax loss because it only iterates over K negative examples, instead of looping over the entire vocabulary.

(f)

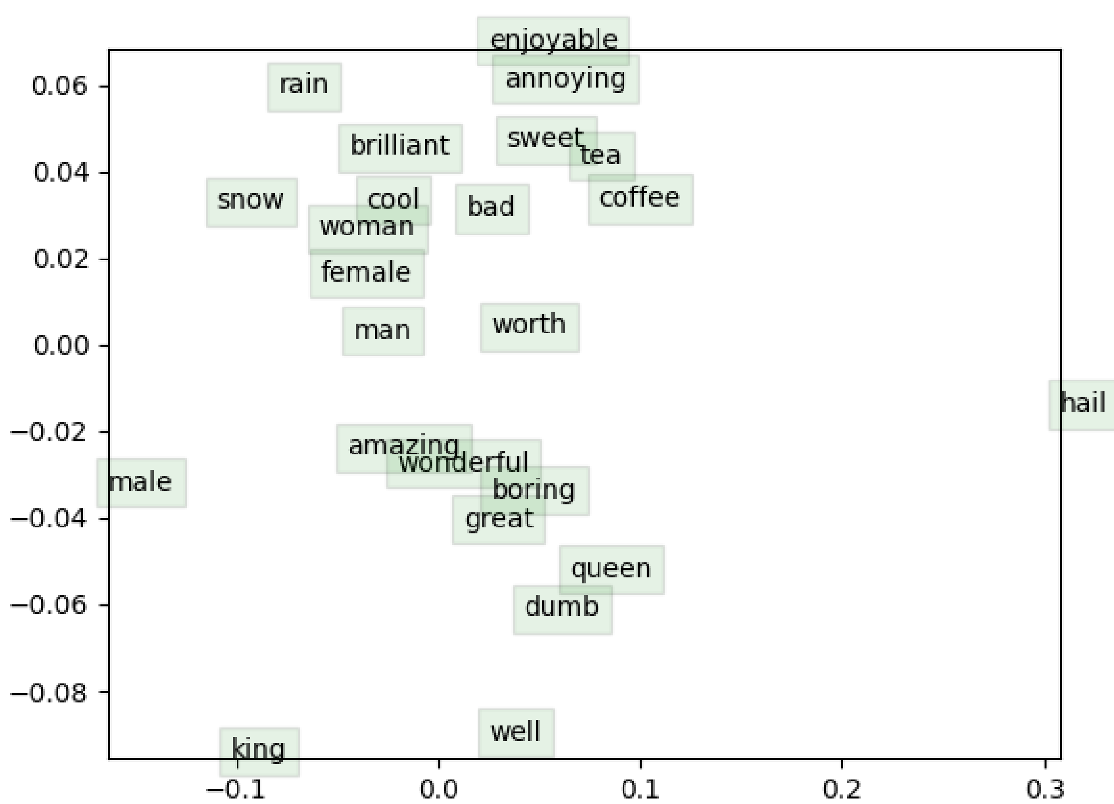
$$\frac{\partial \boldsymbol{J}_{\text{skip-gram}}}{\partial \boldsymbol{U}} = \sum_{\substack{-m \le j \le m \\ j \ne 0}} \frac{\partial \boldsymbol{J}(\mathbf{v}_c, w_{t+j}, \boldsymbol{U})}{\partial \boldsymbol{U}}$$

$$\frac{\partial \boldsymbol{J}_{\text{skip-gram}}}{\partial \mathbf{v}_c} = \sum_{\substack{-m \le j \le m \\ j \ne 0}} \frac{\partial \boldsymbol{J}(\mathbf{v}_c, w_{t+j}, \boldsymbol{U})}{\partial \boldsymbol{v}_c}$$

$$\frac{\partial \boldsymbol{J}_{\text{skip-gram}}}{\partial \mathbf{v}_w}(w \ne c) = 0$$

This last gradient is 0 since these vectors are not used to calculate the loss function, and thus have no influence on the loss.

Code Questions



At first glance, it seems that this 2D visualization has destroyed a lot of spatial relationships between word vectors: for example, *hail* is very far away from the similar words *snow* and *rain*. However, certain small clusters seem to be accurate: *amazing*, *wonderful*, and *great* are nearby, and *tea* and *coffee* are close. Finally, the *king* - *queen* = *male* - *female* analogy holds.