

Robotic Localization and Mapping - Miniset 2

Bayes Filters, Occupancy Grid Mapping, and Expectation/Covariance

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Bayes Filters

BF-Q1. In robotics, we commonly want to keep track of the possible values (or distribution of values) our current state could take on given all previous actions and measurements we have taken or observed up until the current time. We call this our **posterior** belief and often denote it as follows:

$$Bel(x_t) = p(x_t | u_1, z_1, \dots, u_t, z_t). \quad (1)$$

The Bayes filter provides a method for recursively updating our estimate of our state given only the most recent measurement(s), the most recent action, and our belief at the previous time step:

$$Bel(x_t) = \eta p(z_t | x_t) \int p(x_t | u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1} \quad (2)$$

The bayes filter is often broken down into the correction step, the update step, and the prior term. Label the parts of equation 2 that refer to these three main components.

Show how to derive equation 2 from equation 1. Label each step of the derivation with a description of what rule/law/assumption is being used to make it.

BF-Q2. Describe the Markov Assumption. In what situations can we make the Markov assumption and when can we not.

BF-Q3. You recently purchased a robotic vacuum cleaner for your apartment. The vacuum cleaner is able to navigate throughout your home automatically and is supposed to return to its charging station when its battery is low. The robotic system comes pre-programmed with a script that should return it to the docking station, however, you notice that the script only functions part of the time. After running some tests, you determine that the likelihood of the action “return-to-home” working is $p(x_{t+1} = \text{'home'} | u_{t+1} = \text{'return-to-home'}, x_t = \text{'not-home'}) = 0.7$, where x_{t+1} is the state of the robot after the action is taken, u_{t+1} is the action the robot takes, and x_t is the state of the robot before taking the action. Once returning home, if the return action is taken again, the robot stays at the charging station.

Assuming the robot has a 70% chance of being 'not-home' when it first detects a low battery. What is the likelihood that its state is 'home' after executing the 'return-to-home' action once time? What is the likelihood if the robot were to execute the 'return-to-home' action once more after executing it the first time?

Write out your steps and label the equations you use.

- BF-Q4. A¹ robot is equipped with a manipulator to paint an object. Furthermore, the robot has a sensor to detect whether the object is colored or blank. Neither the manipulation unit nor the sensor are perfect. From previous experience you know that the robot succeeds in painting a blank object with a probability of

$$p(x_{t+1} = \text{colored} \mid x_t = \text{blank}, u_{t+1} = \text{paint}) = 0.9,$$

where x_{t+1} is the state of the object after executing a painting action, u_{t+1} is the control command, and x_t is the state of the object before performing the action.

The probability that the sensor indicates that the object is colored although it is blank is given by $p(z = \text{colored} \mid x = \text{blank}) = 0.2$, and the probability that the sensor correctly detects a colored object is given by $p(z = \text{colored} \mid x = \text{colored}) = 0.7$.

Unfortunately, you have no knowledge about the current state of the object. However, after the robot performed a painting action the sensor of the robot indicates that the object is colored.

Compute the probability that the object is blank after the robot has performed an action to paint it. Use an appropriate prior distribution and justify your choice.

Occupancy Grid Mapping

- OGM-Q1. What are the major assumptions of the classic occupancy grid mapping algorithm?
- OGM-Q2. Why do we use log odds notation with occupancy grid mapping.
- OGM-Q3. Write out a short outline of the steps needed to update an occupancy grid map when a new laser range measurement is received.
- OGM-Q4. When can the algorithm fail?

Expectation and Covariance

- EC-Q1. State the definition of $E[x + y]$ in terms of the underlying probability distribution, then prove that $E[x + y] = E[x] + E[y]$, using the definition of expectation. Your proof should be succinctly and clearly written. **Do NOT assume that x and y are independent.**
- EC-Q2. For each of the covariance matrices below, indicate whether it is Valid or Invalid. If it is invalid, give a reason.

(a) $\begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -8 \\ -8 & 9 \end{bmatrix}$

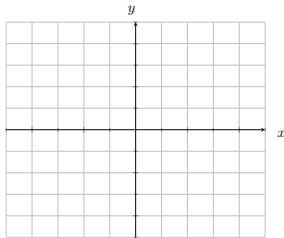
(c) $\begin{bmatrix} 2 & 0 \\ 0 & -8 \end{bmatrix}$

¹Problem courtesy of Ryan Eustice.

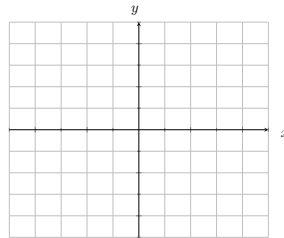
(d) $\begin{bmatrix} 3 & 2 \\ 2 & 10 \end{bmatrix}$

(e) $\begin{bmatrix} 6 & 1 \\ 2 & 10 \end{bmatrix}$

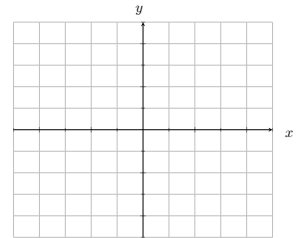
EC-Q3. Sketch the 1-sigma iso-cost contours for the covariance matrices below. Draw them in approximately correct relative scale. Draw the major axis of the ellipse as a dashed line.



$$\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$



$$\begin{bmatrix} 4 & -2 \\ -2 & 8 \end{bmatrix}$$



$$\begin{bmatrix} 4 & 3.9 \\ 3.9 & 4 \end{bmatrix}$$