MeEn 537 Homework #5 Solution

1. See parts a)-c) below.

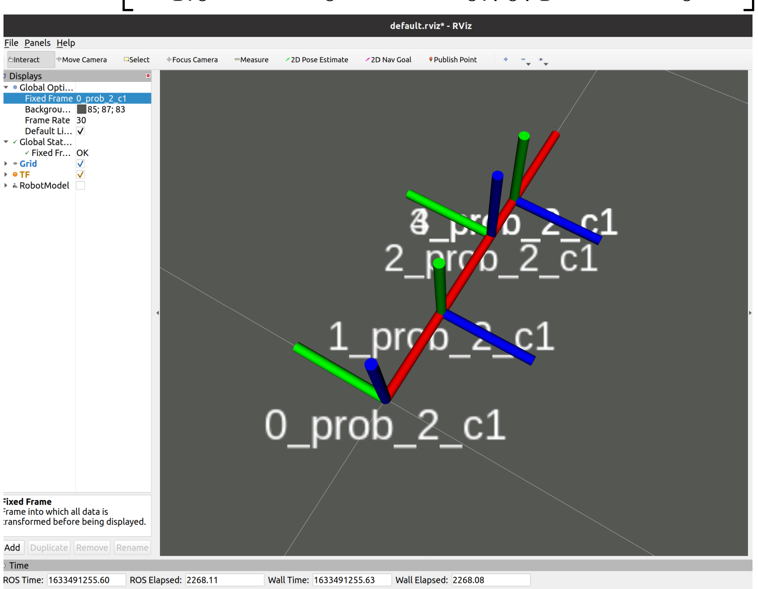
   (a) See code

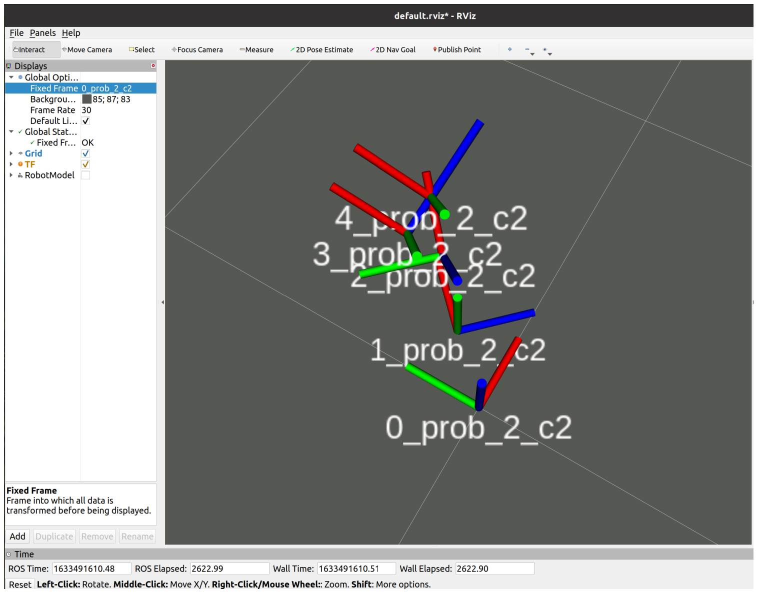
   (b) See code

   (c) See J matrices below corresponding to joint configuration 1 and 2 (and the figures of the DH coordinate frames). In comparing the two, we are looking for how the joint configuration changes the expected contribution of each joint to the end effector velocity. For example, in column 1, we see that in the first joint configuration, the first joint would add velocity at the tip, only in the y_0 direction, but after moving joint 1 by pi/4, we see that joint would now add velocity in equal amounts (for a given $\dot{q}$) in both the x_0 and y_0 directions. Any discussion along these lines in your solution is worth full credit. The main goal is to get you to understand what the columns mean.

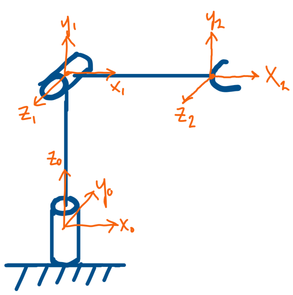
$$J(q_1) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.1 & -1 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.0 \\ 0 & -1 & 0 & 0.0 \\ 1 & 0 & 1 & 0.0 \end{bmatrix}$$

$$J(q_2) = \begin{bmatrix} -0.3121 & -0.1707 & -0.1 & 0.8536 \\ 0.3121 & -0.1707 & 0.1 & -0.1464 \\ 0 & 0.2414 & -6.939 \cdot 10^{-18} & 0.5 \\ 0 & 0.7071 & -0.5 & 0 \\ 0 & -0.7071 & -0.5 & 0 \\ 1.0 & 0 & 0.7071 & 0 \end{bmatrix}$$





2. Given the following robot:



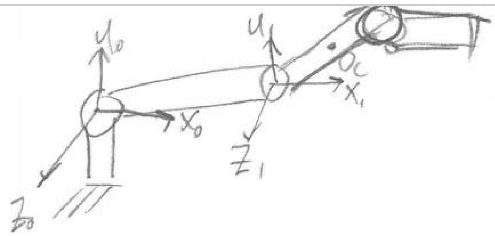
We know that the Jacobian for the tip will be the following:

$$J = \begin{bmatrix} z_0^0 \times (o_2^0 - o_0^0) & z_1^0 \times (o_2^0 - o_1^0) \\ z_0^0 & z_1^0 \end{bmatrix}$$

(a) See code in "hw05_test_jacobian_key.py" to see how we calculated each term in this matrix (as opposed to using the kinematics libary that we've been writing). The objective here was not to cause frustration or busy work. The key was to make sure you knew where each term in the for loop in "kinematics.py" was coming from and to help you see how important good algorithms are, given the difficulty in calculating the Jacobian by hand (even for only 2 joints).

(b) See print out in code for comparing the two Jacobians (calculated more or less manually, versus using the library we've been writing), and comparing them at two different joint configurations. As an example of discussion, it's quite clear that in the [0, 0] joint configuration, the 1st joint only causes instantaneous linear velocity at the tip in the y_0-direction (based on the figure above). This is reflected in the Jacobian where only the 2nd entry in the top three rows is non-zero. When we move the first joint by $\pi/4$, then the 1st column of the Jacobian has equal parts in the $-x_0$ and $+y_0$ directions, which means in this configuration, any joint velocity for the first joint will result in equal parts linear velocity at the tip in the -x and +y directions of the base frame. Being able to look at the Jacobian columns, and interpret them is an important skill from this class.

3. See hand written notes below calculating the Jacobian for a RRR robot where we want to know the Jacobian at point o_c on the second link.



let the first link length be a_1 and the distance from o_1 to o_c be a_c

find:

o_c ; $J_c^0(q) \Rightarrow$

$$\boxed{o_c = \begin{bmatrix} x_c = a_1 c_{\theta_1} + a_c c_{\theta_1+\theta_2} \\ y_c = a_1 s_{\theta_1} + a_c s_{\theta_1+\theta_2} \\ z_c = 0 \end{bmatrix}}$$

done by inspection, could also use forward kinematics

$$J_c^0(q) = \begin{bmatrix} z_0^0 \times (o_c^0 - o_0^0) & z_1^0 \times (o_c^0 - o_1^0) & 0 \\ z_0^0 & z_1^0 & 0 \end{bmatrix}$$

$$J_c^0(q) = \left[\begin{array}{c|c|c} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 c_{\theta_1} + a_c c_{\theta_1+\theta_2} \\ a_1 s_{\theta_1} + a_c s_{\theta_1+\theta_2} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_c c_{\theta_1+\theta_2} \\ a_c s_{\theta_1+\theta_2} \\ 0 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{array}\right] \Rightarrow$$

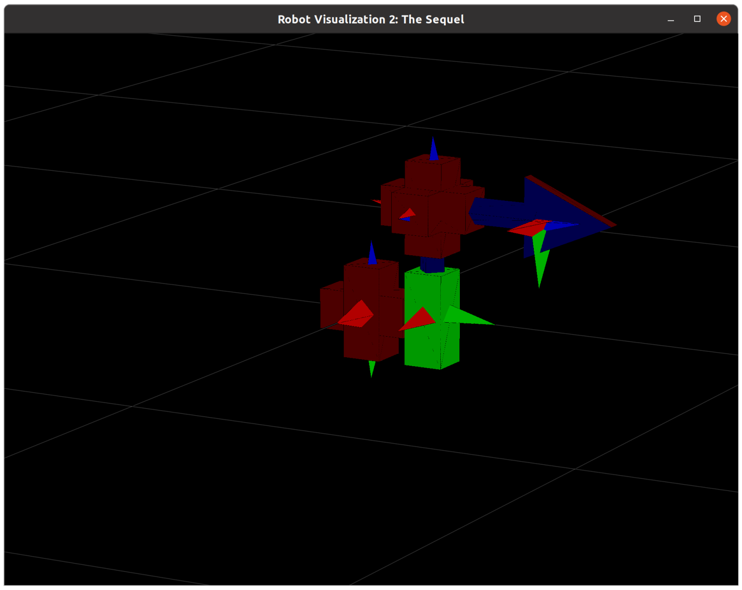
$$\boxed{J_c^0(q) = \begin{bmatrix} -a_1 s_{\theta_1} - a_c s_{\theta_1+\theta_2} & -a_c s_{\theta_1+\theta_2} & 0 \\ a_1 c_{\theta_1} + a_c c_{\theta_1+\theta_2} & a_c c_{\theta_1+\theta_2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}}$$

4. See sections below:

   (a) These DH parameters are correct for the image drawn. See figure below for part c) that matches the coordinate frames on the image from the book.

   (b) See code in "hw05_test_jacobian_key.py"

   (c) See image below:



   (d) In comparing the two jacobians below, it is clear that moving the prismatic joint (which is the 3rd joint) only affected the second column. This is because the direction in which the prismatic joint moved is along the direction of the axis of rotation for the first joint. This means that considering the idea of $\omega \times r$, (where r is the distance from the joint to the point of interest, and ω is the angular velocity caused by $\dot{q}_1$) "r" increasing in the same direction as the vector omega would not increase linear velocity at the tip. However, increasing the vertical distance away from joint 2 (or axis z_1) does increase "r" and thus results in a higher value for that column in the x-direction (first row). Finally, none of the other joints are affected as their distance to the end effector or tip of the robot is unchanged by the prismatic joint moving.

$$J(q=0) = \begin{bmatrix} -0.417 & 0.25 & 0 & -0.263 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0.263 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 1.0 \\ 1.0 & 0 & 0 & 1.0 & 0 & 0 \end{bmatrix}$$

$$J(q=[0,0,0.1,0,0,0]) = \begin{bmatrix} -0.417 & 0.35 & 0 & -0.263 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0.263 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 1.0 \\ 1.0 & 0 & 0 & 1.0 & 0 & 0 \end{bmatrix}$$