11d_to_10d_ansatz

Implications of the dimensional reduction ansatz

11d to 10d ansatz

In 11d, we are measuring length in units of l_P, and in 10d we are using l_S, both of which has dimension of length.

We use 11d coordinates of $(\vec{X}, u)$, where x are Minkowski coordinates with dimension length and $u \sim u + 2\pi R$ is a coordinate with length dimension in the 11th dimension, whose length scale is R.

It is natural to write down a relation between the 11d and 10d metrics

$$ds_{11}{}^2 = ds_{10}{}^2 + (du + A_m dx^m)^2$$

with implicit unit conversions. Because as numerical values $ds_{10}{}^2$ is measured as quantities of $l_S{}^2$, while $ds_{11}{}^2$ is measured in $l_P{}^2$, and du is measured in R. To make the unit conversions explicit, we divide dimensionful displacements such as dx, ds^2, du by the natural length units they are measured in, to obtain a dimensionless equation, for numerical measurereadings that one would see on a ruler

$$\frac{ds_{11}^2}{l_P^2} = \frac{l_S^2}{l_P^2}\left[\frac{ds^2_{(S),10}}{l_S^2} + \left(\frac{R}{l_S}\right)^2\left(\frac{du}{R} + \frac{A}{R/l_S}\cdot\frac{dx}{l_S}\right)^2\right] \tag{1}$$

We can obtain a dimensionless equation by dividing every dimensionful quan

The most general dimensional reduction formula, with the correct units is

$$\frac{ds_{11}^2}{l_P^2} = \frac{l_S^2}{l_P^2}\left[\frac{ds^2_{(S),10}}{l_S^2} + \left(A\cdot\frac{dx}{l_S}\right)^2\right] + \frac{R^2 d\theta^2}{l_P^2} + 2\frac{l_S}{l_P}\frac{Rd\theta}{l_P}\left(A\cdot\frac{dx}{l_S}\right) \tag{2}$$

In 10d string frame metric $ds_{(S),\,10}{}^2$ and $d\vec{X}$ are most naturally measured in l_S. The compact circle $Rd\theta$ originally came from 11d so it is naturally measured in l_P. Then the necessary factors were added from dimensional analysis.

We can extract a dimensionless quantity R/l_S and redefine $\frac{A}{R/l_S} \to A$, then the metric is

$$\frac{ds_{11}^2}{l_P^2} = \frac{l_S^2}{l_P^2}\left[\frac{ds^2_{(S),10}}{l_S^2} + \frac{R^2}{l_S^2}\left(d\theta + A\cdot\frac{dx}{l_S}\right)^2\right]. \tag{3}$$

Letting it be implicit that $ds_{11}{}^2$ is measured in $l_P{}^2$ and $ds_{10}{}^2$ is measured in $l_S^2, we have

$$ds_{11}^2 = \frac{l_S^2}{l_P^2}\left[ds_{(S),10}^2 + \frac{R^2}{l_P^2}(d\theta + A\cdot dx)^2\right]. \tag{4}$$

Substituting in the standard KK reduction ansatz and replacing e^{Φ} with its vacuum expectation g_s, we find the relations

$$l_P{}^3 = g_s l_S{}^3, \quad R = g_s l_S.$$

Torus setup

12d to 10d ansatz

Similar to the above analysis, we write down a general form for 12d spacetime with length unit l_F reduced diagonally on a torus to 10d with length unit l_S:

$$\frac{ds_{12}^2}{l_F^2} = \frac{l_S^2}{l_F^2}\left[\frac{ds_{(S),10}^2}{l_S^2} + \frac{\sqrt{M_s}}{l_S^2\tau_2}\left[(du+\tau_1 dv)^2 + \tau_2^2 dv^2\right]\right]. \tag{5}$$

We are adopting to the same notation in 12d_unification.md, where u, v have dimension of length and the torus has volume $4\pi^2\sqrt{M_s}l_S^2$. One can also choose to measure the torus with l_F, in which case the torus has volume $4\pi^2\sqrt{M_F}l_F^2$, and the two choices are related by $\sqrt{M_s}l_S^2 = \sqrt{M_F}l_F^2$.

The 12d reduction ansatz to 10d written in string frame is

$$ds_{12}{}^2 = e^{-\Phi/2}ds_{(S),\,10}{}^2 + e^{\Phi}[(du+Cdv)^2+e^{-2\Phi}dv^2]$$

The $ds_{12}{}^2$ must be measured in the 12d length unit l_F while the string frame metric is measured in l_S. The coordinates du, dv are measured in l_S had we chosen to work with $\sqrt{M_s}$. Putting the units back

$$\frac{ds_{12}^2}{l_F^2} = e^{-\Phi/2}\left[\frac{ds_{(S),10}^2}{l_S^2} + \frac{e^{\Phi/2}}{l_S^2 e^{-\Phi}}\left[(du+Cdv)^2 + e^{-2\Phi}dv^2\right]\right]. \tag{6}$$

Substituting e^{Φ} with g_s, we obtain the relations

$$l_F{}^4 = g_s l_S{}^4, \quad R = g_s{}^{1/4} l_S = l_F$$

where $R^2 = \sqrt{M_S}l_S^2 = \sqrt{M_F}l_F^2$ gives the volume of the torus $4\pi^2R^2$. So the compactification is done on a torus whose volume coincides with the 12d length scale.