IIB_branes

IIB brane solutions

This is a quick reference for the standard half-BPS Type IIB brane solutions.

Unless stated otherwise, we use the **10d string-frame** metric, and split coordinates into

- worldvolume directions x^{μ}, $\mu = 0, \ldots, p$
- transverse directions y^{i}, $i = 1, \ldots, 9 - p$

with flat line elements

$$ds^2(\mathbb{R}^{1,p}) = \eta_{\mu\nu} dx^{\mu} dx^{\nu}, \quad ds^2(\mathbb{R}^{9-p}) = \delta_{ij} dy^i dy^j.$$

Dilaton and frames

We write the **10d dilaton** as a background value plus a fluctuation,

$$\Phi = \langle \Phi \rangle + \phi, \quad g_s = \langle e^{\Phi} \rangle = e^{\langle \Phi \rangle}, \quad e^{\Phi} = g_s e^{\phi}.$$

Unless otherwise stated, all solutions below give ${ds_{10,str}}^2$ and Φ.

The string-frame and Einstein-frame metrics are related by the Weyl rescaling

$${g_{MN}}^{(E)} = e^{-\Phi/2} {g_{MN}}^{(str)}, \quad {ds_{10,E}}^2 = e^{-\Phi/2} {ds_{10,str}}^2.$$

In terms of g_s and ϕ this is

$${ds_{10,E}}^2 = {g_s}^{-1/2} e^{-\phi/2} {ds_{10,str}}^2, \quad {ds_{10,str}}^2 = {g_s}^{1/2} e^{\phi/2} {ds_{10,E}}^2.$$

For $p < 7$ define $r^2 = \delta_{ij} y^i y^j$ and the harmonic function

$$H(r) = 1 + \frac{Q_p}{r^{7-p}}.$$

For $p = 7$ (codimension 2), H is replaced by a logarithm (see the D7 section).

We also use the shorthand volume form

$$\mathrm{vol}_{1,p} = dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p.$$

Dp (template, *p* odd)

For a Dp-brane with $p = -1, 1, 3, 5$ (and formally $p = 7, 9$ with caveats), the standard ansatz is

$$ds_{10,str}{}^2 = H^{-1/2} ds^2(\mathbb{R}^{1,p}) + H^{1/2} ds^2(\mathbb{R}^{9-p}),$$

$$e^{\Phi} = g_s H^{(3-p)/4}, \quad e^{\phi} = H^{(3-p)/4},$$

$$C_{p+1} = \frac{1}{g_s}(H^{-1} - 1)\,\mathrm{vol}_{1,p}, \quad F_{p+2} = dC_{p+1}.$$

Using $ds_{10,E}{}^2 = e^{-\Phi/2} ds_{10,str}{}^2$, the Einstein-frame metric for a Dp in this normalization is

$$ds_{10,E}{}^2 = g_s{}^{-1/2}(H^{(p-7)/8} ds^2(\mathbb{R}^{1,p}) + H^{(p+1)/8} ds^2(\mathbb{R}^{9-p})).$$

The magnetic dual satisfies $F_{8-p} = (-1)^p *_{10} F_{p+2}$.

F1 (fundamental string)

Worldvolume: $\mathbb{R}^{1,1}$.

$$ds_{10,str}{}^2 = H^{-1} ds^2(\mathbb{R}^{1,1}) + ds^2(\mathbb{R}^8),$$

$$e^{2\Phi} = g_s{}^2 H^{-1}, \quad e^{2\phi} = H^{-1},$$

$$B_2 = (H^{-1} - 1)\,\mathrm{vol}_{1,1}, \quad H_3 = dB_2.$$

NS5

Worldvolume: $\mathbb{R}^{1,5}$.

$$ds_{10,str}{}^2 = ds^2(\mathbb{R}^{1,5}) + H\, ds^2(\mathbb{R}^4),$$

$$e^{2\Phi} = g_s{}^2 H, \quad e^{2\phi} = H,$$

$$H_3 = *_{\mathbb{R}^4} dH.$$

D(-1) (D-instanton)

This is Euclidean and is most cleanly written in Einstein frame, where the metric is flat:

$$ds_{10,E}{}^2 = ds^2(\mathbb{R}^{10}).$$

The axio-dilaton varies with a harmonic function $H(r) = 1 + \frac{Q_{-1}}{r^8}$:

$$e^{\Phi} = g_s H, \quad e^{\phi} = H,$$

and the RR scalar C_0 is sourced with

$$dC_0 = \pm\, d(H^{-1}).$$

Equivalently, in terms of the axio-dilaton

$$au = C_0 + ie^{-\Phi} = C_0 + ig_s{}^{-1}e^{-\phi}$$

one has a BPS relation between C_0 and the dilaton profile.

D1

Worldvolume: $\mathbb{R}^{1,1}$.

$$ds_{10,str}{}^2 = H^{-1/2}ds^2(\mathbb{R}^{1,1}) + H^{1/2}ds^2(\mathbb{R}^8),$$

$$e^{\Phi} = g_s H^{1/2}, \quad e^{\phi} = H^{1/2},$$

$$C_2 = \frac{1}{g_s}(H^{-1} - 1)\,\mathrm{vol}_{1,1}, \quad F_3 = dC_2.$$

D3

Worldvolume: $\mathbb{R}^{1,3}$.

$$ds_{10,str}{}^2 = H^{-1/2}ds^2(\mathbb{R}^{1,3}) + H^{1/2}ds^2(\mathbb{R}^6),$$

$$e^{\Phi} = g_s, \quad \phi = 0,$$

$$C_4 = \frac{1}{g_s}(H^{-1} - 1)\,\mathrm{vol}_{1,3}.$$

The self-dual 5-form field strength is

$$F_5 = (1+*_{10})\,dC_4.$$

D5

Worldvolume: $\mathbb{R}^{1,5}$.

$$ds_{10,str}{}^2 = H^{-1/2} ds^2(\mathbb{R}^{1,5}) + H^{1/2} ds^2(\mathbb{R}^4),$$

$$e^{\Phi} = g_s H^{-1/2}, \quad e^{\phi} = H^{-1/2}.$$

One can write either the electric 6-form potential or the magnetic 2-form potential:

$$C_6 = \frac{1}{g_s}(H^{-1} - 1)\,\mathrm{vol}_{1,5}, \quad F_7 = dC_6,$$

with $F_3 = -*_{10} F_7$.

D7

Worldvolume: $\mathbb{R}^{1,7}$ and transverse space is 2d. A single isolated D7-brane has logarithmic behavior rather than a power-law harmonic function.

A common local form (away from the brane core) is to take

$$H(r) = h_0 + Q_7 \log\frac{r}{r_0}, \quad r^2 = (y^1)^2 + (y^2)^2,$$

and use the same warped metric ansatz

$$ds_{10,str}{}^2 = H^{-1/2} ds^2(\mathbb{R}^{1,7}) + H^{1/2}(dr^2 + r^2 d\theta^2),$$

$$e^{\Phi} = g_s H^{-1}, \quad e^{\phi} = H^{-1}.$$

The D7 is electrically charged under C_8 (and magnetically under C_0):

$$C_8 = \frac{1}{g_s}(H^{-1} - 1)\,\mathrm{vol}_{1,7}, \quad F_9 = dC_8.$$

More precisely, the exact backreacted solution is naturally described in terms of the axio-dilaton $\tau = C_0 + ie^{-\Phi}$ which is holomorphic in the transverse complex coordinate and has $SL(2,\mathbb{Z})$ monodromy around the brane.

D9

Space-filling: there is no transverse space. In practice one takes H = 1, so the fields are constant and the metric is flat.

$$ds_{10,str}{}^2 = ds^2(\mathbb{R}^{1,9}), \quad e^{\Phi} = g_s, \quad \phi = 0.$$

(p,q) multiplets (generated by S-duality)

Type IIB has an $SL(2,\mathbb{Z})$ duality acting on the axio-dilaton τ and the two-form doublet (B_2,C_2). Applying it to F1 and NS5 generates the full family of (p,q) strings and (p,q) fivebranes. In Einstein frame the metric warp factors take the same schematic form as above, while the dilaton and the NS/RR potentials mix into $SL(2,\mathbb{Z})$ covariant combinations.