3.1 $\quad z_{sp} = z_{trans} \times z_{rot} \times z_{vib}$

$z_{trans} \propto V T^{3/2}, \quad z_{rot} \propto T$

$$z_{vib} = \frac{\exp(-h\nu/2k_BT)}{1 - \exp(-h\nu/k_BT)} \qquad \text{if } \frac{h\nu}{k_BT} \text{ is small,}$$

$$\exp\left(-\frac{h\nu}{2k_BT}\right) \approx 1 - \frac{h\nu}{2k_BT} = \frac{2k_BT - h\nu}{2k_BT}$$

$$1 - \exp(-h\nu/k_BT) \approx 1 - \left(1 - \frac{h\nu}{k_BT}\right) = \frac{h\nu}{k_BT}$$

$$z_{vib} \approx \frac{2k_BT - h\nu}{2k_BT} \cdot \frac{k_BT}{h\nu} = \frac{k_BT}{h\nu} - \frac{1}{2}.$$

$$\Rightarrow z_{trans} \times z_{rot} \times z_{vib} \propto V T^{1/2}.$$