

Schwarz 7.20:

To first order in ϕ , compute $g^{\alpha\beta}$

where $g_{\alpha\beta}$ defined by

$$ds^2 = -(1+2\phi)dt^2 + (1-2\phi)(dx^2 + dy^2 + dz^2)$$

clearly, $g_{\alpha\beta}$ is

$$\begin{pmatrix} -(1+2\phi) & & & \\ & (1-2\phi) & & \\ & & (1-2\phi) & \\ & & & (1-2\phi) \end{pmatrix}$$

in (t, x, y, z) basis.

$$g^{\alpha\beta} = (g_{\alpha\beta})^{-1}$$

$$\Rightarrow (1+2\phi)^{-1} = (1+2\phi)^{-1}|_0 + \frac{d}{d\phi}(1+2\phi)^{-1}|_0 \phi$$

$$= 1 - 2\phi.$$

$$(1-2\phi)^{-1} = (1-2\phi)^{-1}|_0 + \frac{d}{d\phi}(1-2\phi)^{-1}|_0 \phi$$

$$= 1 + 2\phi$$

$$\Rightarrow g^{\alpha\beta} = \begin{pmatrix} -1+2\phi & & & \\ & 1+2\phi & & \\ & & 1+2\phi & \\ & & & 1+2\phi \end{pmatrix}$$