Deriving Dirac Eq. and Clifford Algebra

Begin with massive KG: $(\Box^2 + m^2)\psi = 0$.

In p-space, it's $(-p^2+m^2)\psi = 0$, $(p^2-m^2)\psi = 0$.

$$(p^\mu p_\mu - m^2)\psi = 0, \quad (p^\mu p^\nu g_{\mu\nu} - m^2)\psi = 0.$$

$g_{\mu\nu}$ separates into $\varepsilon_\mu \otimes \varepsilon_\nu$, or $\varepsilon_\mu \varepsilon_\nu$ depending on convention. Since $g_{\mu\nu}$ is symmetric, $g_{\mu\nu} = g_{\nu\mu} = \varepsilon_\nu \varepsilon_\mu$,

then $\varepsilon_\nu \varepsilon_\mu = \varepsilon_\mu \varepsilon_\nu$, $\quad \varepsilon_\mu \varepsilon_\nu + \varepsilon_\nu \varepsilon_\mu = 2 g_{\mu\nu}$,

$$\{\varepsilon_\mu, \varepsilon_\nu\} = 2 g_{\mu\nu}. \qquad \text{(Clifford Algebra)}$$

$\Rightarrow$ KG can be rewritten as

$$(p^\mu \varepsilon_\mu p^\nu \varepsilon_\nu - m^2)\psi = 0.$$

$$(p^\mu \varepsilon_\mu + m)(p^\nu \varepsilon_\nu - m)\psi = 0.$$

$$\boxed{(p^\mu \varepsilon_\mu + m)\psi = 0, \quad (p^\nu \varepsilon_\nu - m)\psi = 0.}$$

or $\boxed{(i\partial_\mu \varepsilon_\mu - m)\psi = 0, \quad (i\partial_\mu \varepsilon_\mu + m)\psi = 0}$

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