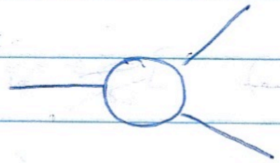
Schwartz

7.1 (c) $\langle 0 | T\{\phi_1 \phi_2 \phi_3 \exp[i\int d^4x \, \mathcal{L}_I[\phi_0(x)]]\} | 0 \rangle$

...



is in 3rd order,

$T\{\exp[i\int d^4x \, \mathcal{L}_I[\phi_0(x)]]\}$

$= 1 + i\int d^4x \, \mathcal{L}_I[\phi_x] + \frac{i^2}{2}\int d^4x \, d^4x' \, T\{\mathcal{L}_I[\phi_x] \mathcal{L}_I[\phi_{x'}]\}$

$+ \frac{i^3}{3!}\int d^4x \, d^4x' \, d^4x'' \, T\{\mathcal{L}_I[\phi_x] \mathcal{L}_I[\phi_{x'}] \mathcal{L}_I[\phi_{x''}]\}$.

$O(g^3)$

$+ \cdots$

$\langle 0 | \frac{i^3}{3!} \int d^4x \, d^4x' \, d^4x'' \, T\{\phi_1 \phi_2 \phi_3 \, \mathcal{L}_I[\phi_x] \mathcal{L}_I[\phi_{x'}] \mathcal{L}_I[\phi_{x''}]\} | 0 \rangle$

Letting $\mathcal{L}_I[\phi_0] = \frac{g}{3!}\phi_0^3$, we have

$\frac{1}{3!}\left(\frac{ig}{3!}\right)^3 \langle 0 | \int d^4x \, d^4x' \, d^4x'' \, T\{\phi_1 \phi_2 \phi_3 \, \phi_x^3 \, \phi_{x'}^3 \, \phi_{x''}^3 \, | 0 \rangle$

$$= \frac{1}{3!}\left(\frac{ig}{3!}\right)^3 \int d^4x\, d^4x'\, d^4x'' \langle 0 | T\{\phi_1\phi_2\phi_3\, \phi_x^3\, \phi_{x'}^3\, \phi_{x''}^3\} | 0 \rangle .$$

$$T\{\phi_1\phi_2\phi_3\, \phi_x^3\, \phi_{x'}^3\, \phi_{x''}^3\}$$

$$\left.\begin{aligned}
&= D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''} && \times 6\\
&+ D_{1x'}\, D_{2x''}\, D_{3x}\; (\cdots) && \times 6\\
&+ \;\vdots && \vdots\\
&\quad D_{1x}\, D_{2x''}\, D_{3x'}\; (\cdots) && \times 6
\end{aligned}\right\} \times 9\times 6\times 3$$

$$= \frac{9\times 6\times 3\times 6}{3!}\left(\frac{ig}{3!}\right)^3 \int d^4x\, d^4x'\, d^4x''\, D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''}$$

$$= \frac{\cancel{9}^{\,3}\times\cancel{6}\times\cancel{3}\times\cancel{6}}{\cancel{6}\times\cancel{6}\times\cancel{6}\times\cancel{6}_{\,2}\;2}(ig)^3 \int \cdots$$

$$= \frac{3}{4}(ig)^3 \int d^4x\, d^4x'\, d^4x''\, D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''}$$

(d) We want to evaluate $\int d^4x\, d^4x'\, d^4x''\, D_{1x} D_{2x'} D_{3x''} D_{xx'} D_{x'x''} D_{xx''}$

$$D_{1x} = \int \frac{d^4k}{(2\pi)^4} \frac{i e^{ik_1(x_1-x)}}{k_1^2 + i\varepsilon}, \quad \text{Let } \int \frac{d^4k}{(2\pi)^4} \text{ be implicit,}$$

we write $D_{1x} = \dfrac{i e^{ik_1(x_1-x)}}{k_1^2 + i\varepsilon}$ for simplicity,

$D_{1x} D_{2x'} D_{3x''} D_{xx'} D_{x'x''} D_{xx''}$

$$= \frac{i e^{ik_1(x_1-x)}}{k_1^2+i\varepsilon} \frac{i e^{ik_2(x_2-x')}}{k_2^2+i\varepsilon} \frac{i e^{ik_3(x_3-x'')}}{k_3^2+i\varepsilon} \frac{i e^{ik_4(x-x')}}{k_4^2+i\varepsilon} \frac{i e^{ik_5(x'-x'')}}{k_5^2+i\varepsilon} \frac{i e^{ik_6(x-x'')}}{k_6^2+i\varepsilon}$$

$$= \frac{(i)^6\, e^{ix(k_6+k_4-k_1)}\, e^{ix'(k_5-k_4-k_2)}\, e^{ix''(-k_6-k_5-k_3)}\, e^{ik_1x_1}\, e^{ik_2x_2}\, e^{ik_3x_3}}{(\cdots)(\cdots)(\cdots)\cdots(\cdots)}$$

$$= \frac{(i)^6\, \delta(k_6+k_4-k_1)\,\delta(k_5-k_4-k_2)\,\delta(-k_6-k_5-k_3)\, e^{ik_1x_1}\, e^{ik_2x_2}\, e^{ik_3x_3}}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(k_3^2+i\varepsilon)(k_4^2+i\varepsilon)(k_5^2+i\varepsilon)(k_6^2+i\varepsilon)}$$

Integrate k_6, so imposing $k_6 = k_1 - k_4$.

$$\frac{(i)^6\, \delta(k_5 - k_4 - k_2)\, \delta(k_4 - k_1 - k_5 - k_3)\, e^{ik_1x_1} e^{ik_2x_2} e^{ik_3x_3}}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(\cdots)\ \cdots\ [(k_1-k_4)^2+i\varepsilon]}$$

Integrate over k_5, imposing $k_5 = k_4 + k_2$.

$$\frac{(i)^6\, \delta(-k_1 - k_2 - k_3)\, e^{ik_1x_1} e^{ik_2x_2} e^{ik_3x_3}}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(k_3^2+i\varepsilon)(k_4^2+i\varepsilon)[(k_4+k_2)^2+i\varepsilon][(k_1-k_4)^2+i\varepsilon]}$$

Applying LSZ, attach $\left[-i\int d^4x_1\, e^{-ip_1x_1}\, p_1^2\right]\left[-i\int d^4x_2\, e^{+ip_2x_2}\, p_2^2\right]\left[-i\int d^4x_3\ \cdots\right]$

$$\Rightarrow (i)^6(-i)^3\int d^4x_1\, d^4x_2\, d^4x_3\ \frac{e^{ix_1(k_1-p_1)}\, e^{ix_2(k_2+p_2)}\, e^{ix_3(k_3+p_3)}\ p_1^2\, p_2^2\, p_3^2}{(k_1^2+i\varepsilon)(k_2^2+i\varepsilon)(k_3^2+i\varepsilon)(k_4^2+i\varepsilon)[(k_4+k_2)^2+i\varepsilon]\,[(k_1-k_4)^2+i\varepsilon]} \times \delta(-k_1-k_2-k_3)$$

$$= (i)^6(-i)^3\, \frac{1}{(k_4^2+i\varepsilon)[(k_4-p_2)^2+i\varepsilon][(p_1-k_4)^2+i\varepsilon]} \times \delta(-p_1+p_2+p_3)$$

Letting k_q be denoted $k \equiv k_q$, then we have

$$\int \frac{d^4k}{(2\pi)^3} \frac{i}{k^2+i\varepsilon} \frac{i}{(k-P_2)^2+i\varepsilon} \frac{i}{(P_1-k)^2+i\varepsilon} \times \delta(P_2+P_3-P_1)$$

Attaching back $\frac{3}{4}(ig)^3$, we have

$$(-1)\frac{3}{4} g^3 \int \frac{d^4k}{(2\pi)^3} \frac{i}{k^2+i\varepsilon} \frac{i}{(k-P_2)^2+i\varepsilon} \frac{i}{(P_1-k)^2+i\varepsilon} \times \delta(P_2+P_3-P_1)$$

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