-Tassani

10.2 (a) $z = x + iy, \quad \Rightarrow \quad x = \frac{1}{2}(z + z^*), \quad y = \frac{1}{2i}(z - z^*)$

$$f(z) = f(x+iy) = u(x,y) + iv(x,y)$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial z} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial z} = \frac{1}{2}\frac{\partial f}{\partial x} + \frac{1}{2i}\frac{\partial f}{\partial y}$$

$$= \frac{1}{2}\left[\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right] + \frac{1}{2i}\left[\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right]$$

$$= \frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial v}{\partial y} + \frac{i}{2}\frac{\partial v}{\partial x} + \frac{1}{2i}\frac{\partial u}{\partial y}$$

$$= \boxed{\frac{1}{2}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] + \frac{i}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]}$$

$$\frac{\partial f}{\partial z^*} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial z^*} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial z^*} = \frac{1}{2}\frac{\partial f}{\partial x} - \frac{1}{2i}\frac{\partial f}{\partial y}$$

$$= \frac{1}{2}\left[\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right] - \frac{1}{2i}\left[\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right]$$

$$= \frac{1}{2}\frac{\partial u}{\partial x} + \frac{i}{2}\frac{\partial v}{\partial x} - \frac{1}{2i}\frac{\partial u}{\partial y} - \frac{1}{2}\frac{\partial v}{\partial y}$$

$$= \boxed{\frac{1}{2}\left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right] + \frac{i}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]}$$

(b) The Cauchy-Riemann condition states that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\Rightarrow \frac{df}{dz} = \frac{1}{2}\left[2\frac{\partial u}{\partial x}\right] + \frac{i}{2}\left[2\frac{\partial v}{\partial x}\right]$$

$$= \boxed{\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}}$$

$$\frac{df}{dz^*} = \frac{1}{2}[0] + \frac{i}{2}[0]$$

$$= [0]$$

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