hassani
10.37

Taylor series: $f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(z_0)}{n!}(z-z_0)^n$

$$f^{(n)}(z_0) = \frac{d^n f}{dz^n}\Big|_{z=z_0} \qquad z_0 = i\pi$$

$$\cancel{\cosh}\ \sinh z = \frac{1}{2}\left[e^{x+iy} - e^{-x-iy}\right]$$

$$= \frac{1}{2}\left\{e^{x}(\cos y + i\sin y) - e^{-x}(\cos(-y) + i\sin(-y))\right\}$$

$$= \frac{1}{2}\left\{e^{x}\cos y + e^{x}\, i\sin y - e^{-x}\cos y - e^{-x}\, i\sin(-y)\right\}$$

$$= \frac{1}{2}(e^{x} - e^{-x})\cos y + \frac{i}{2}(e^{x} + e^{-x})\sin(y)$$

$$\frac{df}{dz} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\Rightarrow \frac{d\sinh z}{dz} = \frac{1}{2}(e^{x} + e^{-x})\cos y + \frac{i}{2}(e^{x} - e^{-x})\sin y$$

$$= \frac{1}{2}e^{x}\left[\cos y + i\sin y\right] + \frac{1}{2}e^{-x}\left[\cos(-y) - i\sin y\right]$$

$$= \frac{1}{2}e^{x}e^{iy} + \frac{1}{2}e^{-x}e^{-iy}$$

$$= \frac{1}{2}\left[e^{x+iy} + e^{-x-iy}\right] = \boxed{\cosh z}$$

$\Rightarrow \quad f^{(0)} = \sinh z \quad \rightarrow 0$

$f^{(1)} = \cosh z \quad \rightarrow -1$

$f^{(2)} = \sinh z \quad \rightarrow 0$

$\vdots$

$$\sinh z_0 = \sinh i\pi = \frac{1}{2}\left[e^{i\pi} - e^{-i\pi}\right] = 0$$

$$\cosh z_0 = \cosh i\pi = \frac{1}{2}\left[e^{i\pi} + e^{-i\pi}\right] = -1$$

$$\Rightarrow \boxed{\sinh z = -(z - i\pi) - \frac{(z-i\pi)^3}{3!} - \frac{(z-i\pi)^5}{5!} - \cdots}$$

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