

Polchinski 2.3 (a)

$$\text{For } n=2, \frac{n}{2!} \sum_{i=1}^2 e^{ik_i} \cdot X(z_i, \bar{z}_i) = :e^{ik_1 X_1} : :e^{ik_2 X_2} :$$

where we have used  $X_1$  to denote  $X(z_1, \bar{z}_1)$ .

Then the exercise states

$$\langle :e^{ik_1 X_1} : :e^{ik_2 X_2} : \rangle = i \mathcal{C}^X (2\pi)^D \delta^D(k_1 + k_2) |z_1 - z_2|^{2' k_1 k_2}$$

Recall (eq. 2.2.14) states -

$$:e^{ik_1 \cdot X(z, \bar{z})} : :e^{ik_2 \cdot X^{(0,0)}} : = |z|^{2' k_1 k_2} :e^{i(k_1 + k_2) \cdot X^{(0,0)}} [1 + o(z, \bar{z})]$$

Apply  $\langle \dots \rangle$  gives

$$\langle :e^{ik_1 X(z)} : :e^{ik_2 X^{(0)}} : \rangle = |z|^{2' k_1 k_2} \langle :e^{i(k_1 + k_2) X^{(0)}} [1 + o(z, \bar{z})] : \rangle$$

To zeroth-order, this is

$$|z|^{2' k_1 k_2} \langle :e^{i(k_1 + k_2) X^{(0)}} \rangle$$

$$= \boxed{|z|^{2' k_1 k_2} (2\pi)^D \delta^D(k_1 + k_2)}$$

The factor of  $(2\pi)^D$  comes from the fact that the  $\delta$ -function is on variables in phase space.