Schutz 8.7

(a). $\bar{h}^\alpha{}_\alpha = \bar{h}^{\alpha\beta} \eta_{\alpha\beta}$

$= \eta_{\alpha\beta} \left(h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h \right)$ by defn

$= h^\alpha{}_\alpha - \frac{1}{2} \eta^{\alpha\beta} \eta_{\alpha\beta} h$ $\eta^{\alpha\beta} \eta_{\alpha\beta} = Tr\left[(^{-1}{}_{,,})(^{-1}{}_{,,}) \right]$
$= 4$

$= h - 2h = \boxed{- h}$

(b). $\bar{h}^{\alpha\beta} = h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h$

$\Rightarrow h^{\alpha\beta} = \bar{h}^{\alpha\beta} + \frac{1}{2} \eta^{\alpha\beta} h$

$= \boxed{\bar{h}^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} \bar{h}}$

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