Hassani
11.1 (a)

$\frac{4z-3}{z(z-2)}$ has 2 isolated singularities, $z_0 = 0, 2$

for $z_0 = 0$, we wish to expand the series around $z_0 = 0$, and find the coefficient for the $(z-z_0)^{-1}$ term which corresponds to $\frac{1}{2\pi i}\oint_C f(z)\,dz$, C encloses z_0

$$\frac{4z-3}{z(z-2)} = \frac{1}{z}\left(\frac{4z-3}{z-2}\right) = \frac{1}{z}\left(\frac{4(z-2)+5}{z-2}\right)$$

$$= \frac{1}{z}\left[4 + \frac{5}{z-2}\right]$$

$$= \frac{1}{z}\left[4 - \frac{5/2}{1-z/2}\right]$$

$$= \frac{1}{z}\left[4 - \frac{5}{2}\frac{1}{1-z/2}\right]$$

$$= \frac{4}{z} - \frac{5}{2}\frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$$

$$= \left(4-\frac{5}{2}\right)\frac{1}{z} + \cdots\cdots$$

$$\Rightarrow \text{Res}[f(z_0=0)] = \boxed{\frac{3}{2}}$$

For $z_0 = 2$:

$$\frac{4z-3}{z(z-2)} = \frac{1}{(z-2)}\left(\frac{4z-3}{z}\right)$$

$$= \frac{1}{(z-2)}\left[4 - \frac{3}{z}\right]$$

$$= \frac{1}{z-2}\left[4 - \frac{3}{(z-2)+2}\right]$$

$$= \frac{1}{z-2}\left[4 - \frac{3/2}{1+\frac{(z-2)}{2}}\right]$$

$$= \frac{4}{z-2} - \frac{3}{2}\frac{1}{z-2}\sum_{n=0}^{\infty}\left[-\frac{(z-2)}{2}\right]^n$$

$$= \left(4 - \frac{3}{2}\right)\frac{1}{z-2} + \cdots\cdots$$

$$\Rightarrow \quad \mathrm{Res}\left[f(z_0=2)\right] = \boxed{5/2}$$

Combine these with Residue Thm. we have

$$\oint_C f(z)\,dz = 2\pi i \sum_k \mathrm{Res}\left[f(z_k)\right] = \boxed{8\pi i}$$

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