Weinberg 2.4 progress

$\cancel{W_\mu W^\mu}$ $\quad W_\mu \equiv \epsilon_{\mu\nu\rho\lambda} J^{\nu\rho} p^\lambda$

$\cancel{W_\mu} = W^\mu \equiv \epsilon^{\mu\nu\rho\lambda} J_{\nu\rho} p_\lambda$

$$\epsilon_{\mu\nu\rho\lambda}\, \epsilon^{\mu\alpha\beta\sigma} = ?$$

$$= \epsilon_{\mu\nu\rho\lambda}\, \epsilon^{\kappa\alpha\beta\sigma}\, \eta^\mu{}_\kappa$$

$\epsilon_{\mu\nu\rho\lambda}\, \epsilon^{\mu\alpha\beta\sigma}$ $\qquad \epsilon^{0123} = 1$

$\epsilon^{0321} = -1$

$\epsilon^{0123} = 1 \qquad \epsilon^{3210} = -1$

$\epsilon^{1230} = 1 \qquad \epsilon^{0321} = -1$

$\epsilon^{2301} = 1 \qquad \epsilon^{1032} = -1$

$\epsilon^{3012} = 1 \qquad \epsilon^{2103} = -1$

Let $\alpha\beta\sigma$ be a permutation such that $\epsilon^{0\alpha\beta\sigma} = 1$

Then $\epsilon^{i\alpha\beta\sigma} = 0$.

Let $\nu\rho\lambda$ be a permutation such that $\epsilon_{0\nu\rho\lambda} = 1$

Then $\epsilon_{i\nu\rho\lambda} = 0$

$\Rightarrow$ if $\alpha\beta\sigma$, $\nu\rho\lambda$ are both all spatial indices, then

$\epsilon_{\mu\nu\rho\lambda}\, \epsilon^{\mu\alpha\beta\sigma} = \pm 1$

$$\boxed{\begin{aligned} \epsilon_{\mu 123}\, \epsilon^{\mu 123} &= 1 \\ \epsilon_{\mu 321}\, \epsilon^{\mu 321} &= 1 \\ \epsilon_{\mu 123}\, \epsilon^{\mu 321} &= -1 \\ \epsilon_{\mu 321}\, \epsilon^{\mu 123} &= -1 \end{aligned}}$$

once you fix $\alpha\beta\sigma$, there is only a single value of $\mu$ such that $\epsilon^{\mu\alpha\beta\sigma}$ is $\neq 0$.

If this value of $\mu$ does not coinside for the value of $\mu$ that makes $\epsilon_{\mu\nu\rho\lambda} \neq 0$, then $\epsilon\epsilon = 0$.

This implies $\alpha\beta\sigma$, $\nu\rho\lambda$ must be sequence of 3 of the same set of differnt #s.

$\boxed{\text{Weinberg 2.4 progress}}$

$$\Rightarrow \varepsilon_{\mu\nu\rho\lambda}\varepsilon^{\mu\alpha\beta\sigma} = \begin{cases} 0 & \alpha\beta\sigma \text{ are not 3 \#s in a row from } \{0,1,2,3\}. \\ 1 & \text{if } \alpha\beta\sigma = \nu\rho\lambda \\ -1 & \text{if } \alpha\beta\sigma = \lambda\rho\nu \end{cases}$$

any ways, $\beta = \rho$.

$$\Rightarrow \varepsilon_{\mu\nu\rho\lambda}\varepsilon^{\mu\alpha\beta\sigma} = \begin{cases} 0 & \text{if } \alpha\beta\sigma \text{ not 3 \#s in a row from } \{0,1,2,3\}. \\ \Rightarrow \rho=\beta \begin{cases} 1 \text{ if } & \alpha=\nu, \ \sigma=\lambda \\ -1 \text{ if } & \alpha=\lambda, \ \sigma=\nu. \end{cases} \end{cases}$$

The $\varepsilon_{\mu\nu\rho\lambda}\varepsilon^{\mu\alpha\beta\sigma}$ uniquely determined by

$\beta$, $\begin{pmatrix}\alpha=\nu \\ \sigma=\lambda\end{pmatrix}$ or $\begin{pmatrix}\alpha=\lambda \\ \sigma=\nu\end{pmatrix}$

$\quad\quad 1 \quad\quad\quad -1$

$\beta=0$: $\alpha=3$, $\sigma=1$ $\rightarrow$ $\nu=3$, $\lambda=1$ $\quad 1$

$\quad\quad\quad\quad\quad\quad\quad\;\; \rightarrow$ $\nu=1$, $\lambda=3$ $\quad -1$

$\beta=1$: $\alpha=0$, $\quad J^{\nu\rho}P^{\lambda} \rightarrow \Lambda^{\nu}{}_{a}\Lambda^{\rho}{}_{b}\Lambda^{\lambda}{}_{c}J^{ab}P^{c}$

$\cancel{J^{\alpha\beta}P^{\sigma} \rightarrow \Lambda}$

$\varepsilon_{\mu\nu\rho\lambda}\varepsilon^{\mu\alpha\beta\sigma}$

$$J_{\alpha\beta}P_{\sigma} \rightarrow \Lambda_{\alpha}{}^{d}\Lambda_{\beta}{}^{e}\Lambda_{\sigma}{}^{f}J_{de}P_{f}$$