$$\Phi(r>a, \theta, \phi) = \sum_{l=0}^{\infty} 2kq\, a^{2l+1}\, r^{-(2l+2)}\, P_l[\cos\theta]$$

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(b) dipole. (?)

$$= \sum_{l=0}^{\infty} \cancel{2kq \frac{2kp}{2a} a^{2l+1} r^{-(2l+1)}}$$

$$2kq \frac{1}{a} a^{2l+2} r^{-(2l+2)} P_l[\cos\theta]$$

$$= \sum_{l=0}^{\infty} 2kq \frac{2q}{p} \left(\frac{a}{r}\right)^{2l+2} P_l[\cos\theta]$$

$$= \sum_{l=0}^{\infty} \frac{4kq^2}{p} \left(\frac{a}{r}\right)^{2l+2} P_l[\cos\theta].$$

$$\Phi(r>a, \theta, \phi) = \sum_{l=0}^{\infty} 2kq \left(\frac{a}{r}\right)^{2l+2} \frac{1}{a} P_l[\cos\theta].$$

$$= \sum_{l=0}^{\infty} 2kq \left[\left(\frac{a}{r}\right)^2\right]^{l+1} \frac{1}{a} P_l[\cos\theta].$$

$$= \sum_{l=0}^{\infty} \frac{4kq^2}{p} \left[\left(\frac{a}{r}\right)^2\right]^{l+1} P_l[\cos\theta]$$

$$= \frac{4kq^2}{p} \left(\frac{a}{r}\right)^2 P_l[\cos\theta] \qquad l=0.$$

$$+ \frac{4kq^2}{p} \left(\frac{a}{r}\right)^4 P_l[\cos\theta] \qquad l=1.$$

$$+ \frac{4kq^2}{p} \left(\frac{a}{r}\right)^6 P_l[\cos\theta] \qquad l=2.$$

$$\Phi = \sum_{l=0}^{\infty} \frac{4kq^2}{p} \left[\left(\frac{a}{r}\right)^2\right]^{l+1} P_l[\cos\theta]$$

$\cancel{\Phi(a=0)=}$

$$\Phi\big|_{a=0} = 0$$

$$\frac{d\Phi}{da}\Big|_{a=0} = 0$$

$$\frac{d\Phi}{da} \quad \cancel{\Phi' =}$$

$$\Phi' = \frac{4kq^2}{p} \sum_{l=0}^{\infty} \cancel{(l+1)(l)}\ (l+1) \left[\left(\frac{a}{r}\right)^2\right]^l \ \frac{2a}{r^2}$$

$$\Phi\big|_{a=0} = 0$$

$$\Phi'\big|_{a=0} = 0$$

$$\Phi''\big|_{a=0} = \frac{4kq^2}{p} \frac{2}{r^2} P_l[\cos\theta]$$

$$\Phi'''\big|_{a=0} = 0$$

$$\Phi''''\big|_{a=0} = \frac{4kq^2}{p} \frac{24}{r^4} P_l[\cos\theta]$$

$$\Phi^{(5)}\big|_{a=0} = 0$$

$$\Phi^{(6)}\big|_{a=0} = \frac{4kq^2}{p} \frac{720}{r^6} P_l[\cos\theta]$$

$$\Phi = \sum_{m=0}^{\infty} \cancel{\frac{\Phi^{(m)} a^m}{m!}} \ \frac{\Phi^{(m)}\big|_{a=0}\ a^m}{m!}$$

$$\Phi^{(m)}\big|_{a=0} = \frac{m!}{r^m} \quad \text{if } m \text{ even.}$$

$$\Rightarrow \quad \Phi = \sum_{m=0}^{\infty} \frac{\cancel{m!}\ a^m}{r^m \cancel{m!}} \quad \text{for even } m$$

f	f'	f''	f'''	f''''
$\left(\frac{a}{r}\right)^2, 0$	$2\left(\frac{a}{r}\right), 0$	$2, 2,$	$0, 0$	$0, 0$
$\left(\frac{a}{r}\right)^4, 0$	$\cancel{4\left(\frac{a}{r}\right)^3}, 0$	$12\left(\frac{a}{r}\right)^2, 0$	$24\left(\frac{a}{r}\right), 0$	$24, 624$
$\left(\frac{a}{r}\right)^6, 0$	$6\left(\frac{a}{r}\right)^5, 0$	$30\left(\frac{a}{r}\right)^4, 0$	$720\left(\frac{a}{r}\right)^3, 0$	$300\left(\frac{a}{r}\right)^2, 0$

$2! = 2.$

$3! = 6$

$4! = 24.$

f	f'	f''		
$\left(\frac{a}{r}\right)^2, 0$	$\frac{2a}{r^2}, 0$	$\frac{2}{r^2}, \frac{2}{r^2}$		$\frac{24}{r^4}$
$\left(\frac{a}{r}\right)^4, 0$	$4\frac{a^3}{r^4}, 0$	$\frac{12a^2}{r^4}, 0$	$\frac{24a}{r^4}, 0$	$\frac{360a^2}{r^6}, 0$
$\left(\frac{a}{r}\right)^6, 0$	$6\frac{a^5}{r^6}, 0$	$\frac{30a^4}{r^6}, 0$	$\frac{120a^3}{r^6}, 0$	$\frac{720}{r^6}$

$2! = 2,$

$4! = 2 \times 3 \times 4 = 24$

$6! = 720 = 4! \times 5 \times 6$

$\frac{a}{\cancel{b}} = \frac{P/2}{a}$, $= \frac{P/2}{a}$, $\frac{a}{a} = P/2$, $a = \frac{P/2}{a}$, $\frac{a}{2} = \frac{2a}{P}$, $\frac{P}{2} = a$, $a = \frac{P}{2}$, $P = 2a.$

$$\Phi = \sum_{m=1}^{\infty} \frac{4kq^2}{p} P_l[\cos\theta] \frac{a^{2m}}{p^{2m}}.$$

~~Fist~~