

13 | 2, 3, 8, 24, 1

1. Show that the Pauli matrices all square to 1.

~~Use exp~~ Prove  $\exp(i\frac{\theta}{2}\sigma_j) = \cos(\theta/2)I + i\sin(\theta/2)\sigma_j$ .

$$\sigma_1^2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \sigma_3^2 = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\sigma_2^2 = \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\exp(i\frac{\theta}{2}\sigma_j) = \frac{(\frac{i\theta}{2}\sigma_j)^0}{0!} + \frac{(\frac{i\theta}{2}\sigma_j)^1}{1!} + \frac{(\frac{i\theta}{2}\sigma_j)^2}{2!} + \frac{(\frac{i\theta}{2}\sigma_j)^3}{3!} + \dots$$

$$= \frac{(\frac{i\theta}{2})^0}{0!} I + \frac{(\frac{i\theta}{2})^1}{1!} \sigma_j + \frac{(\frac{i\theta}{2})^2}{2!} I + \frac{(\frac{i\theta}{2})^3}{3!} \sigma_j + \dots$$

~~$$\cos \frac{\theta}{2} = 1 - \frac{(\theta/2)^2}{2!} + \frac{(\theta/2)^4}{4!} - \frac{(\theta/2)^6}{6!} + \dots$$~~

$$\cos \frac{\theta}{2} = \frac{1}{0!} - \frac{(\theta/2)^2}{2!} + \frac{(\theta/2)^4}{4!} - \frac{(\theta/2)^6}{6!} + \dots$$

$$\sin \frac{\theta}{2} = \frac{\theta/2}{1!} - \frac{(\theta/2)^3}{3!} + \frac{(\theta/2)^5}{5!} - \dots$$

$$\Rightarrow \exp(i\frac{\theta}{2}\sigma_j) = \cos(\theta/2)I + i\sin(\theta/2)\sigma_j$$