Hassani
11.1 (b)

$$\frac{e^z}{z(z-i\pi)} \quad \text{has 2 isolated sing.} : \quad z_0 = 0, i\pi$$

$z_0 = 0$:

$$\frac{e^z}{z(z-i\pi)} = \frac{e^z}{z}\left[\frac{1}{z-i\pi}\right]$$

$$= \frac{e^z}{z}\left[-\frac{1}{i\pi - z}\right]$$

$$= \frac{e^z}{z}\left[-(i\pi)^{-1}\frac{1}{1-\frac{z}{i\pi}}\right]$$

$$= \frac{e^z}{z}\left[\frac{i}{\pi}\frac{1}{1-\frac{z}{i\pi}}\right]$$

$$= \frac{1}{z}\left(\sum_{n=0}^{\infty}\frac{z^n}{n!}\right)\left(\frac{i}{\pi}\right)\left(\sum_{k=0}^{\infty}\left(\frac{z}{i\pi}\right)^k\right)$$

$$= \frac{i}{\pi}\left(\frac{1}{z}\right) + \text{------}$$

$$\Rightarrow \text{Res}[f(z_0 = 0)] = \boxed{i/\pi}$$

$z_0 = i\pi$: $\dfrac{e^z}{z(z-i\pi)} = \dfrac{1}{z-i\pi}\left(\dfrac{e^z}{z}\right)$

$$= \frac{1}{z-i\pi}\left[\frac{e^{(z-i\pi)+i\pi}}{i\pi + (z-i\pi)}\right]$$

$$= \frac{1}{z-i\pi}\left[\sum_{n=0}^{\infty}\frac{(z-i\pi)^n}{n!}\right] e^{i\pi}\left[\frac{1}{i\pi}\,\frac{1}{1+\frac{z-i\pi}{i\pi}}\right]$$

$$= \frac{-1}{z-i\pi}\left[\sum_{n=0}^{\infty}\frac{(z-i\pi)^n}{n!}\right]\frac{1}{i\pi}\left[\sum_{k=0}^{\infty}\left(-\frac{z-i\pi}{i\pi}\right)^k\right]$$

$$= \boxed{\frac{(-1)}{i\pi}\,\frac{1}{z-i\pi}} + \cdots$$

$$\Rightarrow \operatorname{Res}\left[f(z_0 = i\pi)\right] = \frac{-1}{i\pi} = \boxed{\frac{i}{\pi}}$$

By Residue Thm: $\displaystyle\oint_C f(\xi)\,d\xi = 2\pi i \sum_{l} \operatorname{Res}\left[f(z_l)\right]$

we have $\displaystyle\oint_C f(\xi)\,d\xi = (2\cancel{\pi} i)\left[\frac{i}{\cancel{\pi}} + \frac{i}{\cancel{\pi}}\right]$

$$= (2i)(2i) = \boxed{-4}$$

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