

Griffiths 7.30

(a) (a) obvious.

$$\begin{aligned}\text{(b)} \quad \gamma_\mu \gamma^\nu \gamma^\mu &= \gamma_\mu \{2g^{\nu\mu} - \gamma^\mu \gamma^\nu\} \\ &= 2\gamma^\nu - 4\gamma^\nu = -2\gamma^\nu\end{aligned}$$

$$\begin{aligned}\text{(c)} \quad \gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\mu &= \gamma_\mu \gamma^\nu \{2g^{\lambda\mu} - \gamma^\mu \gamma^\lambda\} \\ &= 2\gamma^\lambda \gamma^\nu - \gamma_\mu \gamma^\nu \gamma^\mu \gamma^\lambda \\ &= 2\gamma^\lambda \gamma^\nu - (-2\gamma^\nu) \gamma^\lambda \\ &= 2\{\gamma^\lambda \gamma^\nu + \gamma^\nu \gamma^\lambda\} \\ &= 4g^{\lambda\nu}\end{aligned}$$

$$\begin{aligned}\text{(d)} \quad \gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\sigma \gamma^\mu &= \gamma_\mu \gamma^\nu \gamma^\lambda \{2g^{\sigma\mu} - \gamma^\mu \gamma^\sigma\} \\ &= 2\gamma^\sigma \gamma^\nu \gamma^\lambda - \gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\mu \gamma^\sigma \\ &= 2\gamma^\sigma \gamma^\nu \gamma^\lambda - 4g^{\nu\lambda} \gamma^\sigma \\ &= 2\gamma^\sigma \{2g^{\nu\lambda} - \gamma^\lambda \gamma^\nu\} - 4g^{\nu\lambda} \gamma^\sigma \\ &= 4\gamma^\sigma g^{\nu\lambda} - 2\gamma^\sigma \gamma^\lambda \gamma^\nu - 4g^{\nu\lambda} \gamma^\sigma \\ &= -2\gamma^\sigma \gamma^\lambda \gamma^\nu\end{aligned}$$

(b) These results follow from the fact that a_μ is just an array of numbers and we can commute them with matrices.

$$(7') \quad \not{\epsilon} \not{\epsilon} \not{\epsilon} = \not{\epsilon} \gamma^\nu a_\nu \not{\epsilon} = -2 \gamma^\nu a_\nu = -2 \not{\epsilon}$$

$$(8') \quad \not{\epsilon} \not{a} \not{b} \not{\epsilon} = \not{\epsilon} \gamma^\nu \gamma^\lambda \not{\epsilon} a_\nu b_\lambda \\ = 4 g^{\nu\lambda} a_\nu b_\lambda = 4 a \cdot b.$$

$$(9') \quad \not{\epsilon} \not{a} \not{b} \not{c} \not{\epsilon} = \not{\epsilon} \gamma^\nu \gamma^\lambda \gamma^\sigma \not{\epsilon} a_\nu b_\lambda c_\sigma \\ = -2 \gamma^\sigma \gamma^\lambda \gamma^\nu a_\nu b_\lambda c_\sigma \\ = -2 (\gamma^\sigma c_\sigma) (\gamma^\lambda b_\lambda) (\gamma^\nu a_\nu) \\ = -2 \not{c} \not{b} \not{a}$$