

Polchinski 2.2

$$\begin{aligned} & : X^{\mu_1} X^{\mu_2} X^{\mu_3} X^{\mu_4} : \\ & = X^{\mu_1} X^{\mu_2} X^{\mu_3} X^{\mu_4} \\ & + \frac{1}{2} \alpha' \left[\eta^{\mu_1 \mu_2} \ln |z_{12}|^2 X^{\mu_3} X^{\mu_4} \right. \\ & \quad + \eta^{\mu_1 \mu_3} \ln |z_{13}|^2 X^{\mu_2} X^{\mu_4} \\ & \quad + \eta^{\mu_1 \mu_4} \ln |z_{14}|^2 X^{\mu_2} X^{\mu_3} \\ & \quad + \eta^{\mu_2 \mu_3} \ln |z_{23}|^2 X^{\mu_1} X^{\mu_4} \\ & \quad + \eta^{\mu_2 \mu_4} \ln |z_{24}|^2 X^{\mu_1} X^{\mu_3} \\ & \quad \left. + \eta^{\mu_3 \mu_4} \ln |z_{34}|^2 X^{\mu_1} X^{\mu_2} \right] \end{aligned}$$

Apply $(\partial, \bar{\partial}_1)$:

$$\Rightarrow (\partial, \bar{\partial}_1, X^{\mu_1}) X^{\mu_2} X^{\mu_3} X^{\mu_4}$$

$$\begin{aligned} & + \frac{1}{2} \alpha' \left[\eta^{\mu_2 \mu_3} \ln |z_{23}|^2 X^{\mu_4} (\partial, \bar{\partial}_1, X^{\mu_1}) \right. \\ & \quad + \eta^{\mu_2 \mu_4} \ln |z_{24}|^2 X^{\mu_3} (\partial, \bar{\partial}_1, X^{\mu_1}) \\ & \quad \left. + \eta^{\mu_3 \mu_4} \ln |z_{34}|^2 X^{\mu_2} (\partial, \bar{\partial}_1, X^{\mu_1}) \right] \end{aligned}$$

$$= (\partial, \bar{\partial}_1, X^{\mu_1}) : X^{\mu_2} X^{\mu_3} X^{\mu_4} :$$

$$= (\partial, \bar{\partial}, : X^{\mu_1} :) (: X^{\mu_2} X^{\mu_3} X^{\mu_4} :)$$

$$= \boxed{0}$$

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