Schutz 6.39 (a) (i) $[\vec{U}, \vec{V}]^\alpha = U^\beta \nabla_\beta V^\alpha - V^\beta \nabla_\beta U^\alpha$

$[\vec{V}, \vec{U}] = V^\beta \nabla_\beta U - U^\beta \nabla_\beta V^\alpha = \boxed{-[\vec{U}, \vec{V}]^\alpha}$

(ii) $[\vec{U}, \vec{V}]^\alpha = U^\beta \nabla_\beta V^\alpha - V^\beta \nabla_\beta U^\alpha$.

$= U^\beta \left[V^\alpha{}_{,\beta} + \Gamma^\alpha_{\mu\beta} V^\mu \right] - V^\beta \left[U^\alpha{}_{,\beta} + \Gamma^\alpha_{\mu\beta} U^\mu \right]$

$= U^\beta V^\alpha{}_{,\beta} + \Gamma^\alpha_{\mu\beta} V^\mu U^\beta - V^\beta U^\alpha{}_{,\beta} - \Gamma^\alpha_{\mu\beta} U^\mu V^\beta$

use symmetry of Γ and relabel dummy indices

$\Gamma^\alpha_{\mu\beta} U^\beta V^\mu$

$= \boxed{U^\beta V^\alpha{}_{,\beta} - V^\beta U^\alpha{}_{,\beta}}$

Davidson Cheng

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Schutz 6.39 (b) $[\vec{U}, f\vec{V}] = U^\beta \nabla_\beta (fV^\alpha) - fV^\beta (\nabla_\beta U^\alpha)$

$$= U^\beta (\nabla_\beta f) V^\alpha + U^\beta f (\nabla_\beta V^\alpha) - f V^\beta \nabla_\beta U^\alpha$$

$$= f\left[U^\beta \nabla_\beta V^\alpha - V^\beta \nabla_\beta U^\alpha\right] + V^\alpha U^\beta \nabla_\beta f$$

$$= \boxed{f[\vec{U}, \vec{V}] + \vec{V}(\vec{U} \cdot \nabla f).}$$

Davidson Chen

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