Asymmetric Equilibria in Symmetric Multiplayer Prisoners' Dilemma Supergames

Davidson Cheng

33rd International Conference on Game Theory, Stony Brook

July 19th, 2022

Outline

- Model a **supergame** as a **finite state automaton (FSA)**
- Propose a solution concept on **FSA**s
- Derive properties of such **FSA**s

Background

Definition

A **supergame** is an infinitely repeated game without discounting, whereas the utility of each player is calculated as the limit of their mean payoff [Aumann, 1994].

Background

Definition

A **supergame** is an infinitely repeated game without discounting, whereas the utility of each player is calculated as the limit of their mean payoff [Aumann, 1994].

This implies we are assuming

Background

Definition

A **supergame** is an infinitely repeated game without discounting, whereas the utility of each player is calculated as the limit of their mean payoff [Aumann, 1994].

This implies we are assuming

- the game will eventually reach equilibrium

Background

Definition

A **supergame** is an infinitely repeated game without discounting, whereas the utility of each player is calculated as the limit of their mean payoff [Aumann, 1994].

This implies we are assuming

- the game will eventually reach equilibrium
- we care about the equilibrium payoff only [Rubinstein, 1979]

Why supergames?

We don't expect anything to go on forever, so why supergames?

Why supergames?

We don't expect anything to go on forever, so why supergames?

Sometimes finitely repeating a game won't produce interesting equilibria [Benoit, 1984]

Why supergames?

We don't expect anything to go on forever, so why supergames?

Sometimes finitely repeating a game won't produce interesting equilibria [Benoit, 1984]

- Prisoners' Dilemma

Why supergames?

We don't expect anything to go on forever, so why supergames?

Sometimes finitely repeating a game won't produce interesting equilibria [Benoit, 1984]

- Prisoners' Dilemma
- The "chainstore paradox" [Selten, 1978]

Why supergames?

We don't expect anything to go on forever, so why supergames?

Sometimes finitely repeating a game won't produce interesting equilibria [Benoit, 1984]

- Prisoners' Dilemma
- The "chainstore paradox" [Selten, 1978]

On the other hand, infinitely repeated games may contain a larger set of equilibria [Folk Theorem].

Finite State Automaton (FSA)

What is it?

Finite State Automaton (FSA)

What is it?

- A mathematical model for computation

Finite State Automaton (FSA)

What is it?

- A mathematical model for computation
- Abstracts computation to transitions between **states**

Finite State Automaton (FSA)

What is it?

- A mathematical model for computation
- Abstracts computation to transitions between **states**
- **deterministic** or **non-deterministic** (quantum computation)

Model

We consider infinitely repeated multiplayer prisoners' dilemma without discounting.

Definition

Let the stage game G be defined as

$$G =< \{0, 1\}, S_n, u >,$$

whereas S_n are all **states** of the game.

States

all intermediate/final outcomes of the game.

States

all intermediate/final outcomes of the game.

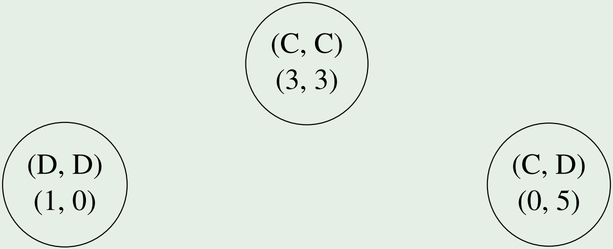
Example

Two Player Prisoner's Dilemma

		Prisoner 1: Cooperate	Prisoner 1: Defect
Prisoner 2	Cooperate	3,3	0,5
	Defect	5,0	1,1

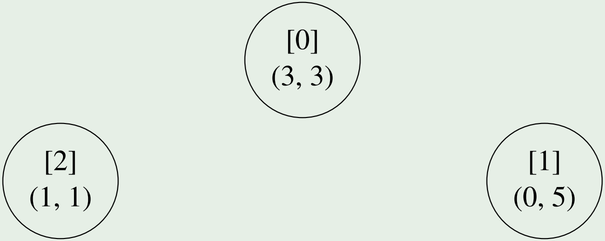
States

Example



States

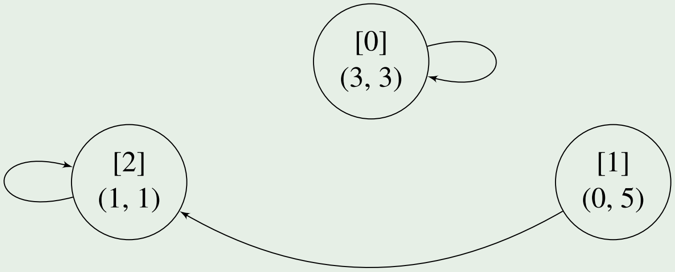
Example



We use the number of defective agents to denote a state, and we use this number to establishing an ordering of states

States

Example



Leading and Chaining

Definition

For some $s_i, s_j \in S_n$, we use $s_i \longrightarrow s_j$ to denote "s_i **leads to** s_j", which means

- **one and only one agent in** s_i **is able to by themself improve and maximize their limit of the means payoff** through inducing a state switch from s_i to s_j,
- or $s_i = s_j$ and s_i is an equilibrium.

Definition

For $s_i, s_j \in S_n$, $s_i \Longrightarrow s_j$ **denotes "**s_i **is chained to** s_j", which means for some $s_i, s_l, s_m, ...s_o, s_j \in S_n$, $s_i \longrightarrow s_l$, $s_l \longrightarrow s_m$, ..., $s_o \longrightarrow s_j$.

This means a state can only lead to itself or its neighbors.

Stage Game Properties

Definition

The game G is **locally non-cooperative** if

- for any state with utility defined for both defective and cooperative agents, defective agents have higher payoff.
- for any $b \in \{0, 1, 2, ..., n-1\}$,

$$u(1, [b+1]) > u(0, [b]).$$

Definition

The supergame G^* has **monotonously decreasing** utility function if for any $s, s' \in S_n$ such that $s' > s$, the utility function satisfy

$$u(0, s') < u(0, s), \text{ and}$$

$$u(1, s') < u(1, s).$$

FSA Properties

Theorem

If G^* has monotonously decreasing utility function and is locally non-cooperative, then every state leads to exactly one state.

FSA Properties

Theorem

If G^* has monotonously decreasing utility function and is locally non-cooperative, then every state leads to exactly one state.

The FSA is deterministic.

FSA Properties

Theorem

If G^* has monotonously decreasing utility function and is locally non-cooperative, then every state leads to exactly one state.

The FSA is deterministic.

Corollary

If G^* has ..., then the sequence of state switches for G^* contains exactly one cycle, and such cycle can only contain one state.

FSA Properties

Theorem

If G^* has monotonously decreasing utility function and is locally non-cooperative, then every state leads to exactly one state.

The FSA is deterministic.

Corollary

If G^* has ..., then the sequence of state switches for G^* contains exactly one cycle, and such cycle can only contain one state.

Equilibria are stationary

FSA Properties

Lemma

Suppose G^* *has ..., then for any* $b \in \{0, 1, \ldots, n\}$,

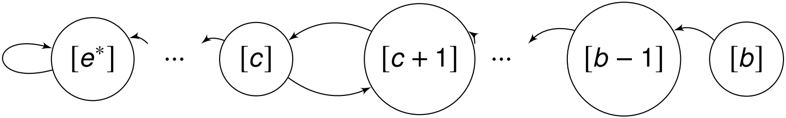
- $[b] \longrightarrow [b-1]$ *implies there exists some state* $[e]$ *such that* $e \leq b-1$, $[e] \longrightarrow [e]$, *and for any* $d \in \{b, b-1, \ldots, e+1\}$, $u(0, [e]) > u(1, [d])$;
- $[b] \longrightarrow [b+1]$ *implies ....*

FSA Properties

Lemma

Suppose G^* has ..., then for any $b \in \{0, 1, \ldots, n\}$,

- $[b] \rightarrow [b-1]$ *implies there exists some state* $[e]$ *such that* $e \leq b-1$, $[e] \rightarrow [e]$, *and for any* $d \in \{b, b-1, \ldots, e+1\}$, $u(0, [e]) > u(1, [d])$;
- $[b] \rightarrow [b+1]$ *implies ....*



FSA Properties

Lemma

If G^* has ..., then the state $[n]$ is an equilibrium.

FSA Properties

Theorem

If G^* *has ..., then for some state* $[b] \in S_n$ *such that* $u(1, [n]) > u(0, [b])$*, it must be true that* $[b] \Rightarrow [n]$.

Symmetric Equilibria Example

Example

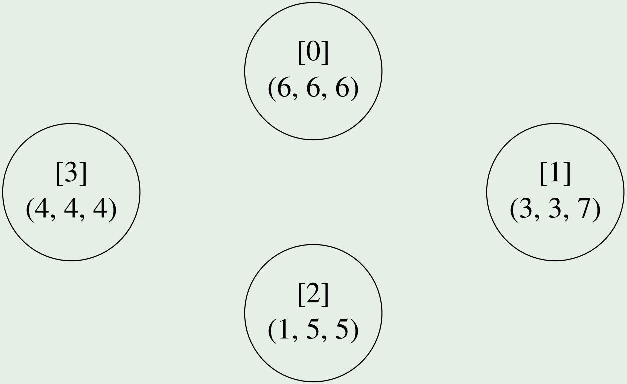
Consider an instance of G^* with three players and stage game payoff function defined as below.

State

Action		[0]	[1]	[2]	[3]
	0 (Cooperate)	6	3	1	-
	1 (Defect)	-	7	5	4

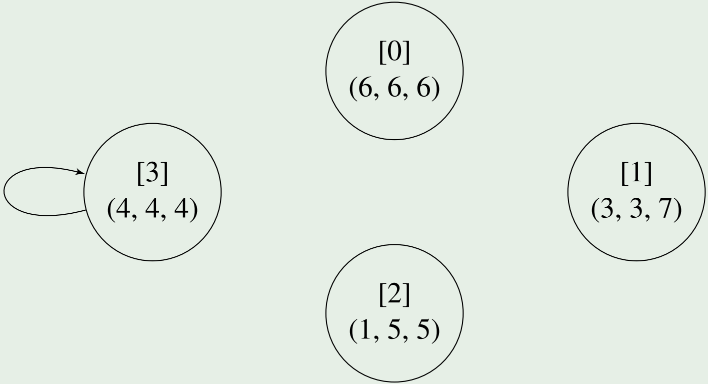
Symmetric Equilibria Example

Example



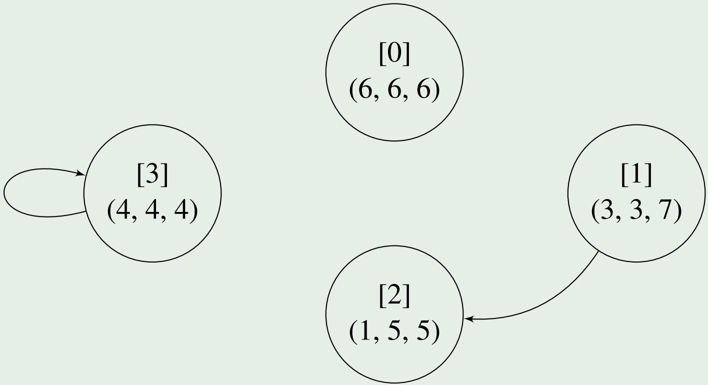
Symmetric Equilibria Example

Example



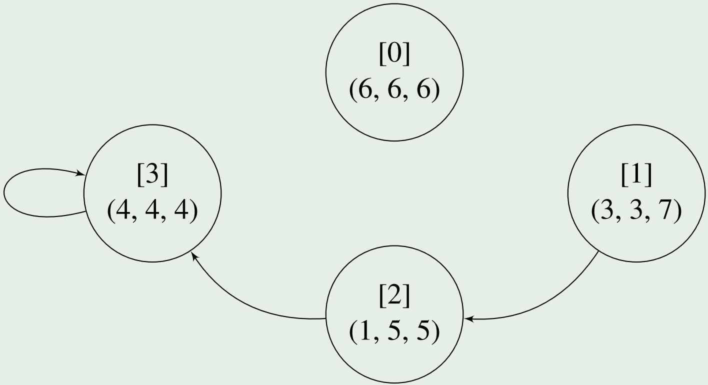
Symmetric Equilibria Example

Example



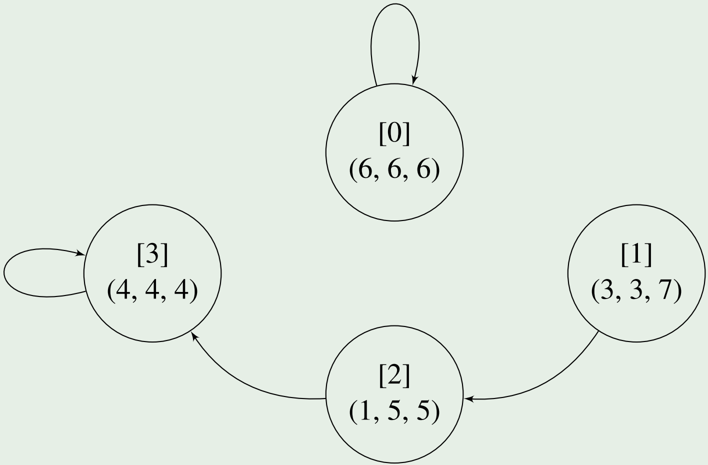
Symmetric Equilibria Example

Example



Symmetric Equilibria Example

Example



Asymmetric Equilibrium Example

Example

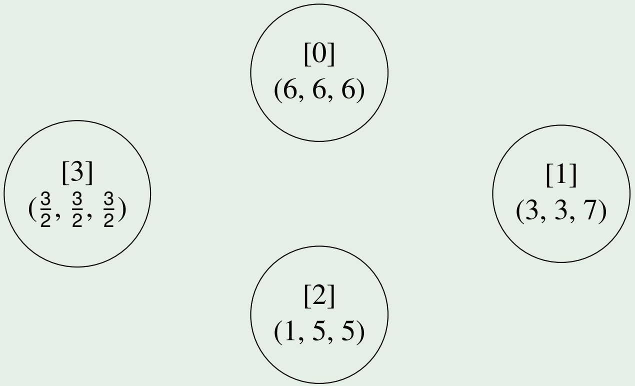
Consider an instance of G^* with three players and stage game payoff function defined as below.

State

Action		[0]	[1]	[2]	[3]
	0 (Cooperate)	6	3	1	-
	1 (Defect)	-	7	5	1.5

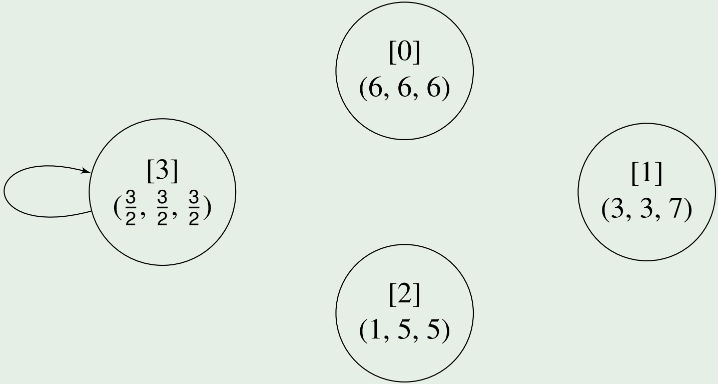
Asymmetric Equilibrium Example

Example



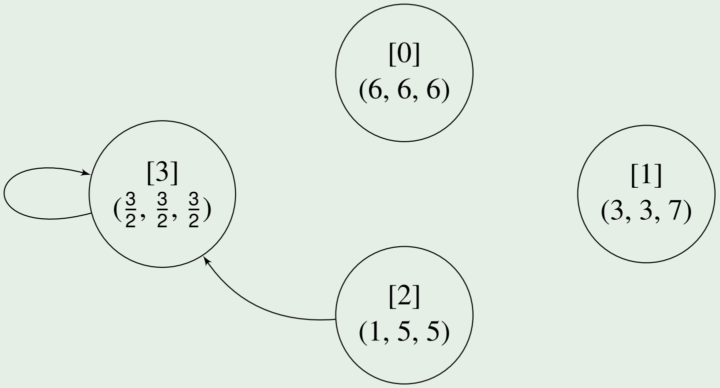
Asymmetric Equilibrium Example

Example



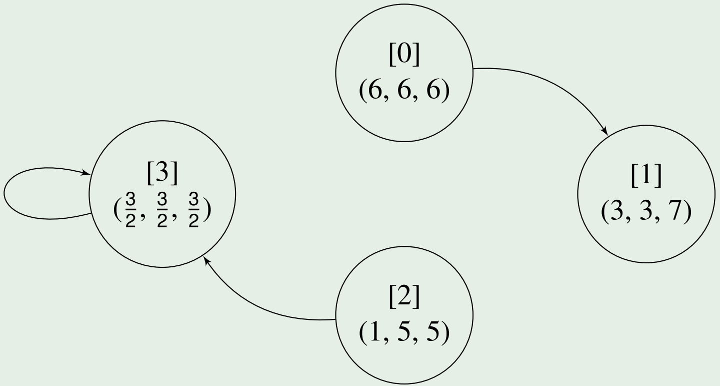
Asymmetric Equilibrium Example

Example



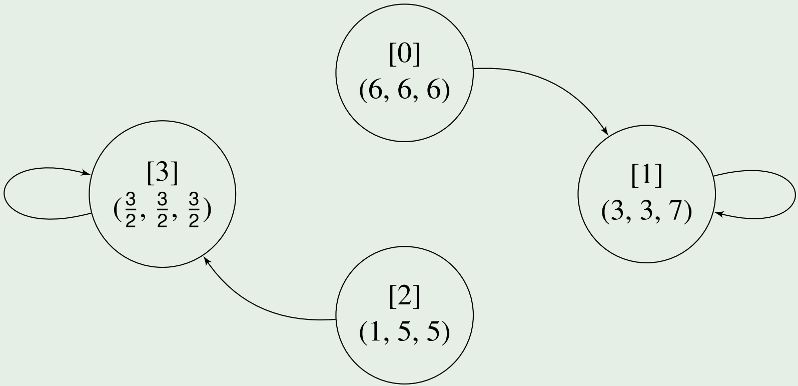
Asymmetric Equilibrium Example

Example



Asymmetric Equilibrium Example

Example



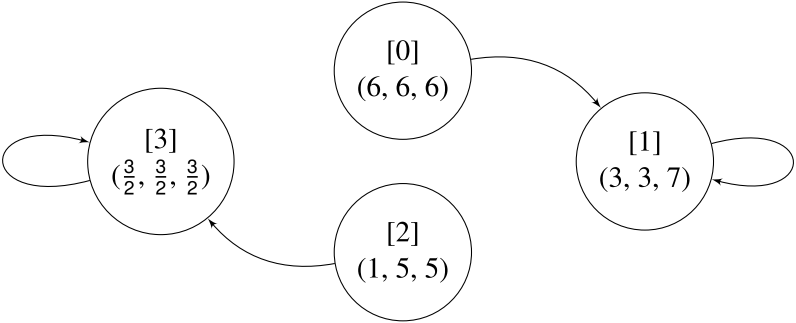
Mechanism Design Implication

Don’t give everyone the same grade for a group assignment.

Mechanism Design Implication

Don't give everyone the same grade for a group assignment.

		State			
		[0]	[1]	[2]	[3]
Action	0 (Cooperate)	6	3	1	-
	1 (Defect)	-	7	5	1.5

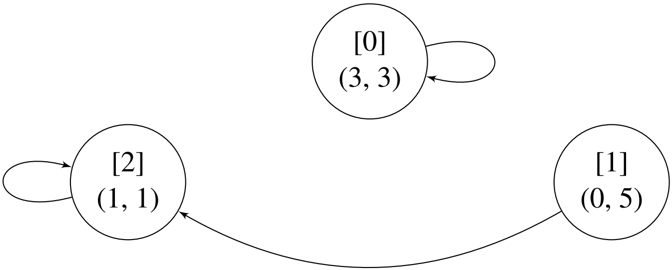


Tit for Tat

		Prisoner 1	
		Cooperate	Defect
Prisoner 2	Cooperate	3,3	0,5
	Defect	5,0	1,1

Tit for Tat

		Prisoner 1	
		Cooperate	Defect
Prisoner 2	Cooperate	3,3	0,5
	Defect	5,0	1,1



Ongoing work

- Generalizing two examples provided
- Identiy when equilibria are symmetric, when asymmetric
- Define subgames

Thank You

Thank You!

Come talk to me about related stuff I am working on now!

Davidson Cheng

email: d_cheng@coloradocollege.edu Paper is on arxiv under

2205.13772