

Guth's.

7.28. Apply Casimir's trick to compute

$$|u|^2 = |u_1|^2 + |u_2|^2 + u_1 \otimes u_2^* + u_2^* \otimes u_1$$

$$\text{where } u_1 = \frac{g_e}{(p_1 - p_3)^2 - m_c^2} [\bar{u}_4 \not{\epsilon}_2 (\not{p}_1 - \not{p}_3 + m_c) \not{\epsilon}_3^* u_1],$$

$$u_2 = \frac{g_e}{(p_1 + p_2)^2 - m_c^2} [\bar{u}_4 \not{\epsilon}_3^* (\not{p}_1 + \not{p}_2 + m_c) \not{\epsilon}_2 u_1].$$

$$\begin{aligned} |u_1|^2 &= (-i) [\bar{u}_4 \not{\epsilon}_2 (\not{p}_1 - \not{p}_3 + m_c) \not{\epsilon}_3^* u_1] u_1^* \not{\epsilon}_3 (\not{p}_1 - \not{p}_3 + m_c) \not{\epsilon}_2^* u_4 \\ &= (-i) [\bar{u}_a T_1 u_b] [\bar{u}_a T_1 u_b]^* \end{aligned}$$

$$\text{where } \bar{u}_a = \bar{u}_4, u_b = u_1, T_1 = \not{\epsilon}_2 (\not{p}_1 - \not{p}_3 + m_c) \not{\epsilon}_3^*.$$

By Casimir's trick, we have

$$\langle |u_1|^2 \rangle = \text{Tr} [T_1 (\not{p}_4 + m_4 c) \bar{T}_1 (\not{p}_1 + m_1 c)] (-i).$$

$$|u_2|^2 = (-i) [\bar{u}_4 T_2 u_1] [\bar{u}_4 T_2 u_1]^*$$

$$\Rightarrow \langle |u_2|^2 \rangle = \text{Tr} [T_2 (\not{p}_4 + m_4 c) \bar{T}_2 (\not{p}_1 + m_1 c)] (-i).$$

$$u_1 u_2^* = (\dots) [\bar{u}_4 \Gamma_1 u_1] [\bar{u}_4 \Gamma_2 u_2]^*$$

$$\Rightarrow \langle u_1 u_2^* \rangle = \text{Tr} [\Gamma_1 (\not{p}_4 + m_q c) \bar{\Gamma}_2 (\not{p}_1 + m_l c)] (\dots)$$

$$u_2 u_1^* = (\dots) [\bar{u}_4 \Gamma_2 u_1] [\bar{u}_4 \Gamma_1 u_2]^*$$

$$\Rightarrow \langle u_2^* u_2 \rangle = \text{Tr} [\Gamma_2 (\not{p}_4 + m_q c) \bar{\Gamma}_1 (\not{p}_1 + m_l c)] (\dots)$$

Adding in the appropriate factors we have

$$\langle M^2 \rangle = \langle M_1^2 \rangle + \langle M_2^2 \rangle + \langle M_1 M_2^* \rangle + \langle M_1^* M_2 \rangle$$

$$= \frac{g_e^4}{(p_1 - p_3^2 - m_e^2 c^2)^2} \text{Tr} \left\{ \Gamma_1 (\not{p}_4 + m_e c) \bar{\Gamma}_1 (\not{p}_1 + m_e c) \right\} \frac{1}{4}$$

$$+ \frac{g_e^4}{(p_1 + p_3^2 - m_e^2 c^2)^2} \text{Tr} \left\{ \Gamma_2 (\not{p}_4 + m_e c) \bar{\Gamma}_2 (\not{p}_1 + m_e c) \right\} \frac{1}{4}$$

$$+ \frac{g_e^4}{(p_1 - p_3^2 - m_e^2 c^2)(p_1 + p_3^2 - m_e^2 c^2)} \text{Tr} \left\{ \Gamma_1 (\not{p}_4 + m_e c) \bar{\Gamma}_2 (\not{p}_1 + m_e c) \right\} \frac{1}{4}$$

$$+ \frac{g_e^4}{((p_1 - p_3^2 - m_e^2 c^2)(p_1 + p_3^2 - m_e^2 c^2))} \text{Tr} \left\{ \Gamma_2 (\not{p}_4 + m_e c) \bar{\Gamma}_1 (\not{p}_1 + m_e c) \right\} \frac{1}{4}$$

$$\text{where } \Gamma_1 = \not{p}_2 (\not{p}_1 - \not{p}_3 + m_l c) \not{p}_3^*, \quad \bar{\Gamma}_2 = \not{p}_3^* (\not{p}_1 + \not{p}_2 + m_l c) \not{p}_2$$