Schwartz
3.5 ab

(a) $\mathcal{L} = -\frac{1}{2}\phi(\partial_\mu^2\phi) + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

eqm: $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial \mathcal{L}}{\partial\phi} = 0.$

$\frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)} = -\phi(\partial_\mu\phi)$ $\quad$ $\frac{\partial \mathcal{L}}{\partial\phi} = m^2\phi - \frac{\lambda}{3!}\phi^3$

$\Rightarrow$ EQM: $-\partial_\mu[\phi\partial_\mu\phi] - m^2\phi + \frac{\lambda}{3!}\phi^3 = 0$

Use ansatz $\phi(x) = c$, we have

$$\frac{\lambda}{6}c^3 = m^2 c$$

$$\frac{\lambda}{6}c^2 = m^2, \qquad c = \pm\frac{6}{\lambda}m$$

The ground state would be $c = -\frac{6}{\lambda}m$, the other soln is $c = +\frac{6}{\lambda}m$.

(b) This $\mathbb{Z}_2$ symmetry is clearly not respected by $\phi = c$, because we would swap the ground and excited state.

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