8. show $(\vec{\sigma} \cdot \vec{w})^2 = \vec{w}^2$

$(\vec{\sigma} \cdot \vec{w})^2 = (\sigma_1 w_1 + \sigma_2 w_2 + \sigma_3 w_3)^2$

$= \sigma_1^2 w_1^2 + \sigma_1 \sigma_2 w_1 w_2 + \sigma_1 \sigma_3 w_1 w_3$
$+ \sigma_2 \sigma_1 w_1 w_2 + \sigma_2^2 w_2^2 + \sigma_2 \sigma_3 w_2 w_3$
$+ \sigma_3 \sigma_1 w_1 w_3 \quad \sigma_3 \sigma_2 w_2 w_3 \quad \sigma_3 \sigma_2 w_3^2$

The off diagonal terms cancel each other by
the anti commutation rule $\{\sigma_i, \sigma_j\} = 2 \delta_{ij}$

$\Rightarrow$ we get $(\vec{\sigma} \cdot \vec{w})^2 = \vec{w}^2$.