Kittel

6.1. Show that the kinetic energy of 3D gas of N free electrons at 0K is $U_0 = \frac{3}{5} N \varepsilon_F$.

At T=0k, the occupation is binary, thus

$$U_0 = \int_0^{\varepsilon_F} \frac{dN}{d\varepsilon} \varepsilon \, d\varepsilon$$

$$= \int_0^{\varepsilon_F} \frac{V}{2\pi^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{3/2} \, d\varepsilon$$

$$= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{2}{5} \varepsilon^{5/2} \Big|_0^{\varepsilon_F}$$

$$= \frac{V}{3\pi^2} \frac{3}{2} \left(\frac{2m\,\varepsilon_F}{\hbar^2}\right)^{3/2} \frac{3}{5} \varepsilon_F$$

$$= \boxed{\frac{3}{5} N \varepsilon_F} \qquad \text{since } N = \frac{V}{3\pi^2} \left(\frac{2m\varepsilon}{\hbar^2}\right)^{3/2}$$

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