

$$\underline{1312.3824.5}$$

5. show equivalence as following

$$e^{-(p/2)\sigma_z} = \cosh(p/2) \mathbb{I} - \sinh(p/2) \sigma_z$$

$$\begin{pmatrix} \cosh p & -\sinh p \\ -\sinh p & \cosh p \end{pmatrix}.$$

2nd-Rank spinors transform as.

$$X \rightarrow \Lambda X \Lambda^\dagger, \text{ so we have}$$

$$X = \sum_{\mu} x^{\mu} \sigma^{\mu} \rightarrow (\cosh p/2 \mathbb{I} - \sinh p/2 \sigma_z) \sum_{\mu} x^{\mu} \sigma^{\mu} (\cosh p/2 \mathbb{I} - \sinh p/2 \sigma_z),$$

$$\text{since } e^{-(p/2)\sigma_z \dagger} = e^{-(p/2)\sigma_z}$$

$$\Rightarrow X \rightarrow (\cosh \mathbb{I} - \sinh \sigma_3) \begin{Bmatrix} x^0 \sigma_0 \\ x^1 \sigma_1 \\ x^2 \sigma_2 \\ x^3 \sigma_3 \end{Bmatrix} (\cosh \mathbb{I} - \sinh \sigma_3).$$

$$= \begin{Bmatrix} x^0 \sigma_0 \\ x^1 \sigma_1 \\ x^2 \sigma_2 \\ x^3 \sigma_3 \end{Bmatrix} \cosh^2 - \sinh \cosh \begin{Bmatrix} x^0 \sigma_3 \\ x^1 (-i) \sigma_2 \\ x^2 i \sigma_1 \\ x^3 I \end{Bmatrix} - \sinh \cosh \begin{Bmatrix} x^0 \sigma_3 \\ x^1 i \sigma_2 \\ x^2 (-i) \sigma_1 \\ x^3 I \end{Bmatrix} \\ + \sinh^2 \begin{Bmatrix} x^0 I \\ x^1 i i \sigma_1 \\ x^2 (-i)(-i) \sigma_2 \\ x^3 \sigma_3 \end{Bmatrix}$$

$$= \sigma^0 (\cosh^2 x^0 + \sinh^2 x^0 - \sinh \cosh 2x^3)$$

$$+ \sigma^1 (\cosh^2 x^1 - \sinh^2 x^1)$$

$$+ \sigma^2 (\cosh^2 x^2 - \sinh^2 x^2)$$

$$+ \sigma^3 (\cosh^2 x^3 + \sinh^2 x^3 - \sinh \cosh 2x^0)$$

$$= \sigma^0 (\cosh_p x^0 - \sinh_p x^3)$$

$$+ \sigma^1 (1)$$

$$+ \sigma^2 (1)$$

$$+ \sigma^3 (\cosh_p x^3 - \sinh_p x^0)$$

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