

Townsend (2.90), II.1

By defn, $R_{\mu\nu\rho}{}^{\sigma} \xi_{\sigma} = D_{\mu} D_{\nu} \xi_{\rho} - D_{\nu} D_{\mu} \xi_{\rho}$

$$= D_{\rho} D_{\nu} \xi_{\mu} - D_{\nu} D_{\rho} \xi_{\mu} \quad (1)$$

obtained by
(anti)symmetries,
also $D_a \xi^b = -D_b \xi^a$
if ξ killing

$$= D_{\mu} D_{\rho} \xi_{\nu} - D_{\rho} D_{\mu} \xi_{\nu}$$

$$= D_{\mu} D_{\rho} \xi_{\nu} + D_{\rho} D_{\nu} \xi_{\mu} \quad (2)$$

$$(1)+(2) = 2 R_{\mu\nu\rho}{}^{\sigma} \xi_{\sigma} = 2 D_{\rho} D_{\nu} \xi_{\mu}$$

$$\Rightarrow \boxed{R_{\mu\nu\rho}{}^{\sigma} \xi_{\sigma} = D_{\rho} D_{\nu} \xi_{\mu}}$$

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