

1312.3824.12.

12. Let $\beta \equiv \gamma^0$, $\vec{\alpha} \equiv \gamma^0 \vec{\gamma}$, show that Dirac eq. can be written as $H\psi = E\psi$, where $H \equiv \vec{\alpha} \cdot \vec{p} + \beta m$.

The convention of $\gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$, $\vec{\gamma} = \begin{pmatrix} \vec{\sigma} & \\ & -\vec{\sigma} \end{pmatrix}$ was adopted, define $\psi = \begin{pmatrix} \phi_R \\ \chi_L \end{pmatrix}$, the original Dirac eq. was written as.

$$\begin{pmatrix} -m & E + \vec{\sigma} \cdot \vec{p} \\ E - \vec{\sigma} \cdot \vec{p} & -m \end{pmatrix} \psi = 0, \text{ equivalent to.}$$

$$(-m\mathbb{I} + E\gamma^0 - \vec{\gamma} \cdot \vec{p})\psi = 0.$$

This equation is purely algebraic, so we can safely apply a change of basis via a nonsingular transformation $\beta = \gamma^0$ to obtain

$$\beta (-m\mathbb{I} + E\gamma^0 - \vec{\gamma} \cdot \vec{p})\psi = 0$$

$$\Rightarrow (-m\beta + E\mathbb{I} - \vec{\alpha} \cdot \vec{p})\psi = 0.$$

$$\Rightarrow E\psi = (\vec{\alpha} \cdot \vec{p} + m\beta)\psi.$$