Deriving the Dirac Equation

- Begin with massive KG and assume vector fields.

$$(\Box^2 - m^2)\psi = 0.$$

$$(p^\mu p_\mu - m^2)\psi = 0.$$

ψ being a vector implies $m^2 = m^2 \mathbb{I}$, $p^\mu p_\mu = p^\mu p_\mu \mathbb{I}$
$= m^2 \mathbb{I}$.

- writing the vector $\vec{p} = p^\mu \vec{e}_\mu$ in component-vector notation, we have

$$p^\mu p_\mu = p^\mu p^\nu g_{\mu\nu}.$$

$$\Rightarrow g_{\mu\nu} = \mathbb{I} \times \mathrm{diag}(1,-1,-1,-1)$$

or equivalently $\mathbb{I} \times \mathrm{diag}(-1,1,1,1)$.

- linear algebra tells us $g_{\mu\nu}$ is a special $\binom{0}{2}$ tensor that is equal to $\vec{e}_\mu \otimes \vec{e}_\nu$ or $\vec{e}_\mu \vec{e}_\nu$. ~~while~~ order matters.

$$\Rightarrow \vec{e}_\mu \otimes \vec{e}_\nu = \mathbb{I} \times \mathrm{diag}(1,-1,-1,-1) = \mathbb{I}\, g_{\mu\nu}$$

~~This is satisfied by any basis obeying dirac~~

Moreover, we need $\{\vec{e}_\mu, \vec{e}_\nu\} = 2g^{\mu\nu}$

(Dirac Algebra) or (Clifford Algebra).

- This allows us to write KG as $\begin{cases}(\vec{e}_\mu p^\mu - m)\psi = 0, \\ (\vec{e}_\mu p^\mu + m)\psi = 0,\end{cases}$

replacing p^μ with $i\partial_\mu$ gives

$$(i\vec{e}_\mu \partial^\mu - m)\psi = 0$$
$$(i\vec{e}_\mu \partial^\mu + m)\psi = 0$$
(Dirac Eq.).