$$\int_0^{\pi} \Phi(a,\theta) P_{\cancel{m}\, l}[\cos\theta] \sin\theta \, d\theta$$

Jackson 3.1

$$= \int_0^{\pi/2} V P_l[\cos\theta] \sin\theta \, d\theta$$

$$= \int_0^{\pi} \left\{ \sum_{m=0}^{\infty} [A_m a^m + B_m a^{-(m+1)}] P_m[\cos\theta] P_l[\cos\theta] \right\} \sin\theta \, d\theta$$

$$= \sum_{m=0}^{\infty} \left\{ [A_m a^m + B_m a^{-(m+1)}] \int_0^{\pi} P_m[\cos\theta] P_l[\cos\theta] \sin\theta \, d\theta \right\}$$

$$= \sum_{m=0}^{\infty} [A_m a^m + B_m a^{-(m+1)}] \frac{2}{2l+1} \delta_{ml}$$

$$= [A_l a^l + B_l a^{-(l+1)}] \frac{2}{2l+1}$$

$$\Rightarrow \left[A_l a^l + B_l a^{-(l+1)}\right] \frac{2}{2l+1} = \int_0^{\pi/2} V P_l[\cos\theta] \sin\theta \, d\theta$$

$$\left[A_l b^l + B_l b^{-(l+1)}\right] \frac{2}{2l+1} = \int_{\pi/2}^{\pi} V P_l[\cos\theta] \sin\theta \, d\theta.$$

$$\boxed{\begin{aligned} A_l a^l + B_l a^{-(l+1)} &= \frac{2l+1}{2} \int_0^{\pi/2} V P_l[\cos\theta] \sin\theta \, d\theta \\ A_l b^l + B_l b^{-(l+1)} &= \frac{2l+1}{2} \int_{\pi/2}^{\pi} V P_l[\cos\theta] \sin\theta \, d\theta \end{aligned}}$$

$$\int_0^{\pi/2} V P_l [\cos\theta] \sin\theta d\theta \quad , \quad \int_{\pi/2}^{\pi} V P_l [\cos\theta] \sin\theta d\theta ,$$

up to $l = 4$,

Jackson 3.

Let $x = \cos\theta$

$\frac{dx}{d\theta} = -\sin\theta$

$dx = -\sin\theta d\theta$.

$P_0 = 1$

$P_1 = \cos\theta = x$

$P_2 = \frac{1}{2}(3(\cos\theta)^2 - 1) = \frac{1}{2}(3x^2 - 1)$

$P_3 = \frac{1}{2}(5x^3 - 3x)$.

$P_4 = \frac{1}{8}(35x^4 - 30x^2 + 3)$.

$$\int_1^0 V P_l [x] [-dx]$$

$$\int_0^{\pi/2} V P_l [\cos\theta] \sin\theta d\theta = \int_1^0 V P_l [x] [-1] dx = \boxed{\int_0^1 V P_l [x] dx.}$$

$$\int_{\pi/2}^{\pi} V P_l [\cos\theta] \sin\theta d\theta = \int_0^{-1} V P_l [x] [-1] dx = \boxed{\int_{-1}^0 V P_l [x] dx}$$

~~Let~~ Let $\boxed{\int_0^1 V P_l [x] dx = C_l^+ ,}$ $\boxed{\int_{-1}^0 V P_l [x] dx = C_l^-}$.

$\boxed{\int_0^1 P_l [x] dx = C_l^+ \quad \int_{-1}^0 P_l [x] dx = C_l^- .}$

$\cancel{C_0^+ = \int_0^1 V(1)\,dx = V}$

$\cancel{C_0^- = \int_{-1}^0 V(1)\,dx = Vx\Big|_{-1}^{0} = 0-(-V) = V.}$

$\boxed{\text{Jackson 3.1}}$

C

$$C_0^+ = \boxed{1} \quad \checkmark$$

$$C_0^- = \boxed{1}$$

$$C_1^+ = \int_0^1 x\,dx = \boxed{\tfrac{1}{2}}. \quad \checkmark$$

$$C_1^- = \boxed{-\tfrac{1}{2}}$$

$$C_2^+ = \tfrac{1}{2} - \tfrac{1}{2} = \boxed{0.} \quad \checkmark$$

$$C_2^- = -\tfrac{1}{2} - \left(-\tfrac{1}{2}\right) = \boxed{0.}$$

$$C_3^+ = \frac{5}{8} - \frac{3}{4} = \boxed{-\frac{1}{8}} \quad \checkmark.$$

$$C_3^- = \frac{5}{8} - \frac{3}{4} = \boxed{\frac{1}{8}}$$

$$C_4^+ = \frac{7}{8} - \frac{5}{4} + \frac{3}{8} = \boxed{0.}$$

$$C_4^- = -\frac{7}{8} - \left(-\frac{5}{4}\right) + \left(-\frac{3}{8}\right). = \boxed{0}$$

$$A_\ell a^\ell + B_\ell a^{-(\ell+1)} = \left(\frac{2\ell+1}{2} V\right) C_\ell^+$$

$$A_\ell b^\ell + B_\ell b^{-(\ell+1)} = \left(\frac{2\ell+1}{2} V\right) C_\ell^-$$

Jackson 3.1

$$\cancel{A_\ell a^\ell + B_\ell a^{-(\ell+1)}}$$

$$\cancel{A_\ell a^\ell + B_\ell a}$$

$$\left[A_\ell b^\ell + B_\ell b^{-(\ell+1)}\right]\left[\frac{a^{-(\ell+1)}}{b^{-(\ell+1)}}\right] = A_\ell b^\ell \left[\frac{a^{-(\ell+1)}}{b^{-(\ell+1)}}\right] + B_\ell a^{-(\ell+1)}$$

$$= \left(\frac{a}{b}\right)^{-(\ell+1)} \left(\frac{2\ell+1}{2} V\right) C_\ell^-$$

$$A_\ell \left[a^\ell - b^\ell \left[\frac{a^{-(\ell+1)}}{b^{-(\ell+1)}}\right]\right] = \left(\frac{2\ell+1}{2} V\right) C_\ell^+ - \left(\frac{a}{b}\right)^{-(\ell+1)} \left(\frac{2\ell+1}{2} V\right) C_\ell^-$$

$$A_\ell \left[a^\ell - b^\ell \left(\frac{a}{b}\right)^{-(\ell+1)}\right] = \left[\frac{2\ell+1}{2} V\right]\left[C_\ell^+ - \left(\frac{a}{b}\right)^{-(\ell+1)} C_\ell^-\right]$$

do ~~this~~ : write a code to do this

$$A_l = \frac{\left[\frac{2l+1}{2}\right][V]\left[C_l^+ - \left(\frac{a}{b}\right)^{-(l+1)} C_l^-\right]}{\left[a^l - b^l\left(\frac{a}{b}\right)^{-(l+1)}\right]}$$

$$\left[A_l a^l + B_l a^{-(l+1)}\right]\left(\frac{b}{a}\right)^l = A_l b^l + B_l a^{-(l+1)}\left(\frac{b}{a}\right)^l$$

$$= \left(\frac{b}{a}\right)^l\left(\frac{2l+1}{2}\right)V C_l^+$$

$$\Rightarrow B_l\left[a^{-(l+1)}\left(\frac{b}{a}\right)^l - b^{-(l+1)}\right] = \left(\frac{b}{a}\right)^l\left(\frac{2l+1}{2}\right)V C_l^+ - \left(\frac{2l+1}{2}\right)V C_l^-$$

$$= \left(\frac{2l+1}{2}\right)V\left[\left(\frac{b}{a}\right)^l C_l^+ - C_l^-\right]$$

$$\Rightarrow B_l = \frac{\left[\frac{2l+1}{2}\right]V\left[\left(\frac{b}{a}\right)^l C_l^+ - C_l^-\right]}{\left[a^{-(l+1)}\left(\frac{b}{a}\right)^l - b^{-(l+1)}\right]}$$