Notes

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1 Realized volatility (RV)

Realized volatility -
So the basic formula is

$$RV_t = \sum_{i=1}^{M} r_{i,t}^2 \tag{1}$$

where $M = \frac{1}{\Delta}$, 1 represents the trading time of day t, Δ represents the sum number of transaction times of trading frequency, and Δ-period intraday return is defined by

$$r_{i,t} = log(P_{t-1+i\Delta}) - log(P_{t-1+(i-1)\Delta}) \tag{2}$$

2 Heterogeneous autoregressive (HAR) models

- HAR
  Basis idea of this model is the different perception of volatility:
  - short-term (day)
  - medium-term (week)
  - long-term (month)

  This can be written by the following equation:

$$RV_t = \beta_1 RV_{t-1} + \beta_2 RV_{t-1|t-5} + \beta_3 RV_{t-1|t-22} + u_t \tag{3}$$

  where

$$RV_{t-j|t-h} = \frac{1}{h-j+1} \sum_{i=j}^{h} RV_{t-i}, j \leq h \tag{4}$$

- HAR-j
  In this model extend HAR model provides by using bi-power variation (BPV) for dividing the total volatility into continuous and discontinuous volatility. The HAR-J model is famous for the fact that it includes a measure of "jump variation". This can be written by the following equation:

$$RV_t = \beta_0 + \beta_1 RV_{t-1} + \beta_2 RV_{t-1|t-5} + \beta_3 RV_{t-1|t-22} + \beta_J J_{t-1} + u_t, \tag{5}$$

  where

$$J_t = max[RV_t - BPV_t, 0] \tag{6}$$

  and

$$BPV_t = u_1^{-2} \sum_{i=1}^{M-1} |r_{t,i}||r_{t,i+1}| \tag{7}$$

  where $u_1 = \sqrt{\frac{2}{\pi}} = E(Z)$, where Z - is a standart normal distribution

- CHAR
  This motivates the alternative CHAR model, which only includes measures of the continuous variation on the right-hand side of Eq. (7):

$$RV_t = \beta_0 + \beta_1 BPV_{t-1} + \beta_2 BPV_{t-1|t-5} + \beta_3 BPV_{t-1|t-22} + u_t \qquad (8)$$

- SHAR
  Semi-variance HAR model (SHAR), the main idea being to decompose the total volatility into positive and negative volatility based on the semi-variance measures.

$$RV_t = \beta_0 + \beta_1^+ RV_{t-1}^+ + \beta_1^- RV_{t-1}^- + \beta_2 BRV_{t-1|t-5} + \beta_3 BRV_{t-1|t-22} + u_t \quad (9)$$

  where

$$\beta_1^+ RV_{t-1}^+ = \sum_{i=1}^{M} r_{t,i}^2 I(r_{t,i} > 0) \qquad (10)$$

  and $\beta_1^- RV_{t-1}^-$ defined similarly.

- HARQ
  This suggests that adjusting for measurement errors in the daily lagged realized volatilities is likely to play a more important role than adjusting for the weekly and monthly coefficients. So to have under that were proposed using simple HARQ and more complicated full-HARQ (HARQF)

$$RV_t = \beta_0 + (\beta_1 + \beta_{1Q} RQ_{t-1}^{1/2}) RV_{t-1} + \beta_2 RV_{t-1|t-5} + \beta_3 RV_{t-1|t-22} + u_t \quad (11)$$

  where

$$RQ_t = \frac{1}{3\Delta} \sum_{i=1}^{1/\Delta} r_{t,i}^4 \qquad (12)$$

- HARQF

$$V_t = \beta_0 + (\beta_1 + \beta_{1Q} RQ_{t-1}^{1/2}) RV_{t-1} + (\beta_2 + \beta_{2Q} RQ_{t-1|t-5}^{1/2}) RV_{t-1|t-5}$$
$$+ (\beta_3 + \beta_{3Q} RQ_{t-1|t-22}^{1/2}) RV_{t-1|t-22} + u_t \qquad (13)$$