Ph22- Assignment 3

Daniel Chica

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PROBLEM 1

I've had to do this assignment multiple times and I am still having issues, but I think the Runge-Kutta integration for the constrained three-body problem gave me the expected results. To begin I rewrote the routine from the previous assignment to avoid relying on strings as that complicated the coding significantly. I set up the variables that represent the physics and created a function that would use the integrator to calculate the velocity. Then I would use this velocity in another integrator to get the position as a function of time. I performed this for two cases, one where the initial position of the asteroid was at L5, so $\alpha = \frac{\pi}{3}$ and another at $\alpha = \frac{\pi}{2}$.

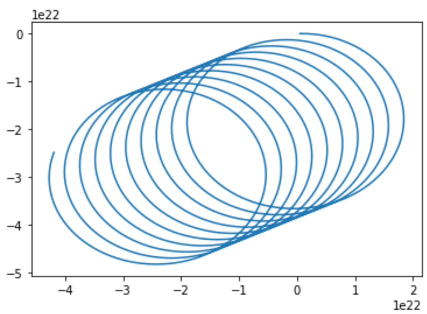

Figure 1: $\alpha = \frac{\pi}{3}$

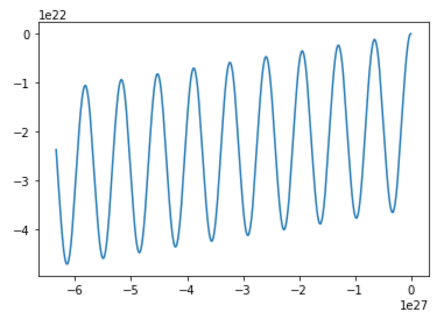

Figure 2: $\alpha = \frac{\pi}{2}$

At the Lagrange Points, we expect oscillatory motion due to the stable position of the asteroid. As a result, we should see orbits that might slightly phase as the other bodies move. Figure 1 shows exactly that, with the position of the asteroid showing stable signs. To contrast this, I looked at the case when the initial position of the body was elsewhere. Figure 2 shows that when this occurs, we get non-orbiting motion across the plane as expected. At this position, the body is unstable.

PROBLEM 2

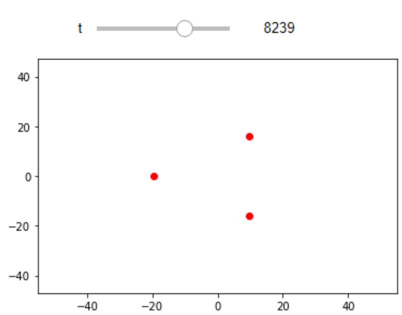

Figure 3: Triangular Orbit Positions

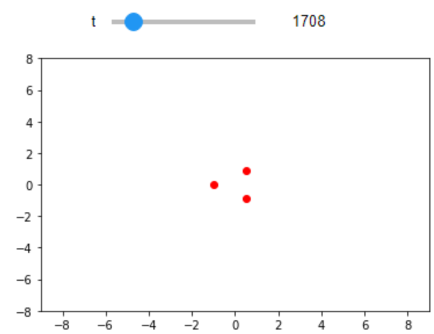

Figure 4: Triangular Orbit Velocities

Here I applied a more general routine for the three-body problem. I set up the initial positions such that they formed a triangle with no initial velocity and saw how the orbits changed over time. According to Figure 3, the system retains its triangular shape but spreads out a bit as time passes. I plotted the velocities to see how they changed as a function of time and noticed that they were very similar to those of the position in terms of orientation, just with a smaller magnitude. According to the write up, the velocities should be related to the masses by the equation $v = \sqrt{GM/d}$ and since all bodies have the same mass, we expect the velocities for all bodies to behave similarly. Unfortunately I expected an inverse relation to the distance as the positions spread out more, but instead the relation seems to be linear. However, all the velocities do seem to have similar magnitudes as expected.

PROBLEM 3

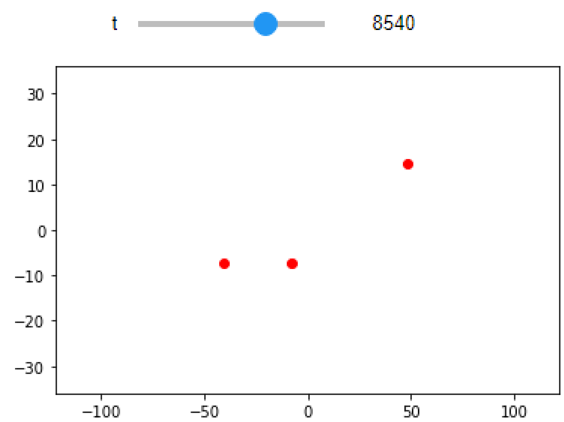

Figure 5: 8-Shape Orbit

I set up the initial positions and velocities according to what the assignment said and ran the integrator. I expected the elliptical motion but instead I see the bodies move apart. I expect that the force vectors are pointing in the opposite direction which is what seems to be pushing the bodies apart, but a change in the magnitude of the force did not yield a solution. It is interesting to note that in this case, one of the bodies stays relatively still while the others move apart.