Appendices

A Notation

Symbol	Description	Value
M	Number of individuals in the sample, we use subscript j to refer to an individual.	200
T	Time over which the study is run.	120 days

Table 1: Table of parameters associated with the simulated data and their values

Symbol *State variables*	**Description**
$Z_{j,t} \in \mathbf{Z}$	Data on the antibody titre at time t for individual j.
Z_j^0	Initial titre (titre at first value of t for individual j).
$X_{j,t} \in \mathbf{X}$	Model estimated antibody titre at time t for individual j.
$\mathbf{E} = \{E_1, \dots, E_j, \dots, E_M\}$	Vector of exposure statuses (binary vector) for each individual j. A superscript (i) specifies a specific value in the Markov chain.
$n_{\mathbf{E}}$	Total number of individuals exposed. . A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E_1} = \{j_1, \dots, j_{j^*} \dots, j_{n_{\mathbf{E}}}\}$	Vector of individuals j^* who are exposed. A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E_0} = \{j_1, \dots, j_{j^*} \dots, j_{M-n_{\mathbf{E}}}\}$	Vector of individuals j^* who are not exposed. A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E}^\tau = \{E^\tau_{j_1}, \dots, E^\tau_{j_{j^*}}, \dots, E^\tau_{j_{n_{\mathbf{E}}}}\}$	Vector of exposure times for each individual j. A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{I} = \{I_1, \dots, I_{j^*}, \dots, I_{n_{\mathbf{E}}}\}$	Vector of infection statuses (binary vector) for each individual j. A superscript (i) specifies a specific value in the Markov chain.
$n_{\mathbf{I}}$	Total number of individuals infected.. A superscript (i) specifies a specific value in the Markov chain.
$\theta_{cop} = \{\beta_0, \beta_1\}$	Fitted parameters for the correlate of protection model
$\theta_{ab} = \{a, b, c, \alpha\}$	Fitted parameters for the antibody kinetics model
$\theta = \{\theta_{cop}, \theta_{ab}, \sigma\}$ *Functions*	All fitted parameters in the model. A superscript (i) specifies a specific value in the Markov chain.
$P(\theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I} \vert \mathbf{Z})$	Posterior distribution function given inputs θ, $\mathbf{E}$, $\mathbf{E}^\tau$, $\mathbf{I}$ and data $\mathbf{Z}$
$\mathcal{L}(\mathbf{Z} \vert \theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I})$	Likelihood function given inputs θ, $\mathbf{E}$, $\mathbf{E}^\tau$, $\mathbf{I}$ and data $\mathbf{Z}$.
$\mathcal{L}_{E_j=0}(Z_j, \vert I_j, E^\tau_j, E_j, \theta)$	Likelihood function for individual j who is not exposed
$\mathcal{L}_{E_j=1}(Z_j, \vert I_j, E^\tau_j, E_j, \theta)$	Likelihood function for individual j who is exposed
$P_t(E^\tau_j \vert E_j)$	Likelihood of an exposure at time E^τ_j given individual j is exposed
$X_{j,t} = F_{ab}(t, I_i, E^\tau_j, \theta_{ab}, Z^0_j)$	Deterministic function for the estimated antibody titre at time t for individual j and starting titre values from the data, Z^0_j for an exposure at time E^τ_j and infection status I_j.
$f^1_{ab}(s, a, b, c)$	The function which determines the antibody titres at time s after E^τ_j
$f^2_{ab}(Z^0_j, \alpha)$	The function which scales the trajectory given pre-titre Z^0_j.
$P_{cop}(I_j \vert X_{j,E^\tau_j}, \theta_{cop})$	Likelihood for the correlate of protection for an individual j with an exposure at time E^τ_j, and estimated titre value X_{j,E^τ_j} and infection status I_j.
$f_{cop}(X_{j,t}, \beta_0, \beta_1)$	Function describing the correlate of protection for infection at time t. (logistic).
$P_{obs}(Z_{j,t} \vert X_{j,t}, \sigma)$	Likelihood of the observation model for the data $Z_{j,t}$ given model-estimated titre values $X_{j,t}$ for individual j at time t.
$\pi(\theta) = \pi(a)\pi(b) \dots \pi(\sigma)$	Prior distributions for all fitted parameters in the model.

Table 2: Symbols used in calculating the posterior distribution

Symbol	Description
N	Length of chain in metropolois hasting algorihtm
$q_\theta\left(\cdot\|\theta^{(i)}\right)$	Proposal distribution for the values of θ at given state i.
$q_I\left(\cdot\|I_j^{(i)}\right)$	Proposal distribution for the values of I_j for individual j at given state i (See XX).
$q_k\left(\cdot\|k\right)$	Proposal distribution for a new model given model is at $\mathcal{M}_k$ (See XX).
$q_\tau\left(\cdot\|E_j^{\tau,(i)}\right)$	Proposal distribution for a new time of exposure for individual j for a given state i.(See XX).

Table 3: Symbols used in the mcmc algorithms