

Appendices

A Notation

Symbol	Description	Value
M	Number of individuals in the sample, we use subscript j to refer to an individual.	200
T	Time over which the study is run.	120 days

Table 1: Table of parameters associated with the simulated data and their values

Symbol	Description
State variables	
$Z_{j,t} \in \mathbf{Z}$	Data on the antibody titre at time t for individual j .
Z_j^0	Initial titre (titre at first value of t for individual j).
$X_{j,t} \in \mathbf{X}$	Model estimated antibody titre at time t for individual j .
$\mathbf{E} = \{E_1, \dots, E_j, \dots, E_M\}$	Vector of exposure statuses (binary vector) for each individual j . A superscript (i) specifies a specific value in the Markov chain.
$n_{\mathbf{E}}$	Total number of individuals exposed. . A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E}_1 = \{j_1, \dots, j_{j^*}, \dots, j_{n_{\mathbf{E}}}\}$	Vector of individuals j^* who are exposed. A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E}_0 = \{j_1, \dots, j_{j^*}, \dots, j_{M-n_{\mathbf{E}}}\}$	Vector of individuals j^* who are not exposed. A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{E}^\tau = \{E_{j_1}^\tau, \dots, E_{j_{j^*}}^\tau, \dots, E_{j_{n_{\mathbf{E}}}}^\tau\}$	Vector of exposure times for each individual j . A superscript (i) specifies a specific value in the Markov chain.
$\mathbf{I} = \{I_1, \dots, I_{j^*}, \dots, I_{n_{\mathbf{E}}}\}$	Vector of infection statuses (binary vector) for each individual j . A superscript (i) specifies a specific value in the Markov chain.
$n_{\mathbf{I}}$	Total number of individuals infected.. A superscript (i) specifies a specific value in the Markov chain.
$\theta_{cop} = \{\beta_0, \beta_1\}$	Fitted parameters for the correlate of protection model
$\theta_{ab} = \{a, b, c, \alpha\}$	Fitted parameters for the antibody kinetics model
$\theta = \{\theta_{cop}, \theta_{ab}, \sigma\}$	All fitted parameters in the model. A superscript (i) specifies a specific value in the Markov chain.
Functions	
$P(\theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I} \mathbf{Z})$	Posterior distribution function given inputs $\theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I}$ and data \mathbf{Z}
$\mathcal{L}(\mathbf{Z} \theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I})$	Likelihood function given inputs $\theta, \mathbf{E}, \mathbf{E}^\tau, \mathbf{I}$ and data \mathbf{Z} .
$\mathcal{L}_{E_j=0}(Z_j, I_j, E_j^\tau, E_j, \theta)$	Likelihood function for individual j who is not exposed
$\mathcal{L}_{E_j=1}(Z_j, I_j, E_j^\tau, E_j, \theta)$	Likelihood function for individual j who is exposed
$P_t(E_j^\tau E_j)$	Likelihood of an exposure at time E_j^τ given individual j is exposed
$X_{j,t} = F_{ab}(t, I_i, E_j^\tau, \theta_{ab}, Z_j^0)$	Deterministic function for the estimated antibody titre at time t for individual j and starting titre values from the data, Z_j^0 for an exposure at time E_j^τ and infection status I_j .
$f_{ab}^1(s, a, b, c)$	The function which determines the antibody titres at time s after E_j^τ
$f_{ab}^2(Z_j^0, \alpha)$	The function which scales the trajectory given pre-titre Z_j^0 .
$P_{cop}(I_j X_{j,E_j^\tau}, \theta_{cop})$	Likelihood for the correlate of protection for an individual j with an exposure at time E_j^τ , and estimated titre value X_{j,E_j^τ} and infection status I_j .
$f_{cop}(X_{j,t}, \beta_0, \beta_1)$	Function describing the correlate of protection for infection at time t . (logistic).
$P_{obs}(Z_{j,t} X_{j,t}, \sigma)$	Likelihood of the observation model for the data $Z_{j,t}$ given model-estimated titre values $X_{j,t}$ for individual j at time t .
$\pi(\theta) = \pi(a)\pi(b) \dots \pi(\sigma)$	Prior distributions for all fitted parameters in the model.

Table 2: Symbols used in calculating the posterior distribution

Symbol	Description
N	Length of chain in metropolis hasting algorihtn
$q_{\theta}(\cdot \theta^{(i)})$	Proposal distribution for the values of θ at given state i .
$q_I(\cdot I_j^{(i)})$	Proposal distribution for the values of I_j for individual j at given state i (See XX).
$q_k(\cdot k)$	Proposal distribution for a new model given model is at \mathcal{M}_k (See XX).
$q_{\tau}(\cdot E_j^{\tau,(i)})$	Proposal distribution for a new time of exposure for individual j for a given state i .(See XX).

Table 3: Symbols used in the mcmc algorithms