TASK LIST NO. 4: Selected Random Variable Distributions

Task 1

The probability of failure of experimental equipment in a single experiment is $p = 0.02$. Experiments can be performed any number of times. Calculate the probability that the second failure:

a) occurs at the tenth experiment,
b) does not occur in the first ten experiments.

Task 2

The probability that a product subjected to a test fails the test is $p = 0.01$. Calculate the probability that among 200 such products (independently tested), at most 2 will fail the test.

Hint: Since $n = 200$ is large and $p = 0.01$ is small, use the Poisson approximation with parameter $\lambda = np$.

Task 3

The time (in minutes) between consecutive subscriber calls at a certain telephone exchange is a random variable with an exponential distribution with parameter $\lambda = 2$. Calculate the average time between consecutive calls and the probability that a call occurs before 3 minutes elapse.

Task 4

The failure-free operation time X of a certain device has an exponential distribution with parameter $\lambda = 5$. Calculate:

a) the average failure-free operation time of the device,
b) the median,
c) the probability that the failure-free operation time of the device is at least 5 hours.

Task 5

The interval between consecutive graduations of a stopwatch scale is 0.1 s. Time on this stopwatch is read with an accuracy of a whole graduation. Assuming a uniform distribution of the time reading error, calculate the probability that the time was measured with an error exceeding 0.02 s.

Hint: The density of the uniform distribution is constant in the interval $(-0.05; 0.05)$.

Task 6

An automated machine produces 10-gram weights. The mass measurement errors of these weights have a normal distribution with an expected value $\mu = 0$ g and a standard deviation $\sigma = 0.01$ g. Find the probability that the mass measurement will be performed with an error not exceeding 0.02 g.

Task 7

Let the random variable X have a distribution $N(\mu, \sigma)$. Calculate the probability $P(|X - \mu| < k\sigma)$ for:

a) $k = 1.96$ (confidence level 0.95),
b) $k = 2.58$ (confidence level 0.99).

Task 8

A certain measuring instrument makes a systematic error of 1 m in the direction of overestimating the measurement and a random error with a distribution $N(0; 0.5)$.

a) Calculate the average value of the measurement error.
b) Determine the probability that the error with which the examined objects are measured does not exceed 2 m.

Task 9

The strength of steel ropes from mass production is a random variable with a distribution $N(1000 \text{ kg/cm}^2, 50 \text{ kg/cm}^2)$. Calculate what percentage of ropes has a strength less than 900 kg/cm^2.

Task 10

Determine and sketch the cumulative distribution function of the Rayleigh distribution, whose density is given by the formula:

$$f(x) = \begin{cases} \frac{2}{\lambda} x \exp(-\frac{x^2}{\lambda}) & \text{for } x > 0 \\ 0 & \text{for } x \leqslant 0 \end{cases}$$

Then calculate the median of this distribution.

Hint: This distribution is often used in telecommunications to model signal fading.