Determinants

Ex 1.

Calculate the determinant of the matrix

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & 0 \\ 7 & 8 & 9 \end{pmatrix} \qquad C = \begin{pmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

using Sarrus' rule.

Ex 2.

Determine the determinants using Laplace expansion:

$$A = \begin{pmatrix} 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 4 & 5 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Ex 3.

Show that if two rows in a matrix are equal, then the determinant is equal to zero. Give an example of a 3×3 matrix with two equal rows and calculate its determinant. Justify why this happens.

Ex 4.

Calculate the determinant of the triangular matrix T with diagonal elements $(3, -2, 5, 1)$.

Ex 5.

For the matrix dependent on parameter t:

$$M(t) = \begin{pmatrix} t & 1 \\ 2 & t \end{pmatrix}$$

calculate $\det(M(t))$ and find the values of t for which the matrix is singular.

Ex 6.

Solve the equation

$$\det \begin{pmatrix} x & 3 \\ 2 & x \end{pmatrix} = 0$$

Ex 7.

$\star$ Solve the equation

$$\det \begin{pmatrix} x & 3 \\ 2 & -x \end{pmatrix} = 0$$

Ex 8.

Calculate the determinant of the matrix

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

Ex 9.

Show that the equality holds

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (z-x)(z-y)(y-x)$$

Prove a similar equality for the determinant

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x & y & z & u \\ x^2 & y^2 & z^2 & u^2 \\ x^3 & y^3 & z^3 & u^3 \end{vmatrix}$$

Ex 10.

Calculate the determinant of the matrix

$$\begin{vmatrix} a & a & a \\ -a & a & a \\ -a & -a & a \end{vmatrix} \quad \& \quad \begin{vmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & a \end{vmatrix}$$

Ex 11.

Check the validity of the following relationships:

a)

$$\begin{vmatrix} a+b & b \\ c+d & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

b)

$$\begin{vmatrix} a+bx & b \\ c+dx & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$