Vectors

Ex 1.

For vectors in space

$$\mathbf{u} = [1{,}2,-1] \quad \text{and} \quad \mathbf{v} = [2,-1{,}3]$$

calculate $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, the dot product $\mathbf{u}\cdot\mathbf{v}$, and the norms $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$. Check if the vectors are orthogonal.

Ex 2.

For points $A(1{,}0{,}2)$, $B(3,-1{,}1)$, and $C(2{,}2{,}0)$, calculate vectors AB and AC and determine the angle between them.

Ex 3.

Calculate the cross product $\mathbf{u}\times\mathbf{v}$ for the vectors from Ex 1 and check if it is orthogonal to both vectors.

Ex 4.

For vectors on the plane: $\mathbf{a} = [3{,}4]$ and $\mathbf{b} = [-4{,}3]$, calculate their dot product and check if they are perpendicular. Determine the projection of vector $\mathbf{a}$ onto $\mathbf{b}$.

Ex 5.

Calculate the length of the vector $\mathbf{c} = [1,1]$ and find the unit vector of this vector.

Ex 6.

Calculate the length of the vector $\mathbf{c} = [1,2,3]$ and find the unit vector of this vector.

Ex 7.

Calculate the area of the triangle spanned by vectors $[2,1,2]$ and $[-1,1{,}1]$.

Ex 8.

Calculate the angle in degrees between vectors [4,2,1] and [1,3,2].

Ex 9.

Find the coordinates of the midpoint of the segment with endpoints $A(-1,2)$ and $B(3,-2)$.

Ex 10.

For three-dimensional vectors: $\mathbf{a} = [a_x, a_y, a_z]$, $\mathbf{b} = [b_x, b_y, b_z]$, $\mathbf{c} = [c_x, c_y, c_z]$, prove that the following identity holds:

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}.$$

Ex 11.

Find the most general form of a vector that is simultaneously perpendicular to

$$\mathbf{v} = [-1{,}3{,}0] \qquad and \qquad \mathbf{u} = [0{,}1{,}1]$$

Ex 12.

For what values of parameters p and q are the vectors $\mathbf{a} = [1-p{,}3,-1]$ and $\mathbf{b} = [-2{,}4-q{,}2]$ parallel?

Ex 13.

For what values of parameter s are the vectors $\mathbf{p} = [s,2,1-s]$ and $\mathbf{q} = [s,1,-2]$ perpendicular?

Ex 14.

Prove that two vectors must have equal lengths if their sum is perpendicular to their difference.

Ex 15.

$\star$ We have 2 people (A and B) walking according to the formulas:

A: $(4,5) + (1,-2)t$

B: $(1,-8) + (2,4)t$

where t denotes time. For what t will the people be closest to each other?