

Vectors

Ex 1.

For vectors in space

$$\mathbf{u} = [1, 2, -1] \quad \text{and} \quad \mathbf{v} = [2, -1, 3]$$

calculate $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, the dot product $\mathbf{u} \cdot \mathbf{v}$, and the norms $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$. Check if the vectors are orthogonal.

Ex 2.

For points $A(1, 0, 2)$, $B(3, -1, 1)$, and $C(2, 2, 0)$, calculate vectors \overrightarrow{AB} and \overrightarrow{AC} and determine the angle between them.

Ex 3.

Calculate the cross product $\mathbf{u} \times \mathbf{v}$ for the vectors from Ex 1 and check if it is orthogonal to both vectors.

Ex 4.

For vectors on the plane: $\mathbf{a} = [3, 4]$ and $\mathbf{b} = [-4, 3]$, calculate their dot product and check if they are perpendicular. Determine the projection of vector \mathbf{a} onto \mathbf{b} .

Ex 5.

Calculate the length of the vector $\mathbf{c} = [1, 1]$ and find the unit vector of this vector.

Ex 6.

Calculate the length of the vector $\mathbf{c} = [1, 2, 3]$ and find the unit vector of this vector.

Ex 7.

Calculate the area of the triangle spanned by vectors $[2, 1, 2]$ and $[-1, 1, 1]$.

Ex 8.

Calculate the angle in degrees between vectors $[4, 2, 1]$ and $[1, 3, 2]$.

Ex 9.

Find the coordinates of the midpoint of the segment with endpoints $A(-1, 2)$ and $B(3, -2)$.

Ex 10.

For three-dimensional vectors: $\mathbf{a} = [a_x, a_y, a_z]$, $\mathbf{b} = [b_x, b_y, b_z]$, $\mathbf{c} = [c_x, c_y, c_z]$, prove that the following identity holds:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}.$$

Ex 11.

Find the most general form of a vector that is simultaneously perpendicular to

$$\mathbf{v} = [-1, 3, 0] \quad \text{and} \quad \mathbf{u} = [0, 1, 1]$$

Ex 12.

For what values of parameters p and q are the vectors $\mathbf{a} = [1 - p, 3, -1]$ and $\mathbf{b} = [-2, 4 - q, 2]$ parallel?

Ex 13.

For what values of parameter s are the vectors $\mathbf{p} = [s, 2, 1 - s]$ and $\mathbf{q} = [s, 1, -2]$ perpendicular?

Ex 14.

Prove that two vectors must have equal lengths if their sum is perpendicular to their difference.

Ex 15.

★ We have 2 people (A and B) walking according to the formulas:

A: $(4, 5) + (1, -2)t$

B: $(1, -8) + (2, 4)t$

where t denotes time. For what t will the people be closest to each other?