TASK LIST NO. 3: Parameters of Random Variable Distribution

(Expected value, variance, moments, correlation)

Task 1

For a random variable X with the probability function given by the table:

x_i	-2	2	4
p_i	0.5	0.3	0.2

Determine:

a) The expected value $E(X)$ (mean).
b) The variance $D^2(X)$ (using the formula $D^2(X) = E(X^2) - (EX)^2$).
c) The standard deviation σ.
d) The median $x_{0.5}$ (middle value).

Task 2

The monthly cost U of running a certain system depends on the number X of active users (employees) according to the formula:

$$U = 15000X + 10000\sqrt{X}$$

The number of employees X is a random variable with the distribution:

x_i	2	3	4	5
p_i	0.10	0.25	0.40	0.25

Calculate the predicted average monthly cost, i.e., the expected value of the variable U.

Hint: Calculate u_i for each x_i, and then apply the formula for the expected value.

Task 3

A random variable X (e.g., measurement error) has a distribution with the density:

$$f(x) = \begin{cases} 6x(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{for other } x \end{cases}$$

Calculate the average (expected) value and the variance of this variable. Then calculate the variance of the linearly dependent variable $Y = 2X - 1$ (use the property of variance: $D^2(aX + b) = a^2 D^2(X)$).

Task 4

A random variable X has a distribution with the density:

$$f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the mode (the value for which the density is greatest) and the median (the value that divides the area under the density graph into two equal halves).

Task 5

The height of people in a certain group is a random variable X with a mean $EX = 170$ cm and a standard deviation $\sigma_X = 5$ cm. The mass of these people is a variable Y with a mean $EY = 65$ kg and a standard deviation $\sigma_Y = 5$ kg.

Which feature (height or weight) is more "stable" (has a smaller relative dispersion)?

Hint: Calculate the coefficient of variation $v = \frac{\sigma}{EX}$ *for both variables.*

Task 6

The probability of not exceeding the daily electricity consumption limit by a certain plant is $p = 0.8$. We observe this plant for $n = 5$ days. Let X denote the number of days in which the limit was not exceeded.

a) What type of distribution is this? Provide the formula for the probability $P(X = k)$.
b) Calculate the expected value and variance of the variable X, using the ready-made formulas for this distribution ($EX = np$, $D^2X = npq$).

Task 7

The time (in minutes) between consecutive subscriber calls at a telephone exchange is a random variable with an exponential distribution with the parameter (expected value) $\lambda = 2$.

a) Calculate the average waiting time for a call (EX).
b) Calculate the probability that the time between calls will be shorter than 3 minutes ($P(X < 3)$).

Task 8

A machine produces weights. Mass measurement errors have a normal distribution with an expected value $\mu = 0$ g and a standard deviation $\sigma = 0.01$ g. Calculate the probability that the measurement error (in terms of modulus) does not exceed 0.02 g.

Hint: Use the cumulative distribution function of the standardized normal distribution $\Phi(u)$*. Note that* $P(|X| < a) = P(-a < X < a)$*.*

Task 9

Given is a two-dimensional random variable (X, Y) with the distribution given in the table (representing, for example, test results at two different moments in time):

$y_k \backslash x_i$	8	9	10	11
1.2	0.10	0.04	0	0
1.3	0.05	0.11	0.20	0
1.4	0	0.10	0.15	0.10
1.5	0	0	0.05	0.10

Calculate the linear correlation coefficient ρ between variables X and Y.

Hint: Calculate sequentially: means EX, EY*, variances* D^2X, D^2Y*, and the mixed moment* $E(XY) = \sum x_i y_k p_{ik}$*. Covariance is* $cov(X,Y) = E(XY) - EX \cdot EY$*.*

Task 10

Let X and Y be independent random variables with zero average values ($EX = 0, EY = 0$). Show that for any $n \in N$, the variables X and X^n as well as Y and $E(X^3)EY$ satisfy the equality: $E(X^3Y) = E(X^3)E(Y)$.

Does the variable $Z = X^3Y$ have an expected value equal to 0? What does this mean in the context of random signals (noise)?