

## Systems of Linear Equations

### Ex 1.

Solve the system of equations:

$$\begin{aligned}2x + 3y &= 5, \\ x - 4y &= -2.\end{aligned}$$

using the methods: Cramer's rule, Gaussian elimination, and inverse matrix.

### Ex 2.

Solve the system of three equations with three unknowns:

$$\begin{aligned}x + y + z &= 6, \\ 2x - y + 3z &= 14, \\ -x + 2y - z &= -2.\end{aligned}$$

using the methods: Cramer's rule, Gaussian elimination, and inverse matrix.

### Ex 3.

★ Consider the parametric system dependent on  $\lambda$ :

$$\begin{aligned}x + \lambda y &= 1, \\ 2x + (1 + \lambda)y &= 3.\end{aligned}$$

Determine the values of  $\lambda$  for which the system has one solution, infinitely many solutions, or no solution.

### Ex 4.

For the coefficient matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

and the right-hand side vector  $b = (4, 1, 3)^\top$ , solve  $Ax = b$  and check the result by substitution.

### Ex 5.

Solve the systems of equations:

a)

$$\begin{cases} x_1 + 4x_2 + 3x_3 = 1, \\ 2x_1 + 5x_2 + 4x_3 = 4, \\ x_1 - 3x_2 - 2x_3 = 5; \end{cases}$$

b)

$$\begin{cases} x_1 - 2x_2 - 3x_3 = 2, \\ x_1 - 4x_2 - 13x_3 = 14, \\ -3x_1 + 5x_2 + 4x_3 = 0; \end{cases}$$

c)

$$\begin{cases} x_1 - 2x_2 - 3x_3 = 2, \\ x_1 - 4x_2 - 13x_3 = 14, \\ -3x_1 + 5x_2 + 4x_3 = 2; \end{cases}$$

d)

$$\begin{cases} -4x_1 + 3x_2 + 2x_3 = -2, \\ 5x_1 - 4x_2 + x_3 = 3; \end{cases}$$

e)

$$\begin{cases} -4x_1 + 3x_2 = 2, \\ 5x_1 - 4x_2 = 0, \\ 2x_1 - x_2 = a; \end{cases}$$

f)

$$\begin{cases} 4x_1 + 5x_3 = 6, \\ x_2 - 6x_3 = -2, \\ 3x_1 + 4x_3 = 3; \end{cases}$$

g)

$$\begin{cases} 3x_1 - x_2 - 2x_3 = 2, \\ 2x_2 - x_3 = -1, \\ 3x_1 - 5x_2 = 3; \end{cases}$$

h)

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 0, \\ x_1 - 4x_2 - 13x_3 = 0, \\ -3x_1 + 5x_2 + 4x_3 = 0; \end{cases}$$

i)

$$\begin{cases} x_1 + x_2 + x_3 = 0, \\ 2x_1 + 4x_2 + 3x_3 = 0, \\ 4x_2 + 4x_3 = 0; \end{cases}$$

j)

$$\begin{cases} x_1 + x_2 + x_3 = -2, \\ 2x_1 + 4x_2 - 3x_3 = 3, \\ 4x_2 + 2x_3 = 2; \end{cases}$$

k)

$$\begin{cases} 4x_1 + 4x_3 = 8, \\ x_2 - 6x_3 = -3, \\ 3x_1 + x_2 - 3x_3 = 3; \end{cases}$$

l)

$$\begin{cases} 5x_1 - 3x_2 = -7, \\ -2x_1 + 9x_2 = 4, \\ 2x_1 + 4x_2 = -2; \end{cases}$$