

TASK LIST NO. 7: Point and Interval Estimation

Task 1

Find the confidence interval for the unknown mean value μ of a population, in the case where σ is known, based on an n -element simple sample X_1, \dots, X_n .

Data: From a population with a standard deviation $\sigma = 0.14$, a sample of $n = 100$ elements was taken (e.g., voltage measurements on a processor). The sample mean was $\bar{x} = 2.5$. Determine the 95% confidence interval for the mean (assume $1 - \alpha = 0.95$, which gives $u_\alpha = 1.96$).

Task 2

The durability of 10 randomly selected structural elements was measured (or e.g., battery life of 10 laptops). The following results were obtained: 383, 284, 339, 340, 305, 386, 378, 335, 344, 346.

Assuming that the distribution of the feature is normal, determine the 95% confidence interval for the mean durability.

Hint: Since $n = 10$ is small ($n < 30$) and we do not know σ , calculate s from the sample and use the Student's t -distribution.

Task 3

To determine the electron charge, 26 measurements were made using the Millikan method. A mean of $\bar{x} = 1.574 \cdot 10^{-19}$ and a standard deviation of $s = 0.043 \cdot 10^{-19}$ were obtained.

Determine the confidence interval for the mean charge at a confidence level of 0.99.

Task 4

A 300-element sample was taken from a population of cotton fibers and their lengths were measured. The mean $\bar{x} = 27.43$ mm and variance $s^2 = 51.598$ were calculated.

Find the 95% realization of the confidence interval for the unknown average fiber length.

Hint: With such a large n ($n = 300$), the Student's t -distribution is practically identical to the normal distribution, so the statistic u_α can be used.

Task 5

Measurements of sea depth (or e.g., network latency) are made in a certain specific location.

How many independent measurements must be made to assume with a confidence level of 0.95 that the absolute error of estimating the mean will not exceed 10 m, if the error distribution is normal with a variance of $\sigma^2 = 180 \text{ m}^2$?

Task 6

Among 120 randomly selected employees of a certain plant, 17 did not meet the work performance standard (in IT: 17 out of 120 servers did not meet SLA requirements).

Determine the 95% realization of the confidence interval for the fraction p of employees not meeting the standard in the entire plant.

Task 7

15 measurements were made of the time to repair yarn breaks on looms. The sample variance was calculated as $s^2 = 134.2$. Assuming that this time has a normal distribution, determine the 90% confidence interval for the variance σ^2 and the standard deviation σ .

Hint: Use the chi-square (χ^2) distribution tables.

Task 8

Two samples were drawn for a certain feature with a normal distribution:

- Sample 1: $n = 25$, mean $\bar{x} = 15$, deviation $s = 5$.
- Sample 2: $n = 100$, mean $\bar{x} = 15$, deviation $s = 5$.

Calculate the lengths of the 95% confidence intervals for both samples. How does a fourfold increase in sample size affect precision (interval width)?

Task 9

A simple sample of size $n = 5$ is given: $\{2, 4, 6, 8, 10\}$. Calculate the value of the unbiased estimator of the expected value (\bar{x}) and the unbiased estimator of the variance (s^2).

Explain why we divide by $n - 1$ instead of n for variance.

Task 10

The table presents the results of the percentage starch content in 80 potatoes (data grouped into a frequency distribution). The sample mean is $\bar{x} = 17.525\%$, and the standard deviation is $s = 1.84\%$.

Assuming a confidence level of 0.95, estimate the average starch content in the entire batch.