Matrix Inversion

Ex 1.

Find the inverse matrix using the formula for a 2×2 matrix

$$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$

Ex 2.

For the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 12 & 5 \\ 7 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 4 & 5 \end{pmatrix}$$

calculate the inverse matrices using the methods:

- augmenting with the identity matrix and performing Gauss-Jordan elimination,
- using the formula with cofactor matrices (adjugate matrix)

So for each matrix provide two methods of calculating the inverse matrix (if it exists).

Ex 3.

Check if the matrix

$$H = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & 1 \end{pmatrix}$$

is invertible. Justify the answer (use the determinant). Could this have been noticed without calculating the determinant? What would have to happen for the matrix to be invertible?

Ex 4.

For a matrix A satisfying $A^2 = I$ (so-called involution), show that $A^{-1} = A$. Give an example of a non-trivial 2×2 matrix satisfying this condition (other than I and $-I$). How many such matrices are there?

Ex 5.

Calculate the inverse of the diagonal matrix $D = \mathrm{diag}(2,5,-3,1)$, if it exists. Discuss the condition for the existence of an inverse for a diagonal matrix.

Ex 6.

Solve the matrix equations:

a)

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \cdot X = \begin{bmatrix} 4 & -6 \\ 2 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

c)

$$X \cdot \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{bmatrix}$$

d)

$$\begin{bmatrix} 3 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$