

## Function Limits

### Ex 1.

Calculate the function limits

- 1)  $\lim_{x \rightarrow 3} \frac{27-x^3}{x-3}$
- 2)  $\lim_{x \rightarrow 3} \frac{x^2-4x+3}{2x-6}$
- 3)  $\lim_{x \rightarrow -1} \frac{x^3-1}{x+1}$
- 4)  $\lim_{x \rightarrow -2} \frac{x+2}{x^5+32}$
- 5)  $\lim_{x \rightarrow 4} \frac{x^2-2x-8}{x^2-9x+20}$
- 6)  $\lim_{x \rightarrow -5} \frac{x^3+125}{2x^2-50}$
- 7)  $\lim_{x \rightarrow -2} \frac{3x^2+5x-2}{4x^2+9x+2}$
- 8)  $\lim_{x \rightarrow 1} \frac{x^n-1}{x-1}$ , n - natural number
- 9)  $\lim_{x \rightarrow 3} \frac{(x-3)(-1)^{[x]}}{x^2-9}$
- 10)  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+mx}-1}{x}$
- 11)  $\lim_{x \rightarrow 1} \frac{x^n-1}{x-1}$  n - natural number.
- 12)  $\lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25}$
- 13)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-\sqrt{x+1}}{1-\sqrt{x+1}}$
- 14)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+25}-5}$
- 15)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{4x}$
- 16)  $\lim_{x \rightarrow 0} \frac{4x}{3 \sin 2x}$
- 17)  $\lim_{x \rightarrow +\infty} \frac{\sin x}{x}$
- 18)  $\lim_{x \rightarrow \pi} \frac{\sin x}{x}$
- 19)  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$
- 20)  $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{4x}$
- 21)  $\lim_{x \rightarrow \pi} \frac{8-x}{\sin x}$
- 22)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$
- 23)  $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\operatorname{tg} x}$
- 24)  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1+\cos x}{\sin^2 x}$
- 25)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \cos \frac{\pi}{4}}{\sin x - \sin \frac{\pi}{4}}$
- 26)  $\lim_{x \rightarrow 1} \frac{|\operatorname{tg}(x-1)|}{(x-1)^2}$

- 27)  $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$
- 28)  $\lim_{x \rightarrow \frac{1}{2}} \frac{\arcsin(1-2x)}{4x^2-1}$
- 29)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x}}{x}$
- 30)  $\lim_{x \rightarrow 0} (1-3x)^{\frac{1}{x}}$
- 31)  $\lim_{x \rightarrow 0} (1+kx)^{\frac{n}{x}}$

**Ex 2.** For the given functions determine if they are continuous at the given points. If not, can they be defined to be continuous.

- 1)  $f(x) = \frac{x^2-25}{x+5}$  for  $x \neq -5$  and  $f(-5) = -10$ .
- 2)  $f(x) = \frac{\sin x}{x}$  for  $x \neq 0$  and  $f(0) = 1$ .
- 3)  $f(x) = \frac{\sin x}{|x|}$  for  $x \neq 0$  and  $f(0) = 1$ .
- 4)  $f(x) = x + \frac{1}{x}$
- 5)  $f(x) = \frac{x^2-x^3}{|x-1|}$
- 6)  $f(x) = x - [x]$
- 7)  $f(x) = [x] + [-x]$
- 8)  $f(x) = \frac{\sqrt{1+x}-1}{x}$
- 9)  $f(x) = x \sin \frac{\pi}{x}$
- 10)  $f(x) = \frac{\sin^2 x}{1-\cos x}$
- 11)  $x \left[ \frac{1}{x} \right]$  at point  $x = 0$ .
- 12)  $x \frac{b}{x} \left[ \frac{x}{a} \right]$  at point  $x = 0$ .
- 13)  $\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$  at point  $x = 0$ .
- 14)  $e^{\frac{1}{1-x^2}}$  at point  $x = 1$ .
- 15)  $xe^{\frac{1}{x}}$  at point  $x = 0$ .
- 16)  $\frac{x}{2x+e^{\frac{1}{x-1}}}$  at point  $x = 1$ .