Function Limits

Ex 1.

Calculate the function limits

1) $\lim\limits_{x\to 3} \frac{27-x^3}{x-3}$

2) $\lim\limits_{x\to 3} \frac{x^2-4x+3}{2x-6}$

3) $\lim\limits_{x\to -1} \frac{x^3-1}{x+1}$

4) $\lim\limits_{x\to -2} \frac{x+2}{x^5+32}$

5) $\lim\limits_{x\to 4} \frac{x^2-2x-8}{x^2-9x+20}$

6) $\lim\limits_{x\to -5} \frac{x^3+125}{2x^2-50}$

7) $\lim\limits_{x\to -2} \frac{3x^2+5x-2}{4x^2+9x+2}$

8) $\lim\limits_{x\to 1} \frac{x^n-1}{x-1}$, n - natural number

9) $\lim\limits_{x\to 3} \frac{(x-3)(-1)^{[x]}}{x^2-9}$

10) $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$

11) $\lim\limits_{x\to 1} \frac{x^n-1}{x-1}$ n - natural number.

12) $\lim\limits_{x\to 25} \frac{\sqrt{x}-5}{x-25}$

13) $\lim\limits_{x\to 0} \frac{\sqrt{x^2+1}-\sqrt{x+1}}{1-\sqrt{x+1}}$

14) $\lim\limits_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+25}-5}$

15) $\lim\limits_{x\to 0} \frac{\sin 3x}{4x}$

16) $\lim\limits_{x\to 0} \frac{4x}{3\sin 2x}$

17) $\lim\limits_{x\to +\infty} \frac{\sin x}{x}$

18) $\lim\limits_{x\to \pi} \frac{\sin x}{x}$

19) $\lim\limits_{x\to \frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}}$

20) $\lim\limits_{x\to 0} \frac{\operatorname{tg}\ x}{4x}$

21) $\lim\limits_{x\to \pi} \frac{8-x}{\sin x}$

22) $\lim\limits_{x\to 0} \frac{\sin 2x}{\sin 3x}$

23) $\lim\limits_{x\to 0} \frac{\operatorname{tg}\ 2x}{\operatorname{tg}\ x}$

24) $\lim\limits_{x\to \frac{\pi}{2}} \frac{1+\cos x}{\sin^2 x}$

25) $\lim\limits_{x\to \frac{\pi}{4}} \frac{\cos x-\cos\frac{\pi}{4}}{\sin x-\sin\frac{\pi}{4}}$

26) $\lim\limits_{x\to 1} \frac{|\operatorname{tg}(x-1)|}{(x-1)^2}$

27) $\lim\limits_{x\to 0} \frac{\operatorname{arctg}\, x}{x}$

28) $\lim\limits_{x\to \frac{1}{2}} \frac{\arcsin(1-2x)}{4x^2-1}$

29) $\lim\limits_{x\to 0} \frac{\sqrt{1+\sin x}}{x}$

30) $\lim\limits_{x\to 0} (1-3x)^{\frac{1}{x}}$

31) $\lim\limits_{x\to 0} (1+kx)^{\frac{n}{x}}$

Ex 2. For the given functions determine if they are continuous at the given points. If not, can they be defined to be continuous.

1) $f(x) = \frac{x^2-25}{x+5}$ for $x \neq -5$ and $f(-5) = -10$.

2) $f(x) = \frac{\sin x}{x}$ for $x \neq 0$ and $f(0) = 1$.

3) $f(x) = \frac{\sin x}{|x|}$ for $x \neq 0$ and $f(0) = 1$.

4) $f(x) = x + \frac{1}{x}$

5) $f(x) = \frac{x^2-x^3}{|x-1|}$

6) $f(x) = x - [x]$

7) $f(x) = [x] + [-x]$

8) $f(x) = \frac{\sqrt{1+x}-1}{x}$

9) $f(x) = x \sin \frac{\pi}{x}$

10) $f(x) = \frac{\sin^2 x}{1-\cos x}$

11) $x\left[\frac{1}{x}\right]$ at point $x = 0$.

12) $x\frac{b}{x}\left[\frac{x}{a}\right]$ at point $x = 0$.

13) $\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$ at point $x = 0$.

14) $e^{\frac{1}{1-x^2}}$ at point $x = 1$.

15) $xe^{\frac{1}{x}}$ at point $x = 0$.

16) $\frac{x}{2x+e^{\frac{1}{x-1}}}$ at point $x = 1$.