TASK LIST NO. 1: Random Events and Probability

Task 1

Let the sample space Ω of elementary events of an experiment consist of five elementary events ω_i: $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$. We define the events: $A = \{\omega_1, \omega_3, \omega_5\}$, $B = \{\omega_2, \omega_3, \omega_4\}$.

Find the events:

a) $A \cup B$ (union of events)
b) $A \cap B$ (intersection of events)
c) $B \backslash A$ (difference of events)
d) $A \backslash B$

Task 2

Consider an electrical circuit in which element a_1 is connected in series with a block consisting of two elements a_2 and a_3 connected in parallel. Let $A_i, i = 1, 2, 3$, denote the event "element a_i is functional at time t".

Using operations on events A_i and the symbols for union ($\cup$) and intersection ($\cap$), describe the event A: "in the time interval t, the current flow through the circuit will not be interrupted".

Task 3

Person X performs a certain job in 4, 5, or 6 hours and may commit 0, 1, or 2 errors. Assuming equal probability for each of the 9 possible elementary events (pairs: time, number of errors), find the probability of the following events:

a) The job will be completed in 4 hours.
b) The job will be completed flawlessly in 6 hours.
c) The job will be completed in at most 5 hours.
d) The job will be completed in at most 5 hours and with at most one error.

Task 4

The figure (block diagram) shows a part of an electrical network consisting of two elements connected in parallel: a_1 and a_2. Let $A_i, i = 1, 2$, denote the event that element a_i remains functional for at least time t.

Calculate the probability of continuous current flow through this system for at least time t, given that $P(A_1) = P(A_2) = p$ and the probability of simultaneous functionality of both elements is $P(A_1 \cap A_2) = p^2$.

Task 5

We consider the volume (in dm^3) of water that a concrete culvert can conduct per second. Past observations allow us to assume that:

- The probability that the volume of water takes a value from the interval $\langle 125, 250\rangle$ is $P(A) = 0.6$.
- The probability that the volume of water takes a value from the interval $(200, 300\rangle$ is $P(B) = 0.7$.
- The probability of the union of these events is $P(A \cup B) = 0.8$.

Calculate the probability:

a) $P(A')$ (complementary event to A)
b) $P(A \cap B)$ (intersection of intervals)
c) $P(A' \cap B')$ (water volume does not fall into either of these intervals)

Task 6

Identical products manufactured by 2 automated machines are placed on a conveyor belt. The quantitative ratio of production of the first machine to the production of the second is 3 : 2. The first machine produces on average 65% of first-grade products, while the second produces 85%.

a) One product is randomly selected from the products on the conveyor belt. Calculate the probability that it will be a first-grade product (use the total probability formula).
b) A randomly selected product turned out to be of first quality. Calculate the probability that it was produced by the first machine (use Bayes' theorem).

Task 7

On a communication line, two types of signals are transmitted in the form of code combinations 111 or 000 with a priori probabilities of 0.65 and 0.35 respectively. The signals are subject to random interference, as a result of which the symbol 1 can be received as 0 with a probability of 0.2, and with the same probability, the symbol 0 can be received as 1. We assume that symbols 1 and 0 are subject to interference independently of each other.

Calculate the probability of receiving the signal:

a) 111
b) 000
c) 010

Task 8

Coded information consists of seven pulses of types A, B, C in quantities: four pulses of A, two pulses of B, and one pulse of C. Assuming a random arrangement of pulses, find the probability that:

a) the first received pulse will be A,
b) the first received pulse will be A or C,
c) the first two pulses will be A and C in that order.

Task 9

A certain good is produced by 3 plants. The probability of producing first-quality goods by these plants is 0.97, 0.90, and 0.86, respectively.

Find the probability that a randomly taken item — from among three items originating (one each) from different plants — is of first quality.

Task 10

Only 3 types of letter sequences are transmitted via a communication channel: $AAAA$, $BBBB$, $CCCC$ with probabilities 0.4, 0.3, and 0.3, respectively. These letters (signals) are subject to independent random interference (errors), as a result of which, e.g., letter A can be received as B or C. The probabilities of correct transmission or error for a single letter are given in the table:

Transmitted	Received	A	B	C
A		0.8	0.1	0.1
B		0.1	0.8	0.1
C		0.1	0.1	0.8

Find the probability of receiving the signal at the output:

a) $AAAA$
b) $ACAA$