

### Problem 1

Find limit of the  $a_n = \frac{2n^2+3n+2}{5-2n^2}$  sequence at  $n \rightarrow \infty$ .

- A)  $-1$
- B)  $1$
- C)  $\infty$
- D)  $\frac{2}{5}$

### Problem 2

Find limit of the function  $f(x) = \frac{\sin(x-\pi)}{x}$  at  $x \rightarrow 0$ .

- A)  $0$
- B)  $1$
- C)  $-1$
- D)  $\infty$

### Problem 3

Calculate the derivative of the function  $y(x) = 2x^3 - 3x^2 + 8x - 9$ .

- A)  $6x^2 - 6x + 8$
- B)  $6x^3 - 6x^2 + 8x$
- C)  $2x^2 - 3x + 8$
- D)  $6x^2 - 6x$

### Problem 4

Calculate the derivative of the function  $y(x) = x \sin(x) + 1$ .

- A)  $\sin(x) + x \cos(x)$
- B)  $\cos(x)$
- C)  $\sin(x) - x \cos(x)$
- D)  $x \cos(x)$

### Problem 5

Calculate the integral  $\int (x^2 + 3x + 5)xdx$ .

- A)  $\frac{x^4}{4} + x^3 + \frac{5x^2}{2} + C$
- B)  $x^3 + 3x^2 + 5x + C$
- C)  $\frac{x^3}{3} + \frac{3x^2}{2} + 5x + C$
- D)  $3x^2 + 6x + 5 + C$

### Problem 6

Calculate the integral  $\int x \sin(x^2) dx$ .

- A)  $-\frac{1}{2} \cos(x^2)$
- B)  $2x \cos(x^2) + C$
- C)  $-\frac{1}{2} \cos(x^2) + C$
- D)  $-\cos(x^2) + C$

### Problem 7

Calculate the integral  $\int \frac{x+2}{x+1} dx$ .

- A)  $x + \ln|x+1| + C$
- B)  $\ln|x+1| + C$
- C)  $x - \ln|x+1| + C$
- D)  $1 + \frac{1}{x+1} + C$

### Problem 8

Find the area bounded by the x-axis and the curve  $y = x^2$  on the interval  $[0, 2]$ .

- A)  $\frac{8}{3}$
- B) 4
- C)  $\frac{4}{3}$
- D)  $\frac{1}{3}x^3 + C$

### Problem 9

A particle moves along a straight line with its position at time  $t$  given by the function  $x(t) = t^3 - 6t^2 + 9t + 2$ . Find the velocity  $V(t) = \frac{dx(t)}{dt}$  and acceleration  $a(t) = \frac{d^2x(t)}{dt^2}$  of the particle at time  $t = 2$ .

- A) Velocity: 0, Acceleration: -3
- B) Velocity: 3, Acceleration: 0
- C) Velocity: -3, Acceleration: 0
- D) Velocity: -3, Acceleration: 12

### Problem 10

Solve the differential equation  $\frac{dy}{dx} = \frac{x+2}{x+1}$  with the initial condition  $y(0) = 1$ .

- A)  $y(x) = x + \ln|x+1| + C$
  - B)  $y(x) = \ln|x+1| + 1$
  - C)  $y(x) = x + \ln|x+1| + 1$
  - D)  $y(x) = \frac{-1}{(x+1)^2}$
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## Answers

1. **A**
2. **C**
3. **A**
4. **A**
5. **A**
6. **C**
7. **A**
8. **A**
9. **C**
10. **C**

## WYNIKI ZADAŃ

Zadanie 1: -1

Zadanie 2: -1

Zadanie 3:  $6x^2 - 6x + 8$

Zadanie 4:  $x \cos(x) + \sin(x)$

Zadanie 5:  $x^4/4 + x^3 + 5x^2/2 + C$

Zadanie 6:  $-\cos(x^2)/2 + C$

Zadanie 7:  $x + \log(x + 1) + C$

Zadanie 8:  $8/3$

Zadanie 9: Prędkość  $v(2) = -3$ , Przyspieszenie  $a(2) = 0$

Zadanie 10:  $x + \log(x + 1) + 1$

## Python code

```
import sympy as sp

# Definiowanie symboli
n = sp.symbols('n')
x = sp.symbols('x')
t = sp.symbols('t')

print("--- WYNIKI ZADAŃ ---")

# Problem 1: Granica ciągu
expr1 = (2*n**2 + 3*n + 2) / (5 - 2*n**2)
limit1 = sp.limit(expr1, n, sp.oo)
print(f"Zadanie 1: {limit1}")
```

```

# Problem 2: Granica funkcji
expr2 = sp.sin(x - sp.pi) / x
limit2 = sp.limit(expr2, x, 0)
print(f"Zadanie 2: {limit2}")

# Problem 3: Pochodna wielomianu
expr3 = 2*x**3 - 3*x**2 + 8*x - 9
deriv3 = sp.diff(expr3, x)
print(f"Zadanie 3: {deriv3}")

# Problem 4: Pochodna iloczynu
expr4 = x * sp.sin(x) + 1
deriv4 = sp.diff(expr4, x)
print(f"Zadanie 4: {deriv4}")

# Problem 5: Całka nieoznaczona
expr5 = (x**2 + 3*x + 5) * x
integral5 = sp.integrate(expr5, x)
print(f"Zadanie 5: {integral5} + C")

# Problem 6: Całka przez podstawienie
expr6 = x * sp.sin(x**2)
integral6 = sp.integrate(expr6, x)
print(f"Zadanie 6: {integral6} + C")

# Problem 7: Całka wymierna
expr7 = (x + 2) / (x + 1)
integral7 = sp.integrate(expr7, x)
print(f"Zadanie 7: {integral7} + C")

# Problem 8: Pole powierzchni (całka oznaczona)
expr8 = x**2
area8 = sp.integrate(expr8, (x, 0, 2))
print(f"Zadanie 8: {area8}")

# Problem 9: Prędkość i przyspieszenie
x_t = t**3 - 6*t**2 + 9*t + 2
v_t = sp.diff(x_t, t)
a_t = sp.diff(v_t, t)
v_2 = v_t.subs(t, 2)
a_2 = a_t.subs(t, 2)
print(f"Zadanie 9: Prędkość v(2) = {v_2}, Przyspieszenie a(2) = {a_2}")

# Problem 10: Równanie różniczkowe
y = sp.Function('y')
diff_eq = sp.Eq(y(x).diff(x), (x + 2) / (x + 1))

```

```
solution = sp.dsolve(diff_eq, y(x), ics={y(0): 1})  
print(f"Zadanie 10: {solution.rhs}")
```