Student Guide: Learning Geometry with Your AI Assistant

Welcome to an interactive guide to the world of analytic geometry! The purpose of this document is to show you how to translate intuitive geometric concepts into the precise language of algebra, using tools such as Gemini.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "Explain what a 'unit vector' (wersor) is in the simplest possible way."
- An example is unclear? Ask for another: "Can you show me graphically how vector addition works?"
- Want to check yourself? Verify your thinking: "If I understand correctly, multiplying a vector by a scalar changes its length but not its direction, right? (Unless the scalar is negative)"

Take responsibility for your learning

Approach this task seriously. The AI assistant is your personal interactive textbook. The goal is understanding, not mechanically copying prompts. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Cartesian space and vectors

Key concepts: In this section you'll learn: Cartesian coordinate system, point vs vector, vector coordinates, free vs position (bound) vector.

- **Step 1: Building intuition**
  - **Prompt 1.1:** "Explain what the Cartesian coordinate system is in 2D and 3D. What is the difference between a point and a vector in this space? Use an analogy, e.g., a treasure map."
  - **Prompt 1.2:** "What does it mean that a vector has direction, sense and magnitude? How do we compute the coordinates of a vector given its starting point A and end point B?"
- **Step 2: Practice and interactive tasks**
  - **Prompt 1.3:** "I have point A=(1, 2) and point B=(5, 5). Ask me to compute the coordinates of vector AB. Then draw (or describe) this vector in the coordinate system. Check my answer."
- **Step 3: Mini-quiz**
  - **Prompt 1.4:** "Give me 3 short questions to check whether I understand the difference between a point and a vector and how to compute vector coordinates."

Topic 2: Addition, subtraction and scalar multiplication

Key concepts: In this section you'll learn: algebraic and geometric vector addition and subtraction, scalar multiplication of vectors.

- **Step 1: Building intuition**
  - **Prompt 2.1:** "How do we add and subtract vectors algebraically (by coordinates)? And how do we interpret it geometrically (triangle or parallelogram method)? Show this on a drawing for vectors u=[1, 3] and v=[4, 1]."

  - **Prompt 2.2:** "What does multiplying a vector by a scalar mean? How does it change its magnitude and direction? Show with the example u=[2, 3] what happens when we multiply by 2, -1 and 0.5."
- **Step 2: Practice and interactive tasks**
  - **Prompt 2.3:** "I have vectors u=[1, 2, 3] and v=[4, 5, 6]. Guide me step by step through computing w = 2*u - v. Check my calculations at each step."
- **Step 3: Mini-quiz**
  - **Prompt 2.4:** "Give me two 3D vectors and ask me to perform two operations on them (e.g., sum and scalar multiplication). Check my result."

Topic 3: Vector length and unit vector

Key concepts: In this section you'll learn: length (norm, magnitude) of a vector, normalizing a vector, unit vector (wersor).

- **Step 1: Building intuition**
  - **Prompt 3.1:** "How do we compute the length of a vector in 2D and 3D? Explain why this formula is essentially the Pythagorean theorem."
  - **Prompt 3.2:** "What is a unit vector (wersor)? What is it useful for? How do we 'normalize' any vector, i.e., find the unit vector with the same direction and sense?"
- **Step 2: Practice and interactive tasks**
  - **Prompt 3.3:** "I have vector v = [3, 4]. Ask me to compute its length. Then guide me through normalizing the vector, i.e., finding the unit vector v. Finally check together that the length of the obtained unit vector is indeed 1."
- **Step 3: Mini-quiz**
  - **Prompt 3.4:** "Give me a vector in 3D. Ask me to compute its length and to find its unit vector. Check my results."

Finale: Test your knowledge and discover applications

Step 1: Final test

- **Prompt 4.1:** "Prepare a combined test on basic vector concepts. I want it to contain 4 tasks: one on computing vector coordinates, one on operations (e.g., 3u - 2v), one on computing length and one on normalization."

Step 2: Why learn this? Applications of vectors

- **Prompt 5.1 (Physics):** "How are vectors used in physics to describe force, velocity and acceleration? Show a simple example, e.g., how two forces acting on an object combine into one resultant force."
- **Prompt 5.2 (Computer Graphics):** "How are vectors used in computer graphics and games? Briefly explain their role in specifying position, movement direction of a character, or lighting."

Step 3: What next? Preview of the next module

- **Prompt 6.1 (Preview):** "I have mastered basic vector operations. But how do you multiply two vectors by each other? Give a short, one-sentence preview of what the 'dot product' is and what it can be used for (e.g., computing the angle between vectors)."

Good luck on your journey through geometry!