Student Guide: Learning Differential Calculus with Your AI Assistant

Welcome to an interactive guide to mathematical analysis! In this part we will learn one of the most powerful tools of mathematics: the derivative, which allows us to measure and analyze change.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "Explain the chain rule with a simple example."
- An example is unclear? Ask for another: "Can you show me how to find extrema of another function?"
- Want to check yourself? Verify your thinking: "If I understand correctly, a positive derivative means the function is increasing, and a negative derivative means it is decreasing, right?"

Take responsibility for your learning

Approach this task diligently. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Definition and interpretation of the derivative

Key concepts: In this section you'll learn: difference quotient, definition of the derivative as a limit, geometric interpretation (tangent) and physical interpretation (instantaneous velocity).

- **Step 1: Building intuition**
  - **Prompt 1.1:** "Explain step by step how we arrive at the definition of the derivative. Start from the secant line and the average rate of change, then show how, via a limiting process (bringing points closer), we obtain the tangent and the derivative (instantaneous rate of change)."
  - **Prompt 1.2:** "What is the geometric interpretation of the derivative at a point? What does its sign (positive, negative, zero) tell us about the behavior of the function near that point?"
- **Step 2: Practice and interactive tasks**
  - **Prompt 1.3:** "Let's compute the derivative of $f(x)=x^2$ at $x=3$ using the definition (the limit of the difference quotient). Guide me through all algebraic steps."
- **Step 3: Mini-quiz**
  - **Prompt 1.4:** "Give me 3 conceptual questions about the definition and interpretation of the derivative. One should concern the relationship between existence of the derivative and continuity of the function."

Topic 2: Differentiation formulas and rules

Key concepts: In this section you'll learn: formulas for derivatives of basic functions (constant, power, trigonometric), differentiation rules (sum, product, quotient), and the chain rule.

- **Step 1: Building intuition**
  - **Prompt 2.1:** "Show me a table of derivative formulas for basic functions such as f(x)=c, f(x)=x^n, f(x)=sin(x), f(x)=cos(x), f(x)=e^x, f(x)=ln(x)."
  - **Prompt 2.2:** "Explain with simple examples how differentiation rules work: derivative of a sum, product and quotient."

  - **Prompt 2.3:** "What is the chain rule for composite functions? Explain it using the example f(x)=sin(x²)."
- **Step 2: Practice and interactive tasks**
  - **Prompt 2.4:** "Compute the derivative of f(x)=x²*sin(x) together. Guide me step by step using the product rule."
  - **Prompt 2.5:** "Now try something harder: f(x)=e^{3x}. Guide me through applying the chain rule."
- **Step 3: Mini-quiz**
  - **Prompt 2.6:** "Give me 3 functions to differentiate, each requiring a different rule (product, quotient, chain). Check my results."

Topic 3: Applications of derivatives and higher-order derivatives

Key concepts: In this section you'll learn: higher-order derivatives, relationship of the first derivative to monotonicity, relationship of the second derivative to concavity, local extrema, inflection points.

- **Step 1: Building intuition**
  - **Prompt 3.1:** "What is the second derivative and what does it tell us about a function? Explain concavity and convexity and their relation to the sign of the second derivative."
  - **Prompt 3.2:** "How do we use derivatives to find local extrema (maxima and minima) of a function? Explain the necessary condition (first derivative zero) and the sufficient condition (testing the sign of the second derivative)."
- **Step 2: Practice and interactive tasks**
  - **Prompt 3.3:** "Let's perform a full analysis of a simple polynomial, e.g., f(x)=x³-3x. Guide me step by step: 1. Compute f'(x) and find monotonicity intervals. 2. Find local extrema. 3. Compute f"(x) and find concavity/convexity intervals. 4. Find inflection points."
- **Step 3: Mini-quiz**
  - **Prompt 3.4:** "Give me a function and ask me to find its local extrema. Check my answer."

Finale: Test your knowledge and prepare for the next step

Step 1: Final test

- **Prompt 4.1:** "Prepare a combined test on derivatives. I want 3 tasks: 1. Differentiate a complicated function using rules. 2. Find monotonicity intervals of a function. 3. Find local extrema of a function."

**Step 2: Why learn this? Applications of derivatives

- **Prompt 5.1 (Optimization):** "Give me a simple real-world optimization problem. E.g., 'We have 100m of fencing and want to fence a rectangular garden of maximal area.' Show how the derivative helps find the solution."
- **Prompt 5.2 (Physics):** "How are derivatives used in physics to describe motion? Explain the relationship between position, velocity and acceleration via differentiation."

Step 3: What next? Preview of the next module

- **Prompt 6.1 (Preview):** "I have mastered derivatives, i.e., the analysis of change. What if I want to reverse the process and reconstruct an original quantity from information about its change? Give me a short, one-sentence preview of what an 'integral' is and how it relates to the derivative."

Good luck on your journey through analysis!