Problem 1

Calculate the value of the expression:

$$\det\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix} - \det\begin{pmatrix}1 & 0 & 2\\ 0 & 3 & 1\\ 0 & 0 & 2\end{pmatrix}$$

- A) -2
- B) -1
- C) 0
- D) 1

Problem 2

Solve the equation:

$$\det\begin{pmatrix}x & x\\ 3 & x\end{pmatrix} = 0$$

- A) $x = 0;\ x = 3$
- B) $x = 3$
- C) $x = 0$
- D) $x = -3; x = 0$

Problem 3

Calculate the value of the expression:

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}^{100}$$

for $\theta = \frac{\pi}{4}$.

- A) $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$
- B) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
- C) $\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$
- D) $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

Problem 4

Let A be a 2×2 matrix with a determinant equal to 5. Calculate the determinant of the matrix $3A$.

- A) 15
- B) 30
- C) 45
- D) 60

Problem 5

The vector $\mathbf{v} = [\sqrt{2}, -\sqrt{2}]$ is rotated by an angle of $\frac{\pi}{2}$. What is the new value of the vector $\mathbf{v}'$?

- A) $[\sqrt{2}, \sqrt{3}]$
- B) $[-\sqrt{2}, \sqrt{2}]$
- C) $[-\sqrt{2}, -\sqrt{2}]$
- D) $[\sqrt{2}, \sqrt{2}]$

Problem 6

If A is a 2×2 matrix with a determinant of 7, and matrix B is a 2×2 matrix with a determinant of 3, what is the determinant of the matrix AB?

- A) $7 + 3$
- B) 7×3
- C) 7^3
- D) 3^7

Problem 7

The vector in the equation

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

has the value:

- A) $[1, 2]^T$
- B) $[2, 1]^T$
- C) $[3, 4]^T$
- D) $[4, 3]^T$

Problem 8

For what value of α does the system have a solution?

$$\begin{cases} -4x_1 + 3x_2 = 2, \\ 5x_1 - 4x_2 = 0, \\ 2x_1 - x_2 = \alpha; \end{cases}$$

- A) $\alpha = 2$
- B) $\alpha = -6$
- C) $\alpha = 0$
- D) The system has no solution for any α.

Problem 9

$$\begin{cases} 4x_1 + 5x_3 = 6, \\ x_2 - 6x_3 = -2, \\ 3x_1 + 4x_3 = 3; \end{cases}$$

- A) $\{x_1 = 9, x_2 = -38, x_3 = -6\}$
- B) $\{x_1 = 0, x_2 = -2, x_3 = 6\}$
- C) $\{x_1 = 1, x_2 = -2, x_3 = 0\}$
- D) The system has no solutions.

Problem 10

$$\begin{cases} 3x_1 - x_2 - 2x_3 = 2, \\ 2x_2 - x_3 = -1, \\ 3x_1 - 5x_2 = 3; \end{cases}$$

- A) $\{x_1 = 1, x_2 = 0, x_3 = 1\}$
- B) $\{x_1 = 0, x_2 = -1, x_3 = 1\}$
- C) $\{x_1 = 2, x_2 = 1, x_3 = 0\}$
- D) The system is inconsistent.