Student Guide: Learning Differential Calculus with Your AI Assistant

Welcome to an interactive guide to mathematical analysis! In this part we will learn the operation inverse to differentiation — integration. We will discover how to reconstruct an original function from information about its change and how to sum infinitely small quantities.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "What is the difference between a definite and an indefinite integral?"
- An example is unclear? Ask for another: "Can you show me another example of integration by parts?"
- Want to check yourself? Verify your thinking: "If I understand correctly, the definite integral of a function is simply the area under its graph, right?"

Take responsibility for your learning

Approach this task diligently. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Indefinite integral and antiderivative

Key concepts: In this section you'll learn: antiderivative, indefinite integral, and the constant of integration C.

- **Step 1: Building intuition**
  - **Prompt 1.1:** "Explain what an antiderivative is. Why is it the 'inverse' operation to differentiation? Why do we always add the constant of integration 'C' to the result?"
  - **Prompt 1.2:** "Show me a table of indefinite integrals for basic functions that inverts the derivative table."
- **Step 2: Practice and interactive tasks**
  - **Prompt 1.3:** "We know the derivative of x^2 is 2x. Guide me through finding the indefinite integral of f(x)=2x. Let's verify we get $x^2 + C$."
- **Step 3: Mini-quiz**
  - **Prompt 1.4:** "Give me 3 simple functions (e.g., polynomials) and ask me to find their indefinite integrals. Check my results."

Topic 2: Definite integral and its geometric interpretation

Key concepts: In this section you'll learn: definite integral, limits of integration, geometric interpretation (area under a curve), and the Fundamental Theorem of Calculus (Newton-Leibniz formula).

- **Step 1: Building intuition**
  - **Prompt 2.1:** "What is the definite integral? Explain intuitively how it relates to computing the area under the graph of a function using the idea of Riemann sums (dividing the area into small rectangles)."

  - **Prompt 2.2:** “What does the Fundamental Theorem of Calculus (Newton-Leibniz formula) state? How does it connect the definite integral (area) with the indefinite integral (antiderivative)? Why is this one of the most important theorems in mathematics?”
- **Step 2: Practice and interactive tasks**
  - **Prompt 2.3:** “Let’s compute together the area under f(x)=2x on the interval from 0 to 3. Guide me step by step applying the Newton-Leibniz formula. Let’s check the result matches the geometric formula for the area of a triangle.”
- **Step 3: Mini-sprawdzian**
  - **Prompt 2.4:** “Give me a simple function and an interval. Ask me to compute the definite integral of that function over the interval. Check my result.”

Topic 3: Basic integration methods

Key concepts: In this section you’ll learn: substitution method (change of variables) and integration by parts.

- **Step 1: Building intuition**
  - **Prompt 3.1:** “Explain what the substitution method is. With which differentiation rule is it related? (Hint: the chain rule)”
  - **Prompt 3.2:** “Explain what integration by parts is. With which differentiation rule is it related? (Hint: product rule)”
- **Step 2: Practice and interactive tasks**
  - **Prompt 3.3:** “Let’s compute the integral of f(x)=2x * cos(x^2) together. Guide me step by step using substitution.”
  - **Prompt 3.4:** “Now compute the integral of f(x)=x * cos(x) together. Guide me step by step using integration by parts.”
- **Step 3: Mini-quiz**
  - **Prompt 3.5:** “Give me 2 problems: one for substitution and one for integration by parts. Check my results.”

Topic 4: Applications of integrals

Key concepts: In this section you’ll see how to use integrals to compute area, the volume of a solid of revolution, and arc length.

- **Step 1: Building intuition**
  - **Prompt 4.1:** “Besides area under a curve, what else can we compute with integrals? Briefly explain how to use the definite integral to compute the volume of a solid obtained by rotating a curve around the x-axis.”
  - **Prompt 4.2:** “What is the formula for the arc length of a curve expressed using an integral?”
- **Step 2: Practice and interactive tasks**
  - **Prompt 4.3:** “We want to compute the volume of a sphere of radius R. We know the sphere is generated by rotating the semicircle y = sqrt(R^2 - x^2) around the x-axis. Guide me through writing the integral that would compute this volume.”
- **Step 3: Mini-quiz**
  - **Prompt 4.4:** “Give me a simple function and ask me to write (without computing) the integral that represents the area under its graph on a given interval and the volume of the solid generated by rotating it.”

Finale: Test your knowledge and prepare for the final step

Step 1: Final test

- **Prompt 5.1:** “Prepare a combined test on integrals. I want 3 problems: 1. Compute a simple definite integral. 2. An integral to solve by parts. 3. An integral to solve by substitution.”

Step 2: What’s next? Preview of the last module

- **Prompt 6.1 (Preview):** “I have mastered derivatives and integrals. What happens if we combine these two worlds and start looking for functions based on relationships between them and their derivatives? Give me a short, one-sentence preview of what ‘differential equations’ are and why they are so important in describing the world.”

Good luck on your journey through analysis!