

# Student Guide: Learning Geometry with Your AI Assistant

Welcome to an interactive guide to analytic geometry! Now that we know vectors, we will use them to describe more complex objects: lines and planes.

## Key to success: your activity and curiosity

### You are in charge of your learning!

- Don't understand a term? Ask the AI: "What is a 'normal vector' and why is it important for a plane?"
- An example is unclear? Ask for another: "Can you show me how to find the intersection point of a line and a plane in another example?"
- Want to check yourself? Verify your thinking: "If I understand correctly, the direction vector of a line and the normal vector of a plane must be perpendicular if the line is parallel to that plane, right?"

### Take responsibility for your learning

Approach this task seriously. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

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## Topic 1: Analytical description of a line (2D and 3D)

**Key concepts:** In this section you'll learn: parametric, directional and general equations of a line, direction vector.

- **Step 1: Building intuition**
    - **Prompt 1.1:** "How can we define a line in space? Explain the idea of having a point and a direction vector. Show how we build the parametric equation of a line in 3D from this."
    - **Prompt 1.2:** "What are other forms of the line equation? Show how to go from the parametric form to the directional form (for 2D and 3D) and the general form (for 2D). How to write the equation of a line passing through two points?"
  - **Step 2: Practice and interactive tasks**
    - **Prompt 1.3:** "Let's find the equation of the line passing through point  $P=(1, 2, 3)$  and parallel to vector  $v=[4, 5, 6]$ . Guide me through forming the parametric equation. Then ask me to find another point on this line."
  - **Step 3: Mini-quiz**
    - **Prompt 1.4:** "Give me two points in 3D. Ask me to write the parametric equation of the line that passes through them. Check my answer."
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## Topic 2: Analytical description of a plane

**Key concepts:** In this section you'll learn: general (normal) equation of a plane, normal vector of a plane, parametric equation of a plane.

- **Step 1: Building intuition**
  - **Prompt 2.1:** "How can we uniquely define a plane in 3D? Explain the concept of a normal vector (perpendicular) to the plane and a point that belongs to it. How do we derive its general equation from this?"
  - **Prompt 2.2:** "How do we find the equation of a plane passing through three non-collinear points? How can the cross product be used for this purpose?"

- **Step 2: Practice and interactive tasks**
    - **Prompt 2.3:** “Let’s find the equation of the plane passing through point  $P=(1, 2, 0)$  and perpendicular to vector  $n=[3, 4, 5]$ . Guide me through substituting into the formula. Then check together whether another point, e.g.,  $Q=(0,0,-2)$ , lies on this plane.”
  - **Step 3: Mini-quiz**
    - **Prompt 2.4:** “Give me three points in 3D. Ask me to find the general equation of the plane that passes through them. Check my result.”
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### Topic 3: Mutual positions of lines and planes

**Key concepts:** In this section you’ll learn: examine parallelism, perpendicularity and intersection points between lines, planes and lines with planes.

- **Step 1: Building intuition**
    - **Prompt 3.1:** “How do we examine the mutual position of two planes in 3D? What role do their normal vectors play in checking whether they are parallel or perpendicular? How to find the line of intersection if they are not parallel?”
    - **Prompt 3.2:** “How to examine the mutual position of a line and a plane? How to use the direction vector of the line and the normal vector of the plane to check parallelism/perpendicularity? How to compute the intersection point?”
    - **Prompt 3.3:** “How to examine the mutual position of two lines in 3D? When are they parallel, intersecting, or skew?”
  - **Step 2: Practice and interactive tasks**
    - **Prompt 3.4:** “I have the plane  $2x + 3y - z = 6$  and the line with parametric equation  $x=1+t, y=2+t, z=3+5t$ . Guide me step by step to find their intersection point.”
  - **Step 3: Mini-quiz**
    - **Prompt 3.5:** “Give me equations of two lines in 3D. Ask me to examine their mutual position. Check my answer.”
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### Finale: Test your knowledge and close the Geometry chapter

#### Step 1: Final test

- **Prompt 4.1:** “Prepare a combined test on analytic geometry. I want 3 tasks: 1. Find the equation of a line passing through 2 points. 2. Find the equation of a plane perpendicular to a vector and passing through a point. 3. Examine the mutual position of a line and a plane.”

#### Step 2: Why learn this? Applications

- **Prompt 5.1 (Computer Graphics):** “How are plane equations used in 3D graphics, for example to define object faces or for frustum culling and clipping?”
- **Prompt 5.2 (Robotics/CAD):** “How is the description of lines and planes fundamental in CAD and in planning motion paths for a robot arm?”

#### Step 3: What next? Preview of the next section

Congratulations! You have completed the second major section: Analytic Geometry.

- **Prompt 6.1 (Preview):** “I can now describe objects in space using algebra. What is the next big area of mathematics we will study? Give me a short, one-sentence preview of what ‘Calculus’ is and why it is key to describing change.”

Good luck on your journey through mathematics!