Student Guide: Learning Differential Calculus with Your AI Assistant

Welcome to an interactive guide to mathematical analysis! In this final part we will combine everything you've learned about functions, derivatives and integrals to solve differential equations — the language in which physics and engineering are often expressed.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "What is the characteristic equation and where does it come from?"
- An example is unclear? Ask for another: "Can you show me another example of solving a separable equation?"
- Want to check yourself? Verify your thinking: "If I understand correctly, the solution of a differential equation is not a single number but a whole family of functions, right?"

Take responsibility for your learning

Approach this task diligently. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Introduction to differential equations

Key concepts: In this section you'll learn: differential equation, order of an equation, general and particular solutions.

- **Step 1: Building intuition**
  - **Prompt 1.1:** "Explain what a differential equation is. Use the analogy of a recipe that specifies not 'how much' but 'how fast' something changes. Give a simple physics example, e.g., the relation between velocity and position in uniform motion."
  - **Prompt 1.2:** "What is the order of a differential equation? How does the general solution (with constants C) differ from a particular solution (after applying initial conditions)?"
- **Step 2: Practice and interactive tasks**
  - **Prompt 1.3:** "I have the equation y' = 2x. Guide me through finding the general solution by simple integration. Then ask me to find the particular solution satisfying y(0)=5."
- **Step 3: Mini-quiz**
  - **Prompt 1.4:** "Give me 3 simple differential equations and ask me to determine their order. Check my answers."

Topic 2: First-order differential equations

Key concepts: In this section you'll learn: separable equations and first-order linear equations.

- **Step 1: Building intuition**
  - **Prompt 2.1:** "Explain step by step the method of separation of variables using the example y' = y * x. What does it mean to 'move' all y terms to one side and all x terms to the other, and then integrate both sides?"
  - **Prompt 2.2:** "What is the general form of a first-order linear equation? Briefly explain the method of variation of constants."

- **Step 2: Practice and interactive tasks**
  - **Prompt 2.3:** "Let's solve y' = -2y together. Guide me step by step using separation of variables. What physical phenomenon does this equation model? (Hint: radioactive decay)"
- **Step 3: Mini-quiz**
  - **Prompt 2.4:** "Give me a simple separable equation and ask me to find its general solution. Check my result."

Topic 3: Second-order linear equations with constant coefficients

Key concepts: In this section you'll learn: homogeneous equations, characteristic equation, and how solutions depend on the roots of the characteristic equation.

- **Step 1: Building intuition**
  - **Prompt 3.1:** "Consider the equation a y" + b y' + c y = 0. What is the characteristic equation and how do we form it?"
  - **Prompt 3.2:** "How does the form of the general solution depend on the roots of the characteristic equation? Explain solutions for: 1) two distinct real roots, 2) one repeated real root, 3) a pair of complex conjugate roots."
- **Step 2: Practice and interactive tasks**
  - **Prompt 3.3:** "Let's solve y" + 5y' + 4y = 0 together. Guide me through: 1) writing the characteristic equation, 2) finding its roots, 3) writing the general solution."
- **Step 3: Mini-quiz**
  - **Prompt 3.4:** "Give me a second-order constant-coefficient equation and ask me to find its general solution. Check my answer."

Topic 4: Applications of differential equations

Key concepts: In this section you'll see how differential equations model the world.

- **Step 1: Building intuition**
  - **Prompt 4.1:** "Show how the harmonic oscillator equation ($y" + \omega^2 y = 0$) models the motion of a pendulum or a mass-spring system. What physical meaning do the parameters in the solution have (amplitude, phase, frequency)?"
  - **Prompt 4.2:** "Give other examples of differential equations used in physics, engineering, biology or economics."
- **Step 2: Practice and interactive tasks**
  - **Prompt 4.3:** "Imagine a bacterial population that grows at a rate proportional to its current size. Help me write this phenomenon as a simple differential equation and find its solution."

Finale: Test your knowledge and finish the math section

Step 1: Final test

- **Prompt 5.1:** "Prepare a combined test on differential equations. I want 2 problems: 1) a separable first-order equation, and 2) a second-order constant-coefficient equation."

**Step 2: What's next? Summary and preview of Physics

Congratulations! You have completed the entire math section, from linear algebra and geometry to differential and integral calculus. You now have a powerful toolkit for describing the world.

- **Prompt 6.1 (Summary):** "How do all these math areas — algebra, geometry and analysis — connect with each other?"
- **Prompt 6.2 (Preview):** "Now that I have mathematical tools, how can I use them to understand physics? Give a short preview of how vectors and derivatives help describe motion in kinematics."

Good luck on your journey through science!