

Student Guide: Learning Differential Calculus with Your AI Assistant

Welcome to an interactive guide to mathematical analysis! We start from the absolute basics on which differential and integral calculus are built — beginning with the concept of a function.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "Explain what the 'domain' of a function is using a simple analogy."
- An example is unclear? Ask for another: "Can you give me an example of a relation that is not a function?"
- Want to check yourself? Verify your thinking: "If I understand correctly, every sequence is a function, but not every function is a sequence, right?"

Take responsibility for your learning

Approach this task diligently. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Relations and Functions — Basic definitions

Key concepts: In this section you'll learn: relation, function, domain, codomain, range, argument, function value.

- **Step 1: Building intuition**
 - **Prompt 1.1:** "Explain what a relation between two sets is. Then explain how a function is a special kind of relation. Use the 'machine' analogy that for each input argument the machine produces exactly one output."
 - **Prompt 1.2:** "Define: domain, codomain and range of a function. Show the difference between codomain and range on a simple example, e.g., $f(x)=x^2$ for $f: \mathbb{R} \rightarrow \mathbb{R}$."
 - **Step 2: Practice and interactive tasks**
 - **Prompt 1.3:** "Give me 3 examples of relations. Ask me to determine for each whether it is a function and justify my answer. Check my reasoning."
 - **Step 3: Mini-quiz**
 - **Prompt 1.4:** "Give me 3 true/false questions about basic definitions of a function, domain and range. Check my answers."
-

Topic 2: Real functions and their graphs

Key concepts: In this section you'll learn: real function of a single real variable ($f: \mathbb{R} \rightarrow \mathbb{R}$), graph of a function, roots (zeros), monotonicity.

- **Step 1: Building intuition**
 - **Prompt 2.1:** "What is a real function of one real variable ($f: \mathbb{R} \rightarrow \mathbb{R}$)? How do we represent its action visually with a graph in the coordinate plane?"
 - **Prompt 2.2:** "What is a root (zero) of a function? What does it mean for a function to be increasing, decreasing, constant, non-decreasing or non-increasing on an interval? Show examples of graphs illustrating these properties."

- **Step 2: Practice and interactive tasks**
 - **Prompt 2.3:** “Draw (or describe) for me the graph of a simple function, e.g., $f(x)=x-2$. Ask me to read from the graph its zero, domain, range and intervals of monotonicity. Check my answers.”
 - **Step 3: Mini-quiz**
 - **Prompt 2.4:** “Show me the graph of a piecewise function. Ask me 3 questions about its properties (e.g., ‘What is $f(\dots)$ ’, ‘Where is the function decreasing?’, ‘What are the zeros?’).”
-

Topic 3: Sequences as a special type of function

Key concepts: In this section you’ll learn: definition of a sequence as a function, term of a sequence, general formula for a sequence.

- **Step 1: Building intuition**
 - **Prompt 3.1:** “Explain why a sequence (e.g., arithmetic or geometric) is a special type of function. What is its domain and typical codomain?”
 - **Prompt 3.2:** “What is the general term a_n of a sequence? Show how to compute the first few terms for a sequence with formula $a_n = 2n + 1$.”
 - **Step 2: Practice and interactive tasks**
 - **Prompt 3.3:** “Give me the general term of a sequence, e.g., $a_n = n^2 - 1$. Ask me to compute the first five terms. Check my results.”
 - **Step 3: Mini-quiz**
 - **Prompt 3.4:** “Give me 2 simple general-term formulas for sequences and ask me to compute particular terms (e.g., a_7). Check my answers.”
-

Finale: Test your knowledge and prepare for the next step

Step 1: Final test

- **Prompt 4.1:** “Prepare a combined test on function basics. I want it to contain 3 tasks: 1. Is a given relation a function? 2. Read properties of a function from its graph. 3. Compute several terms of a sequence.”

**Step 2: Why learn this? Functions around us

- **Prompt 5.1 (Modeling):** “Give me 3 real-world or physics examples that can be described by functions. E.g., distance as a function of time, temperature as a function of time of day, profit as a function of units sold.”

Step 3: What next? Preview of the next module

- **Prompt 6.1 (Preview):** “I have mastered the basics of functions. What is the next step in mathematical analysis? Give me a short, one-sentence preview of what a ‘limit of a function’ is and why it is one of the most important concepts in mathematics.”

Good luck on your journey through analysis!