Student Guide: Learning Differential Calculus with Your AI Assistant

Welcome to an interactive guide to mathematical analysis! In this part we'll study one of the most important and fundamental concepts in all of mathematics: the limit.

Key to success: your activity and curiosity

You are in charge of your learning!

- Don't understand a term? Ask the AI: "Explain intuitively what the limit of a function at a point is."
- An example is unclear? Ask for another: "Can you give me an example of a sequence that does not have a limit?"
- Want to check yourself? Verify your thinking: "If I understand correctly, the limit of a function at a point does not depend on the value of the function at that point, right?"

Take responsibility for your learning

Approach this task diligently. The goal is understanding. Failure to master the material will be your failure. Use this opportunity wisely.

Topic 1: Limits of sequences

Key concepts: In this section you'll learn: proper and improper limits of sequences, convergent and divergent sequences.

- **Step 1: Building intuition**
  - **Prompt 1.1:** "Explain with simple examples what it means for a sequence to converge to a limit. Use the 'approaching a target' analogy. Show how the terms of the sequence a_n = 1/n behave as n becomes large."
  - **Prompt 1.2:** "What does it mean that a sequence diverges to infinity (or minus infinity)? Give an example of such a sequence."
- **Step 2: Practice and interactive tasks**
  - **Prompt 1.3:** "Consider the sequence a_n = (n+1)/n. Ask me to compute some initial terms (a_1, a_2, a_10, a_100). Ask what number these terms seem to approach. Then show me how to compute the limit of this sequence formally as n tends to infinity."
- **Step 3: Mini-quiz**
  - **Prompt 1.4:** "Give me 3 simple sequences and ask me to compute their limits. One of them should be divergent. Check my answers."

Topic 2: Limits of functions

Key concepts: In this section you'll learn: limit of a function at a point, one-sided limits (left and right), limits at infinity, asymptotes.

- **Step 1: Building intuition**
  - **Prompt 2.1:** "How does the limit of a function at a point differ from the value of the function at that point? Show me on the graph of a function with a 'hole' how to understand this."

  - **Prompt 2.2:** "What are one-sided limits and why are they important? When does the limit of a function at a point exist? Also explain what the limit of a function at infinity is and how it relates to a horizontal asymptote."
- **Step 2: Practice and interactive tasks**
  - **Prompt 2.3:** "Show me the graph of f(x) = (x² - 1)/(x - 1). Ask me what the value of this function is at x=1. Then guide me through computing the limit of this function as x approaches 1."
- **Step 3: Mini-quiz**
  - **Prompt 2.4:** "Show me the graph of a function and ask me to read off its one-sided limits at a chosen point and its limit at infinity. Check my answers."

Topic 3: Basic limit theorems (Algebra of limits)

Key concepts: In this section you'll learn: limit rules for sums, differences, products and quotients; indeterminate forms.

- **Step 1: Building intuition**
  - **Prompt 3.1:** "What are the basic 'arithmetic' rules for computing limits? Explain the limit theorems for the sum, product and quotient of functions with simple examples."
  - **Prompt 3.2:** "What are indeterminate forms such as [0/0], [∞/∞] czy [∞ - ∞]? Why can't we just evaluate them directly? Give examples of different limits that lead to [0/0] but yield different results."
- **Step 2: Practice and interactive tasks**
  - **Prompt 3.3:** "I need to compute the limit of f(x) = (2x² - 2)/(x - 1) as x approaches 1. Guide me step by step through the calculation, showing how we first get the indeterminate form [0/0] and then deal with it by simplifying the expression."
- **Step 3: Mini-quiz**
  - **Prompt 3.4:** "Give me 2 limit problems that require applying arithmetic limit theorems and removing an indeterminate form. Check my results."

Finale: Test your knowledge and prepare for the next step

Step 1: Final test

- **Prompt 4.1:** "Prepare a combined test on limits. I want it to contain 3 tasks: 1. Compute the limit of a sequence. 2. Compute the limit of a function at a point (with an indeterminate form). 3. Read one-sided limits from a graph."

**Step 2: Why learn this? Applications of limits

- **Prompt 5.1 (Continuity):** "How does the concept of limit help define the 'continuity' of a function? What does it mean for a function to be continuous at a point?"
- **Prompt 5.2 (Derivative):** "I heard that limits are key to defining the derivative. Give me a very simple, intuitive preview of what sort of limit we will compute to find the 'instantaneous rate of change' of a function."

Step 3: What next? Preview of the next module

- **Prompt 6.1 (Preview):** "I have mastered limits. What is the next natural step in mathematical analysis that rests directly on the concept of limit? Give me a short, one-sentence preview of what the 'derivative' is and what it is used for."

Good luck on your journey through analysis!