

Problem 1

Find limit of the $a_n = \frac{2n^2+3n+2}{5-2n^2}$ sequence at $n \rightarrow \infty$.

- A) -1
- B) 1
- C) ∞
- D) $\frac{2}{5}$

Problem 2

Find limit of the function $f(x) = \frac{\sin(x-\pi)}{x}$ at $x \rightarrow 0$.

- A) 0
- B) 1
- C) -1
- D) ∞

Problem 3

Calculate the derivative of the function $y(x) = 2x^3 - 3x^2 + 8x - 9$.

- A) $6x^2 - 6x + 8$
- B) $6x^3 - 6x^2 + 8x$
- C) $2x^2 - 3x + 8$
- D) $6x^2 - 6x$

Problem 4

Calculate the derivative of the function $y(x) = x \sin(x) + 1$.

- A) $\sin(x) + x \cos(x)$
- B) $\cos(x)$
- C) $\sin(x) - x \cos(x)$
- D) $x \cos(x)$

Problem 5

Calculate the integral $\int (x^2 + 3x + 5)dx$.

- A) $\frac{x^4}{4} + x^3 + \frac{5x^2}{2} + C$
- B) $x^3 + 3x^2 + 5x + C$
- C) $\frac{x^3}{3} + \frac{3x^2}{2} + 5x + C$
- D) $3x^2 + 6x + 5 + C$

Problem 6

Calculate the integral $\int x \sin(x^2) dx$.

- A) $-\frac{1}{2} \cos(x^2)$
- B) $2x \cos(x^2) + C$
- C) $-\frac{1}{2} \cos(x^2) + C$
- D) $-\cos(x^2) + C$

Problem 7

Calculate the integral $\int \frac{x+2}{x+1} dx$.

- A) $x + \ln|x+1| + C$
- B) $\ln|x+1| + C$
- C) $x - \ln|x+1| + C$
- D) $1 + \frac{1}{x+1} + C$

Problem 8

Find the area bounded by the x-axis and the curve $y = x^2$ on the interval $[0, 2]$.

- A) $\frac{8}{3}$
- B) 4
- C) $\frac{4}{5}$
- D) $\frac{8}{3}x^3 + C$

Problem 9

A particle moves along a straight line with its position at time t given by the function $x(t) = t^3 - 6t^2 + 9t + 2$. Find the velocity $V(t) = \frac{dx(t)}{dt}$ and acceleration $a(t) = \frac{d^2x(t)}{dt^2}$ of the particle at time $t = 2$.

- A) Velocity: 0, Acceleration: -3
- B) Velocity: 3, Acceleration: 0
- C) Velocity: -3, Acceleration: 0
- D) Velocity: -3, Acceleration: 12

Problem 10

Solve the differential equation $\frac{dy}{dx} = \frac{x+2}{x+1}$ with the initial condition $y(0) = 1$.

- A) $y(x) = x + \ln|x+1| + C$
 - B) $y(x) = \ln|x+1| + 1$
 - C) $y(x) = x + \ln|x+1| + 1$
 - D) $y(x) = \frac{-1}{(x+1)^2}$
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Answers

1. A
2. C
3. A
4. A
5. A
6. C
7. A
8. A
9. C
10. C

WYNIKI ZADAŃ

Zadanie 1: -1

Zadanie 2: -1

Zadanie 3: $6x^2 - 6x + 8$

Zadanie 4: $x \cdot \cos(x) + \sin(x)$

Zadanie 5: $x^4/4 + x^3 + 5x^2/2 + C$

Zadanie 6: $-\cos(x^2)/2 + C$

Zadanie 7: $x + \log(x + 1) + C$

Zadanie 8: $8/3$

Zadanie 9: Prędkość $v(2) = -3$, Przyspieszenie $a(2) = 0$

Zadanie 10: $x + \log(x + 1) + 1$

Python code

```
import sympy as sp

# Definiowanie symboli
n = sp.symbols('n')
x = sp.symbols('x')
t = sp.symbols('t')

print("--- WYNIKI ZADAŃ ---")

# Problem 1: Granica ciągu
expr1 = (2*n**2 + 3*n + 2) / (5 - 2*n**2)
limit1 = sp.limit(expr1, n, sp.oo)
print(f"Zadanie 1: {limit1}")
```

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# Problem 2: Granica funkcji
expr2 = sp.sin(x - sp.pi) / x
limit2 = sp.limit(expr2, x, 0)
print(f"Zadanie 2: {limit2}")

# Problem 3: Pochodna wielomianu
expr3 = 2*x**3 - 3*x**2 + 8*x - 9
deriv3 = sp.diff(expr3, x)
print(f"Zadanie 3: {deriv3}")

# Problem 4: Pochodna iloczynu
expr4 = x * sp.sin(x) + 1
deriv4 = sp.diff(expr4, x)
print(f"Zadanie 4: {deriv4}")

# Problem 5: Całka nieoznaczona
expr5 = (x**2 + 3*x + 5) * x
integral5 = sp.integrate(expr5, x)
print(f"Zadanie 5: {integral5} + C")

# Problem 6: Całka przez podstawienie
expr6 = x * sp.sin(x**2)
integral6 = sp.integrate(expr6, x)
print(f"Zadanie 6: {integral6} + C")

# Problem 7: Całka wymierna
expr7 = (x + 2) / (x + 1)
integral7 = sp.integrate(expr7, x)
print(f"Zadanie 7: {integral7} + C")

# Problem 8: Pole powierzchni (całka oznaczona)
expr8 = x**2
area8 = sp.integrate(expr8, (x, 0, 2))
print(f"Zadanie 8: {area8}")

# Problem 9: Prędkość i przyspieszenie
x_t = t**3 - 6*t**2 + 9*t + 2
v_t = sp.diff(x_t, t)
a_t = sp.diff(v_t, t)
v_2 = v_t.subs(t, 2)
a_2 = a_t.subs(t, 2)
print(f"Zadanie 9: Prędkość v(2) = {v_2}, Przyspieszenie a(2) = {a_2}")

# Problem 10: Równanie różniczkowe
y = sp.Function('y')
diff_eq = sp.Eq(y(x).diff(x), (x + 2) / (x + 1))

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solution = sp.dsolve(diff_eq, y(x), ics={y(0): 1})
print(f"Zadanie 10: {solution.rhs}")
```