Section 1: Mechanics I

1. Projectile Motion

A projectile is fired from the ground with an initial velocity of 100 m/s at an angle of 37° above the horizontal. Calculate:

a) The maximum height reached.

b) The total time of flight.

c) The horizontal range of the projectile.

2. Range Optimization

For projectile motion, show analytically that the maximum range for a given initial velocity is achieved at a launch angle of 45°.

3. Path Intersection

Alice is moving along a path described by $A(t) = (2+t, 8-3t)$ and Bob is moving along a path $B(t) = (2t-1, 2t+2)$. Determine if their paths intersect. Will they collide at the same time?

4. 2D Collision

Two cannons are aimed at each other. The first cannon at (0,0) fires a projectile with an initial velocity of $[100\text{ m/s}, 80\text{ m/s}]$. The second cannon at $(800\text{ m}, 0)$ fires a projectile with a velocity of $[-100\text{ m/s}, 80\text{ m/s}]$. At what time t and at what point (x,y) will they collide?

5. Relative Velocity

A river flows east at 2m/s. A boat that can travel at 5m/s in still water wants to go directly north across the river. In what direction (angle) should it head? How long will it take to cross the river if it's 200 meters wide?

6. Variable Velocity

An object's velocity is given by $v(t) = t^2 + 2t - 5$. If the object is at $x = 4$ at $t = 0$, what is its position at $t = 3$?

7. Vertical Collision

A stone is thrown vertically upward with a speed of 20m/s. A second stone is thrown vertically upward with a speed of 30m/s exactly one second later. At what time and height do they collide?

8. Circular Motion

Calculate the centripetal acceleration of a person standing on the Earth's equator. The Earth's radius is approximately 6378 km.

9. Momentum Comparison

Which has greater momentum: a 2-gram fly flying at 10 m/s or a 60-gram tennis ball moving at 1 m/s?

10. Vector Calculus

The position of an object is given by $\vec{r}(t) = (3t^2)\hat{i} + (5t - 8t^2)\hat{j}$. Find the object's velocity and acceleration vectors as a function of time.