Section 0: Introduction to Physics

1. Vector Algebra

Given two vectors in 3D space: $\vec{a} = [2, 1, -3]$ and $\vec{b} = [4, -2, 1]$. Calculate:

a) The magnitude of each vector.

b) The dot product $\vec{a} \cdot \vec{b}$.

c) The cross product $\vec{a} \times \vec{b}$.

d) The angle between vectors $\vec{a}$ and $\vec{b}$.

2. Systems of Equations

Find the values of x and y that satisfy both equations: $2x + 3y = 12$ and $x - y = 1$.

3. Proportionality

Consider the Universal Law of Gravitation: $F = G\frac{m_1 m_2}{r^2}$, where F is the gravitational force between two masses m_1 and m_2, r is the distance between their centers, and G is the gravitational constant. Determine the factor by which the force F changes if the distance r is *doubled* and both masses (m_1 and m_2) are *halved*.

4. Rearranging Formulas

The formula for the period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$ Rearrange the equation give formula for g (acceleration due to gravity).

5. Trigonometry

A vector $\vec{A}$ has a magnitude of 15 and makes an angle of $\theta = 60°$ with the horizontal axis. Calculate its horizontal and vertical components.

6. Function Analysis

Consider the function $f(x) = 3x^2 - 12x + 7$. Identify any local maxima or minima.

7. Logic & Series

A bicycle is 10 meters from a wall and moves towards it at a constant speed of 1 m/s. A fly starts from the bicycle's front wheel and flies towards the wall at 2 m/s. When it hits the wall, it instantly turns back and flies to the bicycle, and so on. What is the total distance the fly travels before being crushed?

8. Definite Integrals

Calculate the area under the curve of the function $f(x) = \sin(x)$ from $x = 0$ to $x = \pi$.

9. Optimalization Problem

A rectangle is under the curve $y = 3 - x^2$ in the first quadrant. What are the dimensions of the rectangle with the maximum area?

10. Infinite Series

Determine the final position of an ant that starts at the origin and moves according to the following pattern: 1 m east, 1/2 m north, 1/3 m west, 1/4 m south, 1/5 m east, and so on.