Communication Systems 3 Tutorial 2

1. Sketch the result of the following convolution operations (where $t_0 > 0$) and check your result by examining the product of the Fourier transforms and then taking the inverse transform

   (a) $\delta(t) \otimes \delta(t - t_0)$

   (b) $\mathrm{u}(t) \otimes \delta(t - t_0)$

   (b) $\mathrm{rect}(t) \otimes \delta(t - t_0)$

2. Use the graphical method, or otherwise, to determine the following convolutions

   (a) $\mathrm{rect}(t) \otimes \mathrm{u}(t)$

   (b) $\mathrm{rect}\frac{t}{2T} \otimes \mathrm{rect}\frac{t}{T}$

3. You will have noticed that the convolution of two delta functions results in another delta function. We can handle the convolution of two piecewise linear functions by (1) taking derivatives until delta functions appear, (2) performing the convolution with the delta functions and (3) integrating the result as many times as the total number of differentiations. Use this method to check your answers in question 2. Explain why this technique works by considering the Fourier transform of a derivative.

4. Find the output of a low-pass RC filter for an input $f(t) = \mathrm{rect}\frac{t}{T}$. Check your result by using the method from the previous question for the input waveform.

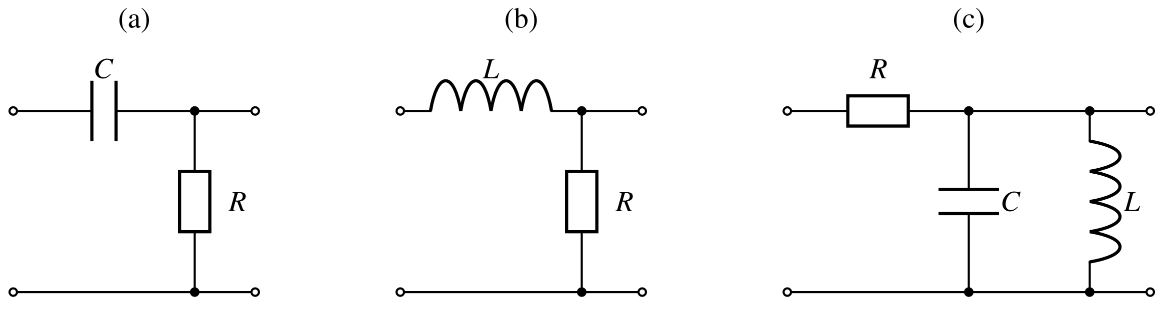
5. Use frequency convolution to prove the trigonometric identity

$$2\cos\omega_1 t \cos\omega_2 t = \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t$$

6. What is the Fourier transform of the digital pulse stream (of period T) where the mark-to-space ratio is 1:3 ?

7. What is the Fourier transform of a train (of period T) of Gaussian pulses $f(t) = \mathrm{e}^{-t^2/(2\sigma^2)}$

8. For the following filters, find the response function $H(\omega)$. For (a) and (b) take the inverse Fourier Transform to find the impulse response function $h(t)$. Hence, determine the output of a step-function input $f(t) = v_0 u(t)$?



What is the resonant frequency of the bandpass filter (c). For cases (b) and (c) find their bandwidths (using -3 dB points, in (c) assume $|\omega - \omega_0| \ll \omega_0$). Write down the quality factor Q (using ratio of resonant frequency to bandwidth) for the LCR bandpass filter (c).