Communication Systems 3 Tutorial 1

1. AM radio stations in the U.K. have a 9 kHz bandwidth. What is the maximum number of AM radio stations which can be received by a tuner which covers 531 to 1611 kHz.

2. Convert a signal-to-noise value of 30 dB to a ratio. Hence find the capacity of a teleprinter channel with this SNR and a bandwidth of 300 Hz.

3. A satellite communications link is required to have a capacity of $1\,\text{Mbit}\,\text{s}^{-1}$. If the bandwidth of this channel is 10 MHz, what is the minimum signal-to-noise ratio which can be tolerated.

4. Show that the set of functions $\cos n\omega_0 t$ are orthogonal by evaluating

$$\int_0^{2\pi/\omega_0} \cos n\omega_0 t \cos m\omega_0 t \, \mathrm{d}t$$

5. Find the trig Fourier series for the rectified cosine wave defined $f(t) = |\cos t|$. Use Parseval's theorem to determine the ratio of the power in the DC term to the total average power.

6. Find the trig Fourier series of the periodic sawtooth function defined $f(t) = t$ over the range $-\pi < t < \pi$. Produce an exponential Fourier series for the same function.

7. Find the exponential Fourier series for the periodic function

$$f(t) = \begin{cases} -1 & -\pi < t < -\pi/2 \\ 0 & -\pi/2 < t < \pi/2 \\ 1 & \pi/2 < t < \pi \end{cases}$$

8. Find the Fourier transform of the single cosine pulse

$$f(t) = \begin{cases} \cos t & |t| < \pi/2 \\ 0 & |t| > \pi/2 \end{cases}$$

9. Find the Fourier transform of the double-sided exponential pulse $f(t) = \mathrm{e}^{-a|t|}$. Check that your answer has the correct symmetry.

10. Using the duality property, find the Fourier transform of the Lorentzian function

$$f(t) = \frac{1}{\pi} \frac{\epsilon}{t^2 + \epsilon^2}$$

    Check your result tends to the Fourier transform of $\delta(t)$ in the limit $\epsilon \to 0$.

11. Using the frequency translation property, use the result for the Fourier transform of a single-sided exponential to find the Fourier transform of

$$f(t) = \begin{cases} 0 & t < 0 \\ \mathrm{e}^{-at} \sin \omega_0 t & t > 0 \end{cases}$$

12. From our example in the lecture of two square pulses, consider the integration of this function to obtain the Fourier transform of a triangular pulse

$$f(t) = \begin{cases} \tau + t & -\tau < t < 0 \\ \tau - t & 0 < t < \tau \\ 0 & |t| > \tau \end{cases}$$