Simple BLDC motor model

This is a simple brushless DC motor model. It has been written with the purpose of generate approximate motor performance charts given a few characteristics numbers and the operating voltage. Often manufacturers of small brushless motors, like those installed on flying model aircraft, do not provide performance charts, but only a few parameters together with two or three propeller as typical installation. If you want to match another torque load, say a propeller with different diameter and/or RPM, you may not know if the motor is suited for your application. As end-user looking for quick and useful info for motor-propeller coupling, you should have a tool to get the motor's steady state performance data with just the basic parameters provided by the manufacturer.

Therefore, the model here provided needs only the following inputs:

- ullet operating voltage V, chosen by the user
- the K_{ν} constant, in RPM/Volt
- the no-load current I_0 , in Ampere
- the internal resistance R_m , in Ohm

The mathematical model in detail

The model is based on a power balance:

$$P_{\text{electric}} - P_{\text{copper}} - P_{\text{iron}} = P_{\text{shaft}}$$

that is, the shaft power, i.e., the power moving the load (propeller) on the shaft, is equal to the motor electric power minus the copper and the iron losses. The copper losses are due to the heat wasted by the internal resistance of the windings (Ohm's law). The iron losses are due to the no-load current, i.e., the current circulating in the motor without an applied load on the shaft.

The electric power is simply the applied voltage times the current absorbed by the motor:

$$P_{\text{electric}} = VI$$

The copper losses are the internal resistance times the square of the motor current:

$$P_{\text{copper}} = R_m I^2$$

The iron losses are the operating voltage times the no-load current:

$$P_{\rm iron} = VI_0$$

The shaft power can be expressed in several ways. By assuming a propeller as motor load, we can write:

$$P_{\text{shaft}} = Q(\omega) \omega$$

that is a torque Q, function of the angular velocity ω , times the angular velocity ω itself.

In principle, the angular velocity may be calculated with the motor constant K_{ν} (in opportune units, since ω is in rad/s):

$$\omega_0 = K_{\nu} V$$

but since the applied load will increase the current absorbed by the motor reducing the useful (shaft) power, the angular speed will also reduce, hence $\omega_0 \neq \omega$. We assume this model:

$$\omega = K_{\nu}(V - R_{m}I)$$

that is the angular speed is linearly decreasing with the motor current, which is an expected result. The higher is the load, the higher will be the current, and the angular speed will be lower with respect to the value achieved with the same voltage in the no-load case.

By detailing the power balance equation by expanding some useful terms we get:

$$VI - R_m I^2 - P_{\text{iron}} = P_{\text{shaft}}$$

where the iron losses and the shaft power are left as known terms. By rewriting this equation in the only unknown *I* we get:

$$R_m I^2 - VI + (VI_0 - P_{\text{shaft}}) = 0$$

that is a quadratic equation which the physical solution is the smaller root:

$$I = \frac{V - \sqrt{V^2 - 4R_m(VI_0 + P_{\text{shaft}})}}{2R_m}$$

Also, the solution is physical if the discriminant is greater or equal than zero. That's why we treat $P_{\rm shaft}$ as a known term: we are interested in motor performance chart, not in a single value of the applied load. Therefore, we calculate the current I for an array of $P_{\rm shaft}$ varying from zero to its maximum possible value, which is derived from the canceling of the discriminant:

$$V^2 - 4R_m(VI_0 + P_{\text{shaft}}) = 0 \longrightarrow P_{\text{shaft max}} = \frac{V^2}{4R_m} - VI_0$$

Hence, the shaft power is varied as:

$$0 \le P_{\text{shaft}} < P_{\text{shaft max}}$$

(theoretically the shaft power could be equal to its maximum, but numerically the discretization error may give imaginary solutions, so in the MATLAB code we stop at 99.9% of this maximum value)

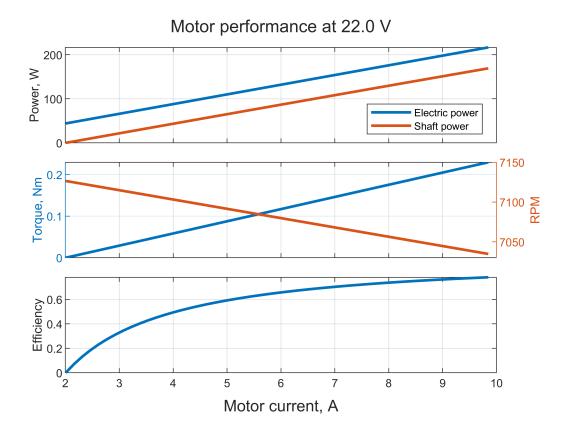
Once known the motor current *I* we can calculate all the other parameters of interest, in particular the electric power, the angular speed, the motor efficiency, and the useful torque. The equations for the first two quantities have been presented before. The last two quantities are simply:

$$eff = \frac{P_{shaft}}{P_{electric}} \quad and \quad Q = \frac{P_{shaft}}{\omega}$$

The MATLAB live script

Here the user assigns the motor constants and change the operating voltage to get a live update of the charts.

The first three input fields are self-explanatory. The max current field Imax is not mandatory, but it is useful to limit the x-axis plot range. Therefore, the user may input the value of interest, even less that the value issued by the motor manufacturer. The voltage V can be quickly changed by the slider.



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