Generalized Thermography: Algorithms, Implementation, and Application to Go Endgames

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Abstract

Thermography [1] is a powerful method for analyzing combinatorial games. It has been extended to games that contain loops in their game graph by Berlekamp [2]. We survey the main ideas of this method and discuss how it applies to Go endgames. After a brief review of the methodology, we develop an algorithm for generalized thermography and describe its implementation. To illustrate the power and scope of the resulting program, we give an extensive catalog of examples of Ko positions and their thermographs.

We introduce a new method related to thermography for analyzing ko in the context of a specific ko threat situation. We comment on some well-known Go techniques, terminology, and "exotic" Go positions from a thermography point of view. Our analysis shows that a framework based on generalized thermography can be useful for the opening and midgame as well. We suggest that such a framework will serve as the basis for future strong Go programs.

1 Analysis of Go Endgames

Generalized thermography [2] is an efficient method for playing well in "loopy" Go endgames containing ko. Solving such endgames by traditional minmax-based methods is feasible only for very tiny boards or extremely late endgames. A program based on combinatorial game theory can solve loopfree endgames containing many independent local positions, where each is of moderate size, up to about 10 local moves [9]. Generalized thermography can deal with endgame positions of comparable size that may contain ko. The technique does not guarantee perfect play, but the imprecision is small, and becomes zero in the case of an "enriched environment".

The method gives good results in all but the rarest ko fights. It gives advice on when to start a ko fight, how big ko threats have to be, and when to finish a ko. There is evidence that the move chosen by generalized thermography is sound in the large majority of real game situations. It is provably optimal in several simplified models of endgame play.

In a few cases of "strange loops" the method fails. These are just the kind of positions which are handled differently by variants of the Go rules, such as Japanese, Chinese or Ing rules. The theory of generalized thermography provides deeper insight into the specific difficulty of such positions.

Computer implementations of thermography need to address several issues: First of all, supporting data structures and detailed algorithms are needed. The speed of algorithms will depend on the size of the game graph and its loop structure. Another important practical issue is the representation of a loopy game. The simplest representation is by a tree, where each terminal node indicates either a terminal position or a loop-closing move that repeats a position on the path to the root.

However, in loopy games transpositions occur very frequently. Using these transpositions reduces the graph size dramatically. Transpositions in effect change the tree into a general graph, where there is no a priori order that can be used for structuring the thermograph computation as a one-pass process. In section 4 we develop an iterative algorithm that allows us to compute thermographs using this efficient data structure.

An implementation of generalized thermography can be used as a building block for the analysis of ko in Computer Go. Ko occurs in every sufficiently complex local fight. In earlier experiments with small corner situations, ko fights were found to affect the local game in all situations of seven or more empty spaces [11]. Current Go programs are still poor at playing ko. Strong future Go programs will need to deal with ko in complex and intricate ways, and generalized thermography can provide the necessary theoretical framework for such programs.

To illustrate the power of the theory, we give a large catalog of examples, proceeding from simple kos to iterated and multi-step ko. We analyze a large number of ko problems taken from textbooks and master games, and discuss exotic cases such as *mannen ko* and *"3 Points Without Capturing"*.

Section 5, by Bill Spight, extends the method to analyze ko in the context of a specific set of threats.

2 Loopfree Combinatorial Games

This section reviews prior work on combinatorial game theory for Go endgames and its implementation. We study models for endgame play based on combinatorial game theory, starting with the loopfree case in preparation for later models that include ko fights. We obtain a framework describing how to decompose a game into a sum of subgames, analyze the resulting local games, and evaluate the sum game to find a good or optimal move.

2.1 The Combinatorial Game Approach to Go Endgames

The steps necessary for the solution of Go endgames by the methods of combinatorial game theory are board partition, local analysis and evaluation, and move selection in the sum game. We give a brief overview of these phases. Detailed descriptions can be found in [9].

2.1.1 Board Partition and Subgame Identification

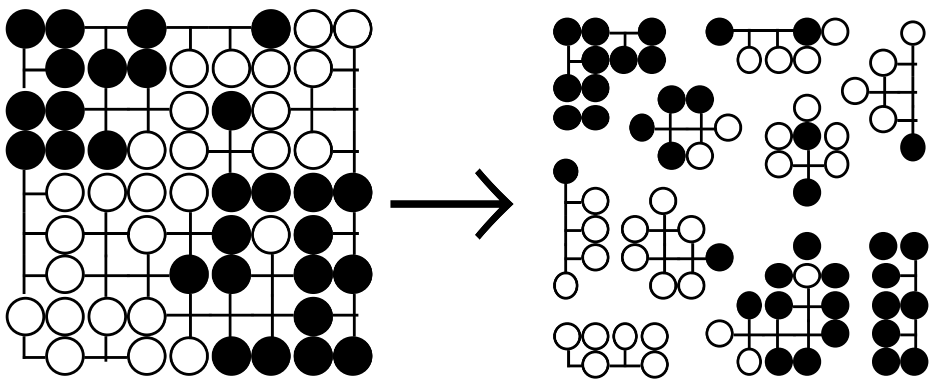

Figure 1: Board partition

The precondition for applying combinatorial game theory is that a game decomposes into a sum of subgames. In Go, this happens when parts of the board are separated by walls of safe stones. Moves in one part have no effect on other parts across such a wall. In the loopfree case, without ko fights, this independence is strong [3, 9]. With ko, a limited kind of interaction is introduced: ko threats played in one subgame affect the status of the ko in another subgame.

2.1.2 Local Search and Evaluation

Local search is exhaustive in principle: all possible moves are generated. Furthermore, in contrast to minmax-style search, successive moves by the same player have to be considered. Capturing a ko restricts the set of legal moves for followup positions. However, the ko ban is a non-local feature of the full board and can be broken by a play elsewhere. Therefore it is necessary to generate all moves without any ko restrictions for local analysis.

Local search is stopped as soon as the value of a local situation can be determined. This happens when there are no more good moves, or when the value of the position is known from transposition into a previously analyzed one.

2.1.3 Sum Game Evaluation and Move Selection

Each local game is analyzed independently at first. Increasingly detailed methods of analysis are computing the "mast value" and the temperature; the thermograph; or the full mathematical game expression of a local game. To select a move, different algorithms compare temperatures, thermographs or incentives of moves.

2.1.4 Exact Solution of Endgames

Loopfree late endgames can be solved exactly [9]:

Board partition Find absolutely safe stones, territories and *dame* points, then partition the rest of the board into connected components.

Local search and evaluation A brute force search inside the local area, with a few refinements to cut down the search space while retaining exactness [11].

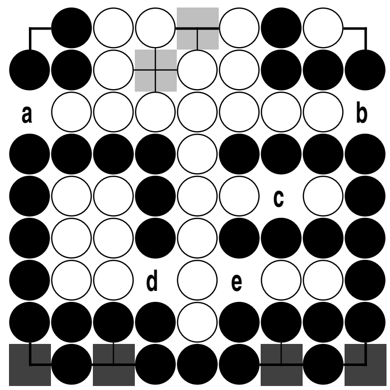

Figure 2: Switches

Move selection in sum game Determine all moves whose incentives are not dominated by other moves. If a unique such move exists it is selected immediately. If there are several nondominated moves, a more expensive summation of games is done to find a move that retains the minmax value of the whole game.

2.2 Combinatorial Game Theory for Loopfree Games

2.2.1 Basic Concepts

The *mean* measures how many points a game is worth on average, when playing sums of many games. The mean is linear:

$mean(A + B) = mean(A) + mean(B)$

Leftscore and *Rightscore* are the minmax values of a game when players move alternately and Left (Right) plays first.

Cooling is a technique for simplifying games by subtracting a tax from every move. Cooling simplifies a game while retaining much of its structure. It does not affect the mean. The *temperature* of a game is the smallest amount of cooling that makes the Leftscore and the Rightscore of the cooled game equal.

Figure 2 shows several games that stop immediately after a single play by either player. Such games are called *switches*, and are of the form $x \mid y$, where x and y are numbers and $x > y$. The mean is $(x + y)/2$ and the temperature $(x - y)/2$.

2.2.2 Simplification of Games and Sums

Loopfree games can be brought into a canonical form by repeatedly removing dominated options and reversing reversals [1]. This process also removes all but one of several options when they have the same value.

The analysis of sum games can be simplified by removing null games, integers, and pairs of games and their inverses $G + (-G)$. All these games can be added to yield a single integer. None of these simplifications may be used directly in sums containing ko. However, thermography provides some related concepts that can still be used (see section 2.4).

2.3 Thermography

Definition 1 *Let G_t be the game G cooled by t. Then the* thermograph *of G is a graph that shows both the* Leftscore(G_t) *and the* Rightscore(G_t) *plotted along the reversed x-axis, as a function of the temperature t on the y-axis.*

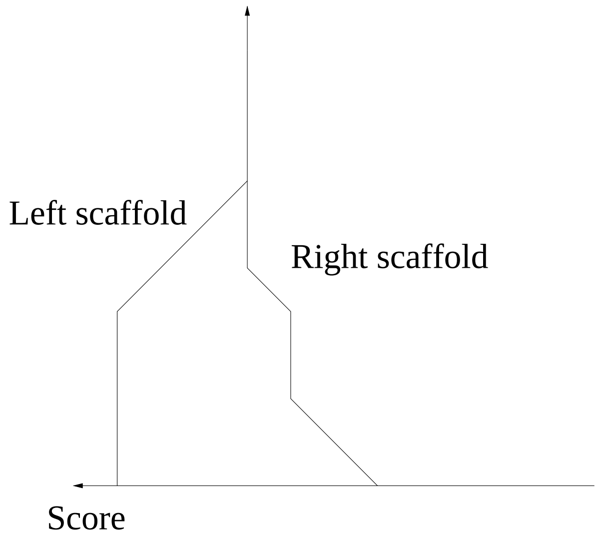

Figure 3: Sample thermograph

Classical thermographs for loopfree games, such as the example in Figure 3, have slopes that are vertical or 1 or -1.

Thermographs provide a powerful tool for determining mean values and temperatures. The thermograph of a game G can be easily computed from the thermographs for G^L and G^R by translating their inner boundaries:

$LeftWall(G) = max_{G^L}(Rightscore(G^L_t) - t)$

$RightWall(G) = min_{G^R}(Rightscore(G^R_t) + t)$

The temperature of G is the value of t at which these walls meet. At values of t less than the temperature of G, the scaffolds become the boundaries of G. At values of t higher than the temperature of G, the tax exemption takes effect, and the left and right boundaries of the thermograph of G coincide to form a *vertical mast*. The *vertical mast* continues upward from the point where the scaffolds meet.

If G is infinitesimally close to a number n, both scaffolds coincide in a mast at n:

$Leftscore(G_t) = Rightscore(G_t) = n$ for all $t \geq 0$

A pass is optimal at t if Left wall and Right wall are the same, and the slope of the wall is vertical or descending in the player's direction (this can occur in *hyperactive* ko, see section 3.3.2). In one-sided sente situations, both playing and waiting will be optimal for one player in some temperature range.

2.4 Temperature Based Pruning Methods

Dominance and reversability are classical concepts of combinatorial game theory. They have the following counterparts in thermography [2]:

Thermographic dominance A move is thermographically optimal at temperature t iff its taxed scaffold coincides with the game scaffold at t. Other moves, which have an inferior scaffold value at t, are thermographically dominated.

Thermographic reversability If a move increases the local temperature, players following a thermography-based strategy will keep playing locally as long as the temperature is greater or equal to the starting temperature.

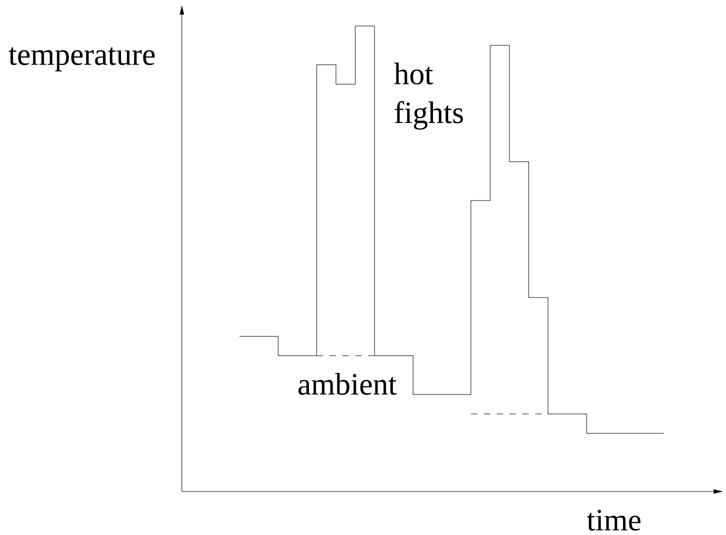

Figure 4: Ambient

2.5 Sum Game Evaluation

2.5.1 Ambient Temperature

Endgame play generally proceeds from the most valuable (hottest) plays, which may be worth 20 points or more, to the smallest moves worth only a fraction of a point. Playing a *sente* move temporarily increases the temperature. Replying to such a move is urgent. Overall, the *ambient* temperature is monotonically non-increasing throughout the endgame.

2.5.2 Sentestrat

The sentestrat algorithm [2] models a slowly decreasing temperature by precisely defining the *ambient temperature*. Often, all endgames have a temperature smaller or equal to the ambient. A player following sentestrat answers any move that increases the local temperature above the ambient. If both players use sentestrat, then at each time there is at most one local game hotter than the ambient. Figure 4 shows such a scenario.

Sentestrat is primarily a local algorithm: there is no complicated global analysis. Information about the global environment is reduced to a single number, the ambient.

2.6 Economic Model

Berlekamp [2] introduces a different model for playing sums of games, where players pay a tax to the opponent for the right to move. The tax rate is determined by auctions. A sum game continues at the same tax rate until both players choose to pass. Then there is a new auction to determine a lower tax rate. Berlekamp shows that in this type of play, always bidding the temperature of the hottest game ensures achieving the game-theoretic value for an arbitrary sum game. Sentestrat is a provably optimum strategy.

2.7 Enriched Environments

The enriched environment is another scenario where sentestrat can guarantee the optimal outcome: This model assumes alternating play, but it augments a given sum game with a dense, uniformly

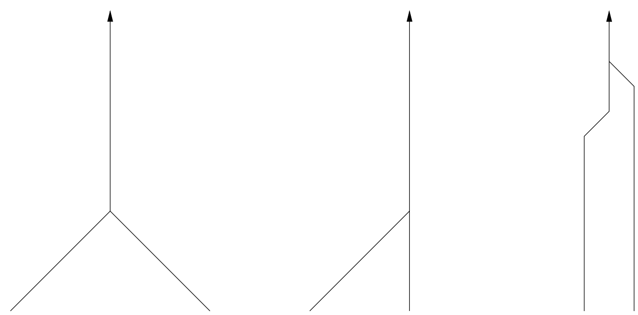

Figure 5: Typical thermographs for gote, one-sided sente, and "double sente" situations

spaced sum of switches (see section 2.2.1).

No matter how complicated a sum, as long as it contains at most one "hyperactive" position one can add to it yet more games to obtain an even bigger sum whose score is known to an arbitrarily high degree of precision. In the loopfree case, the scores of the augmented sum are known exactly. The extra switches provide an "enriched environment" within which even rather wild loopfree games assume angelic behavior under sentestrat's supervision.

Each local position can be studied in an enriched environment. Within this environment, the behavior of a complicated game in a complicated sum is essentially the same as if the other complicated games were not present. Berlekamp [2] gives more details on this algorithm.

2.8 Interpretation of Moves Based on Thermography

Thermography is a method for analyzing local games that summarizes the rest of the board in one parameter, the "ambient temperature" of the sum game. The thermograph shows the value of playing first in a local game for both players and all possible ambient temperatures.

2.8.1 Interpretation of Scaffold Direction

vertical scaffold both players play the same number of moves (or no move at all). The first player keeps *sente*.

diagonal scaffold the first player is also the last, he makes one move more than the opponent and therefore takes *gote*.

If both players' scaffolds are vertical at a temperature t, it indicates a *double sente* situation. The difference in scaffold values at that temperature is available as a free profit to the first player who takes it. Sente and gote are relative to the ambient temperature. At sufficiently high temperatures, any "double sente" situation becomes either one-sided sente or gote. Figure 5 shows an example.

3 Endgames Containing Ko Fights

3.1 Drawing Game Graphs for Ko

Diagrams showing the game graph of loopy games are similar to those used for game trees [1]. Non-loop moves are represented by a straight line. A move always leads from the higher to the lower endpoint of the line. A two move loop is drawn as an arc. It can be traversed in both directions. In Figure 6, players can move back and forth between G and H. Terminal positions such as 0 and 1 in Figure 6 are marked by their value.

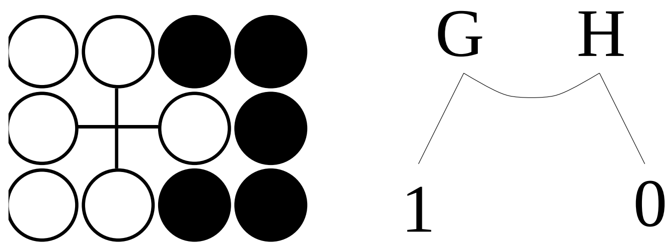

Figure 6: Game graph for ko

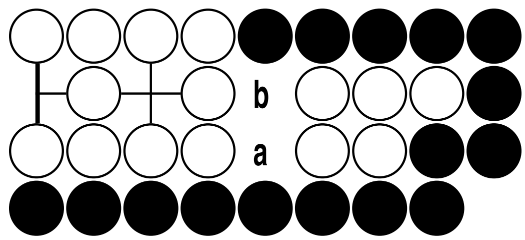

Figure 7: Ko threat for Black

3.2 Ko Threats

Figure 7 shows the game $10 \mid 0 \parallel *$. The mean and the temperature of this game are both 0, since *Leftscore* = *Rightscore* = 0. The thermograph is a vertical mast at 0. In classical combinatorial game theory, the canonical form of this game is also 0. However, it provides one ko threat for Black.

We might similarly construct other games of the form

$$threat_n = n \mid 0 \parallel *$$

for any value of n. By allowing n to be very big, we obtain our standardized black ko threat. By interchanging Black and White, we construct white ko threats $-threat_n = * \parallel 0 \mid -n$.

Arbitrary numbers of ko threats can be added to arbitrary sums of *loopfree* games with no effect on the *Leftscore* and *Rightscore* of the sum. However, it is well known that ko threats *can* affect the scores of positions involving ko.

3.3 Models for Sum Games that Include Ko

Games with loops are harder to analyze than loopfree games: in general such games have no canonical form, no unique mean and no unique temperature.

To study a ko in isolation, or many copies of the same game, as in loopfree theory, gives an incomplete view of its behavior. It is well known that a ko reacts differently in different contexts, so we want to study a spectrum of environments in which a ko could take place.

It is useful to begin with simple models of sums containing ko. The first such model assumes a single ko in the context of an “enriched environment”. There are no explicit ko threats, but both players have enough moves available at each temperature to ensure fair compensation for losing the ko.

Another model adds idealized ko threats of the form $big \mid 0 \parallel *$ to a sum consisting of the ko plus several other, loopfree games, which cannot be used as ko threats.

In his Ph.D. thesis [7], Kim models the urgency of playing ko by *pseudoincentives*. He shows that it is often possible to treat a ko as if it had the same incentive as a carefully chosen loopfree game.

3.3.1 Komaster

Berlekamp [2] models play of a loopy game under the assumption that one player is "komaster", i.e. able to win all kos because she has a surplus of ko threats. We will call the other player the "koloser". The komaster cannot win ko fights for free, however: once she starts playing in a ko she has to continue to play locally. This rule ensures that once a ko is started, it will be won at the same temperature. Komaster has enough ko threats to win the ko but not enough threats to lower the temperature during the ko fight. Such a scenario can be realized by using an enriched environment (see section 2.7).

3.3.2 Placid and Hyperactive Ko

Given a komaster, we can compute the thermograph of a loopy game. The start of the vertical mast defines the temperature of the game. The *mast value* however cannot be interpreted as a mean value in all cases. Many kos have the same mast value independent of who is komaster. Such kos are called "placid", while those whose mast value depends on the komaster are called "hyperactive". Many of the examples in section 11 are hyperactive, for example Figures 76,79,80 and 81.

3.3.3 Decreasing the Temperature During a Ko Fight

Spight and Kim [13, 7] study decreasing the temperature during a ko fight. This changes the thermograph, since komaster may be able to win the ko at a lower temperature, and therefore has to give up less in return. Conversely, koloser may initiate a ko fight at a higher temperature than predicted by thermography. He may even win the ko! This possibility, called "tunnelling" [13], appears in the solution to the "$1,000 Ko" [8].

4 Generalized Thermography

Berlekamp develops a methodology for computing thermographs of loopy games and illustrates it with several examples [2]. In this section we briefly review this methodology, develop details of an algorithm, and describe its implementation.

Trying to compute the thermograph of a loopy game with the equations of loopfree thermography leads to an infinite loop. We consider two closely related loopfree games: the "solidhat" and "hat" games. These games are used to define the thermograph of a loopy game.

4.1 Methodology

Generalized thermography is based on an enriched environment model and a rule for when to win a ko. The algorithm has three crucial steps:

Select the komaster Komaster is able to win all kos because she has a surplus of ko threats. For each loopy game we compute a pair of thermographs, one with Black as komaster and one with White.

The pass ban In classical thermography, players have the option of passing at each move. If Komaster had this option as well, she could win all ko fights for the same price as a normal move: she would move into a position where the opponent could not win in one move, then wait until the temperature is very low to play the remaining moves and resolve the ko. Whenever the opponent tried to play the ko, she would just use her surplus of threats to revert the ko to the previous state. The "komonster" of section 5 models such a player.

To calculate the mast value for a ko, Berlekamp introduces the pass ban rule: once komaster starts playing in a ko she has to go ahead and win it.

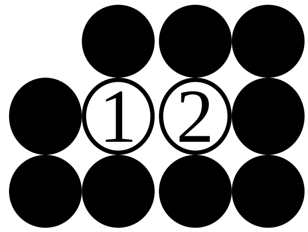

Figure 8: 3 at 1. Suicidal two-move-White-only loop

Construct solidhat and hat graphs The komaster and pass ban rules introduce two different states for some nodes in the game graph: if a node in a loop is entered for the first time, all moves are allowed, and the thermograph of the node is computed from its options in the standard way. However, when a node is entered the second time, both the komaster rule and the pass ban rule change the computation:

The komaster rule removes loop-closing moves by the koloser. The pass ban rule removes the option of passing for komaster. This creates a non-vertical mast. A node with pass ban restrictions is called a *solidhat* node, and a node without restrictions a *hat* node.

4.2 Computing the Thermographs of Loopfree Subgames

If the subgraph below a node in a loopy game contains no ko, i.e. if it is a tree, we can compute thermographs in this subtree using the classical algorithm. Such loopfree subtrees always exist, e.g. the terminal nodes of the game.

4.3 Elimination of Single-Player Loops

From now on, we will consider each loop in a game graph to have moves by both players. In Go, single-player loops are possible when the rules allow suicide (see Figure 8). Such loops contain at least one bad move and can be eliminated in a preliminary step by pruning the loop-closing move [9]. Another preprocessing algorithm that can eliminate bad moves and loops from a game is proposed in section 9.

4.4 Sample Thermograph Computation: One Point Ko

We demonstrate the algorithm that computes solidhat and hat thermographs using the example of the one point ko in Figure 9. This loopy game can be written as follows:

$$\begin{aligned} G &= \{1 \mid H\} \\ H &= \{G \mid 0\} \end{aligned}$$

The only loopfree subgames are the nodes marked 1 and 0. Their thermographs are vertical masts at 1 and 0 respectively.

The following computations assume that Black is komaster. This eliminates White's move from G to H in the solidhat node:

$$\begin{aligned} \overset{\blacktriangle}{\mathrm{G}} &= \{1 \mid \}, \text{ with proviso that Black cannot pass immediately} \\ \hat{H} &= \{\overset{\blacktriangle}{\mathrm{G}} \mid 0\} \\ \hat{G} &= \{1 \mid \hat{H}\} \end{aligned}$$

The thermograph of $\overset{\blacktriangle}{\mathrm{G}}$ is a bent mast starting at 1 with value $1-t$ at temperature t.

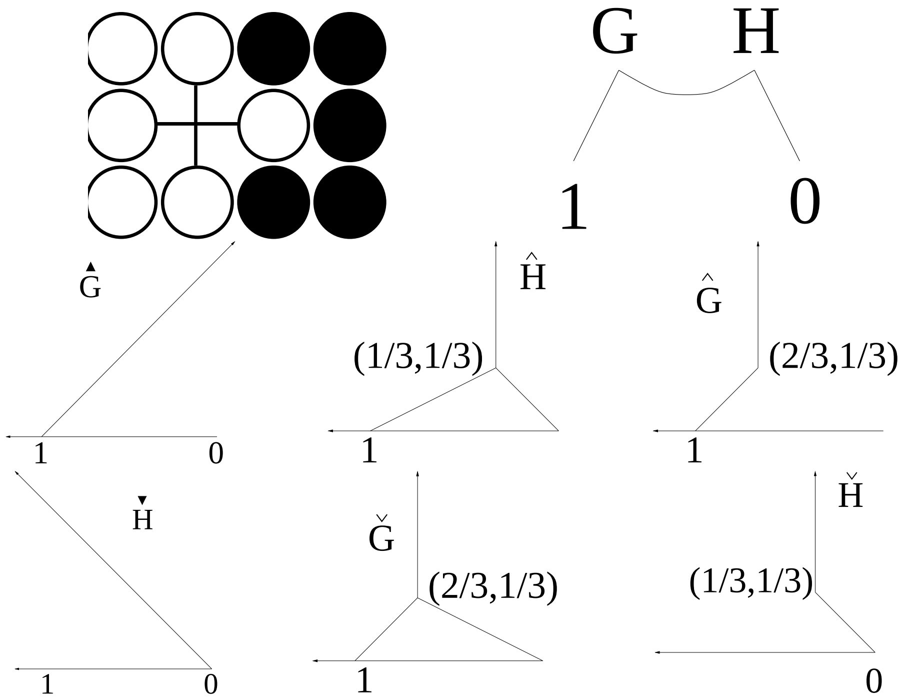

Figure 9: Solidhat and hat thermographs of the one point ko

Given $\overset{\blacktriangle}{\mathrm{G}}$, the hat thermographs $\hat{H}$ and $\hat{G}$ can be computed from the equations above. If White is komaster, the corresponding equations are:

$$
\begin{aligned}
\overset{\blacktriangledown}{\mathrm{H}} &= \{\,|\,0\}, \text{ with proviso that White cannot pass immediately} \\
\check{G} &= \{1\,|\,\overset{\blacktriangledown}{\mathrm{H}}\} \\
\check{H} &= \{\check{G}\,|\,0\}
\end{aligned}
$$

The thermograph for $\overset{\blacktriangledown}{\mathrm{H}}$ starts at 0 and has value t at temperature t.

4.5 Computing Thermographs: Solidhat and Hat Nodes

Before a thermograph can be computed, a loop-breaking move must be selected for each loop in the game. In the most common case of 2-move loops the choice is unique: there is only one move by one player in the loop. The node from which this move is made is called the "solidhat" node.

After marking solidhat nodes, computation of thermograph starts. *ComputeNode* determines if all necessary thermographs of moves needed to compute the hat or solidhat graph are known. It partitions all options into KoOptions and the rest, decides which type of graph should be computed and checks whether the necessary input is available.

When a game graph contains several stages of loops, solidhat and hat values must be computed in turn. ComputeTree traverses the tree representation of the game graph, looking for nodes that can be computed. The main program iteratively calls ComputeTree as long as there is work to do.

```
boolean ComputeNode (node, komaster)
{
   computed = FALSE; // no new graph computed yet.
```

```
   if ( not already computed(node, graph)
      and ( retrieve all komaster options:
         solidhat graphs of nodes in loop,
         hat graphs of all other nodes ) )
   then
   {
      if solidhat of node known then // compute hat graph next
         koOptions = emptyset;
      else
         find koOptions of koLoser;

      if already computed all hat graphs of
         { koLoser options - koOptions }
      then
      {
          computed = TRUE;
          compute Black (White) scaffold from max (min) scaffolds
          of all options, and apply tax;

          select type of new graph:
             if koOptions exist then
                type = solidhat;
             else
                type = hat;

          make graph from scaffolds;
          store(node, graph, type);
      }
   }
   return computed;
}
```

```
ComputeTree traverses the tree underlying the game graph
and calls ComputeNode for each node recursively.

boolean ComputeTree (root, komaster)
{
   changed = ComputeNode (root);
   for all options of root
      if ComputeTree (option) THEN
         changed = TRUE;
   return changed;
}

The main program repeatedly calls ComputeTree
as long as there is work to do:

while (ComputeTree (root));
```

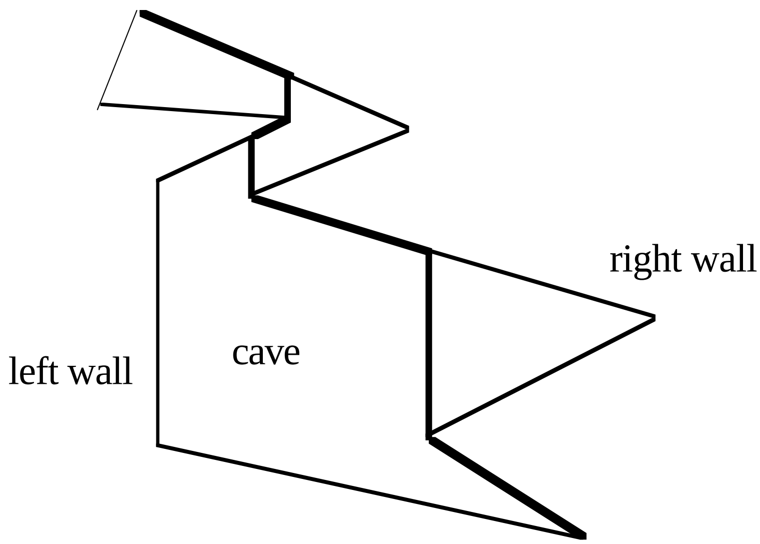

Figure 10: Tracking the balloon's path through a cave

4.6 Discussion of Thermographs for Loopy Games

4.6.1 Differences Between Loopfree and Generalized Thermographs

Generalized thermographs differ from loopfree thermographs: the same position can have two different generalized thermographs depending on who is the "komaster". The mast of pass-banned "solidhat" graphs is non-vertical.

The left and right wall used to construct a thermograph can intersect more than once, leading to several "cave" and "hill" regions [2]. Inside a cave, the thermograph follows a "balloon's path", as shown in Figure 10.

In thermographs of loopfree games, the slopes of the scaffolds indicate whether the difference in the number of moves played by both players is zero or one. In "sente" regions, the difference is zero, and the scaffold is vertical. In "gote" regions, the difference is one, and the scaffold is diagonal. Other variations, in which a player moves several times in a row, influence the shape of the graph: If a move's followup is too big compared to the current temperature, the opponent will prevent it by answering. If answering is too small, local play will stop after the first move.

Generalized thermography allows other differences in the number of moves played by both players at a given temperature. The ko ban rule may force komaster to spend two or more moves in a row to win a ko. Larger slopes, such as $2, 3, 4, \ldots$ indicate this situation.

Generalized thermographs can also bend backwards, in situations where the koloser starts the ko, then forces komaster to spend two or more moves to win and eliminate the ko.

5 Thermographs for Games with Ko Threats

5.1 What is a Ko Threat?

The naive notion of a ko threat is a play which forces a reply, so that the ko may be taken back. Actually, there are different kinds and forms of ko threats. But all threats must raise or maintain the temperature when played. The prototypical ko threat is simply a sente. For simplicity we will confine ourselves to ko threats of the form $\{b \mid 0 \parallel *\}$ for Black, or $\{* \parallel 0 \mid -w\}$ for White, where $b, w > 0$. We call b or w the size of the threat. In combinatorial game theory these threats equal

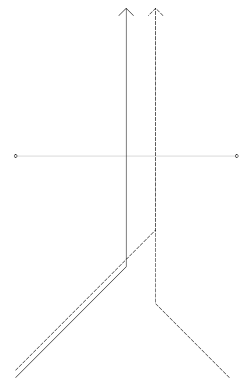

Figure 11: uphat and downhat differ

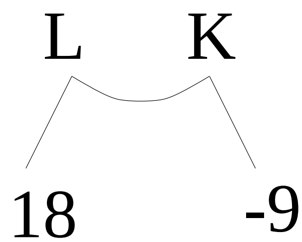

Figure 12: A 27 point ko

zero.

5.2 Primary, Secondary, and Tertiary Threats

We classify ko threats informally on the basis of their use: *Primary threats* have two uses. First, they enable a player to win the ko. Second, since it costs one move to win a ko, kowinner may profit by delaying the win until the temperature drops. *Secondary threats* may be answered or ignored by the the opponent, but they raise the temperature. If kowinner ignores a secondary threat, koloser gains something in exchange. *Tertiary threats* maintain the temperature. They can be used to defend when the opponent is trying to lower the temperature before winning the ko.

5.3 Pseudothermography and Komonster

Since to win the ko the komaster must give up a play in the environment, she may profit by delaying the win until the value of such a play (the ambient temperature) drops [13]. A komaster who has enough primary threats to delay winning the ko until the end of the game is called a *komonster*.

Pseudothermographs [7] show the effect of lowering the temperature before winning the ko. They represent not just the ko, but the ko with threats. The combination of a ko with other plays is called a ko ensemble. (Here we will use a slightly different form of pseudothermograph from Kim's.) Figure 13 shows the pseudothermograph for K in Figure 12 when Black is komonster. The mast value is 4.5 and the temperature is 13.5. To be komonster Black must have enough primary threats to lower

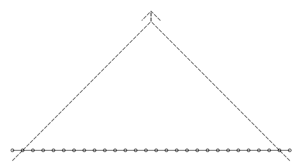

Figure 13: Black is komonster

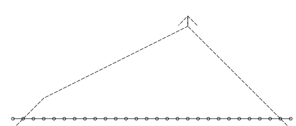

Figure 14: Black can lower temperature by 3

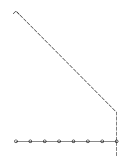

Figure 15: White can lower temperature by 3

the temperature 13.5 points. In a real game that would require a prodigious number of threats. The pseudothermograph when White is komonster is a vertical mast at -9.

More realistically, suppose that Black has enough primary threats to lower the temperature 3 points before winning the ko. Figure 14 shows the pseudothermograph when Black can do that. The mast value m is 1 and the temperature t is 10. When $t < 10$, White should win the ko. When $3 < t < 10$, Black should take the ko and use his extra ko threats to reduce the temperature 3 points before winning the ko. When $t \leq 3$, Black should take the ko and use his threats to wait until the end of the game to win the ko.

Figure 15 shows the pseudothermograph when White has enough primary threats to reduce the temperature 3 points before winning the ko. $m = -2$ and $t = 10$. Note the bent mast. When $t < 10$, Black should keep taking the ko to force White to use up her ko threats. When $t > 3$, White can reduce the temperature 3 points before having to win the ko. When $t \leq 3$, White can wait until the end of the game to win the ko, but Black may still benefit from forcing White to use up her threats.

For a simple ko, the rise in the temperature of the ko compared to the thermograph is $1/3$ the amount the komaster can reduce the temperature before winning the ko. The gain in mast value from delaying winning the ko is $1/3$ the drop in temperature for the komaster who must take two moves to win it and $2/3$ the drop for the komaster who can win in one move.

In reality, the komaster might better save some threats to fight another ko later, or to discourage the other player from creating another ko. Also, the other player could use tertiary threats to cancel the effects of the komaster's extra primary threats by maintaining the temperature.

5.4 No-man's Land and Secondary Threats

In position K of Figure 12, if neither player has a ko threat, Black can take and win the ko; so he is komaster. If White has one primary threat, she can use it to win the ko when Black takes it, and she is komaster. For a simple ko, the difference between Black's being komaster and White's being komaster is one primary ko threat.

There is a no-man's land where neither player is komaster. If, after Black has played his last primary threat, White has a secondary ko threat, this is the case. When White plays her secondary threat, Black wins the ko and White completes her threat.

This line of play is not represented in the komaster thermographs. To represent it, we must draw

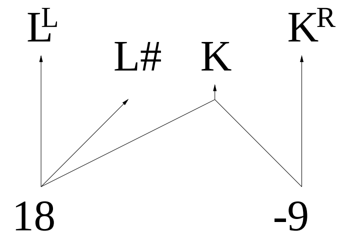

Figure 16: No threats

a thermograph for a ko ensemble including the ko and White's secondary threat. Next let us discuss how to draw thermographs for ko ensembles.

5.5 Methodology for Thermographs of Ko Ensembles

In this section we shall explicate a methodology for drawing the thermographs of ko ensembles and derive the thermographs for a simple ko, that ko with a simple gote, with a threat, and with another ko.

When one player takes a ko, or makes a move which, by the rules governing repetitive situations, prevents the other player from making a certain reply, the resulting position is blocked. The basic rule in our methodology is to treat the masts for blocked positions like pass-banned masts for komaster: the mast within a cave degenerates into the Right scaffold if Left is blocked or the Left scaffold if Right is blocked. (Remember that in a cave, the Right scaffold is to the left of the Left scaffold.)

We also assume that any play in the ko ensemble removes the block (although it may create another one). If the same player moves twice in succession in the ko ensemble, the other player has moved elsewhere in the meantime and removed the block by that play.

We use the # sign to indicate a blocked ko. #A means that position A is blocked for Black, and A# means that it is blocked for White.

5.5.1 Simple Ko with no Threats

We shall use the ko in Figure 12. We have the equations

$$K = L\# \,|\, -9$$

$$L\# = \{18 \,|\, \}$$

Figure 16 shows the thermographs for $L^L = 18$, $L\#$, K, and $K^R = -9$. The thermograph for K is the thermograph when Black is komaster.

5.5.2 Same Ko with Large Gote Play

Consider the game $K + G$ with $G = \{0 \,|\, -28\}$

$$K + G = \{K, L\# + G \,|\, K - 28, G - 9\}$$

We may simplify this by reversal and dominance. The mast value of $K + G$ is the sum of the mast values of K and G: $m = 0 - 14 = -14$. If Black takes the ko and then White plays the gote, we have $L - 28$, with a mast value of $9 - 28 = -19$, which is less than the mast value of $K + G$, which is -14. Therefore Black's taking the ko reverses, and he will go on to win the ko, for a score of -10. Since this is less than K, taking the ko is dominated by playing the gote. Similarly, if White

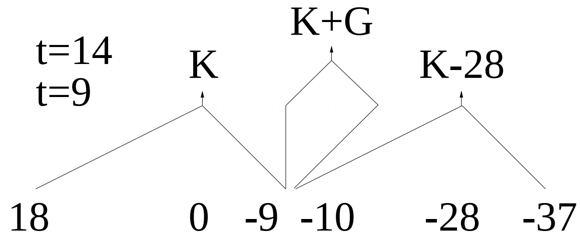

Figure 17: Ko plus large gote play

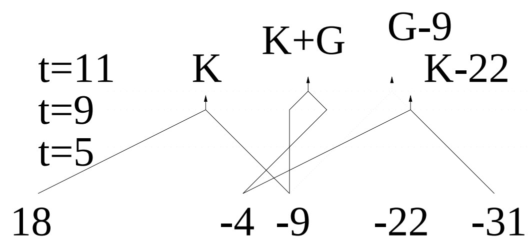

Figure 18: Ko plus medium gote play

wins the ko and Black plays the gote, the score is -9, which is more than $K-28$. So winning the ko is dominated, as well. The equation thus reduces to

$$K + G = K \,|\, K - 28$$

Figure 17 shows the thermographs for K, $K+G$, and $K-28$. When $9 < t < 14$ either player will play the gote. When $t < 9$, the second player will play the ko after the first player has played the gote. Because the second player takes the ko and gets two moves which are worth more than the other plays, the advantage of playing first declines with the ambient temperature.

5.5.3 Same Ko with Medium Gote Play

$$G = \{0 \,|\, -22\}$$

Now both players have two live options. Since, as above, taking the ko reverses to -4, we have the equation

$$K + G = \{K, -4 \,|\, G - 9, K - 22\}$$

Figure 18 shows the thermographs. At temperatures where an option is thermographically dominated, its thermograph is a dotted line. The thermograph for $K+G$ looks like it forms a cave, but it does not. The Left and Right walls kiss at $t = 5$. Above that temperature, play is like the previous example, but below that temperature, each player prefers to play the ko first.

5.5.4 Same Ko with Small Gote Play

$$G = \{0 \,|\, -10\}$$

After reduction we have the equations

$$K + G = \{L\# + G \,|\, G - 9\}$$

$$L\# + G = \{G + 18 \,|\, L - 10\}$$

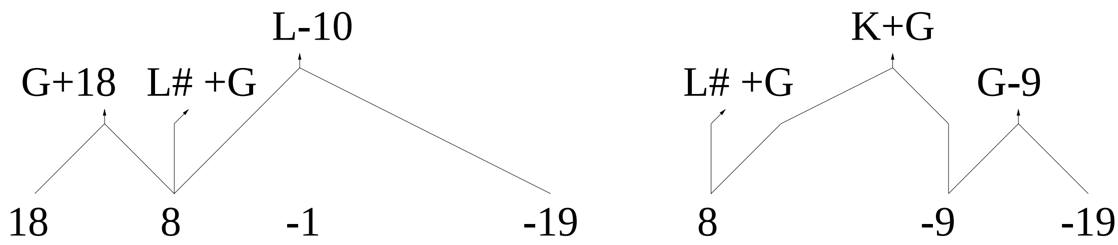

Figure 19: Ko plus small gote play

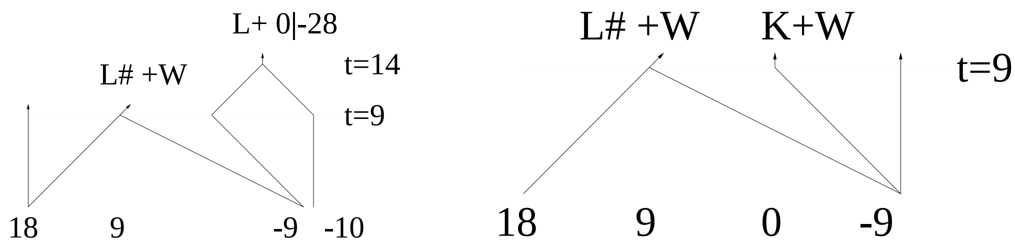

Figure 20: Big threat for White

Figure 19 shows the thermograph for $L\# + G$ on the left. It forms a cave at $t = 5$, and the mast follows the Left scaffold above that point.

The picture on the right shows the thermograph for $K + G$. Each player prefers to move in the ko first, and the second player will respond by playing the gote when $t < 5$.

The thermographs of a simple ko with a simple gote look different depending upon whether the size of the gote is greater than the size of the ko, between the size of the ko and 2/3 the size of the ko, and less than 2/3 the size of the ko.

5.5.5 Same Ko with White Threat

$$W = \{* \,\|\, 0 \,|\, -28\}$$

White's winning the ko dominates playing her threat, and Black's taking the ko dominates eliminating White's threat. So we have the reduced equations

$$K + W = L\# + W \,|\, -9$$

$$L\# + W = 18 \,|\, L + \{0 \,|\, -28\}$$

The left side of Figure 20 shows the construction of the thermograph $L\# + W$. It is the same as that of L, except that the mast follows Left's scaffold. The right side of Figure 20 shows that the thermograph of $K + W$ is the thermograph of K when White is komaster.

5.5.6 Same Ko with White Threat

$$W = \{* \,\|\, 0 \,|\, -22\}$$

After reduction we have the equations

$$K + W = L\# + W \,|\, -9$$

$$L\# + W = 18 \,|\, L + \{0 \,|\, -22\}$$

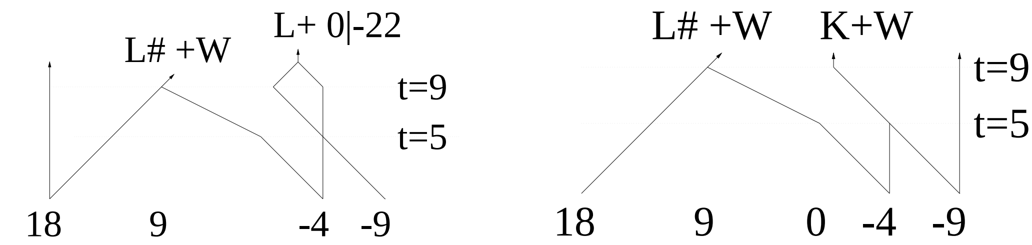

Figure 21: Medium White threat

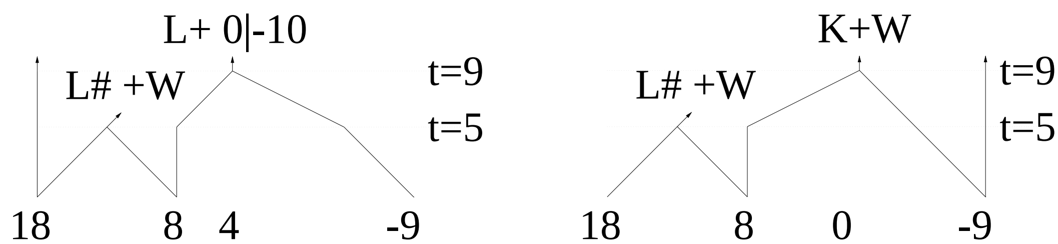

Figure 22: Smaller threat

with thermographs in Figure 21. The vertical line at -4 in the Left wall of thermograph for $K+W$ reflects Black's preference to play the ko in $L+\{0\,|\,-22\}$ when $t<5$.

5.5.7 Same Ko with White Threat

$$W=\{*\,\|\,0\,|\,-10\}$$

After reduction we have the equations

$$K+W=L\#+W\,|\,-9$$

$$L\#+W=18\,|\,L+\{0\,|\,-10\}$$

with thermographs in Figure 22.

5.5.8 Sum of two Kos

Figure 23 shows two simple kos, with the reduced equations

$$K+M=L\#+M\,|\,M-9$$

$$L\#+M=18+M\,|\,12$$

Figure 24 shows the thermographs. For the thermograph of $L\#+M$, White's taking the ko, M, reverses to 12 through Black's winning L and White's winning N.

As the thermograph for $K+M$ indicates, the difference between Black's and White's playing first is constant at 18 points when $t\le 3$. This difference is the difference between the sizes of the two kos. If they were the same size, they would form a miai pair, regardless of ko threats.

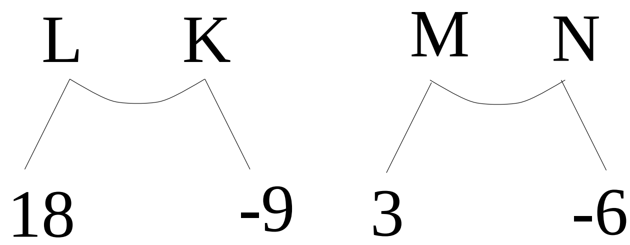

Figure 23: Two simple kos

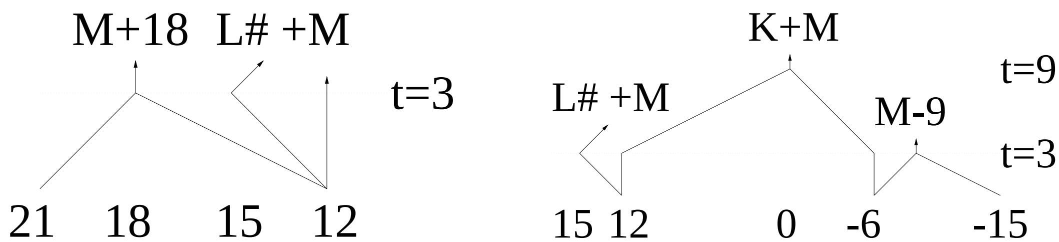

Figure 24: Sum of two kos

5.6 Decreasing Temperature Inside a Ko Fight

Suppose that a player takes a ko, moving from position A to position B. Position B is now blocked, and our basic rule says to treat it like a pass-banned position. However, if the temperature for B, if unblocked, is less than the temperature for A, then the other player will not be inclined to take the ko back, and the block does not matter. In this situation we should not treat B as pass-banned, but draw a vertical mast for it.

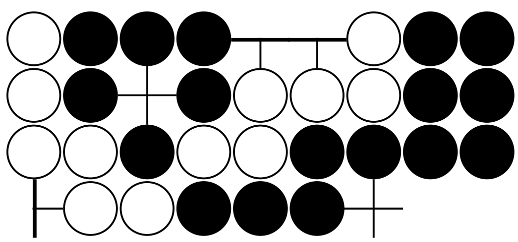

Figure 25: Decreasing temperature in a ko fight

In Figure 25, if neither player has a threat, the basic rule yields the thermograph shown to the left. The fact that the slope of the right wall indicates only one net move for White to resolve the ko, while if White takes the ko, there is still an open ko on the board, implies that the temperature of the ko after White makes that move is less than the temperature in the figure.

Indeed, after White takes the ko in the no-threat case, the position is terminal, and its thermograph is a vertical mast at -16. Using that thermograph we obtain the correct thermograph shown on the right in Figure 25. This is the only known case to require a correction to the basic rule.

With the method outlined here we can draw thermographs for many ko ensembles without designating a komaster. But there are still positions, such as a double ko seki, for which we cannot draw thermographs without otherwise evaluating a position with unfilled kos.

Now let us return to the question of representing secondary threats for a simple ko.

5.7 Secondary Threats

The right sides of Figures 21 and 22 show thermographs for the simple ko in Figure 12 with a secondary threat for White. When Black takes the ko and ignores White's threat, the score after White completes the threat is $18 - w$, where w is the size of the threat. Since each player makes the same number of plays without playing elsewhere, this line of play is indicated by a vertical line at $18 - w$ in each case.

Figure 21 shows the thermograph when $w = 22$. When $t < 5$, the left scaffold is the vertical line at -4. Below that temperature, if Black takes the ko, White plays her secondary threat, and Black exchanges it for the ko. The size of the ko, k, is the difference between the score when Black wins the ko and the score when White wins it. The temperature below which Black should exchange White's threat for the ko is the difference between the size of the ko and the size of the threat, $k - w$. Above that temperature, White's threat is primary.

Figure 22 shows the thermograph of the ko ensemble when $w = 10$. When $t < 5$, the left scaffold is the vertical line at 8. Below that temperature, if Black takes the ko, White plays her secondary threat, and Black exchanges it for the ko. This temperature is $w/2$, the temperature of the right follower of White's threat.

5.7.1 Comparison with Conventional Wisdom

Conventional Go wisdom says that the size of a ko threat must be greater than 2/3 the size of the ko. In Figure 22, right, even though $w < 2k/3$, it is an effective threat when $t < w/2$. But it is only a secondary threat. In Figure 21, right, $w > 2k/3$, and White should always play her threat, but it is primary only when $t \geq k - w$. When $w \leq 2k/3$, the vertical line at $18 - w$ represents White's option to play the threat, and when $w > 2k/3$, it represents Black's option to ignore it.

5.8 Black's Secondary Threat

Black has no need for a secondary threat unless he answers White's primary threat. If Black initiates the ko and then later plays a threat which White ignores, Black will complete his threat at the cost of one move. With the ko in Figure 12 and the threat $b \mid 0 \parallel *$, the line to represent that play is $v(L) = b - 9 - t$.

Figure 26, left, shows the thermograph for the ko ensemble when White has a primary threat and $b = 6$. When $t < 3$, Black can play his threat and complete it when White wins the ko. As above, the temperature at which Black should play his secondary threat is $b/2$.

Figure 26, right, shows the thermograph when $b = 23$. When $t < 4$, White ignores Black's threat and wins the ko. As above, the temperature at which White should do so is $k - b$.

Also as above, when $b \leq 2k/3$, the line $v(L) = b - 9 - t$ represents Black's option to play the threat, and when $b > 2k/3$, it represents White's option to ignore it.

5.9 When is White's Threat Primary?

In the previous two cases, we simply said that White's threat was primary. The next two figures show when it could be either secondary or primary.

Figure 27,left, shows the thermograph when $w = 22$ and $b = 6$. When $1 < t < 5$, White's threat is secondary: Black ignores it and wins the ko. When $t < 1$, it is primary: Black answers it and plays his secondary threat, which White ignores.

Figure 27, right, show the thermograph when $w = 6$ and $b = 23$. When $2 < t < 3$, White's threat is secondary: Black ignores it. When $t < 2$, it is primary: Black answers it and plays his secondary threat.

In both cases, whether and when White's threat is primary depends upon where White's threat line and Black's meet. They meet at $t = b + w - k$, the sum of the sizes of the threats minus the

Figure 26: Left: b=6, right: b=23

Figure 27: Left: w=22, b=6, right: w=6, b=23

Figure 28: Ko ensembles with threats

size of the ko. White's threat is always secondary when $b + w \leq k$. If $b < 2k/3$, White's threat is always primary when $b/2 + w \geq k$. If $b \geq 2k/3$, it is always primary when $b + w/2 \geq k$. Otherwise, it is primary when $t < b + w - k$.

5.10 The Threat Scaffold

Let Black have n threats of sizes $b_0 \geq b_1 \geq \ldots \geq b_{n-1}$ and White have m threats of sizes $w_0 \geq \ldots \geq w_{m-1}$. For the simple ko in Figure 12 we can construct a threat scaffold in the following way. Draw the threat line for White's largest threat, $v(L) = 18 - w_0$. Then draw the threat line for Black's largest threat, $v(L) = b_0 - 9 - t$. They meet at $t_0 = w_0 + b_0 - 27$. If $t_0 > 0$, then the scaffold below t_0 is Black's threat line, and above it is White's. Continue to draw threat lines for the largest White and Black threats alternately, with the last drawn line becoming the bottom of the scaffold, until you run out of White or Black threats or the last line meets the scaffold at $t_i \leq 0$, in which case it does not become part of the scaffold.

If the threat scaffold meets the basic Left scaffold line $v(L) = 18 - 2t$ at $t_j \leq 9$, then the Left scaffold above t_j is that line, and, if $t_j > 0$, the Left scaffold below that is the threat scaffold. If the threat scaffold meets the basicbasic Left scaffold line $v(L) = -9 + t$ at $t_k \leq 9$, then the Left scaffold between temperature t_k and 9 is that line, and, if $t_k > 0$, the Left scaffold below that is the threat scaffold.

Figure 28 shows examples of thermographs of ko ensembles with various threats.

6 A Thermography View of some Exotic Go Positions

In this section we look at long loops, fine points of scoring and eliminating ko, and consider the problem of irremovable ko threats.

6.1 Long Loops

While the majority of loops in Go are two moves long, more complex types of ko with longer loops such as chosei, round robin, triple and quadruple ko do occur. Figures 29 - 31 show some examples.

Traditionally, rules have offered different ways to handle such cases:

- Declare the game "no result" (Japan)
- Forbid full-board repetition by a "super-ko" rule (New Zealand)
- Treat long loops like simple ko (Ing)

Discussing the relative merits of these approaches is beyond the scope of this paper. We will concentrate on the effect of long loops on the computation of thermographs.

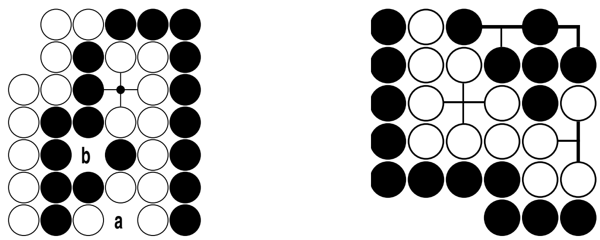

Figure 29: Double ko seki

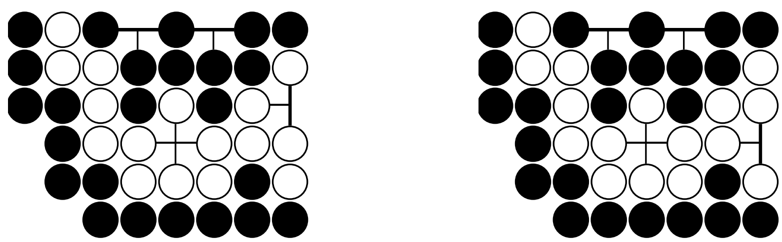

Figure 30: Left: Triple ko. Right: White is dead in Japanese rules

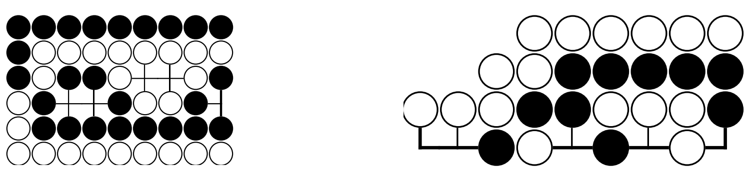

Figure 31: Left: Round robin ko. Right: Chosei

To apply thermography, we must break each loop by applying the ko ban at some node in the loop. In long loops we have a choice. In general, different choices will lead to different thermographs. Koloser wants to minimize the damage from losing the ko, while komaster wants to pay a low tax rate for her successive moves.

Our current algorithm does not attempt to find the optimal cut points for long loops. When it encounters a loop longer than 2 moves, it will issue a warning, then arbitrarily choose one way to break the loop. It is currently unclear what a good way to handle these situations is. Trying all ways to cut each loop, then computing the maximum thermograph could lead to an exponential increase in runtime in the case of many intertwined long loops.

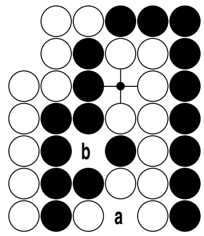

Figure 32: Double ko

6.1.1 Limitations for Multiple Ko

Generalized thermography works well in local games where at most one ko is relevant at any given time. The ko-ban methodology computes a value of playing in this ko by comparing it with the value of moves elsewhere. If there is more than one concurrent ko locally, it must be clear which one is currently being fought: a move in this ko must dominate all moves in other local kos.

The algorithm fails in positions with two or more simultaneous ko, as in Figure 32. In normal ko, the komaster can eventually win any ko or series of ko by ignoring sufficiently many threats. In double ko seki the only good moves for both players are to stay in the loop. A double ko seki can never be resolved into a terminal position. Generalized thermography breaks all loops by force, it does not allow to balance the capture of one ko by taking the other ko.

6.2 The Final Stages of Play

6.2.1 Scoring

In regular terminal Go positions, each point on the board can be classified as alive (stone or empty point that is part of territory), dead (part of opponent territory) or neutral (in a seki). In Japanese rules it costs points to capture dead stones, so competent players will not do it. If players disagree in the end which stones are dead or alive, a special dispute-settling phase follows [3].

In Chinese rules the capture of dead stones in the end is not penalized, allowing play to continue until only alive and neutral (shared liberties in seki) points remain on the board. Scoring is very easy with Chinese rules: all stones are worth one point for their color. Regions of empty points are evaluated as territory for one player if they are surrounded by stones of this color, and as neutral otherwise.

In Japanese rules, fewer positions can be "played out" by capturing dead stones at the end of the game. These positions must be evaluated statically. Seki and unresolved ko are the most frequent examples of problematic positions. Rules differ significantly in the evaluation of such positions. A big collection of interesting positions can be found in Fearnley [5]. Smaller collections appear in the official rules of several countries [4].

6.2.2 Playing One Point Ko and Dame

In Japanese rules, one point ko fights are the smallest plays. Dame are usually filled after the last ko fight to make counting easier, but they are not worth any points.

In Chinese rules, dame and one point ko fights are both valuable but incomparable plays. Playing them in the correct order can already be quite intricate [7]. After all dame are occupied, players can continue to play inside their own territories without losing points. This way they can eliminate most remaining weaknesses such as potential ko or ko threats. This is also the time when positions containing latent ko fights will be played out. Examples are "bent four in the corner" or "three points without capturing".

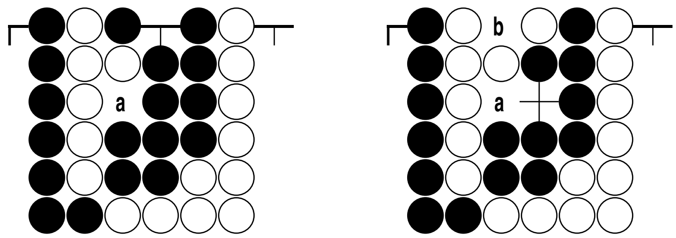

Figure 33: Direct ko and one move approach ko

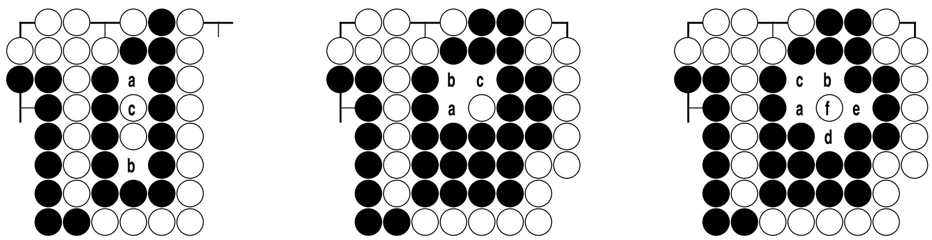

Figure 34: Two, three and ten move approach ko

6.3 Resolving Ko in Japanese Rules

If Black has more ko threats in the position on the left side of Figure 33, he could claim the white stones and the two empty points as his territory. However under Japanese rules, Black must eventually capture at ‘a’, which reduces his area by one point.

On the right side of Figure 33, White has the option to play ‘a’ and make a direct ko. If White does not play, the area becomes Black’s territory without a further play by him.

Figure 34 shows multi-move approach kos. Black does not need to resolve these situations.

White does not gain by playing unless she has a surplus of ko threats. The proverb claims “A three move approach ko is not a ko”. In a three or more move ko, usually Black will just remove the white stones in the end. It would cost White too many moves to make a real ko fight out of such a situation. It is interesting to compare this proverb with the thermographs of approach Ko in Figures 61 to 66 and the iterated Ko in Figures 52 to 56.

6.4 Irremovable Ko Threats

It is well known that not all ko threats can be removed at the end of the game. For example, a seki can be a source of threats. The examples usually given for such irremovable ko threats involve a sacrifice. White ‘a’ in Figure 35 is a threat that loses 8 points if answered, but threatens to gain 18 points. However, irremovable ko threats that do not cost points exist.

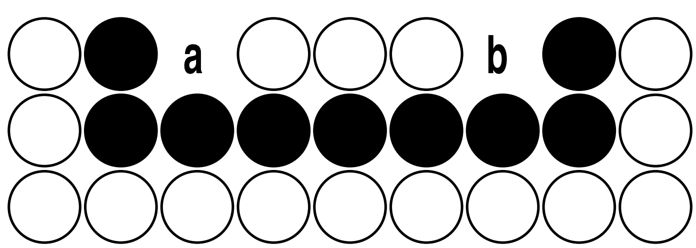

Figure 35: Point-losing irremovable ko threat for White

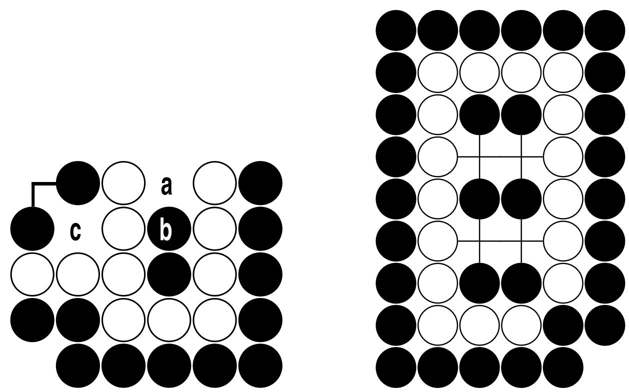

Figure 36: Seki with cost-free irremovable ko threats

This fact becomes important when discussing the differences between Chinese and Japanese rules concerning play at the end of a game in cases such as "Three Points Without Capturing" and "Bent Four in the Corner".

The positions in Figure 36 both contain free white ko threats that Black cannot eliminate [5].

7 Implementation Issues

This section discusses technical aspects of our implementation of generalized thermography: repetition, transposition, underlying arithmetic and geometric operations, the construction of scaffolds and thermographs, and the user interface.

7.1 Repetition and Transposition Detection

To identify loops, we must detect repetitive positions. If a Go position were defined by its history (the sequence of moves from the starting position) there could be no repetition. Therefore a position is defined as the current board state, plus the difference in number of captures when using Japanese rules. The history of moves and captures is used to recognize repetitions. For details on repetition detection see [6].

Loop recognition and transpositions in the game graph are closely related: a loop is a transposition to a previous node in the game history.

A special kind of transpositions unique to Go are *shift transpositions*: transpositions into the same board position, but with a different number of captures. Müller [9] describes an implementation of local search that takes advantage of such transpositions.

7.2 From Thermography to Generalized Thermography

Explorer [9] is a full-featured traditional Go-playing program that has taken part in many computer Go competitions. It contains Wolfe's toolkit for combinatorial games [15] and modules for exact and heuristic endgame calculation. The thermography modules are fully integrated in the program. They use Explorer's display and game handling facilities to generate, store and test problems.

A previous version of Explorer contained an implementation of classical loopfree thermography. The transition from loopfree to generalized thermography brought a significant increase in complexity:

Change of coordinate representation from fixed point binary to rational numbers.

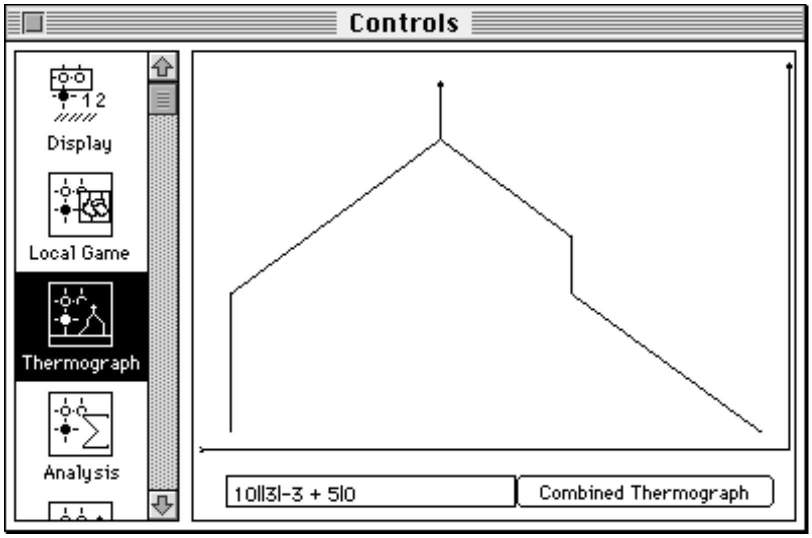

Figure 37: The Thermograph Display of Explorer

Richer structure of thermographs with caves, hills, and bent masts.

Change of algorithm from one pass tree traversal to iteration over game graph.

7.2.1 Rational Arithmetic

Care must be taken to avoid rounding errors in geometric computation: maintaining the identity of values is crucial to avoid additional "phantom" points on the graph, keep the slope of lines exact, and to guarantee correctness of the algorithm.

All coordinate values arising in the calculation of loopfree thermographs are binary fractions. Therefore the old implementation used the same fixed point arithmetic as Wolfe's toolkit [15]. With generalized thermography, the values appearing in calculation can be any rational number. Binary fixed arithmetic leads to unacceptable rounding errors. All calculations now use precise rational arithmetic, with two 16 bit values p/q, $q \geq 1$, $GCD(p,q) = 1$.

Computation of the intersection of two almost parallel lines in rational arithmetic produces astonishingly large intermediate values. It was necessary to reduce these values using GCD and to extend intermediate calculations to 32 bit precision to avoid overflows in some of the problems presented in section 10.

7.2.2 Layers of the Thermography Program

The following table lists the features of the layers of the program for generalized thermography.

Rational Arithmetic Arithmetic operations, comparison operators and conversions.

Points and Line Segments Points in the (value, temperature) plane with rational coordinates, lines and line segments, intersection of lines and line segments, incidence tests for points and line segments.

Scaffolds and Thermographs Data structures for scaffold and thermograph, consisting of line segments plus a mast. Intersection of scaffolds, max/min of two scaffolds, locate segment at temperature t, follow "balloon path" in a cave between two scaffolds, handle bent masts.

Game graphs, Nodes and Properties Tree structure, tree traversal, map game graph to tree, find loops in graph, store thermographs as properties in nodes, handle shift transpositions.

Move generation and evaluation Safe stones and territories, local games, move generation, terminal position detection.

Interface Show local games, game graph, draw and write hat and solidhat thermograph (Figure 37), write mean values, temperatures, temperature range for moves.

8 Applications to Full Board Analysis

8.1 Sum Game Models for Opening and Midgame

In this section, we claim that local games and thermography are natural models not only for the endgame, but for earlier stages of the game as well. The choice of moves during opening and midgame is often affected by endgame considerations. A problem is that at this stage of the game we do not know what the real endgame situation will be later on. Most of the other subgames will only materialize later in the game.

Still, we must choose from local moves leading to different endgames early in the game, while the situation is being "played out" locally. Examples are choosing one of several *joseki* moves in a corner, or making a small living group after a deep invasion of a moyo.

Typically, a player must decide whether to "live small in sente" or "big in gote", whether to leave the possibility of a ko fight behind, or whether to eliminate such danger at the cost of one or more moves. If moves have incomparable values in the classical sense, we cannot decide on a best move without knowing more about the context in the current sum game. If moves lead to different ko situations, again we cannot decide the best move by purely local analysis.

In the opening and midgame, we have no hope to do a full board global analysis. Since we want to make a rational move choice, our only possibility is to play "as well as possible", to make educated guesses or "reasonable assumptions" about the rest of the board. In other words, we look for a suitable model of the environment.

One reasonable-looking method is to find a good estimate of the current ambient temperature, then play sentestrat.

In practice we cannot know which endgame situation will occur in the given environment. It makes sense to look for a kind of "standard environment" against which to compare different local results. The enriched environment [2] seems a good candidate for a non-biased a priori choice of environment.

One problem is choosing a komaster. A pessimist would always choose the opponent as komaster, an optimist would choose himself. A realist would base this assumption on an evaluation of existing ko threats. Further developments of the theory, such as pseudothermography, could investigate how to adapt play to a specific given environment of ko threats and other endgame plays.

8.2 Playing Ko and Ko Threats in Real Games

Since ko fights can be so complex, we have been studying them in simplified models such as the "enriched environment". This section discusses a number of issues that arise in real play.

Besides maximizing the local territory count, players have secondary objectives for choosing their moves: For example, they want to maximize the number and size of own ko threats while minimizing the opponent's.

Games often are not strictly independent, and players seek to create "double threats" or other multi-purpose moves. Müller [9] investigates several kinds of dependencies and surveys methods that have been proposed for dealing with them.

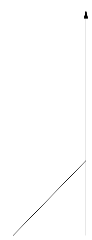

Figure 38: One-sided sente play

Sets of ko threats are clearly not well-ordered: In some situations a few big threats will be superior to many small threats, in other situations it will be the opposite. The possible existance of local threats in a ko fight further affects the value of other ko threats.

Endgame moves create or destroy ko threats as a side effect. Kim [7] introduces *perturbation theory* for solving the simplest problem of handling a ko in a real endgame environment: to play the final one point ko correctly in Japanese rules. He assigns very small values ϵ to ko threats and solves the corresponding combinatorial game.

One-sided sente moves behave differently in loopfree and loopy sum games: These moves have a temperature range in which only the player keeping sente can play them. Above that range both players would pass. Below the range the opponent could play a *reverse sente* move here. Figure 38 shows the game $1 \,\|\, 0 \,|\, -8$: Black can play reverse sente when $t = 0 \ldots 1$, White can play in sente when $t = 0 \ldots 4$.

Loopfree theory advises a player to make her sente move soon. In a real game players tend to keep these moves in reserve as long as possible, because they make very good ko threats.

If a sudden drop of temperature is imminent, sometimes both players will have several of these moves left. That can lead to intricate sequences of forcing moves: both players are concerned about preventing reverse sente plays. This situation can be modeled well by classical combinatorial game theory: the objective is to get the last move at a given temperature, which means winning the cooled game.

Pure ko threats are the ultimate one-sided sente moves: their temperature is zero, so the opponent has no incentive to play there as an endgame move. The player can use these threats across a wide range of temperatures.

Some games work as ko threats for both players. An example is the game $10 \,|\, 0 \,\|\, 0 \,|\, -20$. The player who does not have to play the first ko threat will usually eliminate these threats before the ko starts. If she forgets to do that, the opponent should play out *all* these games as his first threat.

Combinatorial game theory takes a static view of the endgame: the partition into safe territories and remaining "interesting" subgames is done once and for all before starting the analysis. For real play this model sometimes must be adapted: If mutual defiance leads to large-scale exchanges during the endgame, territories will be destroyed and the remains turn into new endgame areas. The whole sum game will change, and parameters such as the ambient have to be recomputed.

9 Open Problems and Future Work

9.1 Is The Algorithm Complete?

The iterative algorithm for generalized thermography terminates when it cannot find any more work to do, when there are no new solidhat or hat graphs to compute in the whole game. If the game graph contains many intertwined loops, it is conceivable that this situation could happen before the hat graphs of all nodes have been computed. We have not yet encountered such an example, and suspect that it cannot happen in the game graphs that occur in Go.

One can easily construct abstract loopy games where our algorithm would fail to compute a thermograph. An example is the game defined by $A = \{B \mid \}, B = \{ \mid A\}$. This is an "eternal ko fight", since no player can ever move to a loopfree position and end it. We believe that there are no "pure" loops without an exit to a loopfree subgame in Go. An interesting open problem is to find good technical conditions that ensure termination of generalized thermography, and prove that (most) Go positions satisfy such a condition.

9.2 Pruning Irrelevant Loopy Moves

As in loopfree thermography, the thermograph of a loopy game can not be affected by dominated moves. Computer-generated local game graphs typically contain many more loops than human-generated graphs. A program looks at many sequences that human players would never consider, but are hard to dismiss beforehand with current techniques.

Most of these variations contain one or more bad moves, so for efficiency they should be eliminated from a game before computing the thermograph.

In [11] we used an iteration technique, based on sidling [1], to find loopfree lower and upper bounds on a loopy game. If both bounds for a node are the same, ko moves do not affect the game value (they are dominated by non-ko moves), and we can remove the loops before computing the thermograph.

Similar techniques will yield bounds on thermographs: loops can be broken in one player's favor by forbidding one of the opponent's moves in the loop. If a move's optimistic thermograph is dominated by another move's pessimistic bound, it can safely be pruned. This reduces the loopyness of the game graph.

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10 Catalog of Examples

The format of the example collection is as follows: Each example shows a picture of the Go position, followed by its game graph. For complex graphs, we show only selected lines of play. Omitted parts of the game graph are indicated by dots. The third picture in each example shows the uphat thermograph using solid lines, and the downhat thermograph using dashed lines. The downhat graph has been slightly shifted to the left, in order to see it in those regions where uphat and downhat graphs coincide.

All examples presented here use Japanese scoring. To give an indication of the values and temperatures involved, a scale with integer unit is drawn at temperature 1.

For many positions, one or more interesting follow-up positions are included. When the situation is symmetrical for both players, as in many iterated ko, thermographs can be obtained from the examples given by taking the mirror image.

10.1 Figures 39 to 41: Simple Ko

Simple kos contain just one loop. They differ in the size of the ko, and possibly in the loopfree parts of the game graph.

10.2 Figures 42 to 47: Sente Ko

Many ko fights are one-sided "hanami ko": one player has many points at stake, while the other has comparatively little to lose. The thermograph of such ko looks similar to the one-sided sente situation of Figure 5.

Figures 42 to 43 are from a problem on p.34 of Kim Yonghoan's thesis [7]. Black's capture is a fairly big threat, and White will usually defend. This exchange leaves a one point ko which will be played much later.

Figures 44 to 45 show a similar real-game position where the threat is smaller.

10.2.1 Figures 46 to 47: Throw-in Ko

A very frequent type of ko is illustrated by these examples. If White is komaster, she can throw in a stone and start a ko fight, threatening a capture. Black will often defend, leaving only a one point ko. The uphat graph of the first example is a mast: White's threat to start the ko fight if Black is komaster is not big enough. However, in the other example White can expect to gain 2/3 of a point by throwing in. Further increasing the stakes by making White's threat bigger does not affect the thermographs.

10.3 Figures 48 to 60: Iterated Ko

10.3.1 Figures 48 to 49: 2-iterated Ko

This ko frequently appears as a subgame of other, more complicated situations, for example in Figures 46,82,88 and 89.

10.3.2 Figures 50 to 52: Corner Iterated Ko

There is only a subtle difference between the game graphs of the 2-iterated and the corner iterated ko, but it leads to quite different thermographs.

10.3.3 Figures 53 to 54: 3-iterated Ko

The 3-iterated ko shows similar thermograph shapes as the 2-iterated one. Only two nodes are shown, the symmetrical cases are omitted.

10.3.4 Figures 55 to 56: Hot 3-iterated Ko

This position is much hotter than the previous one, because big black and white groups are at stake. Notice the slope 4 in the uphat graph of Figure 56, corresponding to the four moves in a row that Black has to play to win. Two symmetrical cases not shown.

10.3.5 Figures 57, 58 and 60: Real 2

A very common case of iterated ko, but with complicated-looking thermographs. Still, the ko is placid.

10.4 Figure 59: Corridor Approaching Ko

The effect of the ko on the thermograph becomes more visible as White pushes on and play approaches the position containing the final ko fight.

10.5 Figures 61 to 66: Approach Move Ko (Yose Ko)

Scoring of multi-move approach ko depends on whether the rules demand capturing the stones. The examples in Figures 61 - 66 were computed under the assumption that stones must be captured eventually.

10.6 Figures 67 to 69: Mannen Ko

In mannen ko, nobody wants to play for most of the game. The position in Figures 68 - 69 is unusual: if Black is komaster then White is as good as dead. Both players prefer to pass even at temperature zero. Scoring depends on the rules of the game, whether Black has to capture white or not and whether this costs any points.

10.6.1 Figures 70 to 75: Murashima's Ko

A very interesting position from Murashima's ko dictionary [12]. White has two options: Figure 72 can lead to mannen ko and Figure 74 to an iterated ko.

10.7 Figures 76 to 81: Small Hyperactive Ko

Hyperactive kos have different mean values depending on who is komaster. They contain a node where one player can choose to increase the stakes by making a hotter ko fight than that present in the previous position.

10.7.1 Figures 76 to 78: The Rogue

The "rogue" position of Figure 76 appears in Berlekamp and Wolfe's catalog of "node rooms" [3]. It was the first hyperactive ko to be studied in detail.

10.7.2 Figure 79: Kao's Ko

Kao's ko is a hyperactive ko in a very small area. After White takes the ko, she can increase the stakes by pushing in from the right side.

10.7.3 Figures 80 to 81: Kato - Fujisawa Ko

An example that illustrates that interesting behavior can occur even in very small local situations in the center of the board. White can capture a stone and create an iterated ko like position. Yet this ko is hyperactive.

10.8 Figures 82 to 105: More Ko From Master Games

Most of the examples in this section have selected from a collection of Kisei title games [10]. Many are directly copied from the game record. In other examples a few stones have been added to reinforce the boundary and therefore limit the size of the local area.

10.8.1 Figure 82: Unnamed Ko

This ko will often end up as a 2-iterated ko (see Figure 48). If Black is komaster, White will just connect. If White is komaster she can do a bit better by fighting the ko. Black's option to extend instead of capturing (the leftmost branch) is thermographically dominated.

10.8.2 Figure 83: Real 4

White can cut to start a ko fight. Above temperature 1/3, this is a good strategy even if Black is komaster.

10.8.3 Figures 84 to 85: Real 5

A very simple ko which looks similar to Real 2 and Kao's ko. It is not hyperactive.

10.8.4 Figures 86 to 87: Real 8

A fairly hot ko of temperature 5 2/3. If White wins she earns the chance to pick up an extra 5 stones. There is a small range between temperature 5 and 5 2/3 where this is not a big enough threat to be answered. The kinked mast in the downhat graph of node 1.2 is also interesting.

10.8.5 Figures 88 to 89: Real 9

A splendid example showing how complicated seemingly simple endgames can be. Who would have predicted the zigzag scaffold?

10.8.6 Figure 90: Real 7

A frequently occuring type of endgame, where the ko plays only a minor role. One of the main lines ends in a one point ko fight.

10.8.7 Figures 91 to 96: Real 10

A hot ko fight that resolves into several interesting low-temperature positions in nodes 1.3.3 and 1.3.3.3.

10.8.8 Figures 97 to 102: Real 11 and 12

If Black cuts in Real 11, White can opt for peace or fight the ko. Real 12 is similar, with an extra outside liberty for White.

10.8.9 Figure 103: Chitoku's Ko

If White is komaster here, the game reduces to a number. White will capture a stone at an appropriate time and Black will respond. If Black is komaster, it is still a one-sided sente situation for White, but at temperatures below 1 Black will fight the ko to gain two points.

10.8.10 Figures 104 to 105: Cho-Yu Ko

Yet another small example with very interesting thermographs.

10.9 Figures 106 to 109: "3 Points Without Capturing"

This is a classical position which appears in every rulebook. A detailed discussion is beyond the scope of this paper.