EXPLORING WILD CHAOS IN A 3D HÉNON-LIKE MAP

Dana C. Julio, Hinke M. Osinga and Bernd Krauskopf | The University of Auckland

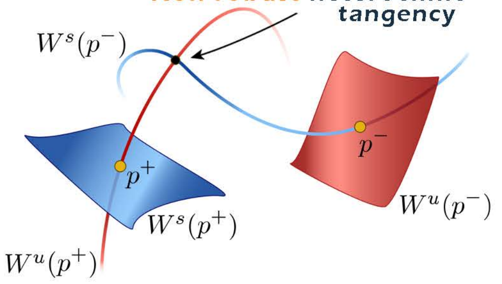
BACKGROUND...

- **Wild chaos** is a new type of chaotic dynamics; a minimum of three dimensions is required for discrete-time dynamical systems.
- Unlike classical chaos, systems that exhibit wild chaos admit robust heteroclinic tangencies leading to **robust chaotic dynamics**.

- Wild chaos can be generated with a geometric structure called a **blender**.
- Blenders have invariant manifolds that behave as geometric objects of higher dimensions than expected.
- We say that such a manifold has the **carpet property**.

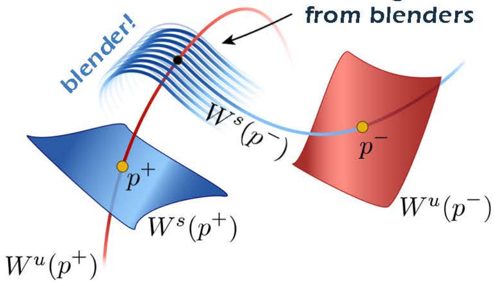
CLASSICAL CHAOS

Non-robust heteroclinic tangency



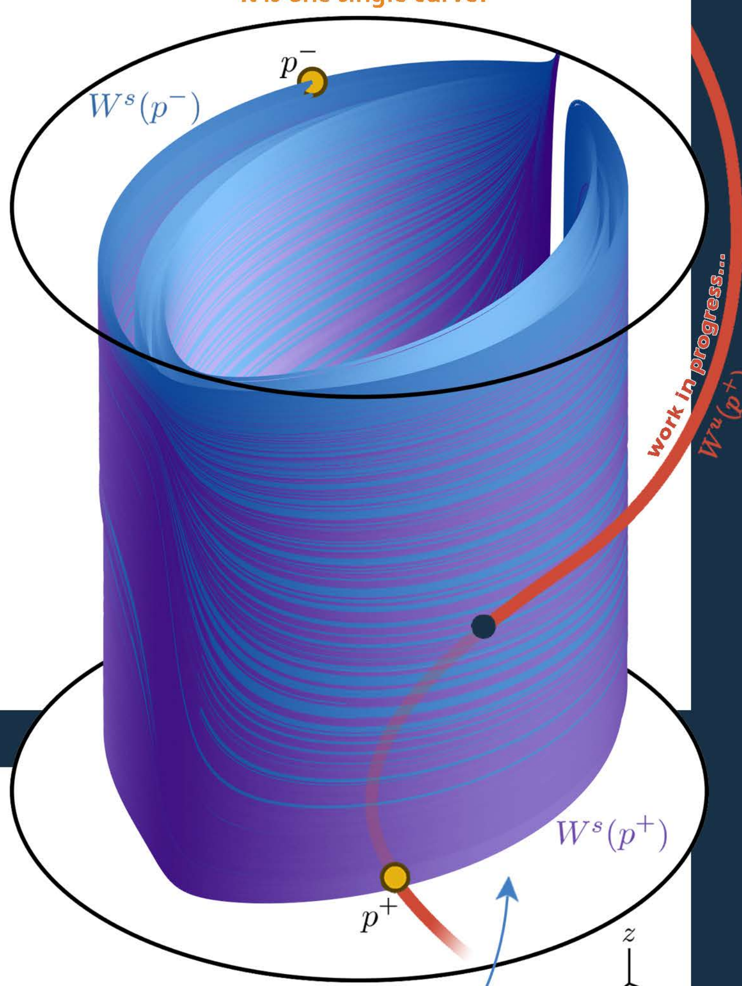
WILD CHAOS

Robust heteroclinic tangencies from blenders



A BLENDER IN A 3D HÉNON-LIKE MAP

It is one single curve!



BLENDERS IN A 3D HÉNON-LIKE MAP...

3D HÉNON-LIKE MAP

$$H(x,\ y,\ z) = (y,\ a - y^2 - b\ x,\ \xi\ z + y),$$

FIXED POINTS

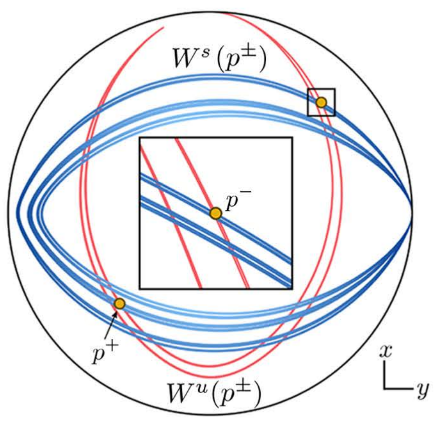
$$p_\pm = \left(\rho_\pm, \rho_\pm, \tfrac{\rho_\pm}{1-\xi}\right) \text{ with}$$

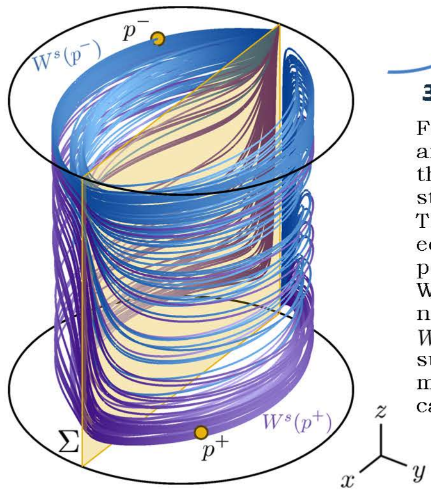
$$\rho_\pm = \tfrac{1}{2}\left(-1 - b \pm \sqrt{4a + (1+b)^2}\right).$$

- With fix parameter values $a = 4.2$, and $b = 0.3$ such that H is orientable and has a full horseshoe.
- H has two saddle fixed points p^- and p^+ with one-dimensional $W^s(p^\pm)$ and two-dimensional $W^u(p^\pm)$.
- Computing long manifolds to test the carpet property is a major challenge since the manifolds make excursions near infinity.

SOLUTION: Compactification of the phase space of H to a cylinder and implementation of advanced and efficient numerical techniques.

TOP VIEW OF THE 3D HÉNON-LIKE MAP





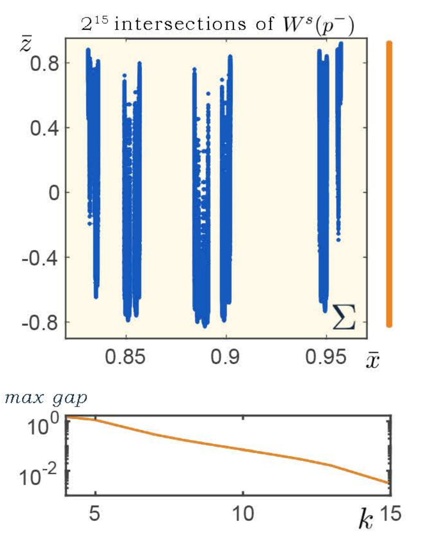
longer $W^s(p^\pm)$

3D HÉNON-LIKE MAP

Fixed points p^- and p^+ are shown, along with their one-dimensional stable manifolds $W^s(p^\pm)$. The manifolds are computed up to 2^7 intersection points with the plane Σ. When we increase the number of intersections, $W^s(p^\pm)$ starts to look like a surface, sugesting that the map has a blender with the carpet property.

IS IT REALLY A BLENDER? THE CARPET PROPERTY...

CARPET PROPERTY TEST (BLENDER FOR $\xi = 1.2$)



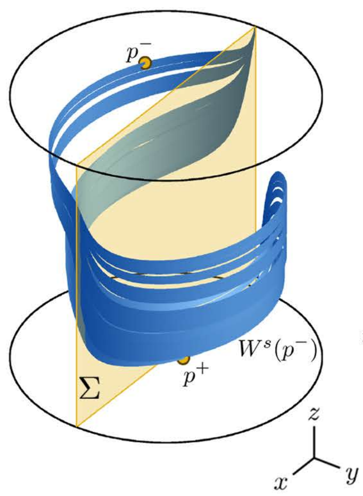
Evidence of a blender for $\xi = 1.2$. The maximum gap converges to 0.

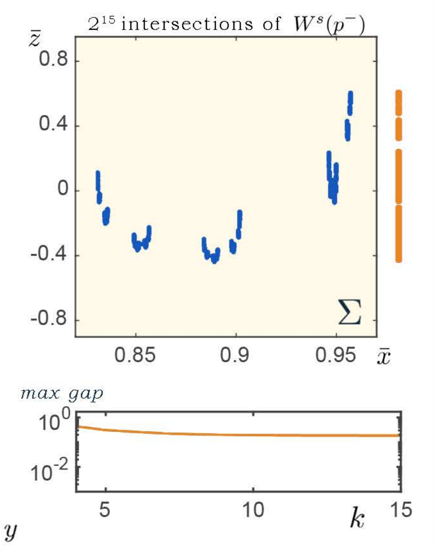
- We intersect the stable manifold with a plane Σ. The system has the carpet property if the **maximum gap** in the z direction converges to zero as we double the amount of (2^k) intersections.
- The generation and disappearance of blenders in the system can be studied as a function of ξ.

FUTURE WORK:

- Generalize the map H to have the situation where the fixed point p^+ has a one-dimensional unstable manifold $W^u(p^+)$ that passes through the blender.
- The goal is to have an explicit example of a system that exhibits robust heterodimensional cycles, which is a route to wild chaos.

CARPET PROPERTY TEST (NON-BLENDER FOR $\xi = 1.8$)





The carpet property has been lost for $\xi = 1.8$. The maximum gap converges to a non-zero value.

[1] C. Bonatti and L.J. Díaz, Persistent nonhyperbolic transitive diffeomorphisms, Annals of Mathematics 143(2): 357–396, 1996 .
[2] C. Bonatti, L.J. Díaz and M. Viana, Dynamics beyond Uniform Hyperbolicity, Encyclopaedia of Mathematical Sciences 102, Springer, 2005.
[3] S. Hittmeyer, B. Krauskopf, H.M. Osinga and K. Shinohara, Existence of blenders in a Hénon- like family: geometric insights from invariant manifold computations, Nonlinearity 31(10) , 2018.
[4] B. Krauskopf and H.M. Osinga, Growing 1D and quasi-2D unstable manifolds of maps, Journal of Computational Physics 146(1):404-419, 1998.