BLENDERS IN A 3D HÉNON-LIKE MAP: IN THE SEARCH FOR WILD CHAOS

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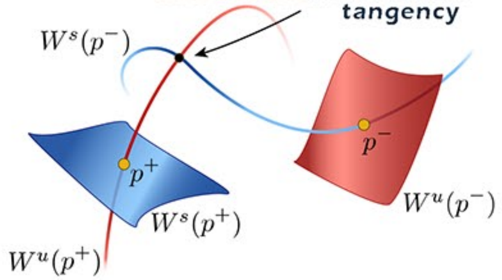
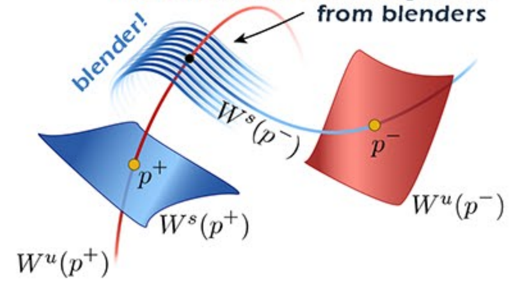
BACKGROUND...

- **Wild chaos** is a new type of chaotic dynamics; a minimum of three dimensions is required for discrete-time dynamical systems.
- Unlike classical chaos, systems that exhibit wild chaos admit robust heteroclinic tangencies leading to **robust chaotic dynamics**.
- Wild chaos can be generated with a geometric structure called a **blender**.
- Blenders have invariant manifolds that behave as geometric objects of higher dimensions than expected.
- We say that such a manifold has the **carpet property**.

CLASSICAL CHAOS
Non-robust heteroclinic tangency



WILD CHAOS
Robust heteroclinic tangencies from blenders



BLENDERS IN A 3D HÉNON-LIKE MAP...

3D HÉNON-LIKE MAP

$$H(x,\ y,\ z) = (y,\ a - y^2 - b\,x,\ \xi\,z + y),$$

FIXED POINTS

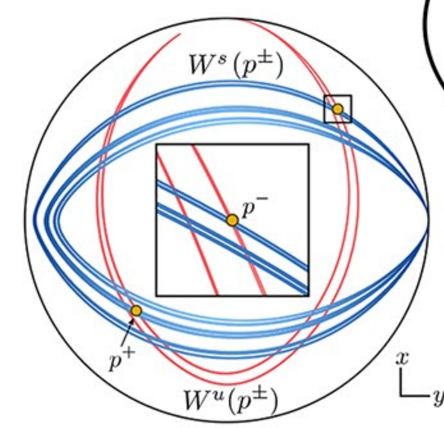
$$p_{\pm} = \left(\rho_{\pm}, \rho_{\pm}, \tfrac{\rho_{\pm}}{1-\xi}\right) \quad \text{with}$$

$$\rho_{\pm} = \tfrac{1}{2}\left(-1 - b \pm \sqrt{4a + (1+b)^2}\right).$$

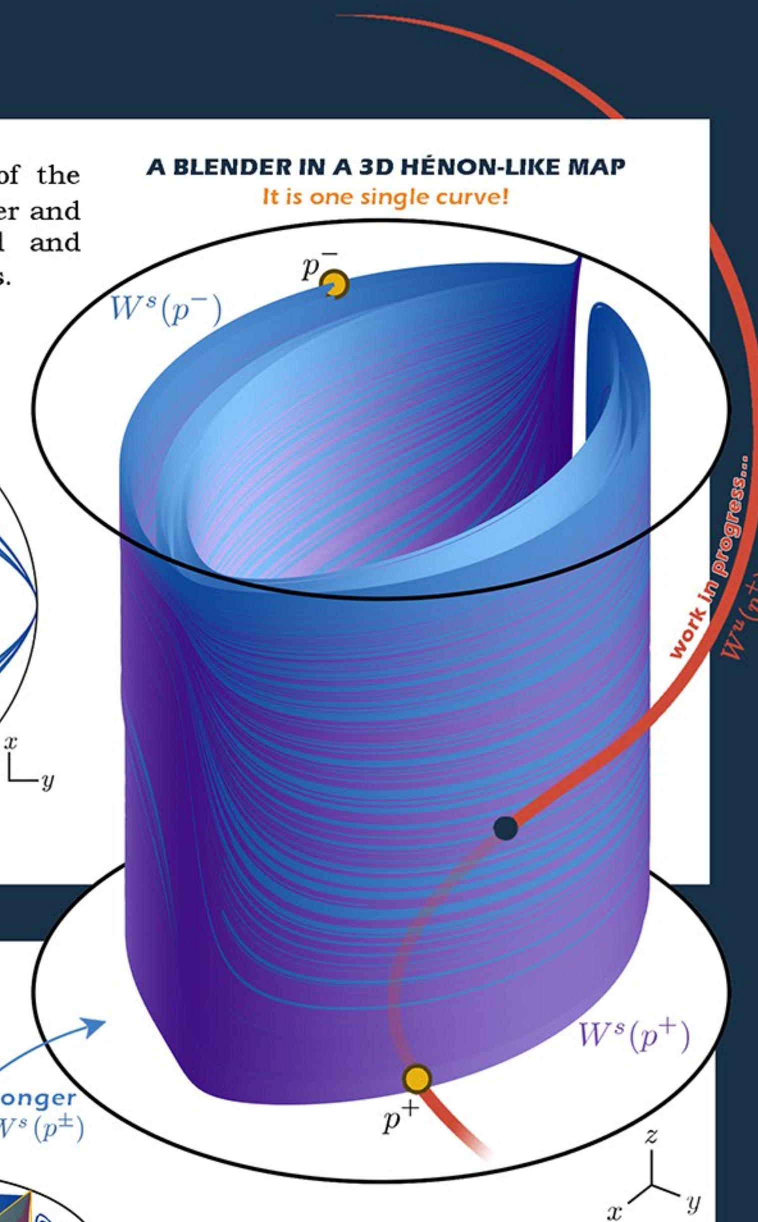
- With fix parameter values $a = 4.2$, and $b = 0.3$ such that H is orientable and has a full horseshoe.
- H has two saddle fixed points p^- and p^+ with one-dimensional $W^s(p^{\pm})$ and two-dimensional $W^u(p^{\pm})$.
- Computing long manifolds to test the carpet property is a major challenge since the manifolds make excursions near infinity.

SOLUTION: Compactification of the phase space of H to a cylinder and implementation of advanced and efficient numerical techniques.

TOP VIEW OF THE 3D HÉNON-LIKE MAP

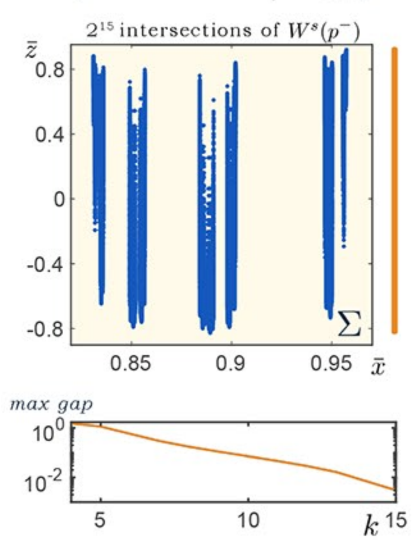


A BLENDER IN A 3D HÉNON-LIKE MAP
It is one single curve!



IS IT REALLY A BLENDER? THE CARPET PROPERTY...

CARPET PROPERTY TEST (BLENDER FOR $\xi = 1.2$)



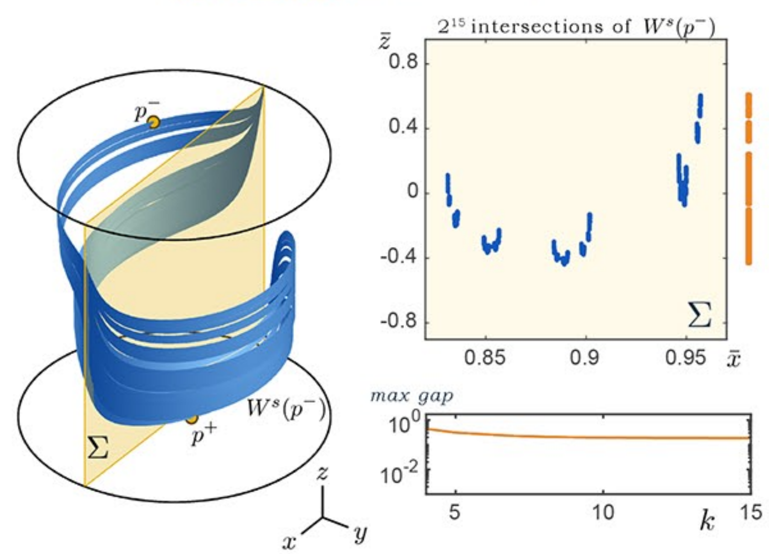
Evidence of a blender for $\xi = 1.2$.
The maximum gap converges to 0.

- We intersect the stable manifold with a plane Σ. The system has the carpet property if the **maximum gap** in the z direction converges to zero as we double the amount of (2^k) intersections.
- The generation and disappearance of blenders in the system can be studied as a function of ξ.

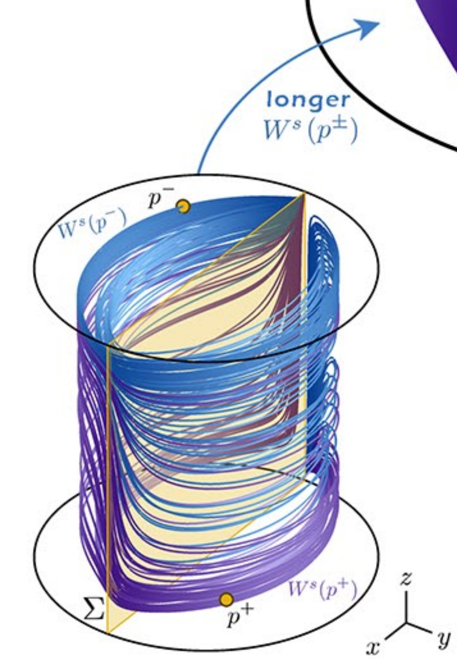
FUTURE WORK:

- Generalize the map H to have the situation where the fixed point p^+ has a one-dimensional unstable manifold $W^u(p^+)$ that passes through the blender.
- The goal is to have an explicit example of a system that exhibits robust heterodimensional cycles, which is a route to wild chaos.

CARPET PROPERTY TEST (NON-BLENDER FOR $\xi = 1.8$)



The carpet property has been lost for $\xi = 1.8$.
The maximum gap converges to a non-zero value.



3D HÉNON-LIKE MAP

Fixed points p^- and p^+ are shown, along with their one-dimensional stable manifolds $W^s(p^{\pm})$. The manifolds are computed up to 2^7 intersection points with the plane Σ. When we increase the number of intersections, $W^s(p^{\pm})$ starts to look like a surface, sugesting that the map has a blender with the carpet property.

[1] C. Bonatti and L.J. Díaz, Persistent nonhyperbolic transitive diffeomorphisms, Annals of Mathematics 143(2): 357–396, 1996 .

[2] C. Bonatti, L.J. Díaz and M. Viana, Dynamics beyond Uniform Hyperbolicity, Encyclopaedia of Mathematical Sciences 102, Springer, 2005.

[3] S. Hittmeyer, B. Krauskopf, H.M. Osinga and K. Shinohara, Existence of blenders in a Hénon- like family: geometric insights from invariant manifold computations, Nonlinearity 31(10) , 2018.

[4] B. Krauskopf and H.M. Osinga, Growing 1D and quasi-2D unstable manifolds of maps, Journal of Computational Physics 146(1):404-419, 1998.