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Optimal delivery routing with wider drone-delivery areas along a shorter truck-route

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ABSTRACT

While convergent technology has been booming recently, drones are being applied in various fields of industry and are expected to be used as a commercial delivery method. In a delivery system, routing becomes one of the major issues, and several studies have attempted to solve drone-based routing problems. In this study, we focus on finding an effective delivery route for trucks carrying drones. To put it concretely, we propose a new approach on a nonlinear programming model to find shift-weights that move the centers of clusters to make for wider drone-delivery areas along shorter truck-route after initial K-means clustering and TSP (Traveling Salesman Problem) modeling. In order to verify the effectiveness of the proposed model with shift-weights, we compare it with two other delivery route approaches. One is a route without shift-weights after K-means clustering and TSP modeling, and the other is a route by TSP for all delivery locations without K-means clustering. Through experimental results of paired *t*-tests on randomly generated delivery locations, we show that our proposed model is more effective than the other two models.

1. Introduction

While convergent technology has been booming recently, drones are being applied in various fields of industry, and are expected to become a commercial delivery method. In December 2013, Amazon released the drone-based PrimeAir service with an ambitious strategy, making drone-based delivery a hot issue (CNN, 2017). At last, in December 2016, the first PrimeAir delivery service was realized in the UK (Amazon, 2017). In 2013, DHL began developing Parcelcopter. From January to March in 2016, the third generation of Parcelcopter completed a delivery test between Parcelcopter SkyPorts in the mountains in Germany (DHL, 2017). In addition, Google's Project Wing is testing food delivery drones in Australia (The Irish News, 2017).

According to the ARK Investment Management's report (Keeney, 2015), the cost to deliver a small parcel within 16.1 km in the USA is \$12.92 by UPS ground using next-day delivery, and \$8.32 by FedEx ground. On the contrary, the cost is \$1 to deliver a parcel within 30 min using Amazon PrimeAir. Therefore, the competition for drones to deliver parcels will become more intense in terms of cost.

In delivery systems, the determination of an effective delivery path is one of the principal events. Although several recent studies have attempted to solve the drone-based routing problem (Ferrandez, Harbison, Weber, Sturges, & Rich, 2016; Murray & Chu, 2015), we also need to consider other simple models to find effective delivery routes. We can use drones departing from distribution centers for delivery service, but we have to consider another approach if the drones are not located within the drones' service range. In that case, a truck carrying drones can be an alternative. That is, after the truck loaded with multiple drones moves near the delivery locations, we can dispatch the drones for delivery service from this optimal point. In this case, it would be meaningful to look for a way to group delivery locations into several drone-delivery clusters and then determine the truck's most efficient delivery path among the centers of the clusters. The clusters are found using the K-means clustering technique (MacQueen, 1967), and the truck's delivery path is found using the TSP (Traveling Salesman Problem) model (Dantzig, Fulkerson, & Johnson, 1954), if we do not consider constraints such as shipping weights and time windows.

Here, it is very significant to find a delivery path that can minimize the total delivery time for a truck's delivery path among the centers of clusters, and the drones' delivery paths within the clusters. In terms of operation cost and speed, drones are more efficient than trucks. With that, it is most efficient to increase

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the drones' flight distance and decrease the trucks' traveling distance. This can be achieved most effectively by moving the centers of clusters more toward or farther away from a depot. Shift-weights used to shift the centers of clusters to reduce the total delivery time will make this situation possible. In this paper, therefore, we propose a new approach to the nonlinear programming model in order to find shift-weights to move the centers of clusters for wider drone-delivery areas along a shorter truck-route, after K-means clustering and TSP (Traveling Salesman Problem) modeling. In this case, we have to consider the constraints on a drone's service range within each cluster. This simple approach can be a basis for easy application to the industry.

To achieve the aim of our study, we review several similar vehicle routing optimization models and drone-based routing models in chapter 2. In chapter 3, we propose a three-step approach to finding effective delivery routes with shift-weights. In chapter 4, we verify the effectiveness of the proposed approach by comparing it with routes not adjusted by shift-weights, and routes decided by TSP only. Finally, in the conclusion, we discuss the usability and limitation of this research and further research issues.

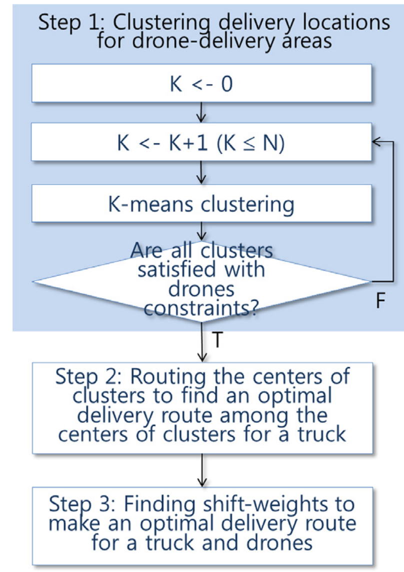


Fig. 1. The three-step approach to problem solving.

2. The evolution of delivery routing models and similar existing models in literature

The TSP (Dantzig et al., 1954) creates the shortest path among several nodes. The VRP (Vehicle Routing Problem) (Dantzig & Ramser, 1959) is a generalization of TSP, and it calls for one or more routes from one depot. The MDVRP (Multi-Depot VRP) (Laporte, Nobert, & Arpin, 1986) treats multi-depots, and The VRPTW (VRP with Time Windows) (Solomon, 1987) considers the constraints of time windows for delivery locations. In addition, there is the PDP (Pickup and Delivery Problem) (Kalantari, Hill, & Arora, 1985), which picks up items from some locations and delivers them to other locations. Furthermore, there is a delivery study, Dynamic PDPTW (Chang & Lee, 2007), which is based on adding new delivery locations to previously planned routes during delivery service.

In recent years, as drones have been spotlighted as a primary delivery method, delivery routing models using drones are being studied. Murray and Chu (2015) formulated FSTSP (Flying Sidekick TSP) and PDSTSP (Parallel Drone Scheduling TSP) with mixed integer linear programming (MILP), and proposed a heuristic algorithm to find their solutions. FSTSP is an optimization model in which trucks and drones perform delivery services for different locations at the same time, with drones departing from a truck traveling along a TSP-based delivery route then returning to the truck. In comparison, PDSTSP is an optimization model for parallel deliveries in which trucks and drones, from a depot, simultaneously deliver parcels along different paths, and come back to the depot.

In another case, Ferrandez et al. (2016) proposed an optimization algorithm to minimize total delivery time - composed of the trucks' traveling time and the sum of drones' average delivery time in each cluster. The delivery points are grouped into a number of clusters by a K-means algorithm (MacQueen, 1967), and the truck moves through the centers of the clusters. They conducted experiments to find the optimal number of clusters, and the appropriate speed of drones relative to the speed of trucks, to make for effective delivery systems.

As we have reviewed, a variety of delivery routing models have been proposed and can be applied to industries as technology advances. In this paper, we propose yet a newer approach to the nonlinear programming model that can further shorten the total delivery time by shifting the centers of clusters after applying a K-means clustering algorithm (MacQueen, 1967) and TSP modeling (Dantzig et al., 1954). Here, we consider a truck's traveling time, drones' flight time, and delivery service time, as factors affecting total delivery time. The truck's traveling time is the time required for moving along shifted centers of clusters. The drones' flight time is the sum of the longest flight times required by drones in each cluster. The delivery service time is the total spent time to deliver items to customers at delivery locations.

3. Research models for wider drone-delivery along shorter truck-routes

3.1. Overview of problem solving approach

The delivery problem covered in this paper is as follows: a truck carrying delivery items departs from a depot and travels the shifted centers of clusters bundled by several delivery locations, and finally returns to the depot. In each cluster, several drones leave the truck at the same time and return to the truck after visiting delivery locations. In these cases, the total delivery time is determined by the longest flight time among drones in each cluster. The proposed model in this paper aims at minimizing the total delivery time by having a truck moving among the centers of clusters as drones operate in each cluster. We assume that delivery items are for small parcels deliverable by drones, and we do not consider time window constraints on the delivery locations.

To solve this problem, our research model is composed of three steps as shown in Fig. 1. In the first step - clustering delivery locations–nearby delivery locations within the drone's service range are clustered using the K-means clustering technique (MacQueen, 1967). At this time, the number of clusters (K) is minimized within the range of the number of delivery locations (N) or less. Because drones can move faster than trucks, deliveries can be completed faster by reducing the number of clusters composed of delivery locations within the drones' service range. In the second step - routing the centers of clusters–a truck's delivery route among the centers of clusters is set up to minimize its traveling time using TSP (Dantzig et al., 1954). In the third step - finding shift-weights - we propose nonlinear programming to find the shift-weights. These correspond to an adjustment factor that shifts

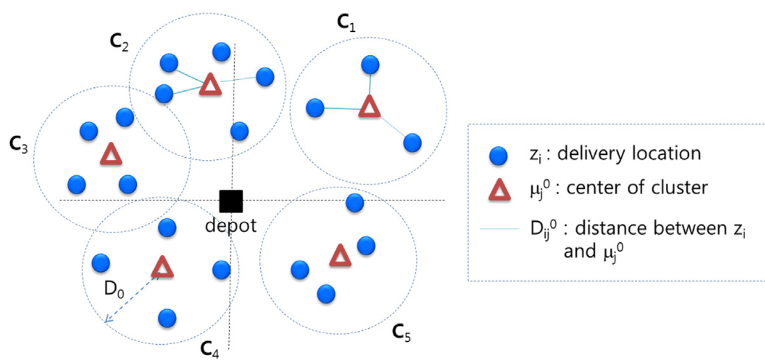


Fig. 2. K-means clustering satisfied with drone constraints for 20 delivery locations (step 1 in Fig. 1).

to the centers of clusters to reduce the total delivery time. The details are described in the next section.

3.2. Problem solving models

As described above, our problem solving approach consists of three steps. We will now illustrate the modeling procedure step-by-step with 1 depot and 20 random delivery locations.

3.2.1. Step 1: clustering delivery locations

The first step is to cluster the 20 delivery locations into areas within drones' delivery range. For detail descriptions, we first define the notations used in this step as follows:

i index of delivery locations, $1 \le i \le N$, where N is the number of delivery locations,
j, k index of clusters, $1 \le j,k \le K$, where K is the number of clusters,
z_i ith delivery location, represented with the coordination of (x_i, y_i),
$\mathbf{C}_j$ jth cluster composed of several nearby delivery locations, and $n(\mathbf{C}_j)$ means the size of the cluster,
$\mu_j{}^0$ a center of jth cluster, represented with the coordination of $(x_{\mu_j{}^0}, y_{\mu_j{}^0})$,
D_0 drone's service range, where the drones can move the farthest from the center of each cluster,
E_0 number of drones in a truck, and
$D_{ij}{}^0$ distance $(\le D_0)$ between the center of jth cluster $(\mu_j{}^0)$ and ith delivery location (z_i).

Delivery locations (z_i) are grouped into K clusters using the K-means clustering algorithm MacQueen, 1967) as shown in Fig. 2. Thin dotted circles represent the clusters. At this time, the delivery locations of each cluster $(\mathbf{C}_j)$ should satisfy the drones' constraints ((1) and (2). Eq. (1) means that each delivery location represented by $z_i = (x_i, y_i)$ should be within the range (D_0). Thin solid lines in clusters refer to distances between the centers of clusters and delivery locations within each cluster $(D_{ij}{}^0)$. Eq. (2) shows that the size of each cluster $(n(\mathbf{C}_j))$ must be less than or equal to the number of drones (E_0) in a truck. The number of clusters (K) is determined by iterating from 1 to N (=30) until it first finds that all clusters satisfy the Eqs. (1) and (2) while K is increased by 1. In this example, this step generates 5 $\mathbf{C}_j$ clusters $(\mathbf{C}_j,\ 1 \le j \le K = 5)$.

$$D_{ij}^0 = \sqrt{\left\{\left(x_i - x_{\mu_j^0}\right)^2 + \left(y_i - y_{\mu_j^0}\right)^2\right\}} \le D_0 \text{ for all } j,\ i \in \mathbf{C}_j \tag{1}$$

$$n\left(\mathbf{C}_j\right) \le E_0 \text{ for all } j \tag{2}$$

3.2.2. Step 2: routing centers of clusters

The second step is to find an optimal route for the depot and the centers of the five clusters generated in step 1. We define the additional notations used in this step as follows:

M number of cluster j or k for a depot, where M equals $K+1$,
$Z_{jk}{}^0$ delivery path among a depot $(\mu_M{}^0)$ and all cluster centers $(\mu_j{}^0)$, where 1 means connected path and 0 disconnected path.

For centers of a depot $(\mu_M{}^0)$ and all clusters $(\mu_j{}^0)$, a TSP model (Dantzig et al., 1954) is used to set up a delivery path (thick solid lines) that minimizes the truck's traveling time as shown in Fig 3. Eventually, this step generates $Z_{jk}{}^0$ values.

3.2.3. Step 3: finding Shift-weights

The third step is to find shift-weights that move the centers of the five clusters in order to make for wider drone-delivery areas along a shorter truck-route. It is based on the approach by moving the centers of the five clusters more toward or farther away from the depot. We add the notations used in this step as follows:

$D_{j,max}$ the longest distance between the center of jth cluster and ith delivery location in the cluster, $\max_i\{D_{ij}\} \le D_0$,
μ_j a shifted center of jth cluster, represented with the coordination of $(x_{\mu_j} = \omega_j x_{\mu_j{}^0},\ y_{\mu_j} = \omega_j y_{\mu_j{}^0})$ where ω_j means a shift-weight $(\omega_j \ge 0)$ and plays a role of shifting $x_{\mu_j{}^0}$ and $y_{\mu_j{}^0}$ into x_{μ_j} and y_{μ_j}, respectively,
S_0 service time at each delivery location,
V_{truck} average speed of truck,
V_{drone} average speed of drone,
t_{truck} truck's traveling time,
t_{drone} drone's flight time, $\Sigma_j D_{j,max}/V_{drone}$,
$t_{service}$ service time, required for parcels to all delivery locations,
t_{travel} total traveling time spent by truck and drones, $t_{truck} + t_{drones}$, and
t_{total} total delivery time including service time, $t_{travel} + t_{service}$.

The center of each cluster $(\mu_j{}^0)$ moves on a straight line $(y_j = a_j x_j + b_j$, where $b_j = 0)$ connected to the depot and becomes a shifted center of the cluster (μ_j), where the total travelling time (t_{travel}) combined with the truck's traveling time (t_{truck}) and the sum of the drones' longest flight times (t_{drone}) determined in each cluster is minimized. In Fig. 4, thick solid lines among shifted centers of clusters (μ_j) are the truck's new traveling route. This route becomes shorter than the route (thick dotted lines) in step 2.

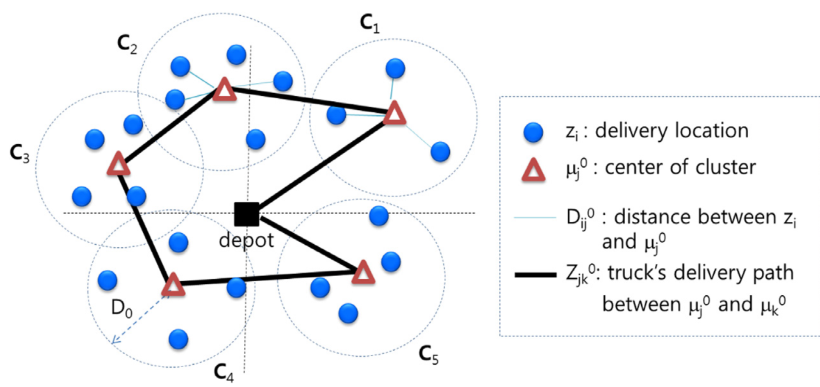


Fig. 3. Routing for the centers of 5 clusters and a depot with TSP model (step 2 in the Fig. 1).

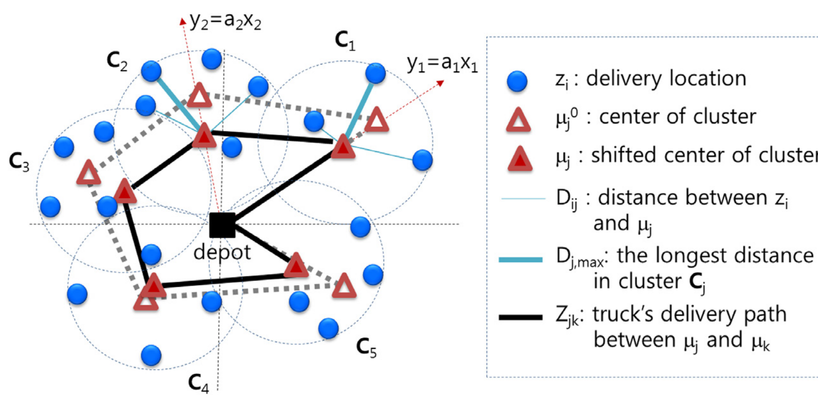


Fig. 4. Finding 5 shift-weights for optimal route (step 3 in the Fig. 1).

The shifted center of *j*th cluster (μ_j) is determined by Eq. (3). If $\omega_j = 1$, the center of *j*th cluster is not moved. If $\omega_j < 1$, it moves in the depot's direction. If $\omega_j > 1$, it moves in the opposite direction of the depot.

$$\mu_j = \omega_j \mu_j^0 \tag{3}$$

At this time, each of the shifted centers of clusters represented by $\mu_j = (x_{\mu_j}, y_{\mu_j})$ should satisfy the constraints of Eq. (4). It is the same situation as Eq. (1) for original centers of clusters.

$$D_{ij} = \sqrt{\left\{ \left(x_i - x_{\mu_j}\right)^2 + \left(y_i - y_{\mu_j}\right)^2 \right\}} \le D_0 \text{ for all } j, i \in \mathbf{C}_j \tag{4}$$

Here, the problem is to find shift-weights (ω_j), that make cluster centers (μ_j) move toward or farther away from the depot. We can derive the solution from the following nonlinear programming:

$$\min \sum\nolimits_{j,k \le M} f(\omega_j, \omega_k) Z_{jk}^0 + \sum\nolimits_{k \le K} g(\omega_k) + KS_0 \tag{5}$$

$$\text{st. } x_{\mu_j} = \omega_j x_{\mu_j^0} \text{ for } j \le K \tag{6}$$

$$y_{\mu_j} = \omega_j y_{\mu_j^0} \text{ for } j \le K \tag{7}$$

$$f(\omega_j, \omega_k) = \sqrt{\left\{ \left(x_{\mu_j} - x_{\mu_k}\right)^2 + \left(y_{\mu_j} - y_{\mu_k}\right)^2 \right\}} \Big/ V_{truck} \text{ for } j, k \le M \tag{8}$$

$$D_{ij} = \sqrt{\left\{ \left(x_i - x_{\mu_j}\right)^2 + \left(y_i - y_{\mu_j}\right)^2 \right\}} \le D_0 \text{ for } j \le K, i \in C_j \tag{9}$$

$$g(\omega_j) = \max_i \{D_{ij} | i \in C_j\} / V_{drone} \text{ for } j \le K \tag{10}$$

$$\omega_j \ge 0 \text{ for all } j \le K, \omega_M = 1 \tag{11}$$

Eq. (5) shows an objective function that minimizes total delivery time (t_{total}). Total delivery time (t_{total}) is composed of total traveling time (t_{travel}) by a truck and drones, and service time ($t_{service}$). The first term represents a truck's traveling time (t_{truck}) along a delivery path among the centers of clusters. Z_{jk}^0 refers to the delivery path between clusters *j* and *k*. It is 1 if a path is set, and 0 otherwise. The values are given by the second step. The second term shows drones' flight time (t_{drone}) determined by all clusters. The third item refers to service time ($t_{service}$) in clusters. t_{drone} is determined by the longest distance ($D_{j,max}$) among the delivery locations from the shifted centers of every cluster. Eqs. (6) and (7) show the shifted centers (μ_j) moved from the original centers of clusters (μ_j^0). Eq. (8) is the term used in the objective function, which refers to the truck's traveling time among clusters. Eq. (9) represents a constraint on the maximum allowable distance (D_0) between each delivery location and a shifted center of each cluster (μ_j). The cluster $\mathbf{C}_j$ is determined in the first step. Eq. (10) determines drones' longest flight time in each cluster. Eq. (11) shows the decision variables. We set ω_M as 1 because the depot location does not change.

4. Performance evaluation through experimentation

4.1. Experimental situation and evaluation

Now, we try to evaluate the proposed model through experimentation. The experiment compares the delivery times for ran-

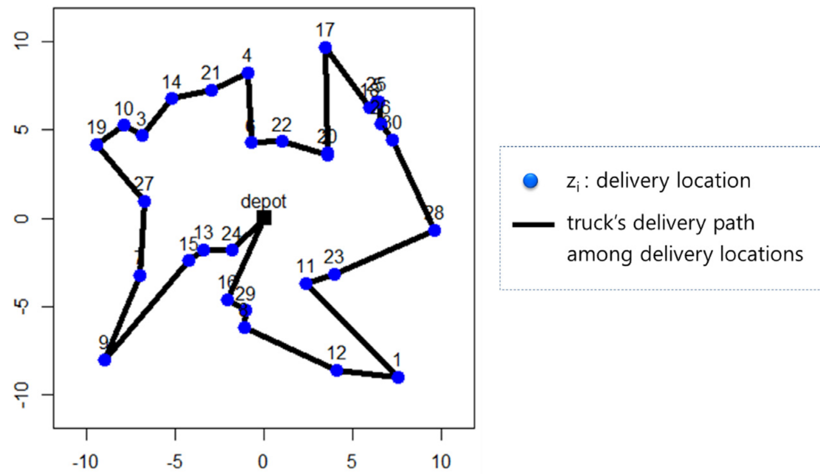


Fig. 5. A delivery route by TSP for N = 30.

Table 1
Evaluation indicators.

Models	Total delivery Time	Null hypothesis For paired *t*-test	Effectiveness
M_{TSP}	$t_{travel} = t_{truck}$ $t_{total} = t_{travel} + t_{service}$		
$M_{cluster}$	$t_{travel} = t_{truck} + t_{drone}$	$\mu_{cluster} > \mu_{TSP}$	$(\mu_{TSP} - \mu_{cluster})/\mu_{TSP} \times 100(\%)$
M_{shift}	$t_{total} = t_{travel} + t_{service}$	$\mu_{shift} > \mu_{cluster}$	$(\mu_{cluster} - \mu_{shift})/\mu_{cluster} \times 100(\%)$

μ_{TSP}, $\mu_{cluster}$, and μ_{shift} refer to the average times of t_{travel} or t_{total} for the M_{TSP}, $M_{cluster}$, and M_{shift}, respectively.

domly generated delivery locations in three ways. One is the method proposed in this study, applying clustering and shift-weights (M_{shift} model). This is the result of step 3 of this study. The second is to not apply shift-weights after clustering ($M_{cluster}$ model). This is the result of step 2 of this study. The third uses only trucks, not drones. In this case, the TSP model (Dantzig et al., 1954) is applied to all delivery locations (M_{TSP} model). The optimization algorithm (Ferrandez et al., 2016) is based on the truck's traveling time and the sum of drones' average delivery time in each cluster, which we reviewed in chapter 2. It is similar to the $M_{clustering}$ model. In this paper, we apply the drones' longest flight time within each cluster ($D_{j,max}$), which is more realistic and relevant than the sum of drones' average delivery time in each cluster.

The experimental situation is as follows: Delivery locations (z_i) are randomly generated in areas within 10 km from the depot to the east, west, south and north. The number of delivery locations (N) is increased from 10 to 100 by intervals of 10. The Average speed of trucks (V_{truck}) and drones (V_{drone}) are 60 km/hr and 90 km/hr, respectively. The number of drones (E_0) in the truck is 10. Drones' service range (D_0) is limited to 5 km, and each service time (S_0) at the delivery location is assumed to be 5 minutes.

Table 1 depicts the evaluation indicators for a null hypothesis in a paired *t*-test and effectiveness between models in terms of total delivery time (t_{total}) to show the accessibility of this study. t_{total} consists of the truck's traveling time (t_{truck}), drones' flight time (t_{drone}), and service time ($t_{service}$). t_{drone} is excluded in the M_{TSP} model. In the second step in Fig. 1, t_{travel} and t_{total} of the $M_{cluster}$ model are compared with those of M_{TSP} for a paired *t*-test and effectiveness. In the third step in Fig. 1, t_{travel} and t_{total} of the M_{shift} model are compared with those of the $M_{cluster}$ model for a paired *t*-test. μ_{model} refers to the average time of t_{travel} or t_{total} for the model. In another case, $M_{cluster}$ and M_{shift} models are compared with M_{TSP} and $M_{cluster}$ models for effectiveness, respectively.

Through this experimentation, we show that the M_{shift} model is the best among the three models.

The experimental system is implemented with R language (www.r-project.org). In the first step, we use a kmeans() function in a stats package for K-means clustering. As a result, we get clusters (**C**j) satisfied with constraints of Eqs. (1) and (2). In the second step, we use ETSP() and solve_TSP() functions in a TSP package to calculate the Euclidean distance, setting a truck's delivery route among the centers of clusters based on the TSP model Dantzig et al., 1954). Here, we get the truck's traveling paths ($Z_{jk}{}^{0}$). In the third step, we use slsqp() functions in an nlopts package to obtain the shift-weights for the M_{shift} model as nonlinear programming composed of Eqs. (5) ~ ((11), in order to minimize the total delivery time (t_{total}).

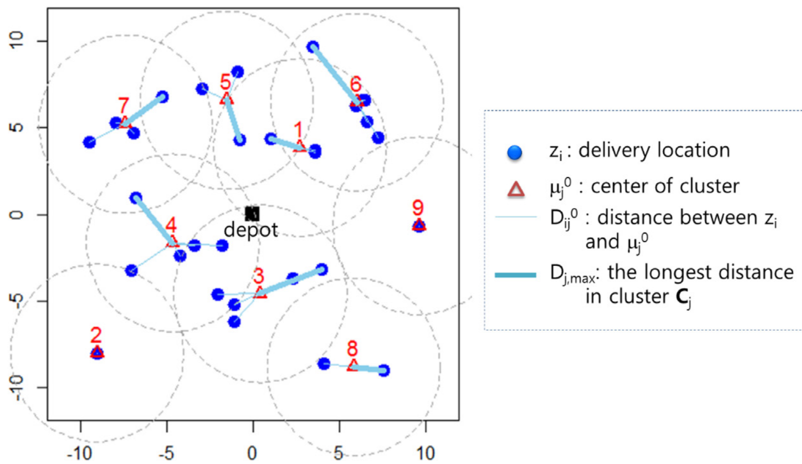
We performed two types of simulations. First, we examined the change of the delivery route according to M_{TSP}, $M_{cluster,}$ and M_{shift} models through 30 iterations, where N is 30, and verify the superiority of the M_{shift} model with a paired *t*-test and effectiveness for delivery time of the models. Second, we verified the superiority of the M_{shift} model with a paired *t*-test and effectiveness by increasing N from 10 to 100.

4.2. Hypothesis test and effectiveness for 30 delivery locations

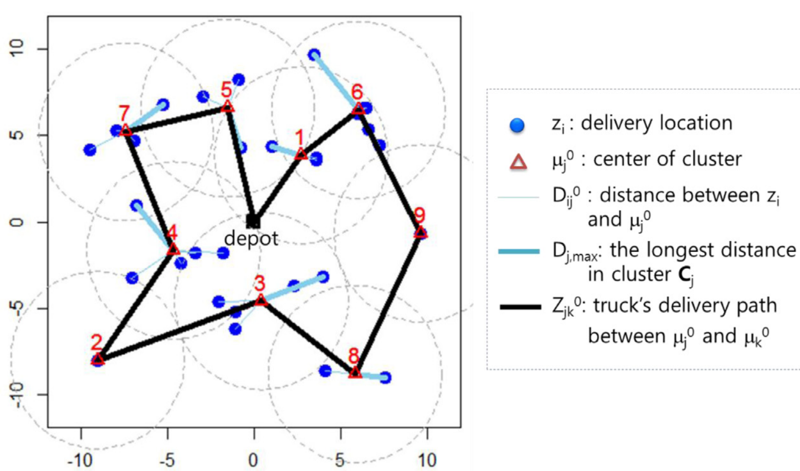
4.2.1. The variance of delivery paths

Fig. 5 shows a delivery path using the M_{TSP} model for 30 delivery locations (N = 30) that are randomly generated. t_{travel} is the same as t_{truck} along the delivery route. If we consider $t_{service}$, it is added to NS_0 minutes because the truck has to visit all delivery locations.

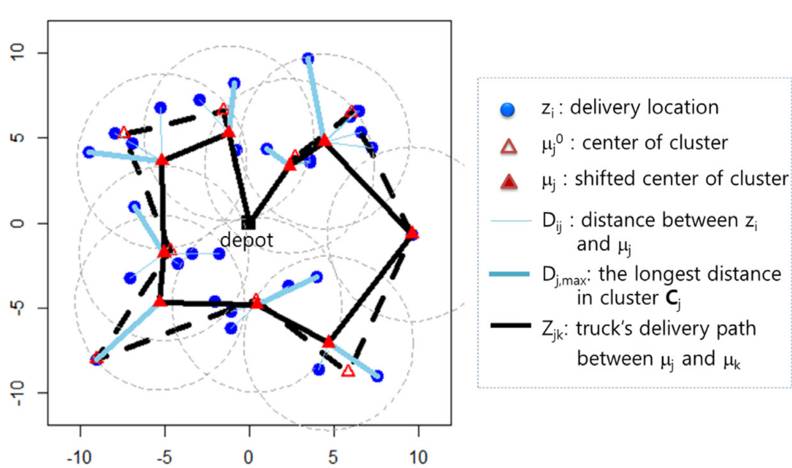
In another case, Fig. 6 (6) depicts the variance of delivery paths according to the steps in Fig. (1). Fig. 6(a) shows clusters and drones' flight paths using K-means clustering of the first step.



(a) Step 1: Centers of clusters and drones' flight paths.

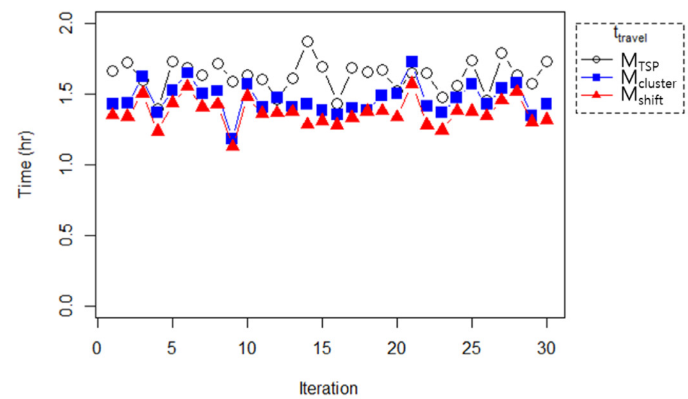


(b) Step 2: A truck's traveling path between the centers of clusters.

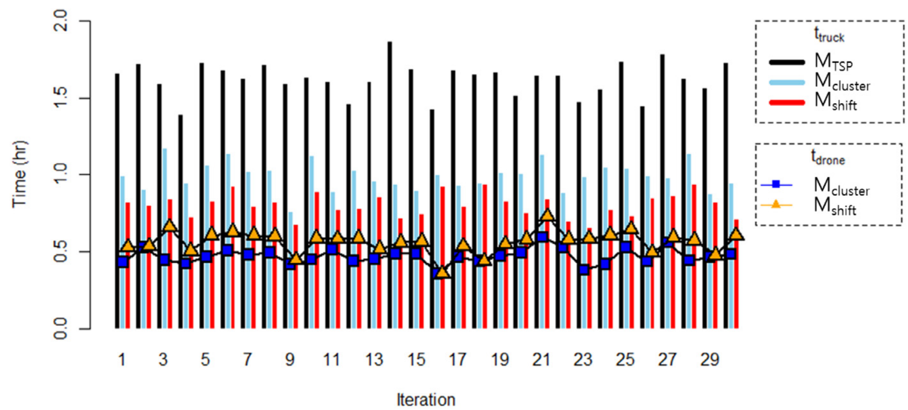


(c) Step 3: An optimal route with shift-weights for a truck and drones.

Fig. 6. Variances of delivery routes based on a truck and drones.



(a) Traveling time (t_{travel}).



(b) Traveling time (t_{truck} and t_{drone}).

Fig. 7. Comparison of traveling times (t_{travel}) by model according to experimental iterations (N = 30).

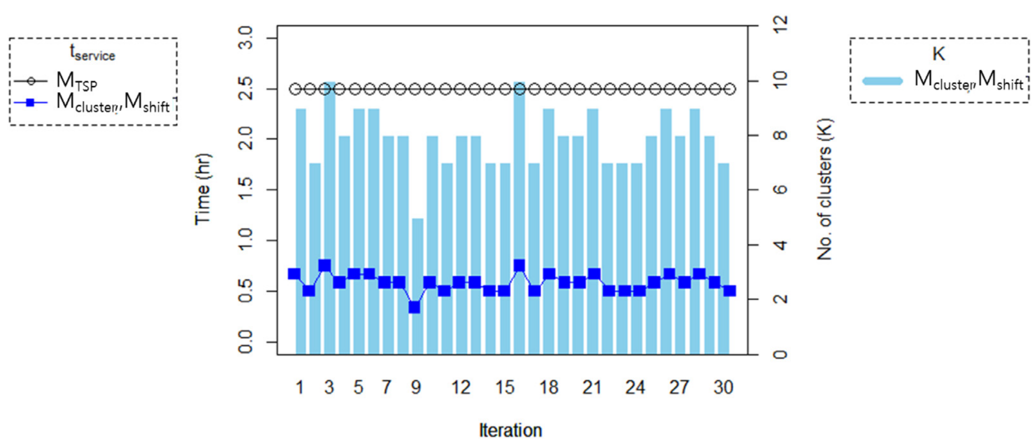


Fig. 8. A comparison of delivery service times ($t_{service}$) by model according to experimental iterations (N = 30).

Fig. 6(b) shows the delivery route among the centers of clusters according to the $M_{cluster}$ model up to the second step. t_{travel} is the sum of t_{truck} among the centers of clusters and t_{drone} in all clusters. If we consider $t_{service}$, KS_0 minutes are added to t_{travel}. Fig. 6(c) depicts the optimal delivery route by the M_{shift} model used in the third step. According to the movement of the cluster centers, t_{travel} is reduced because t_{truck} is smaller than that of the $M_{cluster}$ model, even though t_{drone} is a little larger than that of the $M_{cluster}$ model. Here, $t_{service}$ is the same as that of $M_{cluster}$.

4.2.2. Hypothesis test and effectiveness

When we perform 30 iterations at 30 delivery locations, the comparisons among the three models in terms of traveling time (t_{travel}) are shown in Fig. 7. In Fig. 7(a), t_{travel} of the $M_{cluster}$ model is generally lower than that of the M_{TSP} model, and the M_{shift} model has the lowest distribution. On the other hand, Fig. 7(b) shows t_{travel} divided by t_{truck} and t_{drone}. While t_{truck}s show high distributions in the M_{TSP}, $M_{cluster}$, and M_{shift} models, respectively, t_{drone}s show low distributions in the $M_{cluster}$ and M_{shift} models when they are reversed.

Table 2
Paired-t test and effectiveness for delivery times ($N = 30$, iteration $= 30$).

Model	Traveling time (Total delivery time)		p-value	Effectiveness (%)
	Average (hr)	Standard deviation		
M_{TSP}	1.6268[A] (4.1268)[B]	0.1096 (0.1096)		
$M_{cluster}$	1.4623 (2.0429)	0.1076 (0.1738)	6.9956×10^{-8} (9.2280×10^{-32})	10.1 (50.5)
M_{shift}	1.3656 (1.9462)	0.0971 (0.1672)	6.4729×10^{-14} (6.4641×10^{-14})	6.6 (4.7)

A refers to t_{travel} and B refers to t_{total}, which is the sum of t_{travel} and $t_{service}$.

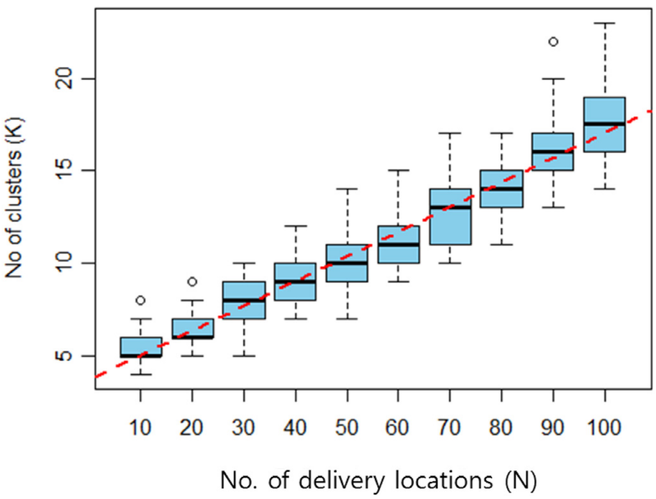


Fig. 9. Variance of No. of clusters (K) according to No. of delivery locations (N).

Fig. 8 compares $t_{service}$s. In the case of the M_{TSP} model, $t_{service}$ is constant, and the time is required by N times of S_0 for all delivery locations visited by the truck, regardless of the number of experiments. On the contrary, $t_{service}$s of the $M_{cluster}$ and M_{shift} models are determined by the number of clusters (K), and are increased by KS_0 minutes. Here, those are the same. At this time, the number of K depends on the geographical locations of delivery locations.

Table 2 shows that the averages of t_{travel}s by the M_{TSP}, $M_{cluster}$, and M_{shift} models are 1.6268 hr (μ_{TSP}), 1.4623 hr ($\mu_{cluster}$), and 1.3656 hr (μ_{shift}), respectively, and the standard deviations are 0.1096, 0.1076 and 0.0971, respectively, after 30 repeated experiments. As a result of one-tailed paired t-tests at significance level $\alpha = 0.05$ for the null hypothesis $\mu_{TSP} > \mu_{cluster}$ and $\mu_{cluster} > \mu_{shift}$, all null hypotheses are rejected. Hence, we can insist that t_{travel} of the $M_{cluster}$ model is shorter than that of the M_{TSP} model, and t_{travel} of the M_{shift} model is shorter than that of the $M_{cluster}$ model. It yields the following results: The efficiency of the $M_{cluster}$ model for M_{TSP} is 10.1%, and that of the M_{shift} model for $M_{cluster}$ is 6.6%. That means that the M_{shift} model is the most efficient among the three models.

4.3. Hypothesis test and effectiveness according to number of delivery locations

We next examine the number of clusters, efficiency, and difference of average delivery times according to the change of N. N is increased by 10 s in the range of 10 to 100, and is repeated 30 times for each N. Fig. 9 depicts the increasing trend of the number of clusters according to the increase of N. As a result of the regression analysis (thick dotted line), we can see that K changes to $K = 0.1335\,N + 3.7067$. That is, the increase of N affects the increase of K, which will have a small impact on trucks' traveling time.

Table 3 shows the change in delivery times (t_{travel} and t_{total}) and the result of paired t-tests for the increase of N. As the results of one-tailed paired t-tests at significance level $\alpha = 0.05$ for the null hypothesis $\mu_{TSP} > \mu_{cluster}$ and $\mu_{cluster} > \mu_{shift}$, all null hypotheses are rejected. That is, it shows that delivery times (t_{travel} and t_{total}) of the $M_{cluster}$ model are shorter than those of the M_{TSP} model, and delivery times of the M_{shift} model are shorter than those of the $M_{cluster}$ model for all N values from 10 to 100.

Fig. 10 shows the change in trucks' traveling times (t_{travel}) with an increase of N. In the case of t_{travel}, in Fig. 10(a), it is generally distributed low in the M_{shift}, $M_{cluster}$, and M_{TSP} models. As per the results of a regression analysis shown in dotted lines, the increased rates of t_{travel}s with increasing N are 0.0182, 0.0126, 0.0123 for the M_{TSP}, $M_{cluster}$, and M_{shift} models, respectively. Fig. 10(b) shows t_{travel} separated by t_{truck} and t_{drone}. As the number of delivery locations (N) increases, those increase. For the M_{TSP} model, the growth rate is the highest, while the $M_{cluster}$ and M_{shift} models show a relatively small increase. The M_{shift} model has a smaller t_{truck} than that of the $M_{cluster}$ model, while the t_{drone} has a slightly higher value.

Delivery service times ($t_{service}$) to delivery locations must be considered realistically. In the case of $t_{service}$, in Fig. 11, the value of the M_{TSP} model, which is sensitive to the increase of N, sharply increases. On the contrary, $t_{service}$s of the $M_{cluster}$ and M_{shift} models are dependent on the number of clusters (K) smaller than N. These increase gradually, compared with those of the M_{TSP} model. Here, the slope in the M_{TSP} is 0.1015, and in the $M_{cluster}$ and M_{shift}, these are all 0.0235.

Fig. 12 shows the effectiveness of the $M_{cluster}$ and M_{shift} models ($Eff_{cluster}$ and Eff_{shift}). It depicts the $M_{cluster}$ model as more effective than the M_{TSP} model. If we consider t_{total} with service time, then the value is very high. In addition, the M_{shift} model is a little more effective than the $M_{cluster}$ model. The conclusion is that the M_{shift} is the most effective among the three models.

In reality, problem solving times are very important. As shown in Fig. 13, the average time using the M_{shift} model also gradually increases as N increases. When N is equal to 100, it requires 6.6 sec. Hence, we can make good use of the M_{shift} model in practical terms.

5. Conclusion

In this study, we focused on the finding an effective delivery route for trucks carrying drones. To do this, we developed a model of optimal nonlinear programming to find shift-weights, and performed experimentation to verify the effectiveness of them. First, we performed 30 iterations at 30 delivery locations and found optimal paths for the $M_{cluster}$ and M_{shift} models for each iteration. As the result of paired t-tests for testing the mean difference of delivery times between models, we confirmed that our proposed M_{shift} model is a more effective model to further reduce total delivery times. In addition, we analyzed paired t-tests and effectiveness between models while increasing the delivery locations from 10 to

Table 3
Paired-t test and effectiveness with increases of N (iteration = 30).

N	Models									
	M_{TSP}		$M_{cluster}$				M_{shift}			
	Avg[A]	SD[B]	Avg.	SD	*p*-value	Eff[C]	Avg.	SD	*p*-value	Eff
10	1.0539[D]	0.1307	1.0327	0.1312	0.0455	2	0.9517	0.1432	2.0097×10^{-12}	7.8
	(1.8872)[E]	(0.1307)	(1.3966)	(0.1892)	(1.6070×10^{-19})	(26)	(1.3156)	(0.2002)	(2.0121×10^{-12})	(5.8)
20	1.3643	0.1356	1.2705	0.1606	1.4388×10^{-6}	6.9	1.1783	0.1544	6.6828×10^{-15}	7.3
	(3.031)	(0.1356)	(1.7261)	(0.2402)	(8.3517×10^{-29})	(43.1)	(1.6338)	(0.236)	(6.6900×10^{-15})	(5.3)
30	1.6268	0.1096	1.4623	0.1076	6.9956×10^{-8}	10.1	1.3656	0.0971	6.4729×10^{-14}	6.6
	(4.1268)	(0.1096)	(2.0429)	(0.1738)	(9.2280×10^{-32})	(50.5)	(1.9462)	(0.1672)	(6.4641×10^{-14})	(4.7)
40	1.8396	0.0843	1.576	0.0941	3.9700×10^{-15}	14.3	1.481	0.0971	2.0517×10^{-15}	6
	(5.1729)	(0.0843)	(2.3426)	(0.1909)	(1.5668×10^{-36})	(56.6)	(2.1477)	(0.1948)	(2.0413×10^{-15})	(4.2)
50	2.0156	0.1184	1.692	0.1252	3.7158×10^{-16}	16.1	1.5806	0.1247	1.0643×10^{-16}	6.6
	(6.1823)	(0.1184)	(2.4365)	(0.2509)	(1.4590×10^{-37})	(60.6)	(2.325)	(0.254)	(1.0608×10^{-16})	(4.6)
60	2.1857	0.0852	1.8199	0.0981	4.6802×10^{-17}	16.7	1.6937	0.0925	2.4525×10^{-18}	6.9
	(7.1857)	(0.0852)	(2.6727)	(0.1957)	(2.5713×10^{-40})	(62.8)	(2.5465)	(0.1943)	(2.4639×10^{-18})	(4.7)
70	2.3597	0.0938	1.9311	0.0963	6.8412×10^{-17}	18.2	1.8262	0.0982	3.6107×10^{-17}	5.4
	(8.1930)	(0.0938)	(2.9117)	(0.2176)	(1.0546×10^{-40})	(64.5)	(2.8068)	(0.2233)	(3.6319×10^{-17})	(3.6)
80	2.4985	0.0881	2.0004	0.0913	5.4622×10^{-20}	19.9	1.8896	0.0878	2.9430×10^{-21}	5.5
	(9.1652)	(0.0881)	(3.0699)	(0.1897)	(3.8181×10^{-44})	(66.5)	(2.959)	(0.1863)	(2.9493×10^{-21})	(3.6)
90	2.6277	0.0835	2.1572	0.1256	6.0155×10^{-20}	17.9	2.0409	0.1218	3.0350×10^{-19}	5.4
	(10.1277)	(0.0835)	(3.4211)	(0.2722)	(1.0598×10^{-42})	(66.2)	(3.3048)	(0.2737)	(3.0475×10^{-19})	(3.4)
100	2.7295	0.1098	2.2293	0.0127	8.4863×10^{-19}	18.3	2.1238	0.0989	4.6585×10^{-21}	4.7
	(11.0628)	(0.1098)	(3.6238)	(0.2572)	(4.6387×10^{-43})	(67.2)	(3.5182)	(0.2506)	(4.6544×10^{-21})	(2.9)

A, B, and C refer to average(hr), standard deviation, and effectiveness(%), respectively.
D refers to t_{travel}(hr), and E refers to t_{total}(hr).

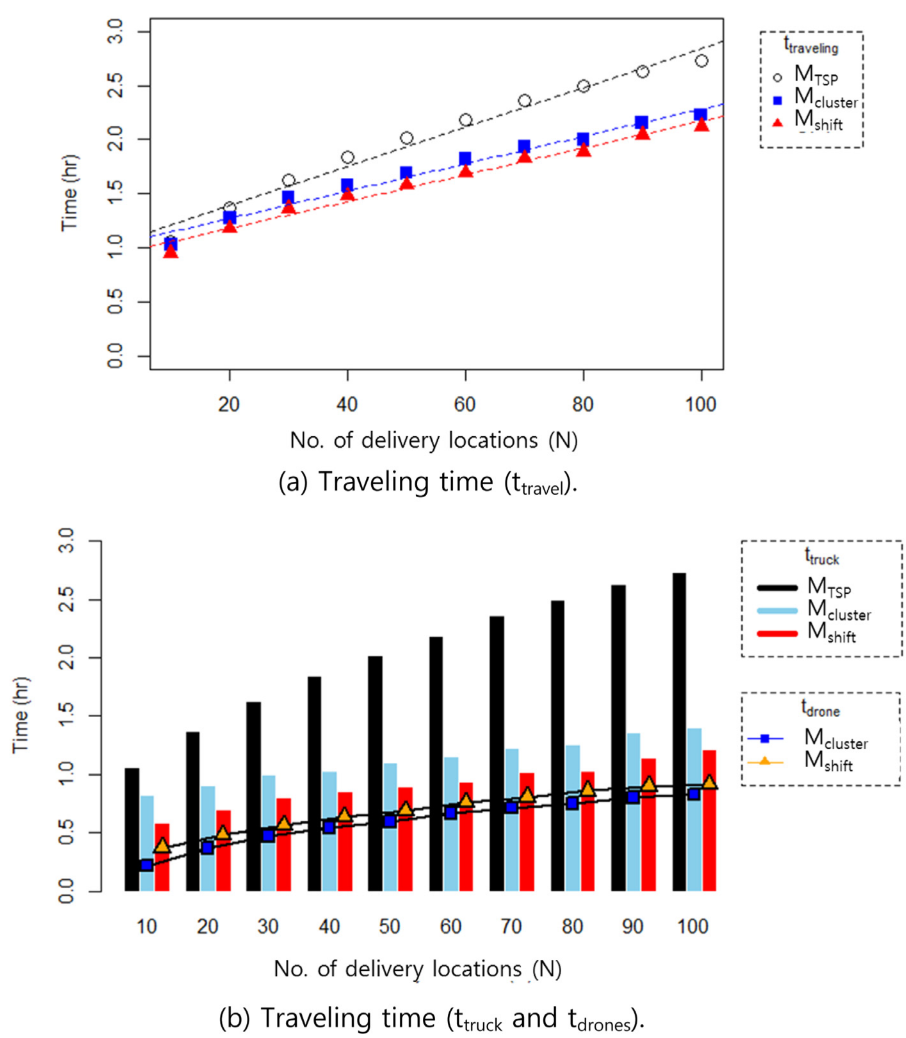


Fig. 10. Comparison of traveling times (t_{travel}) by model according to experimental iterations (N = 30).

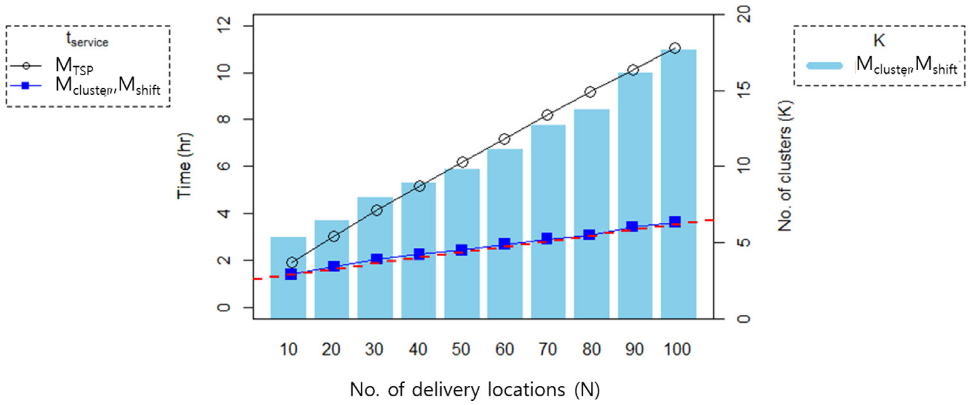


Fig. 11. A comparison of delivery service times ($t_{service}$) by model according to experimental iterations (N = 30).

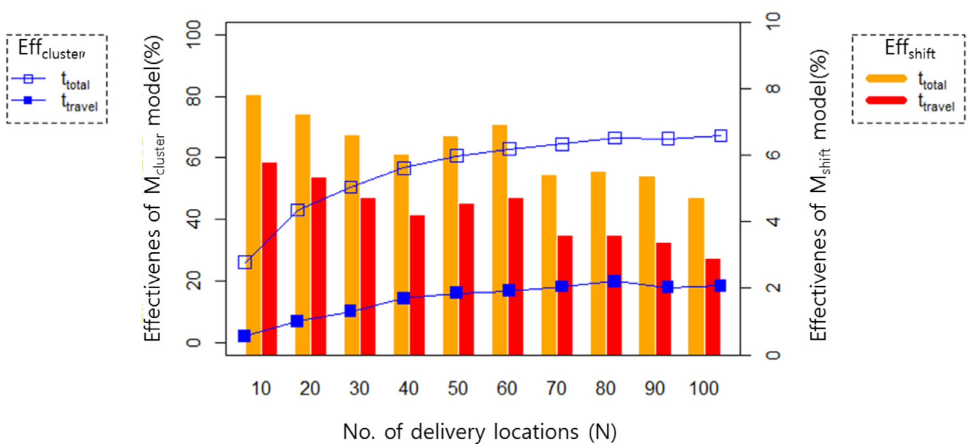


Fig. 12. Effectiveness of traveling times (t_{travel}) and total delivery times (t_{total}) with increases of N.

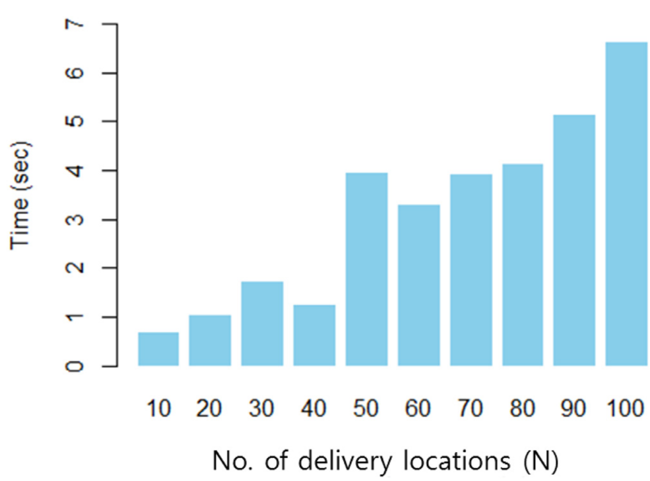


Fig. 13. Average elapsed time for solving M_{shift} model with increases of N.

100 by 10 s, and verified that the M_{shift} model, which increases the use of drones, is more effective. Drones can fly fast in a straight line in the sky. From a cost perspective, in addition, the operating cost of drones is lower than that of trucks. In such an environment, our M_{shift} model will be more effective than the $M_{cluster}$ model.

This study showed that it is possible to deliver parcels efficiently when drones are used in addition to trucks. For example, it will be very useful in complex urban areas where frequent shipment occurs, as well as in mountainous areas where delivery by trucks is difficult. It is also necessary to develop a model that reflects the time window constraints for each delivery location.

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References

Amazon. (2017). Retrieved from https://www.amazon.com/Amazon-Prime-Air/b?node=8037720011.

Chang, Y. S., & Lee, H. J. (2007). Vehicle routing based on pickup and delivery in a ubiquitous environment: U-MDPDPTW. *Journal of Intelligent Information Systems, 13*(1), 49–58.

CNN. (2017). Retrieved from http://money.cnn.com/2013/12/01/technology/amazon-drone-delivery/index.html.

Dantzig, G., Fulkerson, R., & Johnson, S. (1954). Solution of a large-scale traveling-salesman problem. *Operations Research, 2*(4), 393–410.

Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science, 6*, 80–91.

DHL. (2017). Retrieved from http://www.dhl.com/en/press/releases/releases_2016/all/parcel_ecommerce/successful_trial_integration_dhl_parcelcopter_logistics_chain.html.

Ferrandez, S. M., Harbison, T., Weber, T., Sturges, R., & Rich, R. (2016). Optimization of a truck-drone in tandem delivery network using K-means and genetic algorithm. *Journal of Industrial Engineering and Management JIEM, 9*(2), 374–388.

Kalantari, B., Hill, A. V., & Arora, S. R. (1985). An algorithm for the traveling salesman problem with pickup and delivery customers. *European Journal of Operational Research, 22*, 377–386.

Keeney, T. (2015). *ARK analyst, amazon drones could deliver a package in under thirty minutes for one dollars*. ARK Investment Management LLC (ark-invest.com).

Laporte, G., Nobert, Y., & Arpin, D. (1986). An exact algorithm for solving a capacitated location-routing problem. *Annals of Operations Research, 6*, 293–310.

MacQueen, J. B. (1967). Some methods for classification and analysis of multivariate observations. In *Proceedings of the fifth symposium on math, statistics, and probability* (pp. 281–297). University of California Press.

Murray, C. C., & Chu, A. G. (2015). The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery. *Transportation Research Part C: Emerging Technologies, 54*, 86–109.

Solomon, M. M. (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research, 35*(2), 254–265.

The Irish News. (2017). Retrieved from https://www.irishnews.com/magazine/technology/2017/11/06/news/google-s-project-wing-is-testing-food-delivery-drones-in-australia-1180920/.