Solution to Numerical Relativity: Starting from Scratch

September 2, 2023

Contents

1 Newton's and Einstein's Gravity

Exercise 1.1

(a)

$$x = y$$

Exercise 1.2

At constant time t, constant coordinates R and θ, we have $dt = dR = d\theta = 0$. Therefore, line element becomes

$$ds^2 = R^2 \sin^2 \frac{\pi}{2} d\phi^2, \tag{1}$$

and we have proper length

$$C = \int_0^{2\pi} dl = \int_0^{2\pi} R d\phi = 2\pi R. \tag{2}$$

Exercise 1.3

(a) Given $A^a = (-1, 2, 0, 0)$ and $B^a = (-2, 1, 0, 0)$,

$$g_{ab} A^a B^b = -2 + 2 = 0. \tag{3}$$

(b) Consider only in $t - x$ diagram, A and B will be looks like they do make a angle $\theta = \cos^{-1} \frac{4}{5} \approx 36.87°$.

Exercise 1.4

Since four-velocity defined as

$$u^a = \frac{dx^a}{d\tau}, \tag{4}$$

$$\begin{aligned} u_a u^a &= \frac{dx_a}{d\tau} \frac{dx^a}{d\tau} && (5)\\ &= \frac{g_{ab} dx^a dx^b}{d\tau^2} && (6)\\ &= \frac{-ds^2}{d\tau^2} && (7)\\ &= -1. && (8) \end{aligned}$$

2 Foliations of Spacetime: Constraint and Evolution Equations

Exercise 2.1

From

$$\mathcal{C}_E = D_i E^i - 4\pi\rho, \tag{9}$$

$$\partial_t E_i = D_i D^j A_j - D^j D_j A_i - 4\pi j_i, \tag{10}$$

$$\frac{\partial\rho}{\partial t} + D_i j^i = 0, \tag{11}$$

we can get

$$\partial_t \mathcal{C}_E = D_i \partial_t E^i - 4\pi \frac{\partial\rho}{\partial t} \tag{12}$$

$$= D_i (D^i D^j A_j - D^j D_j A^i - 4\pi j^i) + 4\pi D_i j^i \tag{13}$$

$$= D_i D^i D^j A_j - D_i D^j D_j A^i \tag{14}$$

$$= 0. \tag{15}$$

3 Solving the Constraint Equations

4 Solving the Evolution Equations

5 Numerical Simulations of Black-Hole Binaries