

# CS 260: Homework 3

Daniel Lopez

July 21, 2017

16 July 2017

## 1 1

### 1.1 a: Bubble sort

Note: bold denotes bubble.

0: [1 **7** 3 2 0 5 0 8]

1: [1 3 **7** 2 0 5 0 8]

2: [1 3 2 **7** 0 5 0 8]

3: [1 3 2 0 **7** 5 0 8]

4: [1 3 2 0 5 **7** 0 8]

5: [1 3 2 0 5 0 **7** 8]

6: [1 2 **3** 0 5 0 7 8]

7: [1 2 0 **3** 5 0 7 8]

8: [1 2 0 3 0 **5** 7 8]

9: [1 0 **2** 3 0 5 7 8]

10: [1 0 2 0 **3** 5 7 8]

11: [0 **1** 2 0 3 5 7 8]

12: [0 1 0 **2** 3 5 7 8]

13: [0 0 **1** 2 3 5 7 8]

Final sorted array:

[**0 0 1 2 3 5 7 8**]

### 1.2 b: Insertion sort

Using insertionsort:

0: [1 7 3 2 0 5 0 8]

1: [1 3 7 2 0 5 0 8]

2: [1 2 3 7 0 5 0 8]

3: [0 1 2 3 7 5 0 8]

4: [0 1 2 3 5 7 0 8]

5: [0 0 1 2 3 5 7 8]

## 2 2

Using quicksort:

```
[22 36 6 79 26 45 75 13 31 62 27 66 33 16]
[6 13 16 36 79 26 45 75 31 62 27 66 33 22]
[6 13 16 22 79 26 45 75 31 62 27 66 33 36]
[6 13 16 22 26 31 27 33 36 62 45 66 75 79]
[6 13 16 22 26 27 31 33 36 62 45 66 75 79]
[6 13 16 22 26 27 31 33 36 45 62 66 75 79]
```

## 3 3

### 3.1 a

$$T(n) = 4 * T(\frac{n}{3}) + n$$
$$N \sum_{i=0}^{\log_3 n} (\frac{4}{3})^i$$

### 3.2 b

$$\begin{aligned} T(n) &= 4T(n/3) + n^2 \\ &= n^2 + 4(4T(n/9) + n^2/9) \\ &= 2n^2 + 16T(n/9) \\ &= k * n^2 + 4^k * T(\frac{n}{3^k}) \\ k &= \log_2 n \\ T(n) &= O(n^2 \log(n)) \end{aligned}$$

### 3.3 c

$$\begin{aligned} T(n) &= 9T(n/3) + n^2 \\ a &= 9, b = 3, k = 2, P = 0 \\ 9 &< 3^2 \\ T(n) &= \Theta(n^k * \log(n)^p) \\ T(n) &= \Theta(n^2 * \log(n)^0) \\ T(n) &= \Theta(n^2) \end{aligned}$$

## 4 4

Using the master theorem:

### 4.1 a

$$T(n/2) + 1$$

$$a = 1, b = 2, f(n) = 1$$

$$c = \log_2(1) = 0$$

$$\text{Case 2, } 1 = n^0$$

**By definition, the equation is bounded by  $O(\log(n))$  and  $\Omega(\log(n))$**

### 4.2 b

$$2T(n/2) + \log(n)$$

$$a = 2, b = 2, f(n) = \log(n)$$

$$c = \log_2(2) = 1$$

$$\text{Case 2, } \log(n) = \log(n)$$

**By definition, the equation is bounded by  $O(\log^2 n)$  and  $\Omega(\log^2 n)$**

### 4.3 c

$$2T(n/2) + n$$

$$a = 2, b = 2, f(n) = n$$

$$c = \log_2(2) = 1$$

$$\text{Case 2, } n^1 = n^1$$

**By definition, the equation is bounded by  $O(n \log n)$  and  $\Omega(n \log n)$ .**

### 4.4 d

$$2T(n/2) + n^2$$

$$a = 2, b = 2, f(n) = n^2$$

$$c = \log_2(2) = 1$$

$$\text{Case 3, } n^2 = n^{1+\epsilon}$$

**By definition, the equation is bounded by  $O(n^2)$  and  $\Omega(n^2)$ .**