A very brief summary of vector calc in $\mathbb{R}^2$

$(f,g) \longmapsto g_x - f_y$

Scalar functions on $\mathbb{R}^2$ $\xrightarrow{\nabla}$ Vector fields on $\mathbb{R}^2$ $\xrightarrow{\text{curl}}$ Scalar functions on $\mathbb{R}^2$

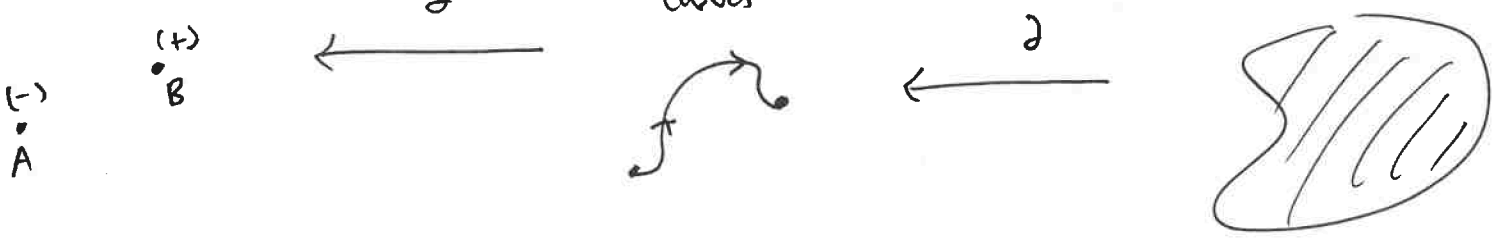
FTC for line integrals

$$\int_C \nabla\varphi \cdot dr = \varphi(B) - \varphi(A)$$

Green's Thm

$$\int_{\partial R} f\,dx + g\,dy = \iint_R (g_x - f_y)\,dA$$

Signed points $\xleftarrow{\partial}$ oriented curves $\xleftarrow{\partial}$ 2d regions



$$\text{curl}(\nabla\varphi) = 0$$

Types of integrals

Flat space

$\iint_R f\,dA$ weighted area

$\iiint_E f\,dV$ weighted volume

$dA = dx\,dy$

$dV = dx\,dy\,dz$

also polar, spherical coord.

Line integrals

$\int_C f\,ds$

weighted arclength

$\int_C f\,dx + g\,dy$

circulation/work

$ds = |r'(t)|\,dt$

$dr = r'(t)\,dt$

Surface integrals

$\iint_S f\,d\sigma$

weighted surface area

$\iint_S F \cdot dn$

flux

$d\sigma = |r_u \times r_v|\,dA$

$dn = r_u \times r_v\,dA$

(careful w/ orientation)