

Transient Stability under Load Perturbation using Small Gain Theorem

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I. INTRODUCTION

The nonlinearities of power system equation is often dealt by bounding techniques on the nonlinear terms in the dynamic equations [1]. By finding the optimal sector bound on the nonlinearity in the system dynamics, our paper proposes a new approach to find maximum perturbation in power injections that certifies stability.

II. TRANSIENT STABILITY UNDER LOAD PERTURBATION

In this paper, we consider the structure-preserving second order swing equation model given by

$$\begin{aligned} m_k \ddot{\delta}_k + d_k \dot{\delta}_k + \sum_{\{k,j \in \mathcal{E}\}} a_{kj} \sin(\delta_k - \delta_j) &= P_k, \quad k \in \mathcal{G} \\ d_k \dot{\delta}_k + \sum_{\{k,j \in \mathcal{E}\}} a_{kj} \sin(\delta_k - \delta_j) &= P_k, \quad k \in \mathcal{L} \end{aligned} \quad (1)$$

where m_k and d_k are inertia and damping of the generator k in equation. $a_{kj} = V_k V_j B_{kj}$ where B_{kj} is the susceptance of the transmission line connecting bus j and k and V_k is the voltage magnitude at bus k . The set of generators and loads are denoted as \mathcal{G} and \mathcal{L} respectively, and \mathcal{E} denotes the edges of the line as a pair of indices. The power injection at generators and loads will be considered as the input perturbation to the system, and output will be the generator angles, δ , and frequency, $\dot{\delta}$, which should remain synchronized.

We consider large perturbations in the load $P_k = P_{k,0} + \Delta P_k$, $k \in \mathcal{L}$, and derive a provable bound on the perturbation that the system remains stable.

Problem Formulation: Consider a nonlinear system $\dot{x} = f(x) + u$. Find the interval $\mathcal{U} = [\underline{u}, \bar{u}]$ such that if $u \in \mathcal{U} \forall t$, then $y \in \mathcal{Y}$ where $\mathcal{Y} = [\underline{y}, \bar{y}]$.

While the above problem is difficult to solve for a general nonlinear system, we will consider the second order swing equation in (1) for the application in transient stability assessment.

Remark. As long as \mathcal{Y} contains only the desirable equilibrium, the system remain stable under any disturbances $u \in \mathcal{U}$.

Suppose the input perturbation and output is defined as $u \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ respectively. Let us linearize our system at the nominal operating point and let this linearized system be G . Then we substitute new variables to any nonlinear terms that are left in Equation 1 with $v \in \mathbb{R}^l$. We define φ as the nonlinear operator that acts on a linear combination of states $w \in \mathbb{R}^l$, which is the input to the nonlinear term. For example, suppose we are given a system $\dot{x} = -2x + \sin(x) + u$ with nominal operating point x_0 and u_0 with output $y = x$. Then the

linearized plant will be defined as $G : \dot{x} = -2x + \cos(x_0)(x - x_0) + v + u_0 + \Delta u$ with $u = \Delta u$ and $\varphi : v = \sin(x) - \cos(x_0)(x - x_0)$ with $w = x - x_0$. This representation can be illustrated with Figure 1 where the nonlinear term is separated out into a closed-loop subsystem.

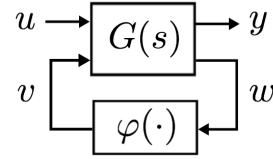


Fig. 1. Block diagram for the system with nonlinearity.

Let the \mathcal{L}_2 gain of G is given by $\gamma_G \in \mathbb{R}^{(m+l) \times (n+l)}$

$$\gamma_G = \begin{bmatrix} \gamma_{G(y,u)} & \gamma_{G(y,v)} \\ \gamma_{G(w,u)} & \gamma_{G(w,v)} \end{bmatrix} \quad (2)$$

where $\gamma_{G(y,u)} \in \mathbb{R}^{m \times n}$, $\gamma_{G(y,v)} \in \mathbb{R}^{m \times l}$, $\gamma_{G(w,u)} \in \mathbb{R}^{l \times n}$, and $\gamma_{G(w,v)} \in \mathbb{R}^{l \times l}$ are the components in the gain of the system. Let γ_φ be the \mathcal{L}_2 gain from the nonlinearity. Note that γ_φ is a function of the bound on the input, $\max_t |w|$.

From the definition of \mathcal{L}_2 gain, we have the following inequalities.

$$\max_t |y| \leq \gamma_{G(y,u)} \max_t |u| + \gamma_{G(y,v)} \max_t |v| \quad (3)$$

$$\max_t |w| \leq \gamma_{G(w,u)} \max_t |u| + \gamma_{G(w,v)} \max_t |v| \quad (4)$$

$$\max_t |v| \leq \gamma_\varphi \max_t |w|. \quad (5)$$

Theorem II.1. (Small-Gain Theorem) The system is finite gain \mathcal{L} stable if $\gamma_G \gamma_\varphi < 1$.

We substitute equation 5 and $\max_t |w| = \max_t |y|$ into equation 4,

$$\max_t |w| \leq \gamma_{G,u} \max_t |u| + \gamma_{G,v} \gamma_\varphi \max_t |w| \quad (6)$$

By rearranging,

$$\gamma_{G,u}^{-1} (1 - \gamma_{G,v} \gamma_\varphi) \max_t |w| \leq \max_t |u| \quad (7)$$

Theorem II.2. If $\bar{u} < \gamma_{G,u}^{-1} (1 - \gamma_{G,v} \gamma_\varphi) \bar{w}$ and $\gamma_{G,v} \gamma_\varphi < 1$, then $\max_t |w| < \bar{w}$ for every u such that $\max_t |u| < \bar{u}$.

Proof. From equation 6,

$$\max_t |w| \leq \gamma_{G,u} \max_t |u| + \gamma_{G,v} \gamma_\varphi \max_t |w|$$

$$\begin{aligned}
(1 - \gamma_{G,v}\gamma_\varphi) \max_t |w| &\leq \gamma_{G,u} \max_t |u| \leq \gamma_{G,u} \bar{u} \\
&\leq \gamma_{G,u} \gamma_{G,u}^{-1} (1 - \gamma_{G,v}\gamma_\varphi) \bar{w} \\
&\leq (1 - \gamma_{G,v}\gamma_\varphi) \bar{w}
\end{aligned}$$

Since $1 - \gamma_{G,v}\gamma_\varphi > 0$, $\max_t |w| < \bar{w}$. \square

Remark. If the trajectories of the solution is desired to stay in \bar{y} , then $\bar{u} = \gamma_{G,u}^{-1} (1 - \gamma_{G,v}\gamma_\varphi) \bar{w}$ is the maximum input perturbation allowed to ensure the stability of the system.

To find the maximum gain, the following optimization problem needs to be solved

$$\max_{\bar{w}} \gamma_{G,u}^{-1} (1 - \gamma_{G,v}\gamma_\varphi(\bar{w})) \bar{w} \quad (8)$$

where $\gamma_\varphi(\bar{w})$ is a monotonically increasing function with respect to \bar{w} . There are two competing effects which are

1. As \bar{w} increases, γ_φ increases. This increases the gain coming from the nonlinearities, so it reduces the certifiable range \bar{u} .
2. If \bar{w} is too small, \bar{u} is proportionally reduced according to Equation 4.

III. SECTOR BOUND

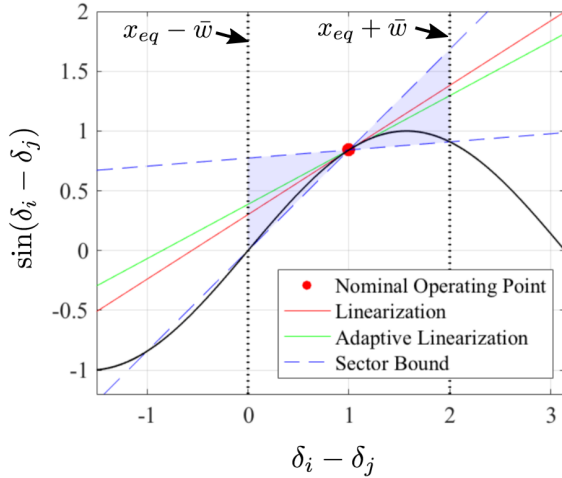


Fig. 2. Sector bound for sin function.

This observation is shown in the optimization problem in (8) as a multiplication of monotonically increasing function \bar{w} and monotonically decreasing function $(1 - \gamma_{G,v}\gamma_\varphi(\bar{w}))$.

Conjecture: With some Lipschitz condition on γ_φ or its derivative, the objective function in optimization problem 8 is a quasi-concave or even concave function of \bar{w}

Lemma III.1. If $g(x)$ is monotonically decreasing function with Lipschitz constant of is 1, then $xg(x)$ is a quasi-concave function.

Proof.

$$\frac{d}{dx} xg(x) = g(x) + x \frac{dg}{dx}$$

\square

Proposed solution for solving optimization problem in 8.

1. Heuristic solution with gradient descent with momentum.
2. If the problem is quasi-concave function, we could solve for the gradient of the problem becoming zero since the problem is unconstrained. If the Lipschitz constant of the derivative of the objective function is less than 1, then the fixed point iteration can be applied.

A. Adaptive Linearization

In this section, the linearization at the nominal operating point is modified to consider the bound on the uncertainties. The adaptive linearization results in less conservative bound on the input perturbation.

IV. CASE STUDIES

A. Single Machine Example

$$m\ddot{\delta} + d\dot{\delta} + a \sin \delta = P \quad (9)$$

Suppose the nominal operating condition is δ_0 and P_0 . By substituting $y = \delta - \delta_0$, $w = E(\delta - \delta_0)$, $u = P - P_0$, and $v = \sin(\delta) - \cos(\delta_0)y$,

$$m\ddot{y} + d\dot{y} + b(v + \cos(\delta_0)y) = u \quad (10)$$

In frequency domain,

$$\begin{aligned}
Y &= \frac{1}{ms^2 + ds + a \cos(\delta_0)} U - \frac{b}{ms^2 + ds + a \cos(\delta_0)} V \\
&= G_u U + G_v V
\end{aligned} \quad (11)$$