The DFT is defined as:

Find a direct expression for the DFT of a rectangular window. Due to the finite size of the DFT, this can be approximated but is not equal to the sinc(x) function.

Truncated to size M, centered:

If M is odd the rectangular window is symmetric around x=0, so the complex phase reduces to zero:

Note that this is an even function of n.

Try direct convolution of Lorentzian in FT space:

For n>k:

It worked! Encouraged by this, let’s just try the Voigt function in its entirety:

Start with Lorentzian:

Understanding why the infinite sum is equal to the hypergeometric function:

For

Now sum with a lower limit of m:

Finally:

We could also look directly at the definition of the hypergeometric function and plug in a non-zero starting index m:

The pochhammer symbol can be written as:

Continue with the definite sum:

This ultimately leads to the same result.

The following we can’t use because it’s -n (not n):

The sum:

Compare it to integral:

But this function may not have real(a)>0, so we rather use the following: