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A Procedure for Highly Reproducible Measurements of ADC Spectral Parameters

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Abstract *– The evaluation of spectral parameters characterizing analog–to–digital converters (ADC) can be addressed by employing both parametric and non–parametric techniques. Whatever the adopted testing techniques, the IEEE standards 1057 and 1241, which list the most effective ADC testing procedure, recommend the use of coherent sampling. Such a condition can not be guaranteed a priori with respect to spurious tones eventually present in the ADC output spectrum and data windowing is usually employed to reduce these phenomena. However, standards do not provide clear criteria for choosing the window that assure the maximum parameter estimation accuracy. The European draft standard Dynad suggests the employment of one out of seven optimal windows, in accordance to the ADC resolution. However, each sequence is characterized by distinctive parameters. Thus test automation may be reduced. In this paper, the use of a class of windows that are defined by setting the mainlobe width value and that can be easily calculated by employing standard computational tools is proposed. Such sequences maximize also the estimation accuracy for any given ADC resolution, thus improving both measurement reproducibility and test automation. Experimental results that validate the effectiveness of the proposed window class are presented.*

Keywords *– Digitizer FFT test, FFT test procedure, DPSS windows*

I. INTRODUCTION

Methodologies usually employed for the parameter estimation of analog–to–digital converters (ADCs) are based on the analysis of the ADC numerical output when a single or dual tone is used as input signal [1], [2]. The observed ADC output data can be often modeled as a set of single–tone components embedded in white zero–mean Gaussian noise. Therefore, the problem of evaluating the ADC frequency–domain performance reduces to the estimation of multiple sine parameters and of the broad–band noise–level.

Such an estimation problem, widely investigated and detailed in the scientific literature, can be addressed by employing both parametric and non–parametric methodologies [3], [4]. Although parametric procedures present an high frequency selectivity and statistical efficiency, the determination of the model coefficients needed for estimating the signal parameters is often a difficult task and iterative procedures must be used. Conversely, non–parametric testing procedures, which are not affected by the model order issue, are characterized by slightly lower selectivity and statistical efficiency [3], [4]. It should be noticed, however, that such performances reduction can be compensated by increasing the observation interval length or the number of analyzed samples. The main advantages of such methods are the low computational effort and the robustness of the procedure. Thus, non–parametric based estimation procedures can be widely applied for waveform digitizer testing.

Non–parametric methodologies employed for the estimation of ADC spectral parameters are usually based on the Discrete Fourier Transform (DFT). Accordingly, the figures of merit of interest are evaluated from the ADC output spectrum. Whatever the adopted testing methodology, the IEEE standards 1057 and 1241 [1], [2] recommend the use of coherent sampling in order to guarantee maximum estimation accuracy. However, such a condition can not be guaranteed a priori with respect to spurious tones eventually present in the output spectrum. In such a situation, spectral granularity and leakage may affect the accuracy with which parameters are estimated. Output–data windowing has been classically employed to reduce the effect of such phenomena. However, at this regard, [1] and [2] do not provide clear criteria for the choice of the most appropriate window. Some suggestions are given in the draft standard Dynad [5], where it is recommended the selection of one out of seven optimal windows, in accordance to the resolution of the tested ADC. Nevertheless, criteria to be followed for choosing the most effective window are not explicitly given. Moreover, the proposed windows attain to different classes, each characterized by several distinctive parameters. As a consequence, test automation may be reduced.

Previous works on this subject have shown that the employment of the zero–order Discrete Prolate Spheroidal Sequences (DPSS) maximizes the measurement accuracy when non–coherent sampling applies [3], [7]. Windows belonging to this class of sequences are completely defined by the mainlobe width value and can be used for testing any given resolution ADC. Thus, testing automation procedure can be controlled easily without reducing estimation accuracy. However, samples of the DPSS windows can be calculated only on the basis of numerical algorithms. As a consequence, different implementation applied to the same set of data can result in different estimates of ADC figures of merit, and measurement reproducibility may be reduced.

In this paper, the use of a class of windows that can easily be calculated by using standard computational tools and that well approximate the DPSS sequences, is proposed for ADC testing under non–coherent conditions. In particular, such windowing sequences have been employed for estimating spectral parameters of a generic b–bit ADC by using a DFT–based method.

In order to improve measurements reproducibility, criteria for selecting the window mainlobe width that maximizes the parameter estimation accuracy are given. This is an important feature, also because free software exists for the application of standard testing techniques [8]. Thus, the availability of a criterion for choosing the optimal window may automate the window selection process.

In the following, the DFT-based testing method is briefly described and the estimators of the main ADC spectral figures of merit are proposed. The algorithm is then step–by–step detailed for estimating spectral parameters of a generic b–bit ADC. Finally, the procedure has been applied to a 16–bit acquisition board and the theoretical and experimental standard deviations have been compared in order to validate the proposed estimation process.

II. DFT–BASED METHOD

The spectral figures of merit usually employed for characterizing ADCs in the frequency domain are the spurious–free dynamic range ($SFDR$), the signal–to–noise–and–distortion–ratio ($SINAD$), the signal–to–random–noise–ratio ($SRNR$) and the total harmonic distortion (THD). They require the power estimation of the wide–band noise σ_R^2 and of L narrow–band components, $\sigma_{X_i}^2$, $i = 1, ..., L$, which are composed by the fundamental tone, H harmonics and S spurious, i.e. $L = H + S + 1$.

When non–coherent sampling applies, the DFT–based method evaluates the power of the narrow– and wide–band components from the windowed ADC output spectrum obtained by applying the FFT algorithm. To this aim, as described in detail in [7], [9] and [10], the output spectrum is divided in sets of frequency bins, each associated with one kind of spectral component, indicated with $\mathcal{B}_{X_i}$ for the L narrow-band components and with $\mathcal{B}_{\mathcal{R}}$ for the wide-band noise.

Expressions for the power estimators of the wide–band noise, $\widehat{\sigma}_R^2$, and of the i–th narrow–band component, $\widehat{\sigma}_{X_i}^2$, are:

$$\widehat{\sigma}_R^2 \triangleq \frac{1}{N_R N^2} \frac{1}{NNPG} \sum_{k \in \mathcal{B}_{\mathcal{R}}} |Y[k]|^2, \qquad (1)$$

$$\widehat{\sigma}_{X_i}^2 \triangleq \frac{2}{N^2} \frac{1}{NNPG} \sum_{k \in \mathcal{B}_{X_i}} |Y[k]|^2 - 2\frac{N_{X_i}}{N} \widehat{\sigma}_R^2. \qquad (2)$$

In (1) and (2) N is the number of acquired samples, $Y[\cdot]$ is the DFT of the windowed acquired data samples, N_{X_i} is the number of samples in $\mathcal{B}_{X_i}$ and N_R is the number of samples in $\mathcal{B}_{\mathcal{R}}$. Moreover, $NNPG \triangleq \frac{1}{N}\sum_{n=0}^{N-1} w^2[n]$ is the window Normalized Noise Power Gain.

By defining $\widehat{\sigma}_{X_1}^2$, $\widehat{\sigma}_H^2 \triangleq \sum_{i=2}^{H+1} \widehat{\sigma}_{X_i}^2$ and $\widehat{\sigma}_S^2 \triangleq \sum_{i=H+2}^{S+1} \widehat{\sigma}_{X_i}^2$ as the estimators of the power of the fundamental, of the harmonics and of the spurious components, respectively, the spectral parameters estimators are [9]:

$$\widehat{SRNR} \triangleq \frac{N_R}{N_R + ENBW_0} \frac{\widehat{\sigma}_{X_1}^2}{\widehat{\sigma}_R^2}, \qquad (3)$$

$$\widehat{SINAD} \triangleq \frac{\widehat{\sigma}_{X_1}^2}{\widehat{\sigma}_R^2 + \widehat{\sigma}_H^2 + \widehat{\sigma}_S^2}, \qquad (4)$$

$$\widehat{SFDR} \triangleq \frac{\widehat{\sigma}_{X_1}^2}{\max_{i>1} \widehat{\sigma}_{X_i}^2}, \qquad (5)$$

$$\widehat{THD} \triangleq \frac{\widehat{\sigma}_H^2}{\widehat{\sigma}_{X_1}^2}, \qquad (6)$$

where $ENBW_0 \triangleq N\sum_{n=0}^{N-1} w^4[n] / (\sum_{n=0}^{N-1} w^2[n])^2$ represents the equivalent–noise bandwidth of the squared window, $w^2[\cdot]$.

The accuracy of each estimator can be separately optimized by suitably choosing test parameters and, in particular, by carefully selecting the window. Draft standards [5] suggests the employment of one out of seven optimal windows, in accordance with the tested ADC resolution, but the sequence selection criterion is not explicitly given. As a consequence, both test automation and measurement reproducibility may be reduced. Previous works on this subject have shown that the zero–order DPSS sequence can be employed to obtain maximum estimation accuracy [9]. In fact, it has been proved that the variance of (3) and (4) is almost entirely due to the variance of $\widehat{\sigma}_R^2$, while the contribution from the tone power estimator variance is negligible [9]. It follows that the window maximizing the accuracy of the broad–band noise estimator leads to the optimum estimation of the spectral figures of merit. Simulated data show that by using the same window, variances of estimators (5) and (6), are lower than variances of (3) and (4). As a consequence, $w[\cdot]$ has been selected on the basis of the statistical properties of $\widehat{\sigma}_R^2$ [3]. At this regard, it has been demonstrated that bias and variance of $\widehat{\sigma}_R^2$ are proportional to the spectral leakage and to the $ENBW_0$ of the employed window, respectively. Since different windows exhibit similar values of $ENBW_0$, the criterion has been followed of choosing the window that minimizes spectral leakage. Therefore, zero–order DPSS result as the optimum window sequences [3], [6]. However, the procedure for calculating their samples requires the application of iterating numerical algorithms.

The class of windows proposed in this paper is defined in the frequency domain on the basis of the Dirichelet kernel as follows [11]:

$$W[k, \Lambda] \triangleq \frac{\sin\left[\frac{N}{2}\cos^{-1}\left(\gamma\cos\left(2\pi k\right) + (\gamma - 1)\right)\right]}{\sin\left[\frac{1}{2}\cos^{-1}\left(\gamma\cos(2\pi k) + (\gamma - 1)\right)\right]}, \quad k = 0, ..., N-1 \qquad (7)$$

where $W[\cdot, \Lambda]$ represents the DFT of $w[\cdot]$, $\gamma \triangleq (1 + \cos(2\pi/N)) / (1 + \cos(2\Lambda\pi/N))$, and Λ is the window mainlobe width expressed in bins.

Such class of windows may easily be calculated from the knowledge of the record length N and of the needed mainlobe width Λ by using standard computational tools. Moreover, it has been demonstrated that it well approximates the zero–order DPSS sequences [11]. It follows that the window samples obtained by applying an inverse FFT (IFFT) algorithm to (7) assure, maximum estimators accuracy, measurement reproducibility improvement and an easy development of the test automation procedure.

In the next section, the DFT–based testing procedure is step-by-step described in order to characterize ADCs with any given resolution, by optimizing the available test–bench resources. Moreover, a criterion for choosing the mainlobe window which guarantees the maximum estimation accuracy is explicitly given.

III. THE PROPOSED PROCEDURE FOR ADC TESTING

Fig. 1 shows the steps to follow for estimating spectral parameters of a generic b-bit ADC by using the proposed method.

The *algorithm input parameters* are related to the characteristics of the measurement bench set–up and of the device under test. The procedure requires knowledge of the available memory–depth of the employed testing bench, N_{max}, the operating full scale range (FS) of the device under test, its sampling rate f_s and an a priori estimate of the expected $SRNR$ at the ADC output, γ_1. If γ_1 is not available, the expression of the $SRNR$ of an ideal b–bit converter can be employed.

The *input sinewave parameters* are then set. In particular, the amplitude A has to be chosen in order not to overload the ADC and a proper frequency f_{in} has to be selected.

The *data acquisition parameters* are then selected as follows: the number of acquired samples N_T, must satisfy the condition $N_T > \pi 2^b$ [1]. If $N_T > N_{max}$ results, the testing algorithm can be successively applied to R data records, each of length N, such that $RN \geq N_T$. Thus, the value of R can be calculated as $R = \lceil N_T/N \rceil$, where the operator $\lceil x \rceil$ rounds x to the closest upper integer. The parameters of interest can then be derived by averaging the R resulting estimates. It should be noticed that the employed number of samples is often set to a power of two in order to reduce the FFT execution time.

The optimum *window parameter*, Λ_{opt}, can then be calculated as:

$$\Lambda_{opt} = 0.607 + 0.189 \log_{10} N_R + 0.378 \log_{10} \gamma_1, \qquad (8)$$

where usually N_R can approximately be set equal to $N/3$ [9]. It is then possible to calculate the number of samples N_{X_i} associated to the estimate of each narrow–band component. The optimum value, obtained as a compromise between maximum estimator accuracy, maximum frequency selectivity and lower computational effort, is:

$$N_{X_i} = 2[\Lambda_{opt} + 1], \qquad (9)$$

where the operator $[x]$ rounds x to the nearest integer [3].

In order to estimate accurately the narrow–band spectral components, it should be verified that the distance between the two closest components is greater than $2\Lambda_{opt} + 1$ bins. The condition to verify is then [9]:

$$N > \frac{2\lceil \Lambda_{opt} \rceil + 1}{\min_{i \neq j} |f_{X_i} - f_{X_j}|} f_s, \quad i,j = 1, ..., H + S + 1 \qquad (10)$$

where f_{X_i} represents the frequency of the i–th narrow–band component expressed in Hz.

It should be also verified that the total number of frequency bins associated to the L narrow–band components and to the wide–band noise do not exceed the number of available frequency bins, i.e. that

$$N_R \leq N - [2(H + S) + 1] N_{X_i} \qquad (11)$$

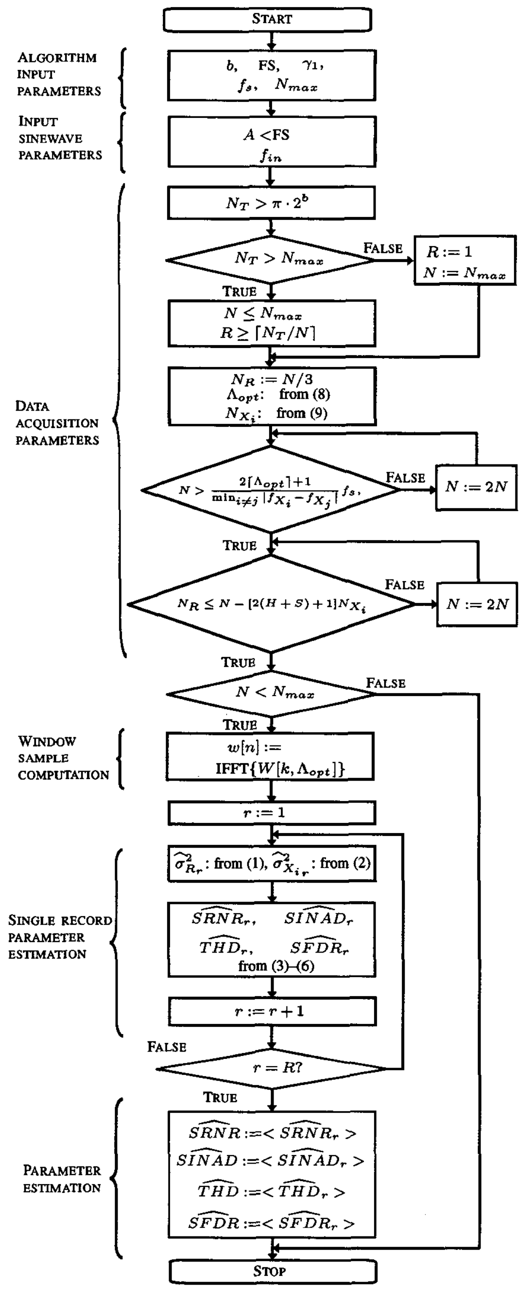

Figure 1. Flow chart of the energy–based DFT testing algorithm. N is set to a power of two in order to apply the FFT to the windowed acquired data.

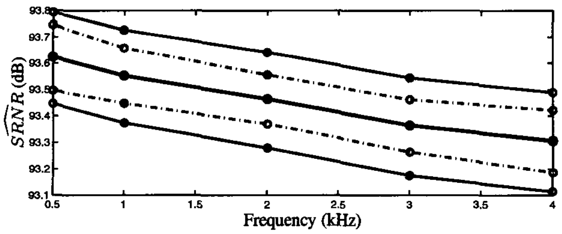

Figure 2. Bolded line represent the $\widehat{SRNR}$ of the AT–MIO–16–XE–50 data acquisition board estimated with the energy–based algorithm by employing parameters of Tab. I and the proposed window (7). Solid and dash–dotted lines represent the average plus and minus the theoretical and experimental standard deviations, respectively.

If (10) or (11) is not satisfied, N must be increased. Whenever the number of samples needed to perform the test is greater than the available memory–depth, i.e. $N > N_{max}$, the test can not be executed because of limits on hardware resources.

The window coefficients $w[\cdot]$ can then be calculated by substituting (8) in (7) and by applying the IFFT algorithm to the resulting expression. Once the DFT–based algorithm parameters have been set, it is possible to proceed with the estimation of the desired ADC spectral parameters based on the r-th data record. In particular, the powers of the wide– and narrow–band components, $\widehat{\sigma}^2_{R_r}$ and $\widehat{\sigma}^2_{X_{i_r}}$, are evaluated by using (1) and (2), respectively. Finally (3)–(6) can be applied to estimate the corresponding figures of merit. The overall ADC spectral performances can then be evaluated by taking the arithmetic average over R records, as indicated in Fig. 1 by the operator $< x_r > \stackrel{\triangle}{=} 1/R \sum_{r=1}^{R} x_r$.

IV. EXPERIMENTAL RESULTS

The algorithm described in section III has been applied to the 16–bit data acquisition board AT-MIO-16XE-50, developed by National Instruments, for evaluating the $SRNR$ at various input frequency values. Such a board has a sampling rate of 20 ksample/s and a full-scale range equal to ± 10 V. The signal generator used as input stimulus is the DS360 generator of the Stanford Research System, which exhibits an $SFDR$ larger than 96 dBc. The amplitude of the input sinewave has been set equal to 9.85 V, the value for which the maximum $\widehat{SRNR}$ has been experimentally obtained by employing a 1 kHz input sinewave. The corresponding γ_1 value is 97.96 dB. The energy-based algorithm has been applied for input frequencies equal to 0.5, 1, 2, 3 and 4 kHz, and the $SRNR$ has been estimated.

The algorithm parameters employed for the specific case are reported in Tab. I and the various $SRNR$ estimates have been graphed in Fig. 2. In particular, the bolded line represents the average based on $R = 100$ data records, and dash-dotted lines represent the average plus and minus the corresponding standard deviation.

In order to validate the proposed procedure, the value of the theoretical standard deviation of $\widehat{SRNR}$ have been calculated by means of [10]:

TABLE I
ALGORITHM PARAMETERS EMPLOYED FOR OBTAINING EXPERIMENTAL RESULTS OF FIG. 2

Algorithm input parameters	***b***	***FS***	f_s	
	16	10 V	20 ksample/s	
Input sinewave parameters	**A**	f_{in}	γ_1 ***ideal***	
	9.85 V	variable	97.96 dB	
Data acquisition parameters	N=N_{max}	N_T	N_R	R
	2^{14}	2^{18}	14448	100
Window parameters	Λ_{opt}	***NNPG***	$ENBW_0$	N_{X_1}
	5.02	0.23	3.14	11

$$\mathrm{std}\left\{\widehat{SRNR}\right\} = \sqrt{\frac{ENBW_0}{N_R}}\gamma_1. \qquad (12)$$

The average plus and minus such a theoretical standard deviation has been also plotted in Fig. 2 with solid lines. The good agreement between the experimental and the theoretical lines confirms the effectiveness of the testing procedure and of the proposed class of windows.

V. CONCLUSIONS

In this paper the DFT–based testing algorithm has been detailed from a procedural point of view to exemplify its application for the evaluation of ADC spectral parameters under non–coherent conditions.

It has been proposed the use of a class of windows that can be calculated by using standard computational tools and are completely characterized by the mainlobe width value. Thus, their use in testing algorithm increases measurements reproducibility. Moreover, in order to improve the test automation, it has been described the criterion of choosing the window mainlobe width value that maximizes the spectral estimator accuracy for any given ADC resolution and for any number of acquired samples. The proposed procedure has been applied to a 16–bit data–acquisition board and experimental results, which validate the testing method, have been presented.

References

[1] *Standard for Digitizing Waveform Recorders*, IEEE Std. 1057, Dec. 1994.

[2] *Standard for Terminology and Test Methods for Analog-to-Digital Converters*, IEEE Std 1241, Oct. 2000.

[3] P.Carbone, E.Nunzi, D.Petri, "Windows for ADC Dynamic Testing via Frequency–Domain Analysis," *IEEE Trans. Instrum. and Meas. Tech.*, pp. 1679–1683, Dec. 2001.

[4] S.L.Marple,Jr., "Digital Spectral Analysis with Applications", Prentice-Hall, Englewood Cliffs, N.J., 1987.

[5] European Project DYNAD, "Methods and Draft Standards for the DYNamic Characterisation and Testing of Analogue to Digital Converters," published in internet at *http://www.fe.up.pt/~hsm/dynad.*

[6] D. Slepian, "Prolate Spheroidal Wave Functions, Fourier Analysis and Uncertainty–V: The Discrete Case," *Bell Syst. Tech. Journ.*, vol. 57, pp. 1371–1430, May–June 1978.

[7] P.Carbone, E.Nunzi, D.Petri, "Characterization of a Data Acquisition System in a Remotely Controlled Environment," *Proc. 11th IMEKO TC–4, 6th EuroWorkshop on ADC modelling and testing*, Lisbon, Portugal, pp. 66–70, 13–14 Sept. 2001.

[8] I.Kollár, "Evaluation of Sinewave Tests of ADC's from Windowed Data," Proc. *4–th Intern. Workshop ADC Modelling and Testing*, pp. 64–68, Bordeaux, France, Sept. 9–10, 1999.
[9] P.Carbone, D.Petri, "Effective Frequency-Domain ADC Testing," *IEEE Trans. on Circ. and Sys.–II: Analog and Digital Signal Processing*, vol. 47, no. 7, pp. 660–663, July 2000.
[10] D.Petri, "Frequency–Domain Testing of Waveform Digitizers," accepted for publication on *IEEE Trans. Instrum. and Meas. Tech.*, June 2002.
[11] T. Saramaki, "Adjustable Windows for the Design of FIR Filter–A Tutorial," *Proc. 6th Mediterranean Electrotechnical Conference,*, vol.1, pp. 28–33, 1991.