Project: "Differential estimators distance transformation"

Introduction

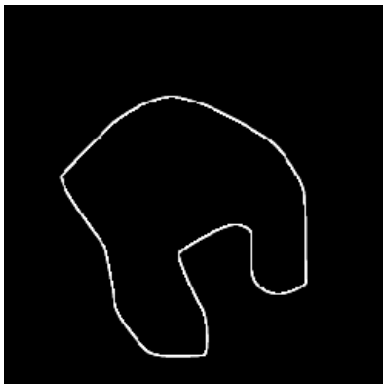
The objective of this project is to estimate differential estimators from an implicit representation of a digital surface.

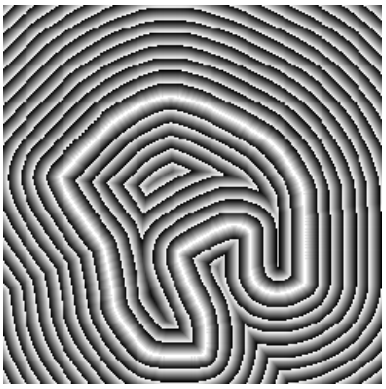
We expect from you:

- A short report with answers to the "formal" questions and a description of the your implementation choices and results.
- A C++ project (`CMakeLists.txt` plus several **commented** cpp program files).

1 Project Description

The idea is to model a digital surface as the zero-crossing of an implicit function and to estimate differential quantities from this implicit parametrization.





Question 1 *Implement a function that digitizes a mathematical shape (see for instance the parameteric shapes described in the MicroTutorial project) on a digital grid with step h. Extract the contour of the digital shape (denoted ∂X in the following).*

Question 2 *Using a distance transformation on the shape and its complement, construct an implicit function $f(x, y)$ such that:*

- $f(x, y) = 0$ *if* $(x, y) \in \partial X$*;*
- $f(x, y) < 0$ *(resp.* $f(x, y) > 0$*) if* $(x, y) \in X$ *(resp.* $(x, y) \in \bar{X}$*) and* $f(x, y)$ *is the distance between* (x, y) *and* ∂X*.*

Such function is called a *signed distance transformation* of ∂X.

Question 3 *From this parametrization, implement differential estimators such as normal vector field and curvature on boundary points. For example, you could consider finite difference analysis of the implicit function. As you would see, the estimated quantities could become noisy if computations are performed too close to ∂X. Propose and evaluate a method to smooth the computations (*e.g. *for instance using Gaussian convolution with a given parameter σ or using the Voronoi diagram –see* `DGtal::VoronoiMap`*– to propagate quantities inside the object).*

2 Multigrid Analysis

Question 4 *Perform a complete multigrid convergence evaluation with comparison to both expected quantities (available in* DGtal *for implicit shapes, cf documentation) and estimated ones from other estimators (*e.g. *estimators based on maximal DSS computations, cf documentation).*

Question 5 *Please discuss about the quality of implicit parametrization based estimators (speed, quality...).*

3 Extensions

Question 6 *Implement the same estimation on volumetric objects (*i.e. *in* $\mathbb{Z}^3$*). For comparison, please consider normal vector and curvature estimators on digital surface implemented in* DGtal.