# Project: "Volumetric analysis"

## Introduction

The objective of this project is to implement and evaluate some volumetric analysis tools of digital objects.

We expect from you:

- A short report with answers to the "formal" questions and a description of your implementation choices and results.

- A C++ project (`CMakeLists.txt` plus several **commented** cpp program files).

## 1    Step 1: Distance Transformation

Let us consider a 2D binary object $X$ (for instance, a parametric or implicit shape from DGTAL digitized at a given resolution $h$).

**Question 1** *Using `DistanceTransformation` and `VoronoiMap` classes available in the library, implement tool that compute both distance transformation and Voronoi map of $X$.*

## 2    Step 2: Discrete $\lambda$-medial Axis

In computational geometry, the $\lambda$-medial axis is an approximation of the medial axis of a continuous shape with both geometric and topological guarantees [CL05]. Its definition can be described as follows: given a point $x$ in the plane, let $\delta$ be the closest distance between $x$ and $\partial X$ (the boundary of $X$). Let $S_x = \{s_i\}$ be the set of points in $\partial X$ such that

$$d(x, s_i) = \delta \tag{1}$$

If $S$ contains more than one point, $x$ belongs to the continuous medial axis of $X$. Now, a point $x$ belongs to the $\lambda-$medial axis of $X$ if and only if the radius of the *minimum enclosing circle* of $S_x$ is greater than $\lambda$.

In digital geometry, we will use the following approximation

1. At a point $p \in X$, we collect closest background points $S_p$ from the VoronoiMap in a 3x3 window around $p$. Points in this set may not be exactly equidistant to $p$, this is where the approximation comes from.

2. We compute the minimum enclosing circle of $S_p$ and $p$ belongs to the discrete $\lambda-$medial axis if the radius is greater than a given paramter $\lambda$

**Question 2** *Implement a function that computes the minimum enclosing circle of a point set (see Appendix).*

**Question 3** *Implement a function that computes the $\lambda-$medial axis of $X$. Evaluate the quality of the medial representation according to paramter $\lambda$ and gridstep $h$.*

# 3 Step 3: Thickness function

Thickness function is an important tool for the analysis of porous material. The function can be defined as follows for $p \in X$:

$$\tau(p) = \max\{r \mid \forall B(c,r) \subset X, p \in B(c,r)\} \tag{2}$$

In other words, the thickness at $p$ is the radius of the largest circle containing $p$ inside the shape $X$[1]. Instead of checking all balls contained in $X$, it is sufficient to only consider balls from the medial axis of $X$.

**Question 4** *Implement a function that computes the thickness function of a shape $X$. First start by a $\lambda-$medial axis approximation and use the obtained medial balls to compute the thickness for all points $p \in X$.*

**Question 5** *Export the thickness function as a graph with the abscissa is the thickness value and the ordinate, the number of pixels with thickness less than $x$. How can this spectra characterize the geometry of the shape. Can you guess what this has been used on porous material analysis ?*

# A   Minimum Enclosing Circle

The following section describes a randomized algorithm for the minimum enclosing circle problem whose expected time is linear.

# References

[CL05] F. Chazal and A. Lieutier. The $\lambda$-medial axis. *Graphical Models*, 67(4):304–331, July 2005.

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[1]$B(c,r)$ is the Euclidean ball with center $c$ and radius $r$.