Project: “Volumetric analysis”

Introduction

The objective of this project is to implement and evaluate some volumetric analysis tools of digital objects.

We expect from you:

- A short report with answers to the ”formal” questions and a description of your implementation choices and results.
- A C++ project (`CMakeLists.txt` plus several **commented** cpp program files).

1 Step 1: Distance Transformation

Let us consider a 2D binary object X (for instance, a parametric or implicit shape from DGtal digitized at a given resolution h).

Question 1 *Using `DistanceTransformation` and `VoronoiMap` classes available in the library, implement tools that compute both distance transformation and Voronoi map of X.*

2 Step 2: Discrete λ-medial Axis

In computational geometry, the λ-medial axis is an approximation of the medial axis of a continuous shape with both geometric and topological guarantees [CL05]. Its definition can be described as follows: given a point x in the plane, let δ be the closest distance between x and ∂X (the boundary of X). Let $S_x = \{s_i\}$ be the set of points in ∂X such that

$$d(x, s_i) = \delta \tag{1}$$

If S contains more than one point, x belongs to the continuous medial axis of X. Now, a point x belongs to the $\lambda-$medial axis of X if and only if the radius of the *minimum enclosing disc* of S_x is greater than λ.

In digital geometry, we will use the following approximation

1. At a point $p \in X$, we collect closest background points S_p from the VoronoiMap in a 3x3 window around p. Points in this set may not be exactly equidistant to p, this is where the approximation comes from.
2. We compute the minimum enclosing disc of S_p and p belongs to the discrete $\lambda-$medial axis if the radius is greater than a given parameter λ

Question 2 *Implement a function that computes the minimum enclosing disc of a point set (see Appendix).*

Question 3 *Implement a function that computes the $\lambda-$medial axis of X. Evaluate the quality of the medial representation according to parameter λ and gridstep h.*

3 Step 3: Thickness function

Thickness function is an important tool for the analysis of porous material. The function can be defined as follows for $p \in X$:

$$\tau(p) = \max\{r \mid \forall B(c,r) \subset X, p \in B(c,r)\} \tag{2}$$

In other words, the thickness at p is the radius of the largest disc containing p inside the shape X[1]. Instead of checking all balls contained in X, it is sufficient to only consider balls from the medial axis of X.

Question 4 *Implement a function that computes the thickness function of a shape X. First start by a $\lambda-$medial axis approximation and use the obtained medial balls to compute the thickness for all points $p \in X$.*

Question 5 *Export the thickness function as a graph such that the abscissa is the thickness value and the ordinate, the number of pixels with thickness less than x. How can this spectra characterize the geometry of the shape. Can you guess what this has been used on porous material analysis ?*

A Minimum Enclosing Disc

The following section describes a randomized algorithm for the minimum enclosing disc problem whose expected time is linear [Wel91]. Given a set of points P, the following recursive algorithm computes the minimum enclosing disc B containing all points P and the set R of points on B. The code is given for points in $\mathbb{R}^2$ and must be called with $R = \emptyset$.

References

[CL05] F. Chazal and A. Lieutier. The λ-medial axis. *Graphical Models*, 67(4):304–331, July 2005.

[Wel91] E Welzl. *Smallest enclosing disks (balls and ellipsoids)*. Springer, 1991.

[1] $B(c,r)$ is the Euclidean ball with center c and radius r.

Algorithm 1: minimumEnclosingDisc(P,R)

Input: $P \subset \mathbb{R}^2$, $R \subset \mathbb{R}^2$.

Result: the (closed) minimum enclosing disc B, updated sets P and R.

if $P = \emptyset$ **then**
- **if** $|R| < 2$ **then**
  - B is an empty disc; `//` $p \in B$ `will always be false,` $\forall p \in P$
  - **return** B;
- **else**
  - `//` R `has two or three points which uniquely determines` B`, the circumscribing disc of either a segment or a triangle`
  - Compute B;
  - **return** B;

else
- Choose $p \in P$ at random;
- $B =$ minimumEnclosingDisc($P \setminus \{p\}$, R);
- **if** $p \notin B$ **then**
  - $B =$ minimumEnclosingDisc($P \setminus \{p\}$, $R \cup \{p\}$);
- **return** B