

# Project: “Volumetric analysis”

## Introduction

The objective of this project is to implement and evaluate some volumetric analysis tools of digital objects.

We expect from you:

- A short report with answers to the “formal” questions and a description of your implementation choices and results.
- A C++ project (CMakeLists.txt plus several **commented** cpp program files).

## 1 Step 1: Distance Transformation

Let us consider a 2D binary object  $X$  (for instance, a parametric or implicit shape from DGTAL digitized at a given resolution  $h$ ).

**Question 1** *Using `DistanceTransformation` and `VoronoiMap` classes available in the library, implement tool that compute both distance transformation and Voronoi map of  $X$ .*

## 2 Step 2: Discrete $\lambda$ -medial Axis

In computational geometry, the  $\lambda$ -medial axis is an approximation of the medial axis of a continuous shape with both geometric and topological guarantees [CL05]. Its definition can be described as follows: given a point  $x$  in the plane, let  $\delta$  be the closest distance between  $x$  and  $\partial X$  (the boundary of  $X$ ). Let  $S_x = \{s_i\}$  be the set of points in  $\partial X$  such that

$$d(x, s_i) = \delta \tag{1}$$

If  $S$  contains more than one point,  $x$  belongs to the continuous medial axis of  $X$ . Now, a point  $x$  belongs to the  $\lambda$ -medial axis of  $X$  if and only if the radius of the *minimum enclosing circle* of  $S_x$  is greater than  $\lambda$ .

In digital geometry, we will use the following approximation

1. At a point  $p \in X$ , we collect closest background points  $S_p$  from the VoronoiMap in a 3x3 window around  $p$ . Points in this set may not be exactly equidistant to  $p$ , this is where the approximation comes from.
2. We compute the minimum enclosing circle of  $S_p$  and  $p$  belongs to the discrete  $\lambda$ -medial axis if the radius is greater than a given parameter  $\lambda$

**Question 2** *Implement a function that computes the minimum enclosing circle of a point set (see Appendix).*

**Question 3** *Implement a function that computes the  $\lambda$ -medial axis of  $X$ . Evaluate the quality of the medial representation according to parameter  $\lambda$  and gridstep  $h$ .*

### 3 Step 3: Thickness function

Thickness function is an important tool for the analysis of porous material. The function can be defined as follows for  $p \in X$ :

$$\tau(p) = \max\{r \mid \forall B(c, r) \subset X, p \in B(c, r)\} \quad (2)$$

In other words, the thickness at  $p$  is the radius of the largest circle containing  $p$  inside the shape  $X$ <sup>1</sup>. Instead of checking all balls contained in  $X$ , it is sufficient to only consider balls from the medial axis of  $X$ .

**Question 4** *Implement a function that computes the thickness function of a shape  $X$ . First start by a  $\lambda$ -medial axis approximation and use the obtained medial balls to compute the thickness for all points  $p \in X$ .*

**Question 5** *Export the thickness function as a graph with the abscissa is the thickness value and the ordinate, the number of pixels with thickness less than  $x$ . How can this spectra characterize the geometry of the shape. Can you guess what this has been used on porous material analysis ?*

## A Minimum Enclosing Circle

The following section describes a randomized algorithm for the minimum enclosing circle problem whose expected time is linear [Wel91]. Given a set of points  $P$ , the following recursive algorithm computes the minimum enclosing circle  $B$  containing all points  $P$  and the set  $R$  of points on  $B$ . The code is given for points in  $\mathbb{R}^d$  and must be calls with  $R = \emptyset$ .

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**Algorithm 1:** minimumEnclosingCircle( $P, R$ )

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**Input:**  $P \subset \mathbb{R}^d, R \subset \mathbb{R}^d$   
**Result:** the minimum enclosing circle  $B$

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1 if  $P = \emptyset$  or  $|R| = d + 1$  then
2   | compute  $B$  which is uniquely defined by points in  $R$ ;
3   | return  $B$ ;
4 else
5   | Choose  $p \in P$  at random;
6   |  $B = \text{minimumEnclosingCircle}(P \setminus \{p\}, R)$ ;
7   | if  $p \notin B$  then
8   |   |  $B = \text{minimumEnclosingCircle}(P \setminus \{p\}, R \cup \{p\})$ ;
9 return  $B$ 
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## References

- [CL05] F. Chazal and A. Lieutier. The  $\lambda$ -medial axis. *Graphical Models*, 67(4):304–331, July 2005.
- [Wel91] E Welzl. *Smallest enclosing disks (balls and ellipsoids)*. Springer, 1991.

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<sup>1</sup> $B(c, r)$  is the Euclidean ball with center  $c$  and radius  $r$ .