

Project: “Volumetric analysis”

Introduction

The objective of this project is to implement and evaluate some volumetric analysis tools of digital objects.

We expect from you:

- A short report with answers to the “formal” questions and a description of your implementation choices and results.
- A C++ project (CMakeLists.txt plus several **commented** cpp program files).

1 Step 1: Distance Transformation

Let us consider a 2D binary object X (for instance, a parametric or implicit shape from DGTAL digitized at a given resolution h).

Question 1 *Using `DistanceTransformation` and `VoronoiMap` classes available in the library, implement tools that compute both distance transformation and Voronoi map of X .*

2 Step 2: Discrete λ -medial Axis

In computational geometry, the λ -medial axis is an approximation of the medial axis of a continuous shape with both geometric and topological guarantees [CL05]. Its definition can be described as follows: given a point x in the plane, let δ be the closest distance between x and ∂X (the boundary of X). Let $S_x = \{s_i\}$ be the set of points in ∂X such that

$$d(x, s_i) = \delta \tag{1}$$

If S contains more than one point, x belongs to the continuous medial axis of X . Now, a point x belongs to the λ -medial axis of X if and only if the radius of the *minimum enclosing disc* of S_x is greater than λ .

In digital geometry, we will use the following approximation

1. At a point $p \in X$, we collect closest background points S_p from the VoronoiMap in a 3x3 window around p . Points in this set may not be exactly equidistant to p , this is where the approximation comes from.
2. We compute the minimum enclosing disc of S_p and p belongs to the discrete λ -medial axis if the radius is greater than a given parameter λ

Question 2 *Implement a function that computes the minimum enclosing disc of a point set (see Appendix).*

Question 3 *Implement a function that computes the λ -medial axis of X . Evaluate the quality of the medial representation according to parameter λ and gridstep h .*

3 Step 3: Thickness function

Thickness function is an important tool for the analysis of porous material. The function can be defined as follows for $p \in X$:

$$\tau(p) = \max\{r \mid \forall B(c, r) \subset X, p \in B(c, r)\} \quad (2)$$

In other words, the thickness at p is the radius of the largest disc containing p inside the shape X ¹. Instead of checking all balls contained in X , it is sufficient to only consider balls from the medial axis of X .

Question 4 *Implement a function that computes the thickness function of a shape X . First start by a λ -medial axis approximation and use the obtained medial balls to compute the thickness for all points $p \in X$.*

Question 5 *Export the thickness function as a graph such that the abscissa is the thickness value and the ordinate, the number of pixels with thickness less than x . How can this spectra characterize the geometry of the shape. Can you guess what this has been used on porous material analysis ?*

A Minimum Enclosing Disc

The following section describes a randomized algorithm for the minimum enclosing disc problem whose expected time is linear [Wel91]. Given a set of points P , the following recursive algorithm computes the minimum enclosing disc B containing all points P and the set R of points on B . The code is given for points in \mathbb{R}^2 and must be called with $R = \emptyset$.

References

- [CL05] F. Chazal and A. Lieutier. The λ -medial axis. *Graphical Models*, 67(4):304–331, July 2005.
- [Wel91] E. Welzl. *Smallest enclosing disks (balls and ellipsoids)*. Springer, 1991.

¹ $B(c, r)$ is the Euclidean ball with center c and radius r .

Algorithm 1: minimumEnclosingDisc(P, R)

Input: $P \subset \mathbb{R}^2$, $R \subset \mathbb{R}^2$.

Result: the (closed) minimum enclosing disc B , updated sets P and R .

```
1 if  $P = \emptyset$  then
2   if  $|R| < 2$  then
3      $B$  is an empty disc;           //  $p \in B$  will always be false,  $\forall p \in P$ 
4     return  $B$ ;
5   else
6     //  $R$  has two or three points which uniquely determines  $B$ , the
        circumscribing disc of either a segment or a triangle
7     Compute  $B$ ;
        return  $B$ ;
8 else
9   Choose  $p \in P$  at random;
10   $B = \text{minimumEnclosingDisc}(P \setminus \{p\}, R)$ ;
11  if  $p \notin B$  then
12     $B = \text{minimumEnclosingDisc}(P \setminus \{p\}, R \cup \{p\})$ ;
13  return  $B$ 
```
