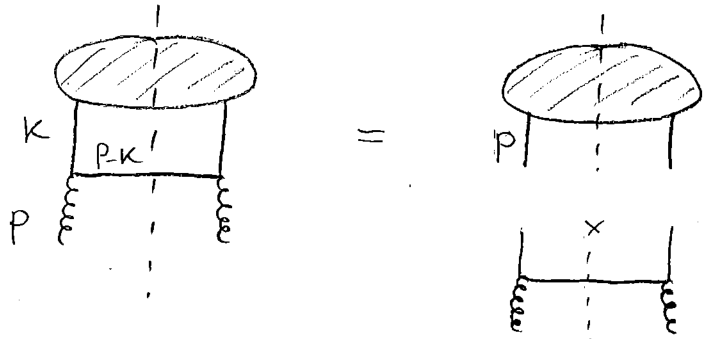
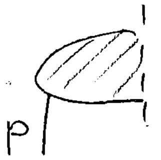
# CALCOLO DI $P_{qg}$

$n^2=0 \qquad K = zp - \bar{z}n + K_T$

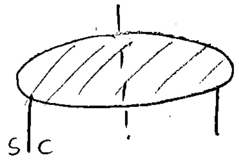
se $p^2=0, \quad K \mapsto zp$



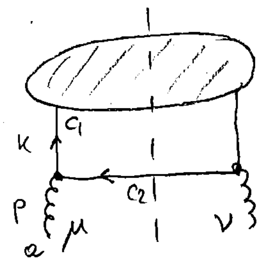
$$\longrightarrow \frac{\alpha_s}{2\pi} P_{qg}(z)\, dz \frac{dK_T^2}{K_T^2}$$



$$= M u_s(p)$$



$$= \frac{1}{N_c} \sum_{\substack{spin\\col}} M_c u_s(p) \left[M_c u_s(p)\right]^* = \mathrm{Tr}\left(\hat{p} M^+ M\right)$$



$$\to \frac{1}{N_c^2-1} \sum_{\substack{pol.\\spin\\colori\ c_1,c_2,a}} \left(M_{c_1} \frac{i\hat{k}}{K^2} \left(-ig\gamma^\mu T^a_{c_1c_2} v_s(p-k)\, \epsilon^{(\lambda)}_\mu(p)\right)\right)\left(\qquad\right)^*$$

$$= \frac{1}{N_c^2-1} \mathrm{tr}(T^aT^a)_{c_1c_1'} \mathrm{Tr}\left[\hat{k}\gamma^\mu(\hat{p}-\hat{k})\gamma^\nu\hat{k}\, M^+_{c_1'}M_{c_1}\right] \frac{g^2}{(K^2)^2} \underbrace{\sum_\lambda \epsilon^{(\lambda)}_\mu(p)\, \epsilon^{*(\lambda)}_\nu(p)}_{d_{\mu\nu}(p)}$$

dovremo tenere i termini $\propto K^2$ a numeratore

In gauge fisica, p. es. $n_\mu A^\mu = 0 \;:\; n^2=0$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{n\cdot p} \quad ; \quad d^\mu{}_\mu(p) = -2 \quad ; \quad p^\mu d_{\mu\nu}(p) = 0$$

Inoltre c'è da integrare nello spazio delle fasi

euclideo

$$\widetilde{d(p-k)} = (2\pi)^{-3} d^4k\, \delta(p-k)^2 = (2\pi)^{-3}\left[\, p\cdot n\, dz\, d\bar{z}\, d^2k_T\right]\delta\left(2p\cdot n(1-z)\bar{z} - k_T^2\right)$$

$$= \frac{dz\, dk_T^2\, d\varphi}{(2\pi)^3\, 2\cdot 2\,(1-z)} = \frac{dz\, dk_T^2}{(4\pi)^2(1-z)} \qquad \text{integrando in } d\varphi \to 2\pi$$

Esercizio: $K^2 = -2p\cdot n\, z\bar{z} - K_T^2 = -\dfrac{K_T^2}{1-z}$

Valutiamo il prodotto di matrici di Dirac

$\hat{K}\gamma^\mu(\hat{P}-\hat{K})\gamma^\nu\hat{K}\; d_{\mu\nu}(p)$

- $\underbrace{\hat{K}\gamma^\mu\hat{P}}_{a.c.}\gamma^\nu\hat{K}\, d_{\mu\nu} = \Big(\hat{K}\,\underbrace{2p^\mu}_{\hookrightarrow 0}\gamma^\nu\hat{K} - \hat{K}\hat{P}\underbrace{\gamma^{(\mu}\gamma^{\nu)}}_{g^{\mu\nu}}\hat{K}\Big)d_{\mu\nu}$

per scelta gauge fisica — perchè $d_{\mu\nu}$ è simm.

$$= -g^{\mu\nu}d_{\mu\nu}\,\underbrace{\hat{K}\hat{P}\hat{K}}_{a.c.} = 2\left(2p\cdot K\,\hat{K} - \hat{P}K^2\right) = 2K^2(\hat{K}-\hat{P})$$

$\hookrightarrow 0 = (p-K)^2 = -2pK + K^2 \Rightarrow 2p\cdot K = K^2$

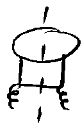
$$= 2\frac{K_T^2}{1-z}(\hat{P}-\hat{K}) = 2\frac{K_T^2}{1-z}\hat{P}\left(1-z+O(K_T^2)\right) = 2K_T^2\,\hat{P} =$$

- $\underbrace{\hat{K}\gamma^\mu\hat{K}}_{a.c.}\gamma^\nu\hat{K}\, d_{\mu\nu} = \Big(2K^\mu\underbrace{\hat{K}\gamma^\nu}_{a.c.}\hat{K} - K^2\underbrace{\gamma^{(\mu}\gamma^{\nu)}}_{g^{\mu\nu}}\hat{K}\Big)d_{\mu\nu}$

$$= \left(4K^\mu K^\nu\hat{K} - 2K^2K^\mu\gamma^\nu - g^{\mu\nu}K^2\hat{K}\right)d_{\mu\nu}$$

- □ $K^\mu d_{\mu\nu}K^\nu\,\hat{K} = K_T^\mu d_{\mu\nu}K_T^\nu\,\hat{K} = K_T^2\,\hat{K} \simeq -z(1-z)K^2\hat{P}$
- □ $-g^{\mu\nu}K^2\hat{K}\,d_{\mu\nu} = 2K^2\hat{K} \simeq 2K^2 z\hat{P}$
- □ $K^2K^\mu\gamma^\nu d_{\mu\nu} \simeq K^2 z\,p^\mu\gamma^\nu d_{\mu\nu} = 0$

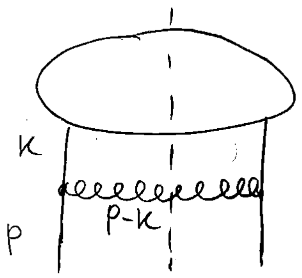
$$\Rightarrow \hat{K}\gamma^\mu(\hat{P}-\hat{K})\gamma^\nu\hat{K}\,d_{\mu\nu} = -2K^2\left[1-2z+2z^2\right]$$

Otteniamo quindi che  =  per la quantità

$$\underbrace{\frac{1}{N_c^2-1}\mathrm{tr}(T^aT^a)_{cc'}}_{T_R\,\delta_{cc'}/N_c \;\rightsquigarrow\; \frac{1}{N_c}M_c^\dagger M_c}\left[-2K^2(1-2z+2z^2)\right]\frac{g^2}{(K^2)^2}\,\frac{dz\,dK_T^2}{(4\pi)^2(1-z)}$$

$$= \frac{\alpha_s}{2\pi}\underbrace{T_R\left[z^2+(1-z)^2\right]}_{P_{qg}(z)}\,dz\,\frac{dK_T^2}{K_T^2}$$

# CALCOLO DI $P_{qq}$

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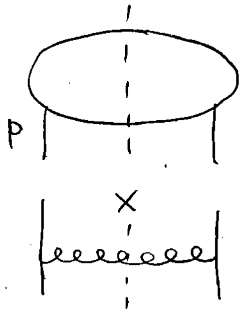
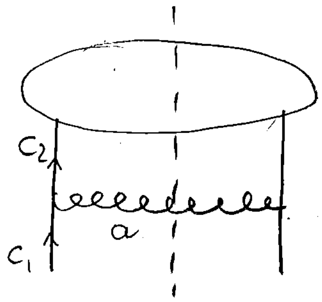



$\rightarrow \frac{1}{N_c}\mathrm{Tr}[\hat p M^\dagger M]$

$\rightarrow \frac{\alpha_s}{2\pi}P_{qq}(z)\,dz\,\frac{dk_T^2}{k_T^2}$

Con le premesse di $P_{gq}$ abbiamo




$$\frac{1}{N_c}\sum_{\substack{\text{pol}\\ \text{spin}\\ \text{colori } c_1c_2a}}\left(M\,\frac{i\hat k}{k^2}\,(-ig\gamma^\mu T^a_{c_2c_1})\,u_s(p)\,\epsilon^{(\lambda)}_\mu(p-k)\right)\left(\qquad\right)^*$$

$$= \underbrace{\frac{1}{N_c}\mathrm{tr}(T^aT^a)_{c_2c_2'}}_{C_F\frac{\delta_{c_2c_2'}}{N_c}}\,\mathrm{Tr}\left[\hat k\gamma^\mu\hat p\gamma^\nu\hat k\,M^\dagger_{c_2'}M_{c_2}\right]\frac{g^2}{(k^2)^2}\underbrace{\sum_\lambda \epsilon^{(\lambda)}_\mu(p-k)\,\epsilon^{*(\lambda)}_\nu(p-k)}_{d_{\mu\nu}(p-k)}$$

Calcoliamo il prodotto di matrici di Dirac:

$$\hat k\underbrace{\gamma^\mu\hat p}_{\text{a.c.}}\gamma^\nu\hat k\,d_{\mu\nu}(p-k) = \left(\hat k\,2p^\mu\underbrace{\gamma^\nu\hat k}_{\text{a.c.}} - \underbrace{\hat k\hat p}_{\text{a.c.}}\underbrace{\gamma^{(\mu}\gamma^{\nu)}}_{g^{\mu\nu}}\hat k\right)d_{\mu\nu}$$

$$= 2p^\mu(2k^\nu\hat k - \gamma^\nu k^2)\,d_{\mu\nu}(p-k) - \underbrace{g^{\mu\nu}d_{\mu\nu}(p-k)}_{-2}\,(\underbrace{2p\cdot k}_{k^2}\hat k - k^2\hat p)$$

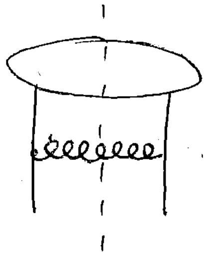
Siccome $d_{\mu\nu}(p-k)(p-k)^\nu = 0 \Rightarrow d_{\mu\nu}k^\nu = d_{\mu\nu}p^\nu$

$$= 4p^\mu p^\nu d_{\mu\nu}\hat k - 2k^2p^\mu\gamma^\nu d_{\mu\nu} + 2k^2(\hat k - \hat p)$$

- $p^\mu p^\nu d_{\mu\nu} = -p^2 + 2\frac{p\cdot(p-k)\,p\cdot n}{(p-k)\cdot n} = -2pk\frac{p\cdot n}{(p-k)\cdot n} \overset{+O(k^2)^2}{\simeq} -k^2\frac{1}{1-z}$
- $p^\mu\gamma^\nu d_{\mu\nu} = -\hat p + \frac{p\cdot(p-k)\hat n + (\hat p - \hat k)\,p\cdot n}{(p-k)\cdot n} = -\hat p + O(k^2) + \frac{(1-z)\hat p}{1-z} \simeq 0$

Otteniamo così

$$\hat{k}\gamma^{\mu}\hat{p}\gamma^{\nu}\hat{k}\, d_{\mu\nu} = -2k^2\,\frac{1+z^2}{1-z}\,\hat{p}$$

$\Rightarrow$ 

$$= \frac{C_F\,\delta_{c_2c_2'}}{N_c}\left(-2k^2\,\frac{1+z^2}{1-z}\right)\mathrm{Tr}\left[\hat{p}M^{\dagger}_{c_2'}M_{c_2}\right]\frac{g^2}{(k^2)^2}\,\frac{dz\,dk_T^2}{(4\pi)^2(1-z)}$$

$$= \frac{\alpha_s}{2\pi}\,C_F\,\frac{1+z^2}{1-z}\,dz\,\frac{dk_T^2}{k_T^2}\,\underbrace{\frac{1}{N_c}\mathrm{Tr}\left[\hat{p}M^{\dagger}M\right]}$$

$$\Rightarrow P_{qq}(z<1) = C_F\,\frac{1+z^2}{1-z}$$

I contributi virtuali con supporto a $z=1$ si possono valutare imponendo la conservazione del sapore: $\int_0^1 P_{qq}(z)dz = 0$

$$P_{qq}(z) = C_F\left\{\frac{1+z^2}{(1-z)_+} + A\,\delta(1-z)\right\} \qquad (*)$$

$$0 = \int_0^1\left[\frac{1+z^2-(1+z^2)|_{z=1}}{1-z} + A\,\delta(1-z)\right]dz$$

$$= \int_0^1\frac{(z+1)(z-1)}{1-z}\,dz + A = -\frac{3}{2} + A \Rightarrow A = \frac{3}{2}$$

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right] = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

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(*) Si definisce $\int_0^1 [f(z)]_+\, g(z)\,dz := \int_0^1 f(z)\,[g(z)-g(1)]\,dz$

$$\int_x^1 [f(z)]_+\, g(z)\,dz := \int_0^1 [f(z)]_+\, g(z)\,\Theta(z-x)\,dz = \int_0^1 f(z)\left[g(z)\,\Theta(z-x) - g(1)\right]d$$

$$= \int_x^1 f(z)\,[g(z)-g(1)]\,dz - \int_0^x f(z)\,g(1)\,dz$$