Structural Estimation of Habits in Consumption

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Abstract

I solve and estimate a consumption-savings model with rational expectations and habits formation in consumption. I show that the habits-in-consumption parameter is identified in a standard fixed effects regression, and compare the estimated coefficient with that resulting from a simulated method of moments.

Keywords: Habits in consumption, Structural Estimation.

1 Introduction

In a textbook consumption-savings problem, a rational agent with concave utility seeks to minimize the volatility of consumption over his lifetime. Under quadratic preferences, in particular, this finding is known as the "random-walk" hypothesis, and has long been studied and tested (Hall, 1978). A number of empirical studies documented two regularities that seem to contradict this simple result. First, consumption appears to be "excessively sensitive": factors other than past consumption tend to be important predictors for current consumption expenditures (Flavin, 1981). Second, consumption is "excessively smooth" meaning that changes in permanent income are not reflected in one-to-one changes in consumption as the model would imply (Campbell & Deaton, 1989).

Excess sensitivity of consumption has been largely abandoned since it is not observed in individual data (Baxter & Jermann, 1999). This seems not to be the case for excess smoothness. Individual-level data do in fact show excess smoothness, and researcher are still studying the possible underlying causes (Attanasio & Pavoni, 2011).

Habit formation in consumption was originally conceived to explain some surprising facts on asset prices, but came to be one of the most interesting explanations for excess smoothness (Abel, 1990; Fuhrer, 2000). Perhaps relevantly, habit formation in consumption was shown to be a real rigidity that can allow money to

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have real effects, despite absence of nominal rigidities in business cycle models, and generate "hump-shaped" impulse-response functions.

In this short essay, I write down a baseline consumption-savings model with habit in consumption in section 2. Then, I show that the parameter governing the inertia in consumption choices can be estimated in reduced form and provide the results of the estimation in section 3. Last, in section 4 I estimate the structural model and show that simulated method of moments yields an estimate of this parameter that is consistent with the regression estimate.

2 The Model

I study a partial equilibrium model in which a rational agent maximizes his intertemporal utility subject to a budget constraint. The agent seeks to solve

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} \quad & \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left(\tilde{c}_t\right)\right] \\ \text{s.t.} \quad & c_t + b_{t+1} = R_t b_t + w_t \\ & w_t = \rho w_{t-1} + \varepsilon_t \\ & \tilde{c}_t = c_t - \alpha c_{t-1} \end{aligned} \tag{1}$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ for all t, and $u(c) = c^{1-\sigma}/(1-\sigma)$. Then, c is consumption, b are nominal risk-less bonds, R is the gross real interest rate and w is an exogenous income process. The α parameter captures the importance of past consumption for current utility and qualitatively measures inertia in consumption decisions. It will be the main object of interest in this study.

The recursive formulation of the problem highlights the fact that habits in consumption inherently enlarge the state space of the problem. The Bellman equation associated to (1) is in fact

$$\begin{aligned} v\left(b, c_{-1}\right) = \max_{b'} \quad & u\left(\tilde{c}\right) + \beta \mathbb{E}[v(b', c)|w] \\ \text{s.t.} \quad & \tilde{c} = w + Rb - b' - \alpha c_{-1} \end{aligned} \tag{2}$$

Equation (2) is key to the numerical approach to tackle the problem. However, reduced-form identification of α does not rely on the recursive formulation.

It should be noted that we impose a very stringent assumption on the shape of habits in consumption. In fact, we assume that consumption accruing to current utility only stems from current and past consumption expenditures. Fuhrer (2000) allow for a more general shape of the process. However, we follow Dynan (2000) in imposing this restriction and furthermore point out that a more general process would be more likely to hinder identification in the structural estimation approach. Specifically, it would be harder to find moments that are informative about all the parameters governing habits in consumption.

Below we report the calibration of the model. To estimate it, we simulated a panel data of N individuals, each observed for T periods. Income are discretized as a Markov chain. Notice that due to the income process being persistent, we followed Kopecky & Suen (2010) and used the Rouwenhorst method, instead of the classic Tauchen, to solve for the Markov transition matrix.

Parameter	**General Description**	**Value**
σ	Risk aversion.	0.850
α	Habit in consumption.	0.20
β	Discount factor.	0.965
R	Gross real rate.	1.035
ρ	Persistence of income process.	0.80
σ_ε	Variance of innovations in income.	10.0
N	Number of simulated individuals.	1000
T	Time of observation.	100
n	Dimension of the transition matrix.	10
$\dim(A)$	Number of points in the assets grid.	20

Table 1: Calibration of the model and simulations.

Note that once the income process is discretized and the resulting Π transition matrix is obtained, then equation (2) is written as

$$\begin{aligned} v\left(b, c_{-1}, w=w_i\right) = \max_{b'} \quad & u\left(\tilde{c}\right) + \beta \sum_{j=1}^{n} \pi_{ij} v\left(b, c_{-1}, w=w_j\right) \\ \text{s.t.} \quad & \tilde{c} = w + Rb - b' - \alpha c_{-1} \end{aligned} \tag{3}$$

where n is the number of states of the Markov chain, and π_{ij} is the generic entry in the stochastic matrix.

3 Reduced-Form Estimation

In this section, we exploit (1) and show that α can be estimated on the data we simulated, in a reduce-form fashion. Begin by writing the lagrangian associated to the problem:

$$\mathcal{L} = \mathbb{E}_t\left[\sum_{t=0}^{\infty} \beta^t \left(u(\tilde{c}_t) + \lambda_t(w_t + R_t b_t - b_{t+1} - c_t)\right)\right]$$

Take the first-order conditions, which read out as follows:

$$\begin{aligned} \frac{\partial L}{\partial c_t} = 0 \quad & \Leftrightarrow \quad \frac{\partial u}{\partial \tilde{c}_t} \cdot \frac{\partial \tilde{c}_t}{\partial c_t} - \lambda_t + \beta \frac{\partial u}{\partial \tilde{c}_{t+1}} \cdot \frac{\partial \tilde{c}_{t+1}}{\partial c_t} = 0 \\ \frac{\partial L}{\partial b_{t+1}} = 0 \quad & \Leftrightarrow \quad \lambda_t = \beta \mathbb{E}_t\left[R_t \lambda_{t+1}\right] \end{aligned}$$

Using the two, we get to the following Euler equation:

$$\frac{\partial u}{\partial \tilde{c}_t} - \alpha\beta\mathbb{E}_t\left[\frac{\partial u}{\partial \tilde{c}_{t+1}}\right] = \beta\mathbb{E}_t\left[R_{t+1}\left(\frac{\partial u}{\partial \tilde{c}_{t+1}} - \alpha\beta\frac{\partial u}{\partial \tilde{c}_{t+2}}\right)\right] \tag{4}$$

Assume that the real rate is constant, at level R. Hayashi (1985) provides an approximation for equation (4), which reduces to the following, simpler expression:

$$\mathbb{E}_t\left[\beta R\frac{\partial u/\partial \tilde{c}_{t+1}}{\partial u/\partial \tilde{c}_t}\right] = 1 \tag{5}$$

We follow Dynan (2000) and use (5) to derive the following proposition.

Proposition 3.1. *Assume T large and constant real interest rate R. Then, the Euler equation (5) implies the following estimation equation:*

$$\Delta \log c_t = \alpha\Delta\log c_{t-1} + \gamma_0 + e_t$$

where e_t is zero-meaned iid error term.

Proof. From equation (5), we have

$$R\beta\frac{\partial u/\partial \tilde{c}_t}{\partial u/\partial \tilde{c}_{t-1}} = 1 + \eta_t$$

where η_t is uncorrelated if rational expectations hold.

Assume that the utility function is

$$u(\tilde{c}) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma}$$

Then, the approximated Euler equation reads out as

$$R\beta\left(\frac{\tilde{c}_t}{\tilde{c}_{t-1}}\right)^{-\sigma} = 1 + \eta_t$$

Applying logs and some manipulations yield:

$$\Delta\log\left(c_t - \alpha c_{t-1}\right) = \frac{1}{\sigma}\log R\beta + \frac{1}{\sigma}\log(1+\eta_t)$$

Note that if η is zero-mean, then $\log(1+\eta)$ is zero-mean. Let $\gamma_0 \equiv \frac{1}{\sigma}\log R\beta$ and $e_t \equiv \frac{1}{\sigma}\log(1+\eta_t)$. Following Dynan (2000), we can approximate $\Delta\log\left(c_t - \alpha c_{t-1}\right) \approx \Delta\log c_t - \alpha\Delta\log c_{t-1}$, so that we finally have

$$\Delta \log c_t = \alpha\Delta\log c_{t-1} + \gamma_0 + e_t$$

and the proof is complete. □

A corollary of the proposition is that α can be identified and estimated through a simple panel regression. If we do that, we obviously obtain $\hat{\alpha} = 0.301$ with a standard error of 0.003. This exercise is trivial, but is nonetheless useful to show that α is identified without resorting to a more complicated structural estimation approach.

Notice, however, that another corollary of the same proposition is that σ, β and R are not identified in reduced-form. Even though we will only seek to estimate α and σ in the next section, the proposition nonetheless provides a motive for undertaking a more involved structural estimation exercise that would in principle allow to estimate the other parameters, which could not be estimated in simple reduced-form framework.

4 Structural Estimation

We estimate the model through simulated method of moments. A first step is of course to simulate the dataset. To do so, we solve the problem in (3) via value function iteration, feeding the algorithm with the calibrated parameters whereof table 1. Given the policy function for assets, which we plot in figure 1, we can simulate assets and consumption given the exogenous realization of the income process for N individuals.

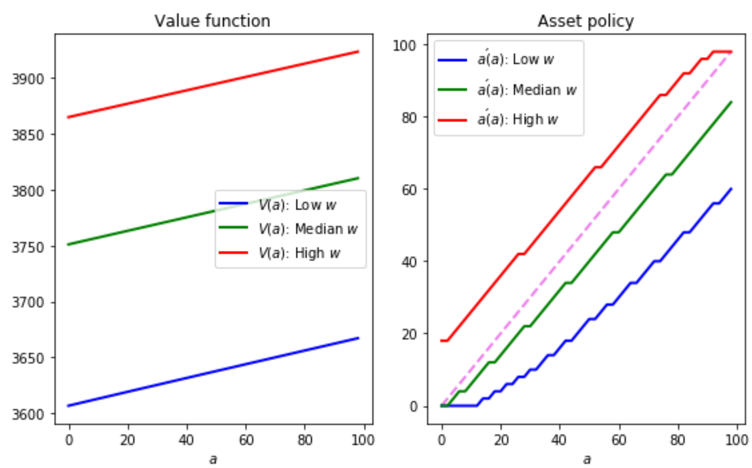


Figure 1: Value function and policy function. NOTES: value and policy for different values of income, holding consumption fixed.

To estimate α given the simulated data, we adopt a similar perspective. We set up a grid of points $\mathcal{A} = \{\alpha_0, \ldots, \alpha_M\}$. In the actual estimation routine, $\alpha_0 = 0.0$ and $\alpha_M = 0.7$. For each value of α, we compute the following:

$$L(\alpha) = \boldsymbol{\mu}(\alpha)^T \boldsymbol{I} \boldsymbol{\mu}(\alpha)$$

where $\boldsymbol{\mu}_\alpha$ is a vector of moments. In particular, let sample moments be

$$\boldsymbol{\mu}^d = \begin{pmatrix} \frac{1}{N}\frac{1}{T}\sum_{i=1}^N \sum_{t=1}^T c_{it} \\ \frac{1}{N}\frac{1}{T}\sum_{i=1}^N \sum_{t=1}^T (c_{it} - \bar{c}_i)^2 \\ \frac{1}{N}\sum_{i=1}^N \gamma_i(1) \\ \frac{1}{N}\sum_{i=1}^N \gamma_i(2) \end{pmatrix}$$

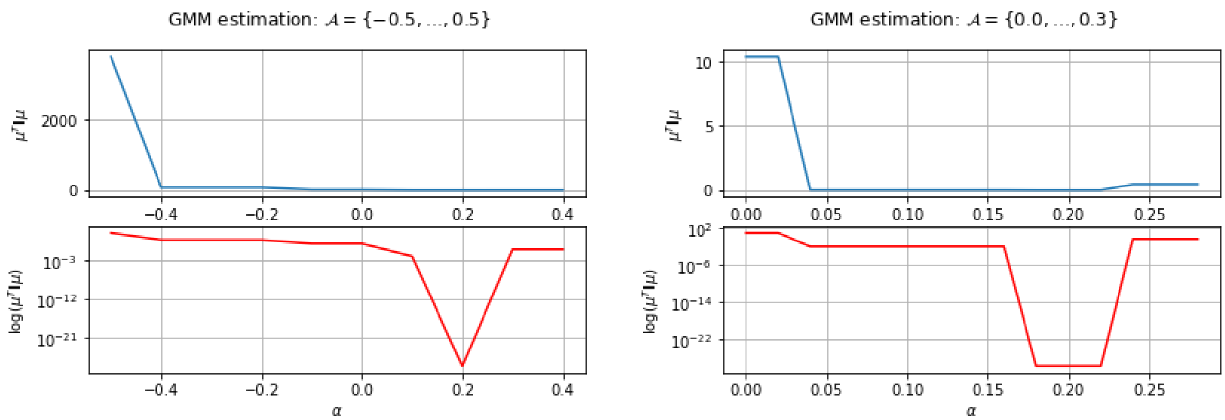
where $\gamma(\cdot)$ is the autocovariance function. Similarly, α-moments of the simulated data are

$$\boldsymbol{\mu}^s(\alpha) = \begin{pmatrix} \frac{1}{N}\frac{1}{T}\sum_{i=1}^N \sum_{t=1}^T c_{it}(\alpha) \\ \frac{1}{N}\frac{1}{T}\sum_{i=1}^N \sum_{t=1}^T (c_{it}(\alpha) - \bar{c}_i(\alpha))^2 \\ \frac{1}{N}\sum_{i=1}^N \gamma_i(1,\alpha) \\ \frac{1}{N}\sum_{i=1}^N \gamma_i(2,\alpha) \end{pmatrix}$$

Hence, we define $\boldsymbol{\mu}(\alpha) = \boldsymbol{\mu}^s(\alpha) - \boldsymbol{\mu}^d$, and pick $\hat{\alpha}$ that solves:

$$\hat{\alpha} \equiv \arg\min_{\alpha \in \mathcal{A}} L(\alpha)$$

In figure 2 we provide a graph showing the value of the loss function $L(\cdot)$ for various values of α. As we would expect, since the DGP comes to coincide with the model as α converges to its true value, the loss function attains its minimum at the calibrated value.

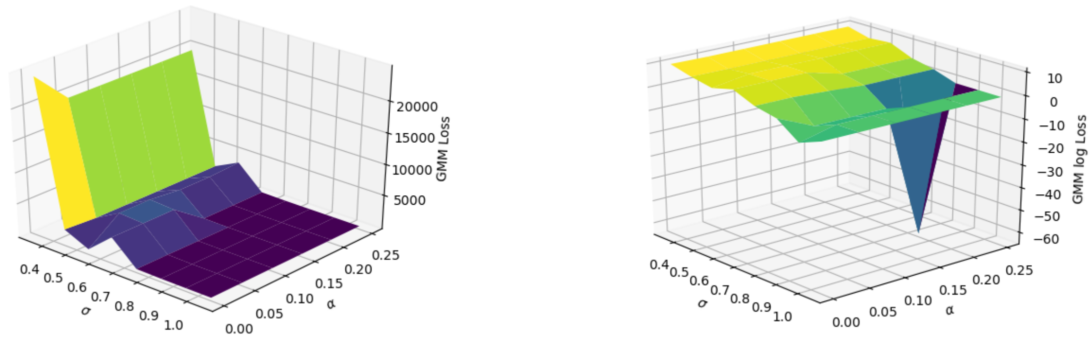


(a) Sparse grid $\mathcal{A}$. **(b)** Dense grid $\mathcal{A}$

Figure 2: Structural estimation of a single parameter for different grid values.

In figure 2a we plot the loss function, in levels and log-levels, and show in a sparse grid, the parameter is identified, *i.e.* the loss function reaches a minimum at $\hat{\alpha} = 0.2$, that is the calibrated parameter.
In figure 3b we provide the same plot, but evaluate the GMM loss function on a denser grid. In particular, we define $\mathcal{A} = \{0.0, 0.05, 0.1, \ldots, 0.3\}$. This second plot attests that α is identified in ± 0.02, that is the *approximate* standard error of $\hat{\alpha}$ to be 10% of the estimate. We provide these plot separately to make two different points clear. First, habits in consumption are informative about the moments we consider, as the graphs show. In particular, this suggests habit formation to induce autocorrelation in the consumption process, thus confirming it to be intuitively related to *excess smoothness*. Second, and related, the parameter governing habit formation is estimated with a relatively big standard error, thus suggesting it not to be very well related to the moments we considered. This of course does not imply that other moments could in principle be considered, in order to improve the uncertainty of the estimate.

For the sake of the exercise we also try to simulate the model and estimate σ, *i.e.* risk aversion, and α jointly. This inevitably implies a heavier computational burden. Therefore, we only provide a graphical qualitative evidence on the estimated coefficients on a sparse grid. In particular, we define $\sigma \in \mathcal{S} = \{0.4, 0.5, \ldots, 1.0\}$ and $\alpha \in \mathcal{A} = \{0.0, 0.05, \ldots, 0.25\}$. We provide the result of the estimation in figure 3.



(a) Levels. **(b)** Logs.

Figure 3: Structural estimation of $(\sigma, \alpha) \in \mathcal{S} \times \mathcal{A}$.

The results are similar to that in 2. Both parameters are well identified. However, given the flattening of the objective function, it is necessary to take it in log to see its minimum. This suggests that had we evaluated the loss function on a finer grid, we would notice that standard errors of the estimates are likely to be comparable to those whereof the single-parameter estimation.

5 Conclusion

In this work we estimated a simple habits in consumption model. We showed that if one is willing to assume that habit formation is governed by a single parameter, then that parameter will be identified in reduced-form, and estimation can be performed using a simple linear regression.

Furthermore, we showed that that same parameter can be estimated using a simulated method of moments, conditional on considering moments that are informative about the persistence of the consumption process. From this perspective, habits in consumption are clearly related to the phenomenon of excess smoothness, suggesting that consumption reacts to changes in permanent income in less than one-to-one sense. In our simple framework, the underlying intuition is that consumption accruing to current utility comprises both current *and* past expenses in consumption.

This exercise was simple and has been implemented naively, but it nonetheless conveys -I believe- the main intuitions on structural estimation of dynamic models.

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