Value-at-Risk and Credit Cycles: an ABM perspective

Candidate: Davide M. Coluccia*
davide.coluccia@unibocconi.it
Bocconi University & SSSUP

Supervisor: Prof. Andrea Roventini
andrea.roventini@santannapisa.it
SSSUP & OFCE, Sciences Po

November 19, 2019

Abstract

This essay studies an Agent-Based model that allows to disentangle the complex links between the real and the financial sides of an economy. We model a financial sector consisting of equity funds and banks. The former trade in bank equity, while the latter hold marked-to-market balance sheets and undertake Value-at-Risk leverage management. We show that these features convey a strong feedback and transmission mechanism to the real side of the economy. We are in particular able to conclude that the amplification dynamics stemming from the pro-cyclical behavior of bank leverage that emerges are related to more volatile business cycles, and harsher recessions. Furthermore, we show that our model is able to successfully replicate a wide array of empirically well-known stylized facts, ranging from macro business cycle facts to relevant firm dynamics regularities.

Keywords: Agent-Based Computational Economics, Financial Cycles, Leverage, Mark-to-Market, Value-at-Risk.

JEL Classification: E32, E44, E51, G01.

Contents

*This essay is submitted in partial fulfillment of the requirements for the *Diploma Magistrale* in Economics at Scuola Superiore Sant'Anna. I wish to thank my supervisor, prof. A. Roventini who also supervised the Master thesis of which this essay is the ideal continuation, for constant guidance and support.

When the music stops (...) things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing.
C. O. Prince, former Citigroup CEO, July 2007

Introduction

The business cycle literature acknowledged the relevance of the financial side of the economy almost three decades ago, following the seminal paper by Kiyotaki & Moore (1997) and the subsequent work by Bernanke, Gertler & Gilchrist (1999). Particularly following the Great Recession mainstream New Keynesian models started embedding financial frictions whose explicit modeling had nonetheless been largely neglected before (see *i.a.* Eggertsson & Krugman (2012)). This notwithstanding, a number of methodological issues are known to affect this class of models, whose theoretical and empirical underpinnings have begun being challenged (Caballero, 2010).
In this paper we adopt a radically different perspective. We study an Agent-Based model (ABM) that seeks to simulate an economy featuring highly interconnected real and financial markets. In doing so, we largely abandon historically important hypothesis, ranging from the representative agent to rational expectations (Kirman, 1989). Instead, we explicitly model heterogenous, boundedly rational agents whose reciprocal interactions prompt aggregate dynamics to emerge in a purely bottom-up perspective.

The main research question that this essay will seek to address is to study the linkages between the real and financial sides of the economy, and to disentangle the feedback and amplification mechanisms that those links imply. The main focus will be to show that a financial system in which banks hold marked-to-market balance sheets and pursue value-at-risk (VaR) leverage targeting is subject to cause business cycles fluctuations stemming from the real side of the economy be amplified and fed back to firms and households, thus causing substantial welfare costs associated to the enhanced fluctuations. Moreover we will show that, in the economy we will model, the VaR rule causes bank leverage to be procyclical, hence standing as a cause in itself of credit-fuelled recessions. In this sense, thus differently from the traditional business cycle literature, the financial sector is not merely an amplifier of real fluctuations, but can in itself be at the heart of their origin.

The empirical stepping stone of the theoretical model we introduce is the stream of literature initiated by Schularick & Taylor (2012). These authors document that recent years witnessed increasing integration between the financial system and the real economy, a finding further confirmed by Ng & Wright (2013). They document a number of results that are relevant for this model, *i.a.*: credit booms tend to be followed by harsher, longer-lasting recessions relative to those that were not preceded by those booms; the conditional moments of the business cycle are decreasing with the credit overheating of the economy, and yet tail events are more frequent the more the financial sector is integrated with the real one.

The other empirical study that provided fundamental evidence about the behavior of financial actors that is embedded in our model is that by Adrian & Shin (2010). They show that, surprisingly, financial agents following VaR-based leverage management plans feature markedly pro-cyclical leverage. In a subsequent work, Adrian & Shin (2013) end up documenting how such a behavior can provide a basis to claim that such rule can be de-stabilizing for the economy as a whole.
Our rather ambitious endeavour will be to embed the empirical findings of these authors in our model, and document their relevance in our environment. Moreover, we will show that the model is capable of replicating more traditional stylized business cycle facts as well, both pertaining to macro dynamics, such as those studied by Stock & Watson (1999), and firm dynamics (Bottazzi & Secchi, 2003, 2006; Dosi, 2006).

The remainder of the essay is organized as follows. In section 1 we briefly present the main relevant literature; 2 discusses the theoretical model we develop; 3 documents the results of the simulated models and deals with the related statistical analysis; in 4 we sketch some conclusions and discuss possible future lines of research.

1 Related Literature

We refer to two streams of literature, the first concerning the use of ABMs for macroeconomic analysis, and the second documenting relevant empirical facts.

1.1 ABMs, macroeconomics and financial markets

The first ABM in the macroeconomic literature that explicitly embeds credit is Delli Gatti *et al.* (2005). Their model replicates some stylized facts, mainly (i) right-skewed firm size distribution, (ii) Laplace-distributed firm growth rate distribution, and (iii) fat-tailed output growth rate distribution. Importantly, the model is capable of endogenously generating tail events with a non-negligible probability. This last feature is essential in our model too, for recessions are a relevant piece of the analysis we undertake in section 3. Delli Gatti *et al.* (2010) take one step ahead and model a financial accelerator in a credit network. Relative to the previous work, this new model (i) explicitly considers the demand side, *i.e.* households, and allows firms to be heterogeneous. They show that a credit network emerges once banks are allowed to differ, and furthermore document that amplification dynamics can stem from such a structure. In our model the amplification mechanism is not due to network effect, but our conclusions for the importance of the financial sector for the overall stability of the economy are close to theirs. Riccetti, Russo & Gallegati (2013) further build on this mechanism but allow banks to be leveraged: credit networks and leverage procyclicality have sizeable effects on the stability of the macro-economy. These models all shar from our perspective e some undesirable features. Banks typically have no balance sheet constraint, hence they can create loans at will with no leverage regulatory limitation. Furthermore, all these models lack a labor market and share a very simplistic production sector.

Riccetti, Russo & Gallegati (2015) partially address these issues, albeit in a very specific framework that heavily relies on the dynamic trade-off theory. Their model is closer to ours in that the authors do not stick to the stochastic price assumption, which all previous models share. However, their conclusions about the moments of GDP, consumption and investment with respect to the amount of credit the economy is fuelled with are at odds with newer empirical evidence that we will discuss shortly.

Aymanns & Farmer (2015) and Aymanns *et al.* (2016) are also close in spirit to our contribution. Neither paper attempts to model an entire economy, hence they are both radically different from our contribution whose focus is instead on the *interaction* between the real and the financial sectors. However, they study how VaR rules can originate pro-cyclical leverage. We borrow their modeling strategy when defining the leverage policy that banks follow. They show that VaR rule lead leveraged investors to manage their leverage procyclically with respect to the price of the equity they mark-up their assets on. As a result, bubbly episodes are more common and more disruptive on the overall stability of the financial market.

Relatedly, Poledna *et al.* (2014) study the impact of three different leverage regulatory regimes. They consider (i) an unregulated system that imposes a simple upper bound; (ii) a Basel II system; and (iii) a hypothetical system allowing perfect hedging of leverage-induced risk. They show that (ii) and (iii) reduce the risk of bank defaults relative to (i) provided leverage be low, but fail to be effective in highly leveraged scenarios.

Our contribution is related to all these, but close to none. In particular, to the best of our knowledge no study has so far attempted to study the consequences of marked-to-market balance sheets and VaR leverage rule for the stability of the whole economy.

1.2 Evidence on procyclical leverage and credit

Adrian & Shin (2010) provided the first piece of evidence suggesting that leverage could be procyclical in the value of assets. Consider the standard definition of leverage:

$$\lambda \equiv \frac{A}{A - L} = \frac{A}{E}$$

where λ is leverage, A, L and E are assets, liabilities and equity in market value. Given the definition, one would expect λ to decrease as A increases, all else being equal, *i.e.* $dL/dA = 0$. However, the authors showed that for *some* agents, namely households, this is in fact true, thus suggesting households to be passive with respect to the value of their assets. Large firms, on the contrary, act so as to keep their leverage constant, whereas large banks are the only one to actively pursue a policy of procyclical leverage whereby leverage is documented to increase following an increase in the value of assets. Adrian & Shin (2013) further show (i) how such a behavior can be rationalized in a theoretical model, but most importantly (ii) argue that procyclical leverage management can be destabilizing for the macroeconomy. Borrowing from Geanakoplos (2010), they

argue that leverage cycles can work as powerful amplification mechanisms for real shocks. Consider an upturn leverage adjustment: following an asset price boom, balance sheets of banks enlarge in their asset value; this leads to an increase in leverage, which in turn prompts demand for assets up, thus making asset prices to raise, hence re-igniting the cycle. The very same mechanism applies to downturn leverage adjustments. These conclusions are well in line with the aforementioned study by Aymanns & Farmer (2015).
In our model we embed VaR and marked-to-market accounting and show that these are sufficient for bank leveraged to be procyclical with respect to the business cycle. Hence, we confirm the leverage cycle hypothesis.

Schularick & Taylor (2012) and follow-up studies by Jordà, Schularick & Taylor (2013, 2015) contributed to shed new light on the integration between the financial and real sectors of modern economies. We could outline their findings as follows. Using historical evidence, they document that the integration of the financial sector has been growing in the last century. For instance, while for the pre-WW2 period the main business cycle indicators were equally correlated with real money growth and real credit growth, after the collapse of the Bretton Woods system the latter became a *much* better predictor for business cycles than money Jordá, Schularick & Taylor (2017). This regime change is so marked that they speak of a *monetary* era, for the years before WW2, and a *credit* era, following those. This is well in line with the evidence found by Ng & Wright (2013), that confirm a substantial regime change in the correlation magnitude between credit and the remainder of the main business cycle indicators.
Furthermore, they show that high excess credit recessions, that is recessions preceded by a credit expansion, after WW2 became significantly heavier and longer lasting than standard ones Schularick & Taylor (2012). The authors further show that the likelihood of experiencing a recession is positively related to the excess-credit that is being created before such a recession actually takes place. Earlier efforts devoted at understanding financial crises and their impact on real variables can be found in Reinhart & Rogoff (2008, 2013); Claessens, Kose & Terrones (2009); López-Salido, Stein & Zakrajšek (2017), and broadly confirm these findings. Downturns that are preceded by such credit boom events are labeled as Financial crises: Jordá, Schularick & Taylor (2017) show that rapid credit growth is associated with deeper subsequent recessions, particularly in the post-WW2 period. Moreover, when the recession hits and the economy is leveraged, the economic slowdown is deeper, and longer-lasting. Last, the authors study the correlation between the conditional moments of the GDP components and a measure of the integration of the financial system with the macroeconomy, *i.e.* an adjusted measure of the credit-to-GDP ratio. They find moments up to the fourth of GDP, consumption and investment are all negatively related with this ratio. Furthermore, they document that the higher the ratio, the higher the probability of tail events, *i.e.* financially driven downturns.
The purpose of our model is to embed these insights in our model and replicate them. We document the results in section 3.

2 The Model

We model an economy that is composed of two macro-sectors. On the one hand, we consider a *financial sector* consisting of mutual funds, *i.e.* unleveraged investors, and banks. The former are endowed with equity that is issued by the latter. For the sake of simplicity and modeling parsimony, we do not allow banks to issue equity beyond that equity-owners are endowed with in the first period. Mutual funds interact between each other by trading their equity in order to maximize the return of their portfolio, and thus implicitly price the object of their trade. Banks, in turn, issue loans to firms given the market value of the equity they maintain. Notice that, since banks can in principle set the size of their balance sheets, we are in tune with the literature dealing with endogenous money (Lavoie, 2014). This notwithstanding banks employ a stylized value-at-risk rule to manage their balance sheet. This heuristic rule can be understood as the outcome of a standard contracting model as in Geanakoplos (2010), and we seek to understand its impact on the overall stability of the rest of the economy. More specifically, we shall see that a VaR rule does imply procyclical lending as argued by Adrian & Shin (2010), and that this negatively affects the stability of the economy.

The *real sector* we consider is composed of two sets of actors. On the one hand, firms obtain credit from the financial sector, more specifically from the banks. Clearly, as there is wide empirical evidence in favor of financial constraints binding firms investment decisions, we do not impose any *a priori* reason why the credit market should clear (Fazzari *et al.*, 1988). Thus, in general firms are either financed constrained or banks are not in the position to satisfy all their lending capacity. Firms all belong to the same sector, thus there is no product heterogeneity, and undertake in-house R&D. Furthermore, they also try to imitate their competitors provided the latter be more productive than the former. Beyond the productive agents, there exists a set of households, whose function is double: on the one hand, they sell labor to firms in exchange for a salary. On the other, they act as customers and buy the homogenous consumption good from the firms. Since each firm is owned, in certain fixed shares, by households, the latter's income is further increased by an equivalent share of the eventual profits firms attain.

Four different *markets* are studied. First, in the equity market, equity owners interact with each other and trade in bank equity. Since, as in the previous model, they are heterogeneous with respect to the expectations they form concerning the future yield of each equity, they will accordingly set the nominal amount of equity they wish to detain in their portfolios. Given that such expectations shall in general differ across agents, trade stems from the heterogeneity that is embedded in the way agents form their expectations. Strategies will be seen to vary, but in general entail a weighted average of a chartist and a fundamentalist component. Equity is thus priced and the market value of the stocks of equity each bank detains is readily solved for.

Differently from the equity market, which is one-sided in the sense that agents belonging to the same class interact, the *credit market* is two-sided. The supply side is composed of the banks, whereas the demand side is determined by the firms asking for external financing. Notice, however, that the market is segmented, for each firm can belong to a single bank's set of customers, and a bank can only lend to those firms belonging to its set of customers. We model these two structures differently on purpose. The equity market is a stylized stock market, in the sense that each actor can interact with virtually each other with negligible transaction costs. While this does not imply information to be symmetric, it nonetheless allows each agent to effortlessly try to engage in a trade with another. The credit market, on the other hand, is the expression of the bank-firm relationship, and as such cannot be thought of in terms of a centralized *marketplace*, and indeed is permeated by linkages of trust, information imperfections and monitoring costs that cannot be accounted for in a centralized structure (Dass & Massa, 2011). Hence, banks try to issue as many loans as their balance sheet preference allows them to, while their customers' demand is determined as a consequence of their interactions with the households. As a result, credit constraints typically emerge and either the firms' investment decisions are bound the supply of credit, or the banks' return on equity is bound by its demand.

The *goods market* is fairly similar to the credit one inasmuch firms, embodying the supply side, face a set of customers, the demand side, whose income may be binding in determining the total sales of the firm. Qualitative considerations on the structure of the market we consider are fairly similar to that of the credit one. Households typically do not consider, as in the neoclassical framework, all viable suppliers of a given good, for transaction and information costs make it unfeasible. As a result, a natural segmentation arises, also as a consequence of geographical clustering -to this end, see the comprehensive contribution by Fujita, Krugman & Venables (2001). Further notice that demand is not solely determined by labor income, for each customer is also the owner of a fixed share of each firm, and thus retains a part of the divided profits, which he spends on consumption.

Last, the *labor market* is extremely simplified, as we assume that the wage can be set centrally and such that each firm agrees to pay such salary. This is indeed a strong simplifying assumption that can be understood as if the wage level was set by a labor union. Still, labor market is not Walrasian in the sense that the wage level thus set is not a market clearing one, and hence some firms may fall short of employees. While the labor market is admittedly the most simplistic, it nonetheless conveys an idea that stretches at least as back as to Marx, that is that of a reserve army of labour: unemployment and wages move one opposed to the other.

One last component that we consider as a distinguishing feature of the model is that it comprises a degree of *evolutionary selection* of actors. Already Brock & Hommes (1997) noted that for a heterogeneous-agent model to be consistent, it needs to be evolutionary consistent, in the sense that it must not merely postulate a set of strategies, but select the best performing and prove them to be stable over time. Still, our framework is not based on strategy selection. Instead, we base the selection procedure upon the outcome of the choice, namely, of firms and banks. In line with the role of market selection outlined, *i.a.* by Dosi & Nelson (2010), firms can switch their bank discriminating on the basis of the interest rate they are charged,

whereas households can switch between firms seeking to minimize the price of the consumption good.
Ideally, one would have preferred to allow both firms and banks to draw from a wider set of strategies, and make the chosen ones evolve according to their fit, possibly through genetic algorithm DosiOrsenigo1995. However, since our main focus is on the macroeconomic relevance of mark-to-market and procyclical leverage policies rather than the microeconomic consistency of different shades of such behaviors, we refrain from overly increasing the complexity of the model and limit the selection algorithm to operate on the basis of the performance of each agent, but do not allow the strategies they adopt to evolve over time.
Thus, while our approach is clearly deficitary with respect to a thorough investigation of the evolutionary consistency of the strategies adopted by the agents, which in our approach are postulated, we nonetheless put forward an attempt aimed at selecting the best performing agents, given their customers' preferences, and study how their dominating position in the market evolves over time.

In the following picture, we provide a schematic representation of the economy described thus far. It should be clear that, while no presumption of realism is claimed, we are nonetheless convinced by the fact that even such a simple structure captures some of the most relevant features of modern post-industrial societies, namely: (i) the degree of integration between financial and real sectors; (ii) the importance of mark-to-market bank accounting for the stability of the macroeconomy; (iii) the relevance of purely financial subsectors, which in our model are stylized as unleveraged investors trading in bank equity, for the purely real sector; and (iv) the proactive role of financial markets in both fostering long-term growth and fuelling short-term business cycle fluctuations.

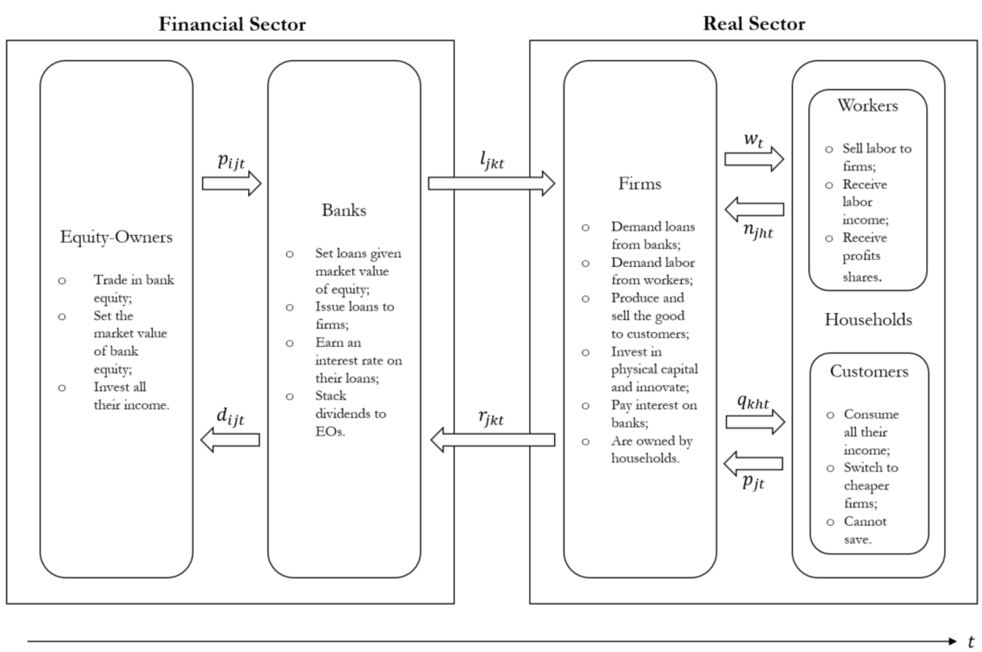


Figure 1: Schematic representation of a mark-to-market economy with two macro-sectors.

2.1 The financial sector

The financial sector is composed of equity owners and banks. Since the two interact only indirectly, we discuss the two groups separately.

2.1.1 Equity owners

Equity owners are unleveraged investors, and can be understood as mutual funds. Thus, unlike banks, they cannot enlarge their balance sheets at will. Also, they can only trade in one asset, *i.e.* bank equity, whose overall stock is fixed, since banks cannot issue equity at will. Equity owners are therefore endowed, in the first period, with a nominal amount of bank equity each, and trade according to the expectations they form concerning the return each bank will attain and reward its owners.

Let $\mathbb{I} = \{i\}_{i=1}^{I}$ be the set of equity owners, and $\mathbb{J} = \{j\}_{j=1}^{J}$ be that of banks. Typically, we shall assume $I > J$. Each equity owners is represented in terms of the composition of its portfolio, that is $n_{it} = \{n_{ijt}\}_{j=1}^{J}$, where n_{ijt} is the nominal amount of equity of bank j that is owned at time t by investor i. Notice that we can also write the portfolio as $w_{it} = \{w_{ijt}\}_{j=1}^{J}$, where w_{ijt} is the weight of bank j in i's portfolio, and $\sum_{j\in\mathbb{J}} w_{ijt} = \sum_{j\in\mathbb{J}} \frac{n_{ijt}}{\sum_{j\in\mathbb{J}} n_{ijt}} = 1$ for each $i \in \mathbb{I}$.
We also assume $\sum_{i\in\mathbb{I}} n_{ij0} = E$ for each $j \in \mathbb{J}$, that is we assume that each bank issues the same amount of equity at $t = 0$, and therefore $w_{ij0} = \{1/J\}_{j=1}^{J}$ for each i. Since no equity issuance is in order, $\sum_{i\in\mathbb{I}} \sum_{j\in\mathbb{J}} n_{ijt} = J \cdot E$ for each t.

In order to solve the *portfolio optimization*, each equity owner evaluates the conditional expected return and conditional expected variance of each bank's equity in terms of a MA(1) process, whose horizon is defined by parameter $\delta = \{\delta_i\}_{i=1}^{I}$[1]:

$$\mu_{ijt} = \delta_i d_{jt-1} + (1 - \delta_i)\mu_{ijt-1} \tag{1a}$$

$$\sigma^2_{ijt} = \delta_i (d_{jt-1} - \mu_{ijt})^2 + (1 - \delta_i)\sigma^2_{ijt-1} \tag{1b}$$

where $\delta_i > 0$ for each i and d_{jt} is the return of one equity of bank j at time t. Notice that equations (1a) and (1b) are the most simple specification of a chartist investor, who forms the expectation current returns given past returns only.
Parameter δ to a large extent embodies the heterogeneity of equity owners. Since each one has a different horizon in the formation of its expectations, then it follows that he shall also differently evaluate the portfolio weights attached to any given bank's equity and, hence, trade occurs because of the heterogeneous beliefs agents form given the different degree of Chartism they feature.

[1] An increasing number of studies dealing with unleveraged investors' investment studies concluded that they are intrinsically chartist, see Braun-Munzinger, Liu & Turrell (2018) and Shek, Shim & Shin (2015). Also, we derive the order of the MA process given the fundamentally sluggish nature of equity prices (Adrian & Shin, 2013).

Following Aymanns & Farmer (2015), we can thus solve the portfolio optimization of the equity owners in terms of a simple rule of thumb[2]

$$w_{ijt} = \frac{\exp(\beta_i s_{ijt})}{\sum_{j \in \mathbb{J}} \exp(\beta_i s_{ijt})} \tag{2}$$

where $\beta_i > 0$ is a portfolio responsiveness parameter, and $s_{ijt} = \mu_{ijt}/\sigma_{ijt}$ is just the Sharpe ratio associated to equity j given i's conditional expectations, specified in (1a) and (1b).

In order to evaluate the orders that they want to place in the market, equity owners also need to estimate the income they will receive from the currently owned equity. Indeed, orders are placed *before* the actual income is known, hence the investors act in a situation in which their budget set is not known with certainty. Albeit seemingly abstruse, this assumption is justified in a twofold manner. On the one hand, it is intended to capture the pervasive uncertainty in common stock markets, in which orders are placed continuously and edging instruments have been developed in order to provide insurance against credit and liquidity risk. On the other, since our modelling approach is different from either an equilibrium pricing or a market maker structure, uncertainty on the payoff of equity is needed in order to make equity owners interact, that is trade, one another.

To this end, and inspired by Braun-Munzinger, Liu & Turrell (2018), we let each equity owner be hit by a noisy signal conveying information on the current return of equity, that is let $f_{ijt} = d_{jt} + \varepsilon_{ijt}$, where $\varepsilon_{ijt} \sim \mathcal{N}(0, \sigma^2_{ijt})$, then the expected return conditional on the signal is computed as

$$d^e_{ijt} = \mathbb{E}[d_{jt}|f_{ijt}] = \gamma_i f_{ijt} + (1 - \gamma_i)\mu_{ijt} \tag{3}$$

where $\gamma_i \in (0, 1)$ for each $i \in \mathbb{I}$. Term γ embodies the degree of fundamentalism agent i features, that is if $\gamma_i = 1$ the agent is purely fundamentalist inasmuch it only values the signal, albeit noisy, and discards all information conveyed by previous returns, whereas an i such that $\gamma_i = 0$ is a purely chartist investor.

The expected income is thus $Y^e_{it} = \sum_{j=1}^{J} d^e_{jt} n_{ijt-1}$ so that one can write the expected excess demand (supply) function as

$$z^e_{ijt} = w_{ijt} \frac{Y^e_{it}}{p_{jt}} - w_{ijt-1} \frac{Y_{it-1}}{p_{jt-1}} \tag{4}$$

where $w_{ijt} Y^e_{it}/p_{jt} = n^e_{ijt}$ is the amount of j-th bank's equity that the i-th investor would like to detain at time t, while $w_{ijt-1} Y_{it-1}/p_{jt-1} = n_{ijt}$ is the amount of equity that he already detained. Thus, $z^e_{ijt} > 0$ implies that i aims at increasing the nominal amount of equity to hold, *i.e.* an excess of demand, whereas $z^e_{ijt} < 0$ is by the same token an excess of supply.

Therefore, summing across i the excess demand (supply) functions one has that the equilibrium price for the

[2] It is possible to show that equation (2) can be obtained as a result of a conventional portfolio optimization.

j-th bank is such that the market clears:

$$\begin{aligned} z^e_{jt} &= \sum_{i=1}^{I} z^e_{ijt} = \frac{1}{p_{jt}} \sum_{i=1}^{I} w_{ijt} Y^e_{it} - \frac{1}{p_{jt-1}} \sum_{i=1}^{I} w_{ijt-1} Y_{it-1} = 0 \Leftrightarrow \\ p_{jt} &= p_{jt-1} \frac{\sum_{i=1}^{I} w_{ijt} Y^e_{it}}{\sum_{i=1}^{I} w_{ijt-1} Y_{it-1}} \end{aligned} \tag{5}$$

The pricing equation (5) is an interesting feature of the trading model we propose. The market clearing price is computed to clear the market in terms of the *expected* excess demand. Hence, if by chance each investor had rightly estimated its income, then it would also clear the actual excess demand. Still, since this is unlikely to be the case, p_{jt} will in general differ from a standard equilibrium pricing function. Therefore, our market structure is crucially different from a standard equilibrium pricing environment. Yet, we fully endorse the critique by LeBaron (2006) concerning an alternative modeling strategy that has been employed, namely, that of a market maker fixing the price as $p_t = p_{t-1} + \lambda ED_t$, where ED_t is previous excess demand and λ is a tuning parameter. The market maker approach, albeit simple, is problematic in that it implies (i) no market clearing, and hence accommodation of eventual residual excess demand/supply, and (ii) a heavy reliance upon the tuning parameter λ. The first point, in particular, makes this approach invalid, for we do not allow further issuance of equity and, even if that was not the case, it would have been problematic to assume automatic accommodation of equity mismatch.

Once the orders are placed and the actual income is known, *i.e.* $Y_{it} = \sum_{j=1}^{J} d_{jt} n_{ijt-1}$, we can evaluate the excess demand function at the market clearing price thus computed, that is we have

$$\begin{aligned} z_{jt} &= \sum_{i=1}^{I} z_{ijt} = \frac{1}{p_{jt}} \sum_{i=1}^{I} w_{ijt} Y_{it} - \frac{1}{p_{jt-1}} \sum_{i=1}^{I} w_{ijt-1} Y_{it-1} = \\ &= \frac{\sum_{i=1}^{I} w_{ijt-1} Y_{it-1}}{\sum_{i=1}^{I} w_{ijt} Y^e_{it}} \frac{1}{p_{jt-1}} \left[\sum_{i=1}^{I} w_{ijt} (Y_{it} - Y^e_{it}) \right] \neq 0 \end{aligned} \tag{6}$$

which confirms our previous assertion on the equilibrium price being consistent with market clearing only in the expected income. Therefore, letting $\tilde{z}_{jt}$ be the scaled excess demand function, we can compute the fraction of orders that are delivered at the equilibrium price that are consistent with market clearing as

$$\begin{aligned} \phi_{jt} \text{ s.t. } \tilde{z}_{jt} &= \frac{1}{p_{jt}} \sum_{i=1}^{I} \phi_{jt} w_{ijt} Y_{it} - \frac{1}{p_{jt-1}} \sum_{i=1}^{I} w_{ijt-1} Y_{it-1} = \\ &= \frac{\phi_{jt}}{p_{jt}} \sum_{i=1}^{I} w_{ijt} Y_{it} - \frac{1}{p_{jt-1}} \sum_{i=1}^{I} w_{ijt-1} Y_{it-1} = 0 \Leftrightarrow \\ \phi_{jt} &= \frac{\sum_{i=1}^{I} w_{ijt} Y^e_{it}}{\sum_{i=1}^{I} w_{ijt} Y_{it}} \end{aligned} \tag{7}$$

which is clearly seen to be unique. The set $\phi_t = \{\phi_{jt}\}_{j=1}^{J}$ thus embodies a correction term which adjusts for the discrepancy between estimated and realized income and, to this end, it can be interpreted as a percentage of fulfilled orders. Indeed, as $\phi_{jt} \to 1$ the fraction of fulfilled orders approaches 100%. On the contrary, the more $\phi_{jt} \to 0$, the more 'stagnant' the market is, that is the less trade occurs.

Once the set of ϕ_t is known, one can analytically solve, and exceptionally since we are in an ABM framework, the new portfolios for each agent, since it shall be

$$n_{ijt} = \phi_{jt} w_{ijt} \frac{Y_{it}}{p_{jt}} = \frac{w_{ijt} Y_{it}}{p_{jt}} \frac{\sum_{i=1}^{I} w_{ijt} Y_{it}^{e}}{\sum_{i=1}^{I} w_{ijt} Y_{it}} \tag{8}$$

for each $i \in \mathbb{I}$ and $j \in \mathbb{J}$.

2.1.2 Banks

The banks represent the supply side of the credit market. Also, they can enlarge their balance sheets according to their own preference, thus we endorse the theory of endogenous money in that banks do not face any constraint stemming from liquid reserves in deciding the size of the asset size of their balance sheets. Banks are also key to understand the mark-to-market nature of the economy we are describing, in that they set the asset size of their balance sheets given the market value of their equity that is retrieved as the outcome of the equity market. Last, banks are further fundamental for their leverage policy, that is based upon a stylized VaR rule, conveys the nature of procyclical leverage.

As said, let $\mathbb{J} = \{j\}_{j=1}^{J}$ be the set of banks and $\mathbb{K} = \{k\}_{k=1}^{K}$ that of firms. Furthermore, let $\Omega_{jt} \subseteq \mathbb{K}$ be the set of firms belonging to the customer set of bank j at time t. By construction, $\Omega_{jt} \cap \Omega_{j't} = \varnothing$ for each $j \neq j' \in \mathbb{J}$, meaning that no firm can belong to more than one customer set, and at the same time $\cup_{j \in \mathbb{J}} \Omega_{jt} = \mathbb{K}$, meaning that each firm always belongs to a given customer set. Each bank is represented, in analogy to the equity owners, in terms of the composition of its portfolio, that is $\ell_{jt} = \{\ell_{jkt}\}_{k \in \Omega_{jt}}$ where ℓ_{jkt} is the loan that bank j grants to firm k at time t. Similarly to the previous class of agents, we can once more write the portfolio as $w_{jt} = \{w_{jkt}\}_{k \in \Omega_{jt}}$ in terms of the weights, where $\sum_{k \in \Omega_{jt}} w_{jkt} = \sum_{k \in \Omega_{jt}} \frac{\ell_{jkt}}{\sum_{k \in \Omega_{jt}} \ell_{jkt}} = 1$ for each $j \in \mathbb{J}$. Notice that the weights we consider here are the *ex-post* ones, that is those after the credit market is solved.

The *portfolio optimization* problem that banks face is slightly different from that of the equity funds, for the return of a given loan is at best equal to the interest rate they charge on that loan. Still, it may be lower, depending on the performance of the beneficiary $k \in \Omega_{jt}$. Let r_{jkt} be the gross nominal interest rate that banks charge on their loans, that is firm-specific and defined as

$$r_{jkt} = r_{jkt-1} \left(1 + \varrho_1 \frac{\Delta A_{kt-1}}{A_{kt-1}} + \varrho_2 \frac{\Delta A_{jt-1}}{A_{jt-1}} + \varrho_3 \frac{\Delta \pi_{t-1}}{\pi_{t-1}} + \varrho_4 \frac{\Delta \bar{\xi}_{t-1}}{\bar{\xi}_{t-1}} \right) \tag{9}$$

where $\varrho_1, \varrho_2 < 0$ and $\varrho_3, \varrho_4 > 0$, A_{kt} is the net worth of firm k at time t, A_{jt} is by the same token bank j's net worth at time t, π_t is the price level and $\bar{\xi}_t$ is the average productivity across firms[3]. Equation (9) is in many respect similar, as will the firms' pricing equations be, to the mark-up equation employed in Dosi, Fagiolo & Roventini (2010), and affirms that the interest rate r_{jkt} grows if either (i) inflation is positive, or (ii) productivity grows, and decreases if either (iii) the net worth of the bank grows, for this makes the bank less in need of liquidity, or (iv) the net worth of the firm grows, meaning that it faces less liquidity risk.

Now, let $\tilde{r}_{jkt}$ be the *realized* gross nominal interest rate that bank j exerts from firm k upon the latter has fulfilled its debt obligation, that is, after production takes place and the firm attains eventual profits. Clearly, $\tilde{r}_{jkt} \leq r_{jkt}$, thus the bank evaluates the conditional mean and variances of the loans associated to any given firm in its customer set as

$$\mu_{jkt} = \delta_j \tilde{r}_{jkt-1} + (1 - \delta_j)\mu_{jkt-1} \tag{10a}$$

$$\sigma^2_{jkt} = \delta_j(\tilde{r}_{jkt-1} - \mu_{jkt})^2 + (1 - \delta_j)\sigma^2_{jkt-1} \tag{10b}$$

where $\delta_j > 0$ for each $j \in \mathbb{J}$ is again the horizon of the MA(1) process governing the rule of expectation formation.

Clearly, equations (10a) and (10b) are the symmetric to (1a) and (1b). Still, while equation (2) can be applied as well, here we consider

$$w_{jkt} = \frac{\exp(\beta_j s_{jkt})}{\sum_{k \in \Omega_{jt}} \exp(\beta_j s_{jkt})} \tag{11}$$

where $\beta_j > 0$ for each $j \in \mathbb{J}$ and $s_{jkt} = \mu^{\gamma_j}_{jkt} A^{1-\gamma_j}_{kt}$. Hence, in this case s_{jkt} is not a Sharpe ratio, but is instead a geometric average, since $\gamma_j \in (0, 1)$ for all j, between the expected return of ℓ_{jkt} and the net worth of the beneficiary of such loan. Notice that, since the net worth shall be seen to be substantially important as a shield against credit risk, once more reverting back to the work by Brunnermeier (2009), it is rational for a bank to take such variable into account when deciding the proportion of loans to allocate to a given firm.

To specify the overall amount of loans a bank issues, assume that the bank consider stock returns to be normally distributed[4]. In this case, the per-dollar Value-at-Risk is given by

$$VaR_{a,jt} = \sqrt{2}\sigma_{jt}\mathrm{erf}^{-1}(2a - 1) \tag{12}$$

where a is the VaR quantile, $\mathrm{erf}(\cdot)$ is the error function, defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{13}$$

and σ_{jt} is the estimated variance of j's portfolio. Given the firm-specific estimates of the variances whereof (10b), we can write $\sigma^2_{jt} = \sum_{k \in \Omega_{jt}} w^2_{jkt}\sigma^2_{jkt}$. A bank following a VaR rule tries to ensure its nominal VaR not to exceed its equity in market value, that is $\ell_{jt} VaR_{jt} \leq p_{jt}E$, where ℓ_{jt} is the size of the asset size of the

[3]Since productivity is firm-specific, we need to take an average across firms.

[4]Aymanns & Farmer (2015) show that this formulation is actually applicable also for non-Gaussian distributed returns.

balance sheet, *i.e.* the total amount of loans the bank issues. Hence, assuming the constraint to be binding, then letting $\alpha_{jt} = \alpha \equiv [\sqrt{2}\mathrm{erf}^{-1}(2a-1)]^{-1}$ one has that the target leverage λ_{jt} is the inverse of the VaR_{jt}, that is $\lambda_{jt} \equiv \min\{VaR_{jt}^{-1}, \bar{\lambda}\} = \min\{\alpha/\sigma_{jt}, \bar{\lambda}\}$, where $\bar{\lambda}$ is a regulatory threshold and, assuming it not to be binding for explanatory purposes, one has[5]

$$\ell_{jt} = \lambda_{jt} p_{jt} E = \frac{\alpha p_{jt} E}{\sigma_{jt}} \tag{14}$$

Equation (14) is crucially important for it embodies a twofold conclusion. First, $\partial \ell_{jt}/\partial p_{jt} > 0$, meaning that each bank increases the amount of loans it issues whenever the market value of its equity increases. This in turn can be understood as the fuelling mechanism of a secular credit growth: provided the returns of bank equity to be on average slightly higher than their prices, then p_{jt} shall increase over time and, all else being equal, the overall supply of credit increases. As originally noted by Schularick & Taylor (2012), in the long period credit tends to grow and make the economy more reliant on the financial sector.

Second, the less risky the environment is perceived, the more credit grows, since $\partial \ell_{jt}/\partial \sigma_{jt} < 0$. Albeit intuitive, and indeed quite tautologic, this conclusion is remarkable. Indeed, there is no reason why to suppose *perceived* risk to move in the same direction of actual credit risk. Hence, credit can in principle increase even though the financial conditions of firms are deteriorating. In other words, banks can prefer to lend more, thus increasing the returns they gain, than reduce the size of their balance sheet, thus reducing their return on equity, if firms, albeit financially less reliable, have not fallen short of paying their debt through outstanding net worth.

In their original contribution, Aymanns & Farmer (2015) further generalize (14) and write

$$\lambda_{jt} = \alpha_{jt}(\sigma_{jt}^2 + \sigma_{j0}^2)^b \tag{15}$$

where $\alpha_{jt} \to \alpha$ determines the scale of bank j's leverage, σ_{0j} is intended as embodying perceived risk, while b is a cyclicality parameter: if $b = -1/2$ and $\sigma_{0j} = 0$, we retrieve the baseline case whereby (14). More generally, the authors label $b < 0$ as a procyclical leverage policy, in which $\partial \ell_{jt}/\partial \sigma_{jt} < 0$, whereas $b > 0$ is termed as procyclical leverage targeting, in which $\partial \ell_{jt}/\partial \sigma_{jt} > 0$. Thus, we may say that we limit our analysis to the procyclical, more reasonable, case.

Given that $\ell_{jt} = \sum_{k \in \Omega_{jt}} \ell_{jkt}$, we can close the supply side of the credit market, for

$$\ell_{jkt} = w_{jkt} \ell_{jt} = \frac{\exp\left(\beta_j \cdot \mu_{jkt}^{\gamma_j} A_{kt}^{1-\gamma_j}\right)}{\sum_{k \in \Omega_{jt}} \exp\left(\beta_j \cdot \mu_{jkt}^{\gamma_j} A_{kt}^{1-\gamma_j}\right)} \cdot \frac{\alpha p_{jt} E}{\sigma_{jt}} \tag{16}$$

is the supply of bank j to firm k.

Furthermore, let $\tilde{\ell}_{jt}$ be the realized returns on loans. If, by chance, all firms belonging to j's customer set paid their debt obligations in full and the bank met no constraints in allocating its supplied credit,

[5] The rule in (14) is consistent with many studies dealing with the Basel II-III implied leverage procyclicality (Aymanns *et al.*, 2016).

$\tilde{\ell}_{jt} = \tilde{\ell}^M_{jt} = \sum_{k \in \Omega_{jt}} (1 + r_{jkt})\ell_{jkt}$. More in general, however, $\tilde{\ell}_{jt} \leq \tilde{\ell}^M_{jt}$, and bank j's gross dividends to be divided among the equity owners are computed as a fraction of the profits according to

$$d^G_{jt} = \rho_j \cdot \tilde{\ell}_{jt} \tag{17}$$

where $\rho_j \in (0, 1)$, while $A_{jt} = A_{jt-1} + (1 - \rho_j) \cdot \tilde{\ell}_{jt}$. The return on equity is thus evaluated as the ratio $d_{jt} = d^G_{jt}/(E \cdot p_{jt})$, that is the ratio between the unit return and the unit price of bank equity.

2.2 The real sector

The real sector is made up of a productive sector, which is in turn composed of firms, and households. The latter can in turn be workers, customers or both, in which case they respectively work, consume or both earn labor income and spend it in consumption.

2.2.1 Firms

Firms constitute the productive sector of the economy in that they convert financial resources, either them be external, *i.e.* credit, or internal, into the consumption good. Firms thus represent the demand side in both the credit market and the labor market, but are on the supply side of the market for the consumption good. Firms all belong to the same sector in that there exists only one homogeneous consumption good, hence we simply the structure whereof Dosi, Fagiolo & Roventini (2010), on which we nonetheless extensively draw. Since production requires labor and capital, whenever firms wish to enlarge their production they undertake in-house investments to enhance their physical capital endowments. Also, they set out in-house R&D activities, which foster technological progress.

As said, let $\mathbb{K} = \{k\}^K_{k=1}$ be the set of firms, and $\mathbb{H} = \{h\}^H_{h=1}$ that of households. Furthermore, let $\Omega^C_{kt} \subseteq \mathbb{H}$ be the set of customers of firm k at time t and $\Omega^N_{kt} \subseteq \mathbb{H}$ be the set of employees of firm k at time t. In analogy to the customer sets of banks, by construction one has that $\Omega^C_{kt} \cap \Omega^C_{k't} = \emptyset$ for each $k \neq k' \in \mathbb{K}$, meaning that no h can belong to more than one customer set, and $\cup_{k \in \mathbb{K}} \Omega^C_{kt} = \mathbb{H}$, meaning that each household always belongs to a given customer set. With respect to the employees, one has that $\Omega^N_{kt} \cap \Omega^N_{k't} = \emptyset$, meaning that no household can have more than one job, *i.e.* if h is employed, it sells all the labor he is endowed with to its unique employer k. Further let $\mathbb{U}_t \equiv \left(\mathbb{H} \cap \cup_{k \in \mathbb{K}} \Omega^N_{kt}\right)'$, where $'$ is intended as the complement set with respect to $\mathbb{H}$. Then, $\mathbb{U}_t \subseteq \mathbb{H}$ is the unemployment set, and $\mathbb{U}'_t \subseteq \mathbb{H}$ is the employment set, *i.e.* the set of all households employed in whichever firm at time t.

The *production function* is Leontief, that is

$$Y_{kt} = \xi_{kt} \min\{K_{kt}, \zeta N_{kt}\} \tag{18}$$

where ξ_{kt} is the firm-specific productivity, while $N_{kt} = \#\Omega^N_{kt}$ is the labor provided by the employees belonging to k's customer set and ζ is a parameter entailing the productivity of labor with respect to capital. We shall see that technological progress in our framework is Solow-neutral in that it influences ξ_{kt} rather than ζ.

Firms plan production at time t given previous met and unmet demand. More specifically, let Y^D_{kt-1} be past demand, and $\Delta Y_{kt} = Y^D_{kt-1} - Y^S_{kt-1}$ be past excess demand, *i.e.* unmet demand. Notice that if $\Delta Y_{kt} > 0$, firm k was demanded more than it produced, whereas $\Delta Y_{kt} < 0$ implies an overestimation of demand at time $t-1$. Hence, let Y^e_{kt} be the desired production level, one has

$$Y^e_{kt} = Y^D_{kt-1} + \lambda \Delta Y_{kt} \tag{19}$$

Equation (19) states that firm k, short of any optimal decision rule such as a maximization algorithm, adjusts its production plans according to previously observed demands[6]. According to the production plan thus computed and the production technology (18), the firm readily solves its production problem in terms of the desired quantities of labor and capital:

$$K^e_{kt} = \frac{Y^e_{kt}}{\xi_{kt}} \tag{20a}$$

$$N^e_{kt} = \frac{Y^e_{kt}}{\zeta \xi_{kt}} \tag{20b}$$

From equation (20a) one can further retrieve the investment level firm k would like to implement, which is simply $I^e_{kt} = \max\{K^e_{kt} - K_{kt}, 0\}$, assuming no disinvestment is in order. Following Dosi, Fagiolo & Roventini (2010), we assume $RD^e_{kt} = \chi \Pi_{kt-1}$, where RD^e_{kt} is the desired R&D expenditures, $\chi \in (0, 1)$ and Π_{kt-1} is the past level of profits. These three uniquely determine the demand for credit ℓ_{kjt}:

$$\ell_{kjt} = N^e_{kt} + I^e_{kt} + RD^e_{kt} - \Phi^k A_{kt} \tag{21}$$

where Φ^k is the propensity of firm k to use its net worth for routine and investment operations. Equation (21) allows firms to finance their investment and employment decisions with both internal and external sources, the former being their net worth, the latter being loans[7].

Let $\tilde{\ell}_{jkt}$ be the outcome of the credit market with respect to the access to credit for firm k from bank j[8]. The firm uses up this credit and the internal sources according to the proportions computed on the basis of their

[6] We tested more complex specifications for the production plans yielding no relevant differences with respect to that of this simpler version.

[7] One can understand (21) as a precautionary argument: the bigger the firm, the more access to credit it attains, and the more stored wealth it wished to hold to face demand downturns.

[8] Clearly, $\tilde{\ell}_{jkt} = \tilde{\ell}_{kjt}$ so we simply write the former when considering both the bank and the firm. Clearly, $\ell_{jkt} \neq \ell_{kjt}$ on the other hand, the former being the baseline supply of credit, the latter being the demand.

desired levels, *i.e.* $I_{kt} = \alpha_{I,kt} I^e_{kt}$, $RD_{kt} = \alpha_{RD,kt} RD^e_{kt}$ and $N^D_{kt} = \alpha_{N,kt} N^e_{kt}$ where

$$\alpha_{I,kt} = \frac{I^e_{kt}}{\ell_{kjt} + \Phi^k A_{kt}} \tag{22a}$$

$$\alpha_{RD,kt} = \frac{RD^e_{kt}}{\ell_{kjt} + \Phi^k A_{kt}} \tag{22b}$$

$$\alpha_{N,kt} = \frac{N^e_{kt}}{\ell_{kjt} + \Phi^k A_{kt}} \tag{22c}$$

where N^D_{kt} is now the demand for labor firm k can exert in the labor market, given the access to finance it has been conceded and its own preferences.

Technological progress is to a large extend inspired by the K+S model but, since we only consider one sector, innovative activities are supposed to be pursued in-house. Firms employ their R&D expenditures RD_{kt} in imitation and innovation:

$$IN_{kt} = \eta RD_{kt} \tag{23a}$$

$$IM_{kt} = (1-\eta) RD_{kt} \tag{23b}$$

where $\eta \in (0, 1)$. Obviously, for firms on the technological frontier $\eta = 1$, *i.e.* they only try to innovate.
Innovation is a two-step process. First, one determines whether firm k has access to innovation, more specifically this happens if, given

$$\vartheta^{IN}_{kt} = 1 - \exp(-\iota_1 \cdot IN_{kt}) \tag{24}$$

with $\iota_1 \in (0, 1]$, a random draw from a Bernoulli distribution with parameter ϑ^{IN}_{kt} is 1. If this is the case,

$$\xi^{IN}_{kt} = \xi_{kt-1}(1 + x_{kt}) \tag{25}$$

where $x_{kt} \sim \beta(a, b)$ where the distribution has support set over $[-1, 1]$. Notice that, therefore, the more firm k invests in R&D activities, the more likely will it be to access innovation, as exemplified by (26): in the limit case in which $RD_{kt} \to \infty$, also $IN_{kt} \to \infty$ and hence $\vartheta^{IN}_{kt} \to 1$. Also, innovation is generally incremental, in the sense that $x_{kt} > 0$, and thus $\xi^{IN}_{kt} > \xi_{kt-1}$. In this sense, there are positive returns stemming from innovation activities.
Imitation is similar to innovation in that the possibility to access imitation is drawn from a Bernoulli distribution whose parameter is, similarly to (26):

$$\vartheta^{IM}_{kt} = 1 - \exp(-\iota_2 \cdot IM_{kt}) \tag{26}$$

where $\iota_2 \in (0, 1]$, and firm k can imitate a rival provided 1 to be drawn at random. Provided this to be the case, k observes a subset of $\nu \cdot (\#\mathbb{K})$, $\nu \in (0, 1)$, competitors with a probability that is directly proportional to the technological similarity between k and the competitors' technologies, measured as the Euclidean distance between the two:

$$\mathcal{P}(k'|k) = \frac{||\xi_{kt}, \xi_{k't}||}{\sum_{k' \in \mathbb{K}} ||\xi_{kt}, \xi_{k't}||} \tag{27}$$

Once the set of observed firms is known, the imitated technology ξ_{kt}^{IM} is simply the highest among these, provided it to be higher than k's already employed technology.

If k engaged in innovation *and* imitation, its newly employed technology shall feature a productivity given by

$$\xi_{kt} = \max\left\{\xi_{kt}^{IN}, \xi_{kt}^{IM}\right\} \tag{28}$$

Prices are set by the firms as a mark-up over the average cost of production. Indeed, one has that

$$p_{kt} = (1 + \mu_{kt})c_{kt} \tag{29}$$

where μ_{kt} is the firm-specific mark-up and c_{kt} is the unitary cost of production, computed as the fraction between total costs, in turn given by interests and labor costs, and total production. The dynamics of the mark-up are related to the evolution of firm k's market share:

$$\mu_{kt} = \mu_{kt-1}\left(1 + \mu\frac{s_{kt-1} - s_{kt-2}}{s_{kt-1}}\right) \tag{30}$$

where $\mu \in [0, 1]$ and s_{kt} is k's market share at time t. Since the consumption good market is informationally imperfect, we shall see that customers switch across firms on the basis of the prices these charge for the homogeneous consumption good. Still, since another factor of competitiveness is the level of unfilled demand ΔY_{kt}, we evaluate competitiveness of firm k as

$$e_{kt} = -\omega_1 p_{kt} - \omega_2 \Delta Y_{kt} \tag{31}$$

where $\omega_1 > 0$ and $\omega_2 > 0$. Average competitiveness is thus computed as the weighted average between all market competitiveness evaluated through (31), yielding

$$\bar{e}_t = \sum_{k=1}^{K} e_{kt} s_{kt} \tag{32}$$

and thus market share can be solved for in terms of a quasi-replicator dynamics stemming from the competitiveness of any given firm k and the average competitiveness thus computed through (32)[9]. Hence we have

$$s_{kt} = s_{kt-1}\left(1 + \sigma\frac{e_{kt} - \bar{e}_t}{\bar{e}_t}\right) \tag{33}$$

where $\sigma > 0$.

To evaluate the *profits*, let $\tilde{Y}_{kt}$ be the sales of firm k at time t following its participation in the consumption-good market. Then, profits are equal to

$$\Pi_{kt} = p_{kt}\tilde{Y}_{kt} - w_t N_{kt} - r_{jkt}\tilde{\ell}_{jkt} \tag{34}$$

If $\Pi_{kt} \geq 0$, the firm is fully able to repay its debt obligation along with the implied interests. If, however, $\Pi_{kt} < 0$, the firm may still be able to repay the debt along with the interests, provided the net worth to be

[9] We label this a 'quasi'-replicator for a canonic replicator evolves on the unit simplex, and requires all agents to have positive shares. Since (33) allows share to be eventually negative, this terminology is more appropriate (Dosi, Fagiolo & Roventini, 2010).

sufficient. If this was not the case, the firm shall be seen to go bankrupt and its net worth will be primarily devoted to pay the salaries and, marginally, to repay the debt obligation. More in general, we can write that the *realized* return over a loan $\tilde{r}$ is given by

$$\tilde{r}_{jkt} = \begin{cases} r_{jkt} & \text{if } \Pi_{kt} + A_{kt-1} \geq 0 \\ \frac{A_{kt}+\Pi_{kt}}{\tilde{\ell}_{jkt}} & \text{else} \end{cases} \tag{35}$$

For completeness, we can marginally write the equation governing the motion of firms' net worth, that is

$$A_{kt} = A_{kt-1} + \begin{cases} (1-\rho_k)\Pi_{kt} & \text{if } \Pi_{kt} \geq 0 \\ \Pi_{kt} & \text{if } \Pi_{kt} < 0 \end{cases} \tag{36}$$

where $\rho_k \in (0, 1)$ is the share of profits that is retained by the firm for precautionary purposes. Notice that the remainder, *i.e.* $\rho_k \Pi_{kt}$ is distributed across the owners of the firm, *i.e.* the households.

2.2.2 Households

Households maintain a twofold role. On the one hand, as seen, they sell labor to firms in exchange for labor income, that is, salary. On the other, they act as consumers, and are thus the demand side of the goods market. We briefly describe these two functions.

Households as *workers* $h \in \mathbb{H}$ are such that either $\exists! k \in \mathbb{K}$ such that $h \in \Omega^N_{kt}$ or $h \in \mathbb{U}_t$, meaning that each h can either be employed or unemployed. Its labor income $\tilde{w}_{ht}$ is thus

$$\tilde{w}_{ht} = \begin{cases} \bar{w}_t & \text{if } h \in \mathbb{U}_t \\ w_t(1-\tau) & \text{if } h \in \mathbb{U}'_t \end{cases} \tag{37}$$

where $\bar{w}_t < w_t$ is the reservation wage, that is some sort of social security wage, and τ is an income tax. Equation (37) states that if a household h is unemployed, his labor income shall be equal to the reservation wage, which is some fraction of the market wage. If, on the other hand, h is employed, he will enjoy the market wage, whose value we shall see below.

Now, let $\sigma_h = \{\sigma_{hk}\}$ be the vector that associates to household h the share of firm $k \in \mathbb{K}$ he owns. Thus, we shall have

$$\begin{aligned} &\sum_{h\in\mathbb{H}} \sum_{k\in\mathbb{K}} \sigma_{hk} = K_t = K \\ &\sum_{h\in\mathbb{H}} \sigma_{hk} = 1 \\ &\sum_{k\in\mathbb{K}} \sigma_{hk} = 1 \end{aligned} \tag{38}$$

meaning that (i) all households overall possess the same ownership shares, (ii) the shares sum up to one for each k, and (iii) ownership shares are fixed over time, that is they cannot be traded. That said, ownership shares confer the right to enjoy a portion of the profits each firm decides to divide among its owners. Let y_{ht}

be the overall income of household h, then we shall write

$$y_{ht} = \tilde{w}_{ht} + \sum_{k \in \mathbb{K}} \sigma_{hk} \cdot (\rho_k \Pi_{kt}) \mathbf{1}(\Pi_{kt} \geq 0) \tag{39}$$

where $\mathbf{1}(\Pi_{kt} \geq 0)$ is an indicator function that is 1 if and only if the specified condition holds.

Households as *customers* are such that for all $h \in \mathbb{H}$, $\exists! k \in \mathbb{K}$ such that $h \in \Omega^C_{kt}$. Given their income, at most each household can demand y_{ht}/p_{kt}, where p_{kt} is the price of the consumption good that is decided by the firm whose customer set household h belongs to.

Associated to the household side, there is a *Government* that provides the social wage to unemployed workers. The government's budget constraint is Ricardian: unemployed workers are given the minimum between the social security wage by law, and the overall tax income divided by the number of unemployed workers. Let $U = \#\mathbb{U}$, then for each $h \in \mathbb{U}$,

$$\bar{w}_{ht} = \min \left\{ (1 - \bar{w}) w_t, \frac{\tau}{U} \sum_{h \notin \mathbb{U}} w_t \right\} \tag{40}$$

where $\bar{w}$ is the mandatory minimum wage, defined as a share of the current wage. Equation (40) states that the government never runs deficit, and cannot provide unemployed workers with more labor income than the level that is exogenously fixed.

2.3 Market Dynamics

In this section we briefly describe the market structure of the three remaining markets, namely the credit, labor and goods ones. Indeed, we already saw the functioning of the equity market, its clearing and the pricing mechanism it entails. Furthermore, notice that the equity market, unlike the remaining three, is walrasian, albeit correcting for expectations mismatch, whereas the ones we deal with are post-walrasian in that they do not postulate market clearing.

2.3.1 Credit market

In the credit market, banks supply credit to firms. The interest rate cannot act as the market clearing variable, for equation (9) states that banks simply decide the interest rate they charge upon firms given some parameters. Thus, the credit market implicitly entails rationing, whether it be on the supply side, *i.e.* banks being unable to place their loans, or affect the demand side, thus causing credit rationing against the productive sector. Indeed, one may acknowledge the influence of the seminal work by Stiglitz & Weiss (1981) upon our framework: imperfect information prevents the market from clearing and the interest rate does *not* act as an equilibrium variable, hence credit rationing is commonplace.

Equations (16) and (21) respectively state the supply of loans a given bank wishes to place, and the demand a given firm would like to achieve. One has

$$\ell_{jkt} = \frac{\exp\left(\beta_j \cdot \mu_{jkt}^{\gamma_j} A_{kt}^{1-\gamma_j}\right)}{\sum_{k\in\Omega_{jt}} \exp\left(\beta_j \cdot \mu_{jkt}^{\gamma_j} A_{kt}^{1-\gamma_j}\right)} \cdot \frac{\alpha p_{jt} E}{\sigma_{jt}} \tag{41a}$$

$$\ell_{kjt} = N_{kt}^e + I_{kt}^e + RD_{kt}^e - 2[1-\Phi(A_{kt})]A_{kt} \tag{41b}$$

Hence, in principle one would have that

$$\tilde{\ell}_{jkt} = \min\left\{\ell_{jkt}, \ell_{kjt}\right\} \quad \Rightarrow \quad \tilde{\ell}_{jt} = \sum_{k\in\Omega_{jt}} \tilde{\ell}_{jkt} = \sum_{k\in\Omega_{jt}} \min\left\{\ell_{jkt}, \ell_{kjt}\right\} \tag{42}$$

However, this market structure would be rife with imperfections that are too pervasive and prevent banks from placing a significant fraction of their supplied loans, as well as overly increase the burden of credit constraints upon firms. Therefore, we take a step forward and consider $\Delta\ell_{jt} = \ell_{jt} - \tilde{\ell}_{jt}$, that is the amount of loans that bank j would have liked to issue, but that is not consistent with (42). If $\Delta\ell_{jt} > 0$, the bank attaches new weights to all those firms whose demand had not been satisfied by (42) according to the aforementioned heuristic rule (11) and re-allocates the residual credit according to the minimum between the firm-specific supply thus set and the residual demands. This algorithm goes on until either (i) no firm in the customer set is demanding credit, or (ii) the bank no longer wishes to issue loans.

The market structure thus allows for credit rationing but does not impose a burden that would be too heavy either upon firms or banks. For the sake of notation clarity, we shall label $\tilde{\ell}_{jkt}$ as the realized loan from bank j to firm k, which shall in general be equal or greater than the one whereof equation (42), and $\tilde{\ell}_{jt}$ is the overall sum of realized loans bank j places among its customers.

2.3.2 Labor market

In the labor market firms, here on the demand side, seek to employ as many workers as their production plan requires. Let N_{kt}^D be the labor that each firm demands at time t. N_{kt}^D is determined *after* the credit market takes place, for $N_{kt}^D = \alpha_{N,kt} N_{kt}^e$, that is the desired demand for labor for any given firm is corrected by a factor scaling it by the access to credit that firm had been granted as a consequence of the banks' decisions.

The labor market is not Walrasian, in that similarly to the interest rate in 2.3.1, the wage does not serve as a market clearing variable between the overall demand and supply of labor. For simplicity, we assume that the nominal wage is centrally set, for instance as would be the case in a fully unionized industry:

$$w_t = w_{t-1}\left(1 + \psi_1 \frac{\Delta U_{t-1}}{U_{t-1}} + \psi_2 \frac{\Delta\pi_t}{\pi_t} + \psi_3 \frac{\Delta\bar{\xi}_t}{\bar{\xi}_t} + \psi_4 \Pi_{t-1}\right) \tag{43}$$

where $U = \#\mathbb{U}$, $\psi_1 < 0$ and $\psi_{2,3} > 0$ and Π_t is the value-added-adjusted mean profit.

Equation (43) states that the centrally set nominal wage corrects the past one by the change in unemployment, inflation and the change in average productivity. It is noteworthy to notice that unemployment and the wage rate tend to move one opposite to the other. Hence, assuming employment to be procyclical, then unemployment is countercyclical and, thus, nominal wages shall thus be procyclical, for $\psi_1 < 0$. In a sense, equation (43) implicitly states that the expected correlation between nominal wages and output is positive: Marx's theory to some extent relate to the infamous Phillips curve, for inflation in nominal wages is typically related to inflation in the consumption price index.

As said, each firm k has an employees set Ω^N_{kt}. If $N^D_{kt} = \#\Omega^N_{kt}$, then firm k is satisfied with its current employees, and no further adjustment is needed. Still, if $N^D_{kt} < \#\Omega^N_{kt}$, then $\Delta N_{kt} = \#\Omega^N_{kt} - N^D_{kt}$ workers are fired from firm k's employees set and join the unemployment pool $\mathbb{U}_t$. On the contrary, if $N^D_{kt} > \#\Omega^N_{kt}$ the firm seeks to enlarge the number of its employees. Firms looking for more employees are drawn in random order, and each hires a number of workers $\Delta\#\Omega^N_{kt}$ equal to

$$\Delta\#\Omega^N_{kt} = \min\{-\Delta N_{kt}, U_t\} \tag{44}$$

that is the minimum between the demanded labor and the overall unemployed available to firm k at time t.

2.3.3 Goods market

In the goods market firms, on the supply side, try to sell the consumption good to their customers, on the demand side. The goods market too is non-Walrasian in that firms set the prices according to (29) and the related equations determining the mark-ups. However, this by no means entail that an equilibrium consistent with market clearing arises, hence constraints shall affect either the demand, that is the produced quantity cannot match the demand, or the supply. As seen, the goods market is segmented just as the credit one, the difference being that customers are *not* the same households that are also employed by the firm they buy from.

Equations (19) and (39) respectively state the supply and demand of the consumption good for firm k at time t, the former being determined by k's production plans, access to credit and success on the labor market, the latter implied by the firms' past profits and households within Ω^C_{kt} being employed:

$$Y^S_{kt} = \xi_{kt} \min\{K_{kt-1} + I_{kt}, N_{kt}\} \tag{45a}$$

$$Y^D_{kt} = \sum_{h\in\Omega^C_{kt}} \frac{y_{ht}}{p_{kt}} = \frac{1}{p_{kt}} \sum_{h\in\Omega^C_{kt}} \left[\tilde{w}_{ht} + \sum_{k\in\mathbb{K}} \sigma_{hk} \cdot (\rho_k \Pi_{kt}) 1(\Pi_{kt} \geq 0)\right] \tag{45b}$$

where I_{kt} and N_{kt}, as said, depend first and foremost on the credit that firm k enjoys following the credit market takes place. This is in turn influenced by the leverage ratio firm k's bank sets, k's financial solidity and a full set of other state variables we already described. Also, the latter variable is influenced by the success k had on the labor market, that is whether the firm was able to fulfill its labor requirements given the access to credit it had been granted and the production plans it has set.
That said, equations (45a) and (45b) determine the sales $\tilde{Y}_{kt}$ of firm k, yielding

$$\tilde{Y}_{kt} = \min\left\{Y^S_{kt}, Y^D_{kt}\right\} \tag{46}$$

and, since the goods market operates in the aggregate of customers and the good is perishable, it is simpler than the credit market with respect to the 'rolling over' of the unplaced loans procedure we explained for that market structure. Also, once (46) determines the sales for each firm, one can further evaluate profits through (34), through which in turn (i) real returns on debts is computed; (ii) firms award eventual profits to the households for future consumption purposes; (iii) banks pay back dividends on equity to the equity owners; (iv) a new market value of bank equity is attained through trading. Hence, the model is virtually closed, for point (iv) implies that another iteration has begun.

2.4 Evolutionary selection

We now discuss the mechanisms of selection we embody in our model to convey the idea that an ABM needs to be evolutionary consistent, that is, agents cannot pursue strategies that are not only non-optimal, but are strictly dominated by their competitors', and survive *only* because of imperfections preventing their rivals from enlarging their market shares. Indeed, this idea is strictly related to an insight stemming from the evolutionary literature affirming that markets act as selective environments for agents that are inherently heterogeneous and present different competitive capabilities Nelson & Winter (1982). Also, the selection criteria themselves are endogenous, in that they are "a collective property of the rates and directions of learning of each and every firm", something we extend here to banks as well (Dosi *et. al.*, 1995).
As we already mentioned, we do explicitly make strategies evolve. Hence, our model is not evolutionary consistent *lato sensu*. However, we do seek to *select* agents according to some fitness rules in order not to let the less competitive ones stay in business merely in the light of market frictions. Hence, in this section we describe the rules we adopt to prevent this from happening.

First, firms and banks can go *bankrupt*. We assume that firms can go bankrupt if either (i) their net worth approaches zero, *i.e.* $A_{kt} \approx 0$; or (ii) their market share approaches zero, *i.e.* $s_{kt} \approx 0$.
Notice that, while the first condition resembles an accounting identity, the second one is implied by firm k having no customers. This very same criterion could be applied to banks as well. However, we do not delve into the issues related to bank bankruptcies and only apply the selection mechanism to foster bank competition, despite avoiding bank bankruptcies.
Firm bankruptcies due to net worth shortages occur whenever a given firm repeatedly demands credit at an interest rate that implies a unitary price of the consumption good that is too high with respect to that of rivals. This, as we shall see shortly, makes customers switch firm and sales for the given firm fall. Clearly this instance is also related to the second one whereby a firm is declared to have gone bankrupt whenever its market share approaches zero.
Whenever a firm goes bankrupt, it is replaced by an entrant whose net worth is computed as the maximum between outstanding savings by households, which in turn are computed as household income minus sales,

and a fraction of the incumbents' net worth, consistent with the empirical studies suggesting entrants being on average smaller than incumbents (Dunne, Roberts & Samuelson, 1988). Thus, the model is stock-flow consistent in the sense that the aggregate net worth of entrants is always at most equal to the level of outstanding savings, and ownership shares are divided across households in proportion to the contribution they provided to the entrant's net worth[10].

Second, selection is operated through *switching rules*, in the spirit of Delli Gatti *et al.* (2010). Firms and households can decide to switch their partner in, respectively, the credit and goods market based on an index of fitness. In the former relationship, that is the one between a bank and the set of its customers, firms can decide to switch bank if they are charged a price for financial resources that they deem as excessive. By the same token, households decide to switch firm whenever they consider the price of the consumption good excessive with respect to the one set by a subset of observed firms.

More specifically, consider firms switching banks in the credit market. One has that each firm $k \in \mathbb{K}$ and currently belonging to Ω_{jt} observes a subset of n_j banks, $n_j << J$, and we define the probability to switch to a different bank as

$$\mathcal{P}(k \in \Omega_{j't+1} | k \in \Omega_{jt} \wedge r_{jkt} > \bar{r}_{j't}) = \text{Bin}\left(1, 1 - \exp\left(-\lambda_k \frac{r_{jkt} - \bar{r}_{j't}}{r_{jkt}}\right)\right) \tag{47}$$

where $\lambda_k > 0$. Equation (47) states that the probability, for a firm currently belonging to a given customer set to switch to another one, conditional on observing that the average interest rate charged by the other firm is lower than the one it currently pays, decreases if the current interest rate increases and decreases the more competitive rivals of the current bank are. Also, in the limit case of $\bar{r}_{j't} \to 0$, $\mathcal{P}(\cdot) \to 1$, that is the firm switches with certainty. Notice, however, that in (47) we state the probability to switch conditional on $r_{jkt} > \bar{r}_{j't}$: if that is not the case, we implicitly assume such probability to be null, for there would be no reason for a firm to change bank.

Consider now a household $h \in \mathbb{H}$ currently belonging to Ω^C_{kt} who observes a subset of $n_k << K$ firms. We define the probability that household has to switch firm in terms of the price the observed firms set:

$$\mathcal{P}(h \in \Omega^C_{k't+1} | k \in \Omega^C_{kt} \wedge p_{kt} > p_{k't}) = \text{Bin}\left(1, 1 - \exp\left(-\lambda_h \frac{p_{kt} - p_{k't}}{p_{kt}}\right)\right) \tag{48}$$

The analogy between (47) and (48) is quite clear. Households seek to consume the most, and thus pay the lowest possible price. Hence, they are more likely to switch to a new firm the cheaper this new firm is. This is the rationale of (48), which once more states the probability to switch conditional on observing $p_{kt} > p_{k't}$, for we assume it to be zero provided this not to be the case.

The two mechanisms of selection, that is, ruling out firms through bankruptcies and allowing agents to select their partners in the credit and goods markets, are clearly related. A firm that is repeatedly charging

[10] Still, we acknowledge that this is somehow artifact. Savings are only used to finance entrants and not to defer consumption. Still, in the light of Caiani *et al.* (2016), we feel that is important to ensure the stock-flow consistency of any ABM model.

prices above the average shall be likely to loose customers due to (48). However, while also increasing the likelihood of bankruptcies, this shall also increase the excess of demand that firm faces, and thus lower the mark-up it charges and, hence, prices, possibly allowing it to stay in business again through (48).

Overall, the selection criteria hereby described operate *post rem* in the sense that they do not delve into the strategies that agents employ, but rather on the outcomes these strategies imply. This is not to say that agents do not *learn*: simply, the strategy they follow is the same, and update their beliefs according to it. Also, it is noteworthy that albeit simple, these evolutionary rules do not cause the same agents to be driven off the market on and on. Thus albeit entrants are endowed with the same strategy as those being ruled out of the market, this does not imply them to be driven out of business with certainty. Hence, the selection structure is neither automatic nor deterministic, and there is no clear 'optimal' strategy to follow that grants a better performance.

3 Simulation Results

After presenting the stage of the simulations, *i.e.* the initial conditions and parameters on which the simulations themselves are valued, we show one sample realization of the economy we have been describing. This shall be a qualitative presentation, whose purpose is to make the reader grasp the most basic and interesting facts stemming from the model equations. We will thus present some stylized facts that the model is able to replicate in a qualitative fashion.

Moving on, we shall undertake a sounder assessment of the capability of the model to replicate some of the empirical stylized facts we dealt with in the previous chapter. We do so by running a Monte-Carlo simulation as suggested by Windrum, Fagiolo & Moneta (2007), that is we run N independent simulations of the model for a given time span and analyze the time series generated by the model. More specifically, we shall present statistical evidence on the stationarity of the time series and on cross-correlations between different macroeconomic indicators. Indeed, we borrow from the business cycles investigation this methodology in order to assess the coherence of our model with the empirical findings whereof the macroeconometric literature. We also document the facts on firm dynamics we shall be able to replicate, most notably those dealing with financial constraints and size and growth distributions asymmetric skewness.

In table 8 we list the parameters we employ, along with a simple description, and the value they are assigned in the simulations we are to discuss[11]. The simulation runs as a small-to-medium scale model, both in the light of computational constraints *and* given that the dynamics are not substantially enriched by adding more agents. In principle, however, the code would run whichever $\{I, J, K, H\}$, as it is programmed in objects[12].

We briefly comment on some of the choices for the parameters. Concerning the MA(1) expectations formation parameters δ_i and δ_j, equity owners are typically more chartist and trend followers than leveraged investors, as Braun-Munzinger, Liu & Turrell (2018) and Shek, Shim & Shin (2017) pointed out. In turn, given that the former have been shown to mostly be trend-followers, also the weight they attach to the fundamentalist component of the return expectation γ_i is relatively low. Banks value their customers' net worth more than their expected return when evaluating the shares of total loans to allocate, as outlined by the values of γ_j. This is consistent with much of the literature stemming from Stiglitz & Weiss (1981) dealing with informational asymmetries in financial markets, which underlines the importance of collateral as a borrowing constraint, *i.a.* as Kiyotaki & Moore (1997). The interest rates on average equally depends on its four determinants, whereas we borrow the VaR-quantile from Aymanns & Farmer (2015).

Concerning the real sector, firms are more than banks, and much less than households, hence each firm hires some 20 workers at the beginning of the simulation. The value of λ says that firms are wishing to adjust their produce quantity by 70% of the excess demand. We run robustness checks changing λ yielding substantially similar results. The value of χ is inspired to Dosi, Fagiolo & Roventini (2010). Most of the parameters determining the behavior of firms are taken not to be specific to each firm. This notwithstanding, firm heterogeneity naturally stems from the different credit access each bank grants to its customers, hence it is not necessary to assume *a priori* different parameters.

As said, households are more than both firms and equity owners. We assume that the centrally set wage equally weights changes in unemployment, productivity and inflation. This is to say that the unit cost of labor keeps pace with the three at equal rate. Albeit extremely stylized, the rationale of it is that (i) nominal wages are found to be correlated with inflation, that is to some extent they are indexed, for real wages show no correlation with the business cycle (Abraham & Haltiwanger, 1995); (ii) nominal wages tend to move contrary to unemployment, something that is shown theoretically *i.a.* by Mincer & Jovanovic (1981) and empirically in

[11] In the table, $\mathcal{T}(\cdot)$ stands for the triangular distribution, whose p.d.f. reads out as

$$\mathcal{T}(a, c, b, x) = \begin{cases} 0 & \text{if } x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{if } a \leq x < c \\ \frac{2}{b-a} & \text{if } x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{if } c < x \leq b \\ 0 & \text{if } x > b \end{cases}$$

It is employed as a way to populate a set of parameters with sufficiently thick tails, that is, sufficient heterogeneity at the borders of the support set.

[12] Appendix B describes in detail how to run the code, and the Github page which hosts it. All codes are freely available.

the seminal work by Phillips (1958)[13]; (iii) nominal wages are positively correlated with productivity, albeit the phenomenon known as the 'productivity gap' is increasingly worrying (Hellerstein, Neumark & Troske, 1999); and (iv) competition is known to drive profits to zero, hence the indexation of the nominal wage to profits.

We now move on to analyze the 'empirical' evidence the model yields. As said, in 3.1 we discuss one sample realization of the model to provide the reader with a qualitative flavor of the results. In 3.3 we undertake a sounder analysis of a Monte-Carlo 50, to make sure the qualitative results we had sketched hold notwithstanding the random seed of the single simulation.

3.1 A sample realization

We organize this section in three thematic parts. In 3.1.1 we discuss some of the main macroeconomic variables the model presents. In 3.1.2 the financial sector is taken into account both *per se* and in terms of its interactions with the rest of the economy. In 3.2 we show that the model is *qualitatively* able to replicate some empirical findings whereof the firm dynamics literature we already dealt with. All the figures and data we will show are the result of a run-up of the model. The time series are truncated not to let the analysis by flawed by the initial transient, and spans 150 periods.

All of the series are filtered according to the method proposed by Christiano & Fitzgerald (2003): since the model is calibrated on a quarterly basis, the threshold frequencies for the cyclical components are $(\omega_L, \omega_H) = (6, 32)$.

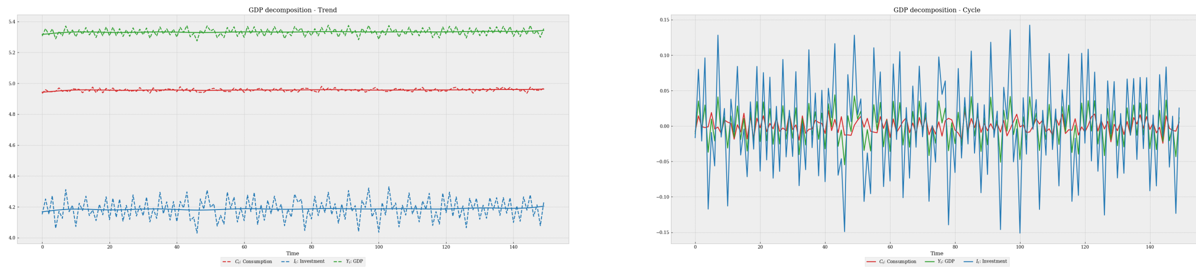
3.1.1 The macro-economy

We first consider some macroeconomic indicators. We defer to the next subsection a thorough analysis dealing with credit and the financial sector, as well as with its interactions with the rest of the macro-economy. Here, we devote the attention on more traditional macroeconomic indicators, such as the GDP and its decomposition, prices and wages and the Phillips Curve.

First, consider a standard *decomposition of the GDP*, that is in our model we can write total GDP, measures as the value of sales, as the sum of consumption and investment, which in turn we consider to be the sum of both expenditures in physical capital and R&D investments, which are both fuelled by credit. In the next figure, we provide one sample plot in which we show both the levels and the cycle, the two components disentangled one from the other using the aforementioned filtering technique.

Some reflection, we believe, are in order. First, figure 2a shows that both consumption *and* investment are trending. We will show this in more depth but at first sight they both appear to be integrated of order 1. This

[13]Notice that, here, we refer to the relation between nominal wages and rates of changes in unemployment. Indeed, as it is widely known, this is different from postulating the existence of the 'Phillips' curve.



(a) Trend.

(b) Cycle.

Figure 2: GDP decomposition for a single realization. Note: 150 iterations, of which the first 50 truncated.

is related to a rather interesting finding, that is that the model supports endogenous, self-sustained growth that, we shall see, is driven by increasing credit fueling the economy. The fact that the economy grows is not trivial, inasmuch in principle one could expect it to stay stationary. Indeed, as it is common in many ABMs, long and short run are not easily disentangled one another. Indeed, *long* run growth coexists with *short* run business cycle fluctuations. While our focus shall be on the latter feature, it is nonetheless important to acknowledge the presence of the former.

Figure 2b is also consistent with a pretty robust finding on the relative volatility of GDP. More specifically, we note that investments are -much- more volatile than both consumption and GDP. In turn, consumption is the least volatile series, consistently with both empirical observations and theoretical presumptions, for instance, dealing with the permanent income hypothesis or other reasons favoring the idea of consumption smoothing. To better grasp the idea about non-normal fluctuations, consider figure 3.

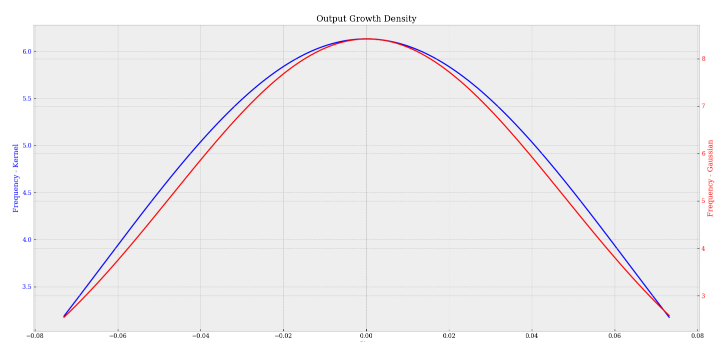
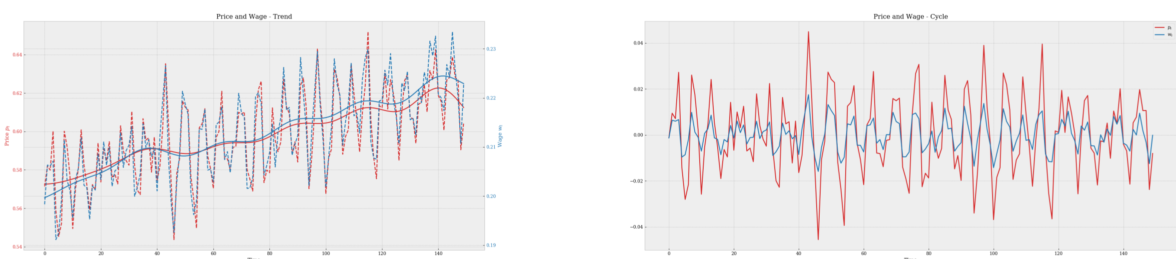


Figure 3: GDP growth rate distribution for a single realization. Note: 150 iterations, of which the first 50 truncated. Epanechnikov kernel density, on the x axis rescaled GDP growth rates.

The distribution of output growth rates features tails that are heavier than one would have expected had it been normally distributed. Indeed, as argued by Fagiolo, Napoletano & Roventini (2008) the distribution of output growth rates is anything but normal, hence our finding here is qualitatively in line with theirs. However, we need to undertake a more thorough investigation of the shape of the distribution to assess whether it is statistically different from Gaussian. However, the model does not match the heavy tails that are found by the authors, in that even though the distribution is more fat-tailed than a Gaussian, it is not quite as skewed as empirical studies find. Non-normality will be further assessed in the subsequent section.

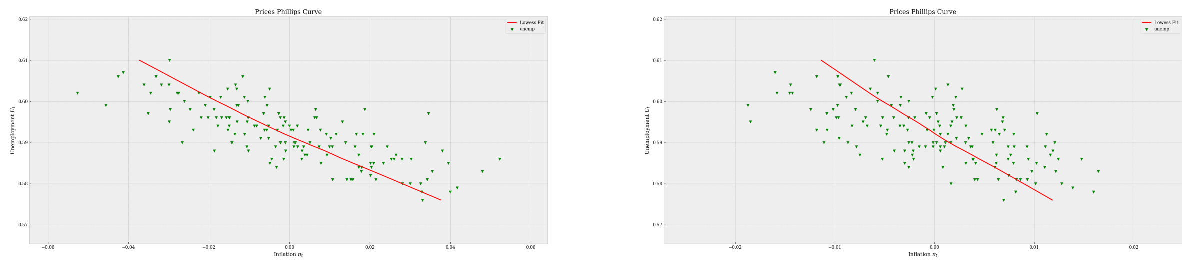
Concerning prices, consider figure 4, in which we plot both the nominal wage and the CPI, computed as the average of the different prices set by the firms, adjusted by their market shares. Clearly, they are both trending, and prices follow the wage, as one expect given the rule of thumb that is employed by firms when setting their price, embodied in (29). Firms set the price of the consumption good as a mark-up over the average cost of production, so whenever one of the determinants of the unit costs, such as the nominal wage, increases for all firms, hence implying price inflation not to cause a loss in competitiveness, vary, so do prices. This mechanism is precisely the one that makes prices and nominal wages coevolve, both in terms of long-run trend and short-run fluctuations. However, as one would expect from theoretical arguments, *i.a.* nominal wage stickiness, and empirical regularities, prices are more volatile than nominal wages, as 2b shows. However, in order to understand the eventual co-movements between these variables and the business cycle we are to look at the correlation structure of the economy, something we defer to the next section.



(a) Trend. **(b)** Cycle.

Figure 4: Prices and wages for a single realization. Note: 150 iterations, of which the first 50 truncated.

One of the most interesting empirical regularities that have been put forward in the literature concerns the so-called *Phillips curve*, that is the 'law' relating changes in inflation to (opposed) changes in unemployment. Stock & Watson (1999) find, using US postwar data, that there is no correlation whatsoever between unemployment and inflation, taken in levels. However, they do find a negative correlation between the cyclical components of the two. In figure 5 we provide two plots showing that in our model such a relation emerges both in terms of price inflation and nominal wage inflation. Note that both are obtained from the filtered cyclical components of these series, thus confirming Stock & Watson (1999) original result.
The LOWESS fit shows that a correlation between inflation and unemployment exists in its original specification, *i.e.* as a negative correlation between wage inflation and unemployment as well as in the formulation pertaining to price inflation and unemployment. Notice that this is not unexpected: firms set their production plans given previous demand and adjust it given the demand they had not been able to fulfill. This in turn

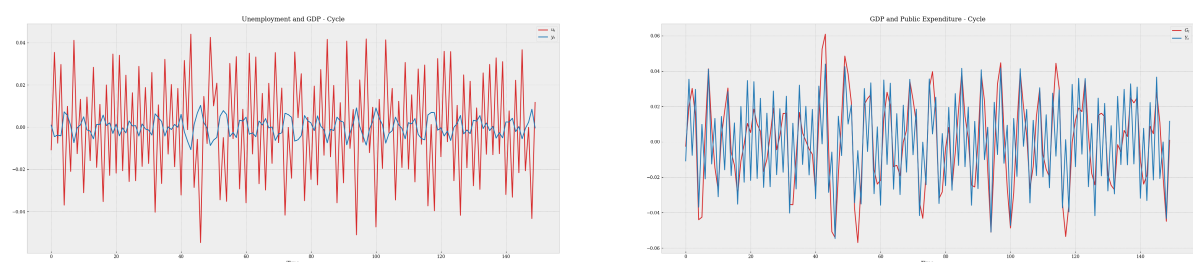


(a) Inflation and unemployment. **(b)** Nominal wage inflation and unemployment.

Figure 5: Different specifications for a Phillips curve in levels. Note: 150 iterations, of which the first 50 truncated.

implies that prices are related to the production choices of the firms, which in turn influence their demand for labor. The nominal wage rate in turn affects aggregate demand and, hence, directly impacts the firms' production plans. Thus the however negative correlation whereof figure 5b is to be interpreted in terms of how firms manage their production choices.

One last point of interest concerns *employment* and its relationship with the business cycle. Typically, unemployment is found to be coincident and strongly countercyclical. Indeed we would too expect this given that labor appears as one of the inputs of the production function and the demand for labor is, as already known with respect to the Phillips curve, influenced by aggregate demand, which in turn, albeit indirectly, influences total output, and hence the business cycle. We provide some qualitative evidence of this in figure 6.



(a) Cycle: Unemployment and GDP. **(b)** Cycle: Public Expenditure and GDP.

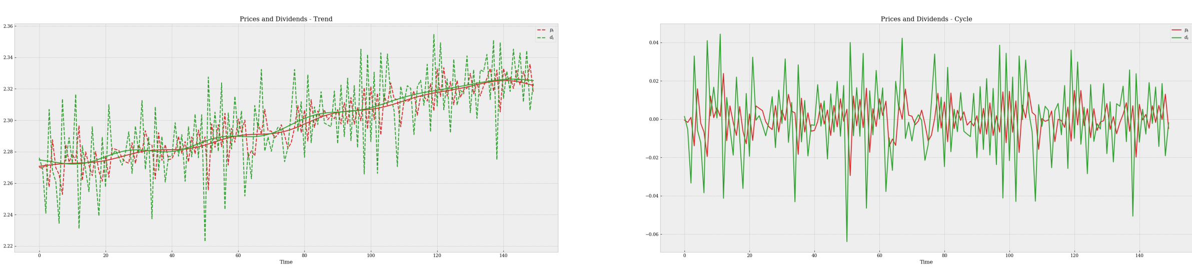
Figure 6: Unemployment, public expenditure and the business cycle. Note: 150 iterations, of which the first 50 truncated.

First, the model delivers credible unemployment levels, and is only occasionally in full employment regimes. Also, as shall be clear afterwards, unemployment is strongly and negatively correlated with changes in GDP, which we use to proxy for the stage of the business cycle. This is consistent with the aforementioned empirical evidence. Therefore, in spite of the imperfections the labor market is affected by, it nonetheless yields a substantially favorable outcome for employment fluctuations accommodate firms' demands enough for such variable to be positively correlated with the business cycle.

3.1.2 Credit and the financial sector

In this paragraph we turn the attention to the more financial side of the model. More specifically, we analyze both the functioning of the financial sector as well as its interactions and integration with the rest of the macro-economy. To this end, we take into account the dynamics of equity prices and returns, the sum of the issued loans and the related leverage level set by the banks and the correlation between credit and investments.

First, consider the functioning of the *equity market*, which yields the price of the equity investors are endowed with, given the return it is expected to yield. We plot these two variables in levels and differences in figure 7.



(a) Trend. **(b)** Cycle.

Figure 7: The equity market: levels and cycle. Note: 150 iterations, of which the first 50 truncated.

In panel 7a we see that prices tend to follow and smooth returns. This is a consequence of the pricing equations (5) and of the expectation formation rule (1a), the former implying current price to be a correction over the past one, the latter making investors weight across time, thus smoothing their expectations volatility. Also, even though the γ_i parameters whereof (3) are 'small', investors behave in a substantially reasonable way, for prices are seen to move with returns, albeit in a probably lagging fashion. This is further clear from panel 7b, which attests that the cyclical component of prices tends to lag behind that of returns, by construction of the expectations of equity traders. Hence we confirm, in the light of the previous model's findings, that with no *a priori* assumption concerning market clearing and rationality and, indeed, allowing traders to be extremely heterogeneous, a price stemming from the endogenous interactions they entertain is on average consistent with standard no arbitrage arguments.

It is further important to notice that the chartist behavior traders feature, embodied in the low δ_i, is relevant insofar it makes equity prices more stable than their returns. Since credit is increasing in equity prices, had prices been unstable so would have credit. Thus, Chartism in the equity market contributes to the stability of the economy by making the supply of credit less volatile.

Let us now turn to banks and, more specifically, to the relationship between *credit* and *leverage*. Notice that we consider credit as the outcome of the credit market, *i.e.* total allocated loans. Indeed, total loans as of (41a) are in principle more than the ones that are actually demanded by the firms, which are given by (41b). The credit market we discussed does not assume market clearing between supply and demand for loans in the equilibrium, hence the *target* leverage ratio whose expression we gave in (14) is in principle correlated with *target* credit. However, figure 8 shows that it is also related to the *realized* amount of loans that banks manage to place. Therefore the credit market, albeit incomplete, is sound enough to allow banks to behave according to their stylized VaR rule.

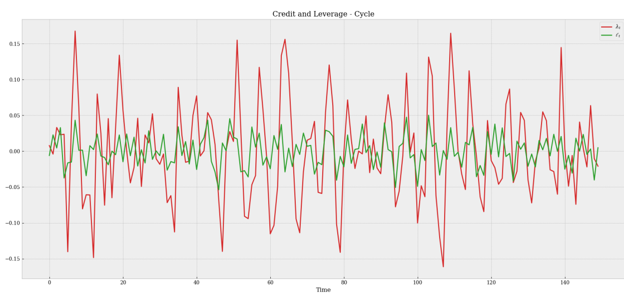
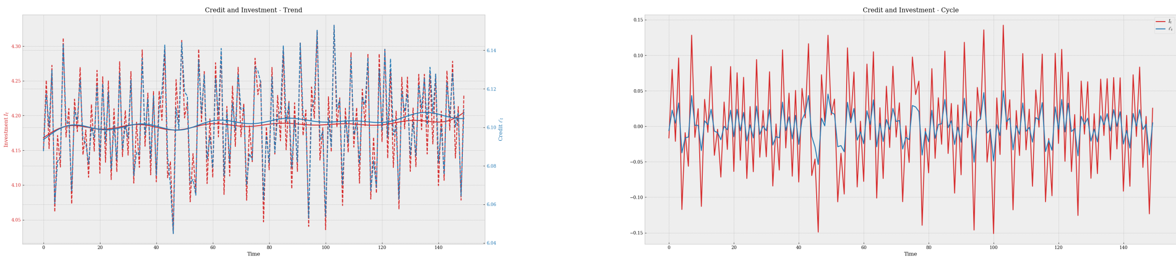


Figure 8: The supply side of the credit market: leverage and realized loans (*i.e.* credit). Note: 150 iterations, of which the first 50 truncated.

In panel 8 we note our previous remark, that is that credit and leverage tend to co-move: higher leverage target prompts banks to increase their balance sheets more with respect to the market value of their equity. Hence, the higher such target, the more credit they will fuel the economy with. Notice that since the amount of nominal equity is fixed, variations in the real value of equity are totally due to changes in prices. Since those prices will be shown to be pro-cyclical, leverage shall be pro-cyclical too, thus prompting amplification mechanisms of the business cycle.

To conclude this overview of the financial sector, let us now consider the integration between it and the real one. Specifically, in figure 9 one can evaluate the comovement between credit and investments, and appreciate the correlation between the two. This should clearly not come as a surprise and, indeed, it is a finding that has empirically been observed by Schularick & Taylor (2012) and we already discussed. In our model, we expect such a correlation to emerge since investment is mainly financed by means of financial external funding. Hence, the less a firm is credit constrained, the more it shall be able to satisfy its investment plans, which in turn are determined by its desired production.
First, form panel 9a it is immediately evident that investments and credit are close one to a downwards translation of the other. This is quite self-evident given our previous reasoning. Notice that the difference between the two is explained by the fact that external resources are further required to pay up the salary bill



(a) Trend. **(b)** Cycle.

Figure 9: Financial and real sectors: credit and investments. Note: 150 iterations, of which the first 50 truncated.

and to undertake R&D activities. Hence the different magnitude. Furthermore, from panel 9b one retrieves the expected correlation between the two. Also, it is interesting to notice that credit is less volatile than investments, albeit slightly so, thus suggesting that firms are credit constrained. Indeed, we are shortly going to see that the size distribution of firms resembles that of financially constrained ones.

The correlation between credit and investment that can be inferred by 9b and will further be assessed in the next section is in line with empirical evidence by Ng & Wright (2013). It is also worth noting that the two series are not coincident, for firms are allowed to finance their production plans with internal financing too. Last, credit is generally more than investment in levels, meaning that investments are not the unique integration channel between the financial and real sectors, but bank lending is also fundamental for wage payments and the like.

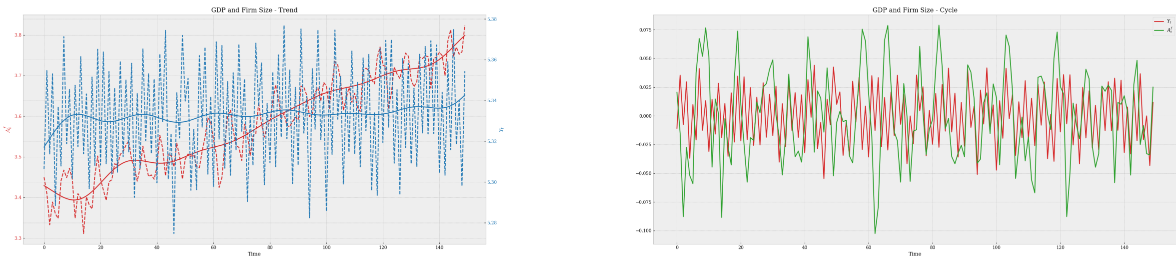
3.2 Firm dynamics

We conclude this short, qualitative presentation of the results the model yields by providing three more insights dealing, this time, with some insights on the dynamics of the productive sector, *i.e.* the firms. We examine three pieces of evidence. First, we look at average net worth, which is known to be procyclical and leading. Concerning firm dynamics proper, we discuss two key dimensions, *i.e.* firm size and growth. We note that the size distributions for selected time periods are consistent with earlier findings on financially constrained size skewness.

First, we evaluate the performance of firms given their net worth. Also, we compare it to the business cycle in order to evaluate the consistency of the results with known empirical regularities.

Firms are successful at increasing their net worth. Notice that entrants' net worth is below that of incumbents rid of those firms which are being substituted by the entrants themselves. Due to savings accumulation, the credit crunch is not evident in terms of firms' net worth for all those firms which go bankrupt are promptly substituted: average net worth does shrink, but there is no evident downfall.

By the same token, panel 10b confirms our previous expectation on the positive correlation between the



(a) Trend.   **(b)** Cycle.

Figure 10: Firms and the business cycle. Note: 150 iterations, of which the first 50 truncated.

business cycle, proxied by the rate of change of the GDP, and firms' net worth. Also, albeit an across-periods correlation is to be computed -and will be, in the next subsection-, we can plausibly identify a leading path of firms' net worth with respect to the business cycle, hence in principle confirming the consistency between the model and the regularities whereof the real world.

We now turn to *firm dynamics* properly defined. For the sake of brevity, we shall only consider firm size and growth distributions. First, in figure 11 we present the firm-size distribution (FSD) for selected times of the simulation.

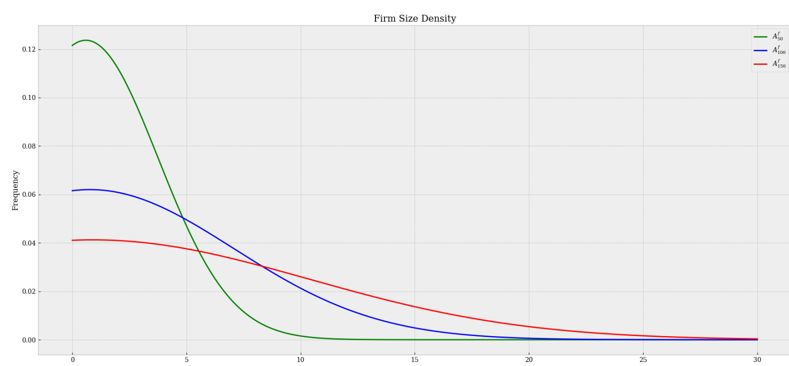


Figure 11: FSD for selected time periods. Note: Epanechnikov kernel densities against re-scaled size values.

Size is measured in terms of real sales, and the FSD is right-skewed. The width of the distribution increases with age too, and moves rightwards as the age of firms increases. In order to state whether the degree of skewness decreases with age we would need to estimate the parameters of the AEPs relative to the different dates, an effort we undertake in the next subsection. However, both the right-skewness, the shift to the right and the increasing width tend to suggest consistency with the empirical findings by Bottazzi, Secchi & Tamagni (2014) on financially constrained firms. Also, the Fligner-Policello test on stochastic dominance confirms that older FSD dominate younger ones, hence a firm randomly drawn from the set of older ones is bigger at more than 50% confidence level than one taken from the set of younger firms[14].

We defer to the next subsection a more quantitative assessment of the features whereby the FSD. However, from a qualitative perspective we can further infer that its non-normality throughout the simulation period suggests persistent heterogeneity across firms, a finding that is sometimes neglected in the mainstream literature and which arises from firm-specific access to credit and success in the labor market, given that from table 8 firms are *ex ante* assigned no different parameters one with respect to the other. Hence, albeit no *a priori* heterogeneity is assumed, the imperfectly competitive structure and the evolutionary pressures firms face make them evolve in persistently different ways.

Let us now turn the attention to the firm growth distribution (FGD). Recall that from the law of proportionate effect, one would expect it to be normally distributed. However, as we already pointed out, a robust finding in the firm dynamics literature is that the FGD features heavy tails, typical of a Laplacian (or hyper-Laplacian) distribution. Bottazzi, Secchi & Tamagni (2014) find that financial constraints are detrimental to explain the heteroskedasticity and autocorrelation of the shocks which make Gibrat's law fail to explain the shape of the FGD. Also, they find that financial constraints have two main consequences: (i) a 'pinioning the wings' effect, that slims down the right tail of the FGD, and (ii) a 'loss reinforcing' one, that entails financially constrained firms that perform badly to worsen their result. Here we do not define a 'financial constraint' indicator, hence we simply look at the FGD for the entirety of the firms in the economy, and provide it in figure 11.

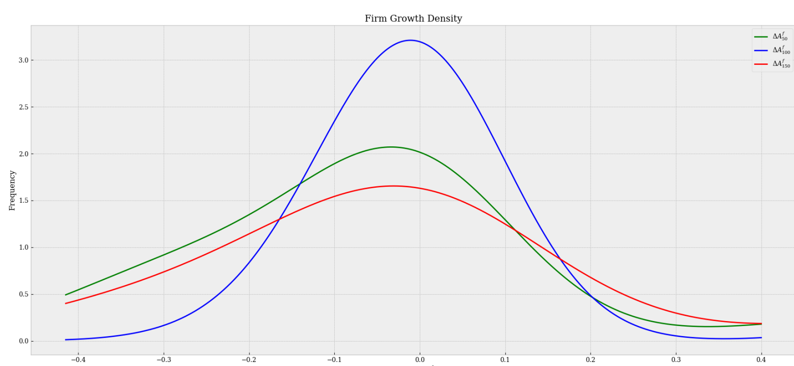


Figure 12: FGD for selected time periods. Note: Epanechnikov kernel densities against re-scaled growth values.

As said, we cannot directly observe the effects of financial constraints from figure 12. However, all the FGDs present tails that are heavier than normal. This is true throughout the sample period, albeit growth variability is higher for earlier periods than for $t = 100$ and $t = 150$. Notice that since firms all begin at the same time period, the wider variability in earlier periods induces that figure 12 is consistent with another typical finding,

[14]The confidence level depends on which group of firms are considered. For instance, it raises to a maximum of 90% when $t = 80$ against $t = 20$ are considered.

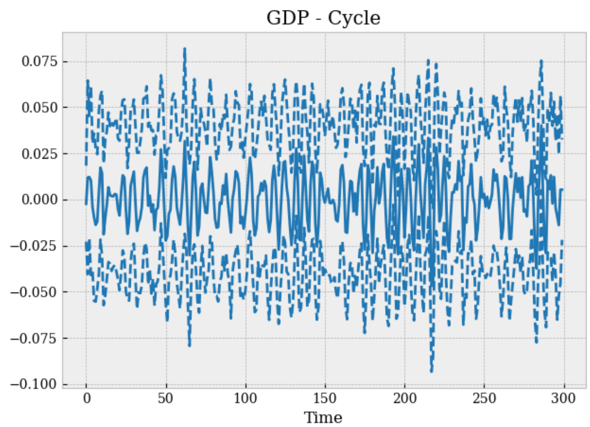
that is age and growth variability tend to be negatively correlated.
Albeit a more quantitative assessment is deferred, the persistent non-normality of the FGDs hints at some correlating mechanism stemming from the competitive environment. Come as it may, this is consistent with the empirical evidence that robustly documents heavy tails of the FGDs, thus further suggesting persistent firm heterogeneity.

3.3 Monte-Carlo experiments

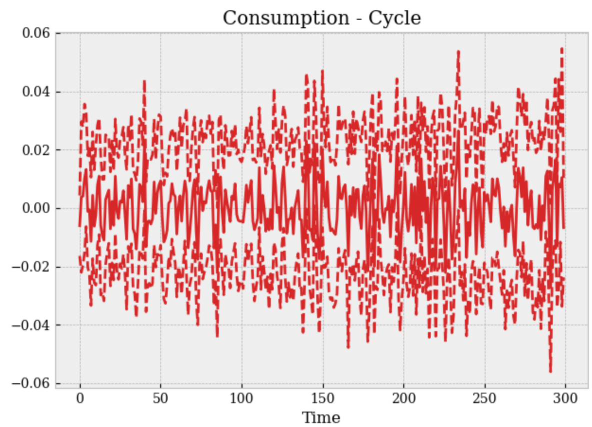
We devote this section to a more formal inquiry on the performance the model attains at replicating a subset of the empirical stylized facts we have been describing the previous section. ABMs are methodologically problematic in that they pose relevant validation issues. Following Windrum, Fagiolo & Moneta (2007), we try to evaluate the model by means of Monte-Carlo simulations. More specifically, we simulate the model $N = 50$ times generating as many time series, and study their properties. Thus, we are able to evaluate the eventual stationarity of the time series and provide descriptive statistics summarizing their behavior. We then study the correlation structure of the economy, and show that it is qualitatively consistent with early analyses by Stock & Watson (1999). Also, we show that the model is consistent with more recent findings by Jordá, Schularick & Taylor (2017) who document the relatively novel integration between the financial and the real sectors.
The section deals with macro-business cycle facts, financial insights and microeconomic evidence, and is thus developed along the lines of the previous. As said, we perform a 50-fold Monte Carlo simulation spanning 350 periods, although we get rid of the first 50 transient-phase iterations. Hence, our final dataset consists of 50 time series 300-periods long each for each of the 26 variables we take into account. Variables are typically band-pass filtered at business cycle frequencies by means of the filter proposed by Christiano & Fitzgerald (2003), who suggest to take a bandwidth of $(6, 32)$ to filter for such frequencies.

3.3.1 Business cycle facts

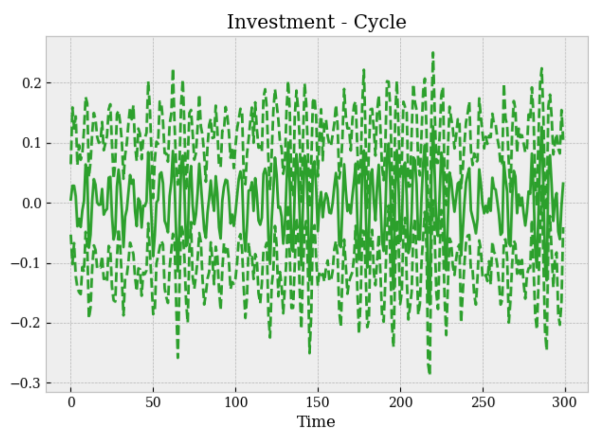
In 13a we plot cross-sectional means for GDP and its components over time, alongside with an error bank indicating the 95% confidence level. From this figure, one understands that the investment component of GDP is the most volatile, as it is confirmed in subsequent quantitative analysis.
The model also supports self-sustained endogenous growth, as we show in the following stationarity tests, that is nonetheless characterized by persistent fluctuations. Therefore, we undertake the same exercise for the band-pass filtered time series, which we show in figure 13. Specifically, in the figure we plot the average cyclical component across the simulations, as well as the 95% confidence intervals derived from the associated Monte-Carlo standard errors, assuming its distribution between simulations to be normal. From 13c relative to the other two panels, we confirm that the model successfully replicates a finding that is commonsense in



(a) GDP.



(b) Consumption.



(c) Investment.

Figure 13: Cross-sectional cyclical component of a the GDP (blue), consumption (red) and investment (green) band-pass filtered time series.

the literature, that is, that GDP is more volatile than consumption, and the two are both less volatile than investments. Indeed, in 1 we see that the relative volatility between the GDP components are consistent with the empirical evidence put forward by Schularick & Taylor (2012). Moreover, the between-simulations volatility is constant over the simulation time-span: this would not be the case had we not dropped a transient phase. Also, the degree of volatility between-simulations is the same for all the components of the business cycle.

In table 1 we report, for the GDP, consumption and investment time series, (i) the average growth rate, (ii) the average standard deviation and its value relative to that of GDP, (iii) the relative volatility of the components of output relative to output, and (iv) the percentage of time series ADF tests resulting in non-rejection of the null hypothesis of unit root in the series.

A first assessment concerns the relative magnitude of the volatility of the GDP components. As said, consumption is the least volatile, investment summing up as much as more than four-fold the volatility of the

Statistic	Output Y	Consumption C	Investment I
Growth Rate	0.033 (0.023)	0.041 (0.026)	0.013 (0.038)
Volatility	0.044 (0.030)	0.015 (0.008)	0.337 (0.149)
Relative Volatility	1.000	0.333	7.637
Stationary (99%)	0.0	0.0	0.0
Stationary (95%)	0.0	0.0	0.0
Stationary (90%)	7.843	23.529	25.490

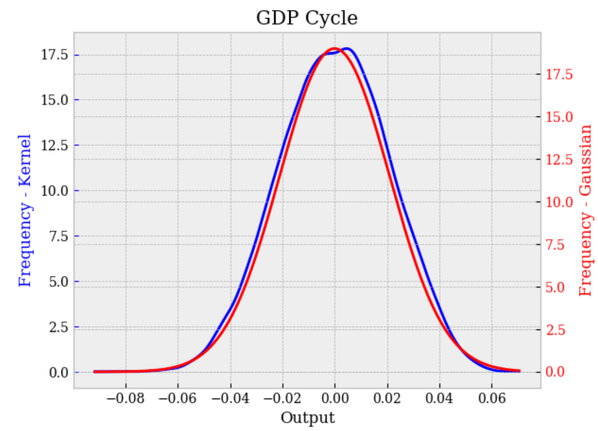
Table 1: Sample statistics and stationarity tests. Monte-Carlo standard errors in parentheses. Lags in ADF are suggested by AIC.

overall business cycle. Notice that this is consistent with empirical findings, and is therefore a first, albeit simple stylized fact the model delivers.

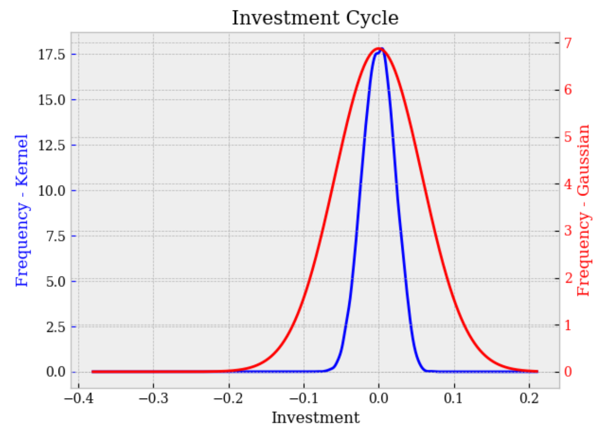
Second, we tested for the presence of a unit root in the GDP components. Indeed, we would expect them not to be stationary, so that once a band-pass filter is applied, its cyclical component can be understood as the business cycle frequency component of the series. Therefore, we report both the ADF χ^2 statistic and the related p-value. In doing so, we exploit the longitudinal dimension of our data to carry out a panel ADF (Im, Pesaran & Shin, 2003). These tests feature the null hypothesis of all the time series within the panel featuring a unit root. Indeed, we find that at 99% confidence level *all* GDP and Consumption series are not stationary, hence we cannot reject the null hypothesis of a unit root in each of the 200 time series the panel consists of. Interestingly, however, we reject the null hypothesis for the investment series, meaning that there is (at least) one stationary series according to the ADF test thus specified. We thus perform a standard ADF test on each of the time series generated through simulation, and report the fraction of those tests non rejecting the null hypothesis of unit root at the 95% confidence level. The table shows the share of ADF tests rejecting the null hypothesis of unit root as applied to the trend component of the series. For the sake of brevity, we did not report the tests on the cyclical components, which are all found to be stationary at 99% confidence level. We will also show that the model supports tail events, namely large drops in GDP, with non-negligible probability, and the likelihood of these to be increasing in the credit-to-GDP ratio.

We can provide some more robust results concerning the non-normality of the GDP growth rates, a fact observed *i.a.* by Ascari, Fagiolo & Roventini (2015). First, consider a qualitative elaboration provided in figure 14a, which essentially is a Monte-Carlo version of 3, in which we provide a Kernel density estimation of the GDP growth rates given all the realizations of the model. In 14b we also provide an estimation of the density distribution of investments, which we expect to be more fat-tailed than that of GDP, in the light of the findings whereof 1.

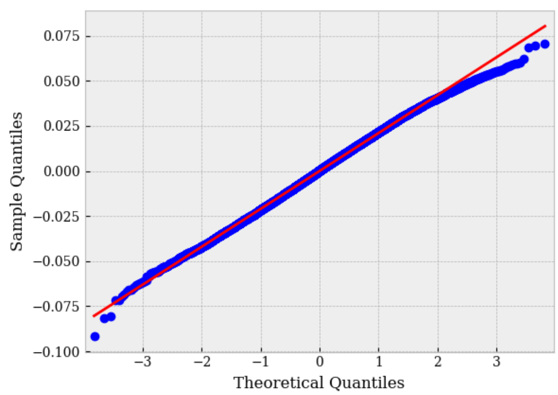
Figures 14 do not provide clear-cut results: the GDP distribution does not substantially depart from normality, whereas the investment one seems to. This is broadly confirmed for the two given the two quantile-quantile



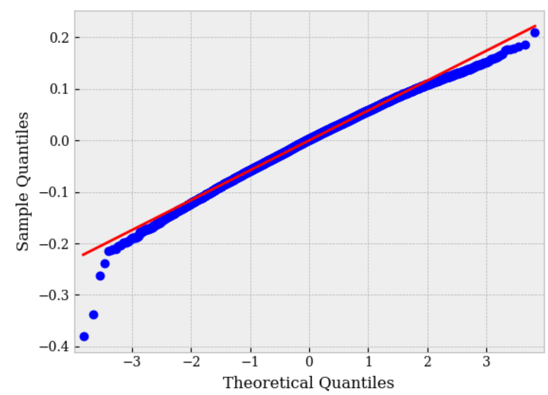
(a) GDP.



(b) Investment.



(c) GDP qq-plot.



(d) Investment qq-plot.

Figure 14: Kernel Densities for Band-Pass filtered time series. Bandwith is suggested by the rule proposed by Silverman (1981). Quantile GDP and Investment BP-Filtered distributions against normal quantile plots.

plots shown in panels 14c and 14d. In particular, the investment distribution appears to feature thinner tails than the ones we would have expected under normality. We provide more quantitative evidence in table 2.

Variable	**Test**					
	Jarcque-Bera		Shapiro-Wilk		D'Agostino	
	99% C.L.	95% C.L.	99% C.L.	95% C.L.	99% C.L.	95% C.L.
GDP	31.373	39.216	35.294	49.019	39.216	43.137
Investment	31.373	56.863	33.333	54.902	31.373	56.863

Table 2: Normality tests on GDP and Investment BP-filtered cyclical components.

Given our sample size, that is over 60, 000 observations, the Jarcque-Bera test would alone yield robust enough substantiation to our previous claim. However, for the sake of solidity, we also perform the Shapiro-Wilk and the D'Agostino tests, which are usually employed with fewer observations. The results are however not conclusive:

most of the GDP and Investment series reject normality. Despite that, a non-negligible share of the distributions is not inconsistent with normality: we deem this feature of the model as an area of possible future work.

We now turn the attention to the correlation structure of the economy depicted so far. In table 9 we show the correlation structure, where each entry is calculated as the average correlation across the 200 iterations of the given correlation coefficient. Thus, for instance, $\mathrm{Corr}(Y, X_{t-k})$, where $\left\{\{X_{it}\}_{t=1}^{300}\right\}_{i=1}^{50}$ and $\left\{\{Y_{it-k}\}_{t=k}^{300}\right\}_{i=1}^{50}$ are two generic time series, is computed as

$$\mathrm{Corr}(Y_t, X_{t-k}) = \frac{1}{N}\sum_{i=1}^{N}\mathrm{Corr}\left(Y_{it}, X_{it-k}\right) = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=k}^{T}\frac{\mathrm{Cov}(Y_{it}, X_{it-k})}{\sqrt{\mathbb{V}[Y_{it}]\mathbb{V}[X_{it-k}]}} \tag{49}$$

with $N = 50$, $T = 300$, $\mathrm{Cov}(\cdot,\cdot)$ is the covariance between the two series, and $\mathbb{V}[\cdot]$ is their variance. In brackets, the reader will find attached the Monte-Carlo standard errors, which are simply computed as the standard deviation of the series generated by the between-iteration cross correlations.

The correlation structure is satisfying, albeit it presents some points that are not fully consistent with the empirics of business cycles[15]. The autocorrelation of the GDP is in line with the evidence, and we believe this to be an improvement with respect to *i.a.* Dosi, Fagiolo & Roventini (2010), and in general to most ABMs, which yield GDP series that are less persistent than the empirically observed ones. Consumption and investments are both procyclical and coincident, the former being prone to lag, the latter to lead, again in line with empirical evidence. Still, here one major issue emerges, that is, consumption is qualitatively fine, but its correlation with GDP is lower than expected. Unemployment is strongly countercyclical and lagging, and the price level is countercyclical and leading.

Concerning the financial sector and its relationship with the business cycle, albeit we take this topic into further considerations in the next subsection, a major finding is that we successfully show that asset prices, that are simplistically embodied as equity prices, are procyclical and leading the business cycle. Returns, which can in turn be understood in terms of yield of stocks, are procyclical and lagging, again in tune with the literature.

One further point of interest concerns the degree of integration between the financial sector and the real one, which one can measure in terms of the correlation between credit and the business cycle. It turns out that such correlation is indeed high, and of the same magnitude as that found by Jordá, Schularick & Taylor (2017) for the floating exchange rates period. Indeed, we shall see this correlation to be driven by that between investment and credit, which in turn also prompts lumpy investment undertakings by firms. Given that our model features no central bank, we are not in the position to evaluate the behavior of *the* interest rate. We can nonetheless observe the lending rate, that is the cost of capital banks charge upon firms, which in the real word

[15] Throughout this paragraph, our main references shall be Stock & Watson (1999) and Jordá, Schularick & Taylor (2017). A more comprehensive discussion on the empirical literature dealing with business cycle is provided in the second chapter of the present work.

is known to be highly, positively and coincidentally correlated with the lending one. Hence, assuming that we can proxy the former with the latter, we find that the interest rate behaves consistently with the empirical evidence, for it is procyclical and lagging.
To conclude, the cyclical components of both firms' and banks' net worth do not appear to be particularly correlated with the business cycle. However, given that all the series display little standard errors, one could in principle claim these two to be procyclical and lagging.

Overall, the model yields data that are quite satisfactorily in tune with real world-evidence. This notwithstanding, some critiques can be moved against the relatively weak correlation between GDP and consumption. However, we nonetheless showed that the consistency between the model's and the real world's data is remarkable, particularly with respect to our initial purpose, that is, to set out a -however simplistic- model that embed both a financial and a real sector, and which could account for the interconnections between the two.

3.3.2 Credit and the financial sector

Along these lines, we devote this section to study the relationship between the financial sector and the real one. As said, a fundamental premise of this work has been to understand such connection, as well as the role of leverage and credit as possible drivers of business cycles. Hence, we study (i) the qualitative relationship between credit, leverage and the business cycle; (ii) the main features of expansions and recessions and the relevance of credit;(iii) the basic features of the equity market.
Notice that from table 9 we already concluded that the correlation between credit and the business cycle, understood in terms of the cyclical component of the output series, is relatively high, credit being procyclical and slightly leading the cycle. Notice, however, that this is mainly due to the extremely high correlation that holds between credit and investment.
It is therefore of interest to understand the impact that credit has on business cycles, if any, and to what extent can the model replicate the basic features on credit-fuelled cycles examined in the second chapter of the present work.
In table 3 we present some statistics on the GDP components in recessions and expansions, distinguishing whether credit growth is above the individual-specific within-time average (HEC) or below it (LEC). We also compute the average duration of both HEC and LEC expansions and recessions.
Concerning recessions, when credit does not collapse, *i.e.* in a HEC recession, the GDP falls less than it would have had the recession been accompanied by a sharp decline in credit, *i.e.* a LEC recession. Also, notice that in HEC recessions consumption falls more than investment. On the other side, in LEC recessions consumption broadly stagnates, while investment collapse, on average, by more than 30%. The sharp decline in credit directly shrinks investment and thus, despite consumption staying flat, such a downfall inevitably causes GDP to fall. Further notice that HEC recessions, which are relatively more common than LEC ones,

Variable	Statistic	Recession			Expansion		
		Average	HEC	LEC	Average	HEC	LEC
GDP	Growth Rate	−0.053	−0.049	−0.066	0.014	0.026	0.010
	Std. Dev.	(0.064)	(0.058)	(0.067)	(0.047)	(0.071)	(0.026)
Consumption	Growth Rate	−0.017	−0.066	0.019	0.010	0.011	0.082
	Std. Dev.	(0.077)	(0.078)	(0.062)	(0.070)	(0.059)	(0.063)
Investment	Growth Rate	−0.201	−0.015	−0.310	0.019	0.173	−0.029
	Std. Dev.	(0.319)	(0.251)	(0.303)	(0.354)	(0.355)	(0.339)
Duration	Mean (years)	1.5	1.4	1.9	7.4	9.3	6.0
	Std. Dev.	(0.6)	(0.5)	(0.8)	(4.8)	(6.7)	(4.1)

Table 3: Duration and sample statistics of the GDP components in Expansions and Recessions. HEC and LEC respectively stand for High and Low excess credit. Recessions are defined if (at least) 3 consecutive quarters feature negative GDP growth rates. Standard errors in parentheses.

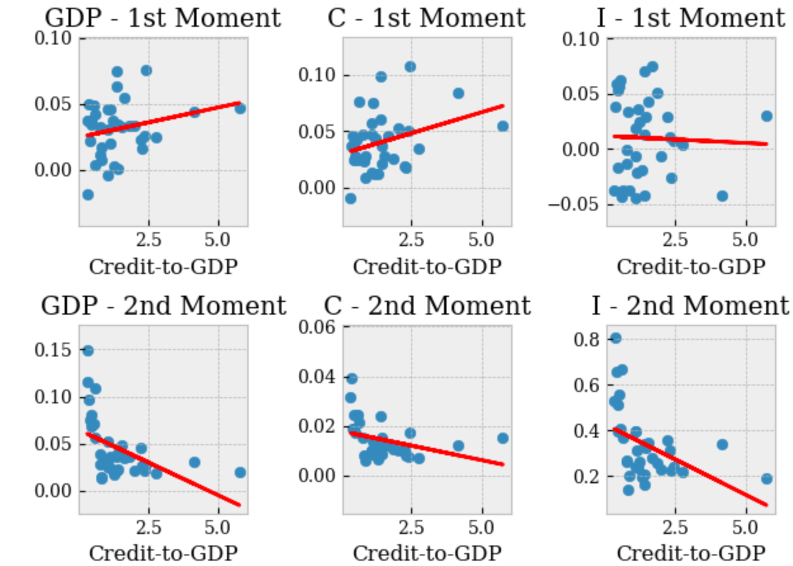
last less than the latter. This in turn implies that whenever a recession features -or is caused by- a steep fall in the credit supplied by the financial sector, the overall cost in terms of GDP losses increases.

Expansions are somewhat similar to recessions when it comes to the role played by credit. Whenever a boom in GDP is accompanied by an increase in credit, *i.e.* a HEC expansion, investment as well as consumption grow, hence fostering GDP growth and thus making HEC expansions sounder in terms of GDP gains. Whenever credit behaves contrary to GDP, however, a LEC expansion occurs, and investment do not contribute to growth, thus making LEC expansions less relevant than HEC ones in terms of GDP gains. Furthermore, notice that HEC expansions are more frequent and last longer than LEC ones, hence the overall GDP gain they imply is higher.

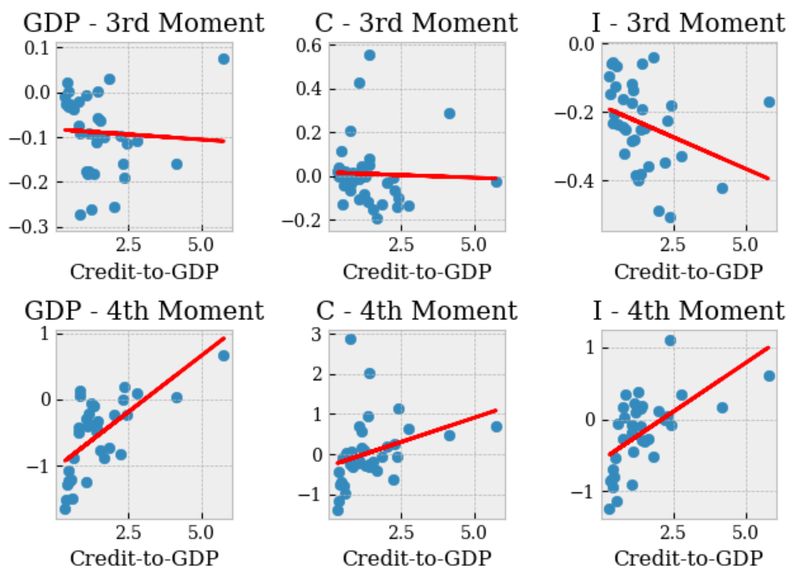
In figure 15 we plot the first three moments of the distributions of GDP, consumption and investment against the credit-to-GDP ratio. Concretely, we take the within-iteration mean, standard deviation and skewness of the three variables and the average credit-to-GDP ratio, and plot each for the 200 iterations. Hence, each panel features 50 data points whose x and y value are each iteration's respectively average credit-to-GDP ratio and within moment of the given variable.

In panel 15a we plot the first two moments of the business cycle against the credit-to-GDP ratio. In the first line of 15a we show that GDP and consumption are increasing in the ratio, whereas investment is not, albeit slightly so. In our model, a higher-than-between-simulation-average credit translates in firms hiring more workers, thus fostering average GDP growth. Note, however, that the relationship is quite noisy: the first moment of the GDP distribution is in fact not very responsive to the credit-to-GDP ratio, as in Schularick & Taylor (2012).

On the other hand, again consistently with the aforementioned authors, the second moment of the GDP distribution is negatively correlated with the ratio. The idea is that more credit dampens the volatility of fluctuations, hence it is beneficial at business cycle frequencies. However, as the further moments show, this comes as no free lunch. However, the correlation is very well in line with empirical evidence.



(a) First and Second Moments.



(b) Third and Fourth Moments.

Figure 15: Within-individual central moments of sample business cycle indicators against Credit-to-GDP ratio.

In the second panel of figure 15 we show the third and fourth moments of the GDP distribution against the credit-to-GDP ratio. The skewness is decreasing, and the compositional effect is decomposed and noted to be mainly driven by the investment series behavior. The idea is that the more credit there is, the more the investment distribution is symmetric, thus prompting the same behavior in the GDP one, whereas consumption is not responsive. This is the same finding as Schularick & Taylor (2012), and it can be rationalized in terms of enhanced capability of the financial sector to absorb mild downturns in the economy: larger balance sheets tend not to amplify small shocks.

The last row of 15b is probably the most interesting. It documents that the kurtosis of all components of GDP is increasing in the credit-to-GDP ratio, and strongly so. This is in line with the empirical evidence, and provides evidence in favor of highly integrated financial sectors fostering the likelihood of deep downturns. If mark-to-market accounting and VaR-based leverage management are found to stabilize mild fluctuations due to size effects, when the economy faces a shock that is large even relative to the balance sheets of banks in highly integrated economies, then the large leverage that financial institutions are faced with prompts deep downturns to be more costly. From this perspective, our model thus provides a theoretical framework that allows to interpret empirical evidence on the interaction between the financial and the real sector in terms of the role that the former has in stabilizing ordinary fluctuations, but dramatically increasing the costs associated with deeper downturns.

To conclude, we now briefly ask whether credit can be *causally* linked to the likelihood of experiencing a recession. To this end, we seek to replicate the findings by Schularick & Taylor (2012), who document that credit is indeed positively correlated with recessions. Thus, we follow the authors and define excessive credit-to-GDP growth ξ_{it} as the difference between the credit-to-GDP growth in any given time period, for each iterations, and the average credit-to-GDP growth experienced in that iteration. Thus, positive ξ_{it} identify periods in which the ratio credit-to-GDP grew at a higher pace than average, thus identifying a period of increasing integration of the financial sector within the economy.

Therefore, we estimate five models, namely

$$d_{it} = \alpha + \sum_{k=0}^{5} \beta_k \xi_{it-k} + \boldsymbol{\delta}' \mathbf{x}_{it} + \varepsilon_{it} \tag{50a}$$

$$d_{it} = \alpha + \alpha_i + \sum_{k=0}^{5} \beta_k \xi_{it-k} + \boldsymbol{\delta}' \mathbf{x}_{it} + \varepsilon_{it} \tag{50b}$$

$$d_{it} = \alpha + \alpha_i + \sum_{k=0}^{5} \beta_k \xi_{it-k} + \sum_{t=1}^{300} \gamma_t \tau_t + \boldsymbol{\delta}' \mathbf{x}_{it} + \varepsilon_{it} \tag{50c}$$

$$\operatorname{logit}(d_{it}) = \alpha + \alpha_i + \sum_{k=0}^{5} \beta_k \xi_{it-k} + \boldsymbol{\delta}' \mathbf{x}_{it} + \varepsilon_{it} \tag{50d}$$

$$\operatorname{logit}(d_{it}) = \alpha + \alpha_i + \sum_{k=0}^{5} \beta_k \xi_{it-k} + \sum_{t=1}^{300} \gamma_t \tau_t + \boldsymbol{\delta}' \mathbf{x}_{it} + \varepsilon_{it} \tag{50e}$$

where d_{it} is a dummy variable that is 1 if at least at time t, $t+1$ and $t+2$ the growth rate of GDP is negative, that is the country is beginning to experience a recession, and 0 otherwise, $\mathbf{x}$ is a vector of controls, τ_t is a time dummy variable equal to 1 for year t and 0 otherwise, and $\operatorname{logit}(d) = \ln(p/(1-p))$ is the log of the odds ratio. Thus, model (50a) is a simple pooled-OLS model. In model (50b) we add country-specific fixed effects to control for unobserved time invariant heterogeneity, whereas in model (50c) we further control for time effects, to get rid of common trends across iterations, which we expect due to the DGP underlying the series being the same. Since linear probability models are known not to constraint the probability of the outcome to belong to the support set [0, 1], we also estimate two conditional panel logit models. First, we estimate a plain vanilla logit in (50d), and then further consider time effects in (50e). We report the estimates of the models in the following table, along with some tests and goodness-of-fit measures.

Model specification	**POLS** (50a)	**FE** (50b)	**Time FE** (50c)	**CL** (50d)	**Time CL** (50e)
ξ_t	-0.063^{*} (0.023)	-0.101^{**} (0.022)	-0.057^{*} (0.027)	-3.888^{**} (1.213)	-1.575^{*} (0.426)
ξ_{t-1}	0.570^{***} (0.067)	0.497^{***} (0.023)	0.545^{***} (0.097)	23.451^{***} (3.146)	19.559^{***} (3.002)
ξ_{t-2}	0.562^{***} (0.044)	0.449^{***} (0.027)	0.487^{***} (0.051)	23.562^{***} (1.974)	27.821^{***} (2.874)
ξ_{t-3}	0.193^{***} (0.021)	0.235^{***} (0.054)	0.249^{***} (0.098)	11.768^{***} (1.982)	14.661^{***} (2.933)
ξ_{t-4}	0.141^{**} (0.068)	0.151^{***} (0.032)	0.252^{***} (0.055)	8.432^{***} (3.767)	8.322^{***} (2.352)
ξ_{t-5}	0.123^{***} (0.046)	0.100^{**} (0.021)	0.161^{**} (0.051)	3.461 (2.682)	3.112 (2.937)
α	-0.002 (0.004)	-0.012^{***} (0.001)	-0.001 (0.006)	/	/
Controls (p-value)	Yes 0.000	Yes 0.027	Yes 0.000	Yes 0.000	Yes 0.000
Lags = 0 (p-value)	41.87 (0.000)	74.22 (0.000)	57.82 (0.000)	$221.15^{\dagger}$ (0.000)	$188.49^{\dagger}$ (0.000)
Individual effects = 0 (p-value)	/	0.93 (0.449)	0.79 (0.601)	$0.034^{\dagger}$ (0.374)	$0.022^{\dagger}$ (0.441)
Time effects = 0 (p-value)	/	/	25.12 (0.00)	/	$166.61^{\dagger}$ (0.00)
Full model F-test (p-value)	2.98 (0.00)	43.15 (0.00)	34.63 (0.00)	$275.28^{\dagger}$ (0.00)	$215.84^{\dagger}$ (0.00)
R^2	0.019	0.014	0.184	0.019	0.301

Table 4: Dependent variable: recession (categorical). Notes: *, ** and *** respectively stand for 10%, 5% and 1% significance level. †: reported statistic is F for OLS, χ^2 for Logit models. Joint significance test are F-tests. Standard errors are robust for OLS, bootstrap for Logit. R^2 is pseudo-R^2 for logit models.

The results shown in table 4 are consistent with those found by Schularick & Taylor (2012) and Jordá, Schularick & Taylor (2017). Indeed, we find that in all model specifications, lags are jointly significant[16] at 1% level. Also, the coefficients are all positive, indicating that credit growth that exceeds the average significantly increases the likelihood of experiencing a recession. Notice that we also include contemporary excess credit-to-GDP growth which, as expected, features a negative coefficient that is however only slightly significant and, more importantly, is noticeably lower than that of the lags.

Individual fixed effects are nowhere significant, indicating that within-iteration heterogeneity is not relevant,

[16]AIC and BIC tests typically suggest a number of lags between 2 and 4. The coefficient for the 6-th lags is also never significant, hence we keep the lag structure simple by including 5.

while time fixed effects are always strongly significant. The estimates once these are included are revised upwards, but this does not affect the robustness of our results.

We believe that, although in an admittedly tentative and probably incomplete manner, we have substantiated some of our *a priori* conjectures in the light of the evidence delivered by the model and their consistency with real world observations. Specifically with respect to the financial sector, we showed that credit is highly correlated with output by means of investment. Furthermore, we showed that while the first moment of the business cycle is decreasing with the degree of integration between financial and real sectors, the second is not. Hence, credit is in general associated with less growth and more volatile business cycles. To better understand the quantitative implications of this finding, we estimated a set of models in order to understand whether the likelihood of experiencing a recession is appreciatively increasing with respect to excessive lending. Indeed we find this to be the case, and thus conclude that *VaR* rules and, more broadly speaking, procyclical leverage behaviors are detrimental to growth and result in unstable fluctuations.

3.3.3 Firm dynamics

We conclude the evaluation of the model with a brief analysis of its capability to reproduce some microeconomic regularity concerning the productive units, the firms. We seek to reproduce some basic facts concerning (i) the firm size distribution (henceforth FSG) and (ii) firm growth distribution (henceforth FGD) which are observed empirically. Concretely, the FSD is typically non-normal, displaying positive kurtosis and heavy tails, thus calling upon wide and persistent heterogeneity to emerge across firms. In our model, firms are *ex ante* potentially identical, but we shall see that they turn out to differ given the environmental conditions they act within, that is, their success in the labor market and their access to credit. Concerning the FGD, we shall see that it features fat tails, and growth variability is typically negatively correlated with age. Furthermore, we shall seek to replicate some of the basic consequences of financial constraints upon firms.

We also report some summary statistics which might be of interest in table 5. We also present the result of the normality tests we perform on the distributions, which clearly reject the null hypothesis of normality. Summary statistics also point out at the striking heterogeneity across firms which, as said, stems from the environment firms are embedded within.

In the following table, we can further appreciate some quantitative facts about the FSDs thus shown. Arguably, the model is successful with some respects, while invalid with others, in replicating real world facts. Most importantly with respect to the latter, FSDs are left-skewed, unlike empirically observed ones, which by contrast are right-skewed.

Hence, we fail to replicate right-skewness, which would imply a positive skewness coefficient, throughout the time span. However, there are nonetheless some promising results. First, we expect older firms to be

Time	0	50	100	150	200
Mean	0.497	0.622	0.804	1.034	1.352
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Variance	0.589	0.723	0.890	1.104	1.414
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Skewness	2.645	2.363	2.447	2.268	2.069
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Kurtosis	9.064	7.452	7.383	6.179	4.834
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Jarque-Bera (95%)	90.196	94.118	94.118	96.078	96.078
Shapiro-Wilk (95%)	92.157	96.078	100.0	100.0	100.0

Table 5: Statistics and normality tests for selected panel-wide FSDs. Notes: Monte-Carlo standard errors in parentheses. Tests report the share (%) rejecting normality at 95% confidence level.

on average bigger than younger ones[17]. The variability of sizes is barely unvaried, but the FSD is clearly leptokurtic, and more so the older firms are. These are both empirically observed stylized facts we are able to replicate. Furthermore, FSDs reject normality throughout the simulation time span. Therefore, we are able to replicate a wide and persistent degree of heterogeneity and increasing conditional mean size given age.

Last, we turn the attention to the distribution of firm growth rates (FGD). The FGD is found to be substantially stable over the simulation time span, and is leptokurtic, consistently with empirical evidence. In the last two rows of 6 we also report evidence suggesting, as usual, non-normality of the FGD at the 5% level for each period, thus replicating a rather robust finding of the literature, namely, heavy tails in the growth rates. However, notice that these are more evidently present in later periods, as the distribution turns out to be clearly leptokurtic.

Time	0	50	100	150	200
Mean	-0.002	-0.002	0.002	0.007	0.004
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Variance	0.159	0.202	0.255	0.358	0.447
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Skewness	0.641	0.702	0.855	0.944	1.171
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Kurtosis	2.914	2.743	2.644	1.526	2.390
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Jarque-Bera (95%)	74.510	72.549	70.588	76.471	76.471
Shapiro-Wilk (95%)	98.039	98.039	98.039	98.039	100.000

Table 6: Summary statistics and normality tests for selected panel-wide FGDs. Notes: Monte-Carlo standard errors in parentheses. Tests report the share (%) rejecting normality at 95% confidence level.

We now sketch a quantitative framework for assessing the validity of Gibrat's law. Let A_{kt} be the size of firm

[17]Notice that since the bankruptcy rate is ≈ 2% per period, later FSDs generally feature older firms.

k at time t, measured in terms of real sales, then according to Gibrat's law one has that

$$\begin{aligned} A_{kt} &= (1 + \varepsilon_{kt})A_{kt-1} \\ a_{kt} &= a_{kt} + \varepsilon_{kt} \end{aligned} \tag{51}$$

where $\varepsilon_{kt} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and $a_{kt} = \log A_{kt}$.

In order to test the law, we consider four possible specifications for the equation above:

$$a_{kt} = \alpha + \beta a_{kt-1} + \varepsilon_{kt} \tag{52a}$$

$$a_{kt} = \alpha + \alpha_k + \beta a_{kt-1} + \varepsilon_{kt} \tag{52b}$$

$$a_{kt} = \alpha + \alpha_k + \beta a_{kt-1} + \sum_{j=0}^{300} \gamma_j \tau_j + \varepsilon_{kt} \tag{52c}$$

$$a_{kt} = \alpha + \beta a_{kt-1} + \sum_{t=0}^{300} \gamma_t \tau_t + \varepsilon_{kt} \tag{52d}$$

Model (52a) is simply a pooled.-OLS regression in which we neither take into account time effect nor individual fixed effects. Model (52b) comprises firm-specific effects and is estimated via fixed effects given the Hausman test rejecting random effects, and also due to the large sample we can exploit. In model (52c) we include a set of quarterly time dummies $\tau_j = 1$ if and only if $t = j$, to control for common trends between firms. Last, model (52d) is estimated via Arellano-Bond, where the number of lags to be taken as instruments is taken so as satisfy (i) residuals of the difference regression between the instrumented regressor and its instruments to be serially correlated of the first order (*i.e.* AR(1)), and (ii) be consistent with the Sargan-Hansen test for overidentifying restrictions.

Country fixed effects are only significant once time effects are included, while the latter always strongly significant. The coefficient $\hat{\beta}$ of the lagged log-size is always strongly significant, and close to .98 in all model specifications. Indeed, we run a Wald test to ensure that $\hat{\beta}$ is actually statistically different from 1, that is the benchmark which would hold had Gibrat's law been valid. Indeed we find this to be the case at 99% confidence level for *all* the iterations of the model. We conclude that Gibrat's law does not hold, albeit it is not critically at odds with the data. What is more, a $\hat{\beta} < 1$ implies a slight reversion-to-the-mean behavior, meaning that younger firms tend to grow more, though not dramatically more, than older ones, a finding that has been shown to be observed in the literature as well.

Admittedly, the consistency between the model and real data concerning microeconomic actors is not entirely convincing. Also, we devoted less effort trying to deal with them, as this is not the main focus of this work. This notwithstanding, for the sake of completeness and intellectual honesty we think that it is worth to outline both strengths and difficulties of the model. We have thus shown that we are able to replicate some key facts on non-normality of both the FSD and the FGD. Also, the FSD is leptokurtic and points out an important and persistent heterogeneity across firms, something that is mostly neglected in mainstream

Model	POLS (52a)	FE (52b)	T-FE (52c)	T-AB (52d)
Sample realization				
Lagged log-size	0.984*** (0.001)	0.981*** (0.001)	0.965*** (0.001)	0.974*** (0.001)
α	0.048*** (0.002)	0.061*** (0.004)	0.111*** (0.019)	0.049*** (0.003)
R^2	98%	98%	98%	/
Test $\hat{\beta} \neq 1$	$135.61^\dagger$	$211.54^\dagger$	$288.69^\dagger$	$214.15^\dagger$
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
Individual effects = 0 (p-value)	/	$1.32^\ddagger$ (0.21)	$1.43^\ddagger$ (0.00)	/
Time effects = 0 (p-value)	/	/	$2.43^\ddagger$ (0.00)	$3.93^\ddagger$ (0.00)
Monte Carlo analysis				
Lagged log-size	0.986 (0.002)	0.982 (0.002)	0.962 (0.002)	0.971 (0.002)
Test $\hat{\beta} \neq 1$	100%	100%	100%	100%

Table 7: Gibrat's regressions. Notes: Dependent variable is log-size. *, **, *** respectively stand for 10%, 5% and 1% significance level. †: reported statistic is χ^2, ‡ is F-statistic.Monte Carlo standard errors features in due section.

macro-models. The FGD too is fat-tailed, and mostly symmetric. Furthermore, Gibrat's law is only partially valid: mean reversion is observed, thus shedding evidence on small firms growing more than old ones, albeit only slightly so. By contrast, the skewness of the FSD is wrong, in that our FSD is left-skewed, whereas empirically observed ones are right-skewed, this being likely to represent the major difficulty the model faces when dealing with firm dynamics.

4 Conclusions

In this essay we developed an agent-based model that seeks to study an economy featuring a real and a financial sector, with the purpose to disentangle the complex links between the two and the feedback and amplification dynamics stemming from such connections. We embedded in the model two empirically documented routines of balance sheet management, namely value-at-risk leverage management and mark-to-market accounting, and showed how these assumptions allow the model to match many of the most recent empirical findings concerning the dynamics of credit, leverage and business cycles.

In our environment, a relatively simple real sector consisting of firms and households finances production through bank lending. The financial sector, which has to some extend been the core of the analysis, is then composed of unleveraged investors trading in bank equity (*i.e.* mutual funds) whose market interactions yield the price of said equity. Given that, banks evaluate their overall loan exposure as a mark-up over the value of their equity using a VaR decision routine. The resulting leverage ratio is therefore a function of the equity market outcome and real activity.

The statistical analysis of the time series generated by the models revealed it to be consistent with a wide array of empirical stylized facts. First, the correlation structure of the model is in line with earlier business cycle studies, both in terms of correlation magnitudes and timing. Furthermore, we find encouraging, albeit incomplete, evidence confirming non-normality of the GDP distribution growth rates. We also document that the model is broadly consistent with basic firm dynamics regularities: both the firm size and size growth distributions reject normality, the FSD is right-skewed while the FGD is persistently fat-tailed. Moreover, Gibrat's law is found to hold in our simulation sample.

What is perhaps more interesting, nonetheless, is that the model generates procyclical leverage, hence standing consistent with the findings whereby Adrian & Shin (2010). Moreover, we replicate some of the main insights of the work by Schularick & Taylor (2012) and related works. Specifically, recessions that are preceded by credit booms are found to be harsher than those that are not. The volatility and skewness of the business cycle are decreasing in the credit-to-GDP ratio even though the kurtosis is increasing, hence suggesting that a more integrated financial sector is associated with more extreme tail events, such are costly financial recessions. Last, we examined whether credit booms can be a predictor for recessions and, once more consistently with empirical evidence, we confirmed the claim.

This notwithstanding, our model has several shortcomings. It fails at robustly replicating the non-gaussian GDP growth rate distribution, which has been documented by Fagiolo, Napoletano & Roventini (2008). Furthermore, in our model the GDP, consumption and investment growth rates are increasing in the credit-to-GDP ratio, even though this contrasts with the empirical evidence. While the model is clearly open to future improvement, we nonetheless regard it as a promising addition to the ABM literature that seeks to study business cycles from a perspective that is crucially different, albeit not antithetical, from traditional neoclassical models.

Davide M. Coluccia

Milan, November 19, 2019

Appendices

A - Calibration and Correlation Tables

Class	Symbol	Description	Value
	I	Number of E.O.s.	100
	δ_i	MA(1) E.O.s' expectation parameter.	$\mathcal{T}(0.1, 0.2, 0.3)$
I	β_i	Portfolio shares resonsiveness.	0.9
	γ_i	Degree of fundamentalism.	$\mathcal{T}(0.2, 0.3, 0.4)$
	σ_{ε_j}	Noise in the signals.	$\mathcal{T}(2.5, 3.0, 3.5)$
	J	Number of banks.	10
	E	Nominal amount of equity per firm.	0.2
	$\bar{\lambda}$	Threshold on bank leverage ratio.	30
	δ_j	MA(1) banks' expectation parameter.	$\mathcal{T}(0.2, 0.5, 0.8)$
	β_j	Portfolio shares responsiveness.	0.9
J	γ_j	Share of net worth in portfolio weights.	$\mathcal{T}(0.8, 1.0, 1.2)$
	ϱ_{1j}	Interest rate weights: firm net worth.	$\mathcal{T}(0.2, 0.3, 0.4)$
	ϱ_{2j}	Interest rate weights: bank net worth.	$\mathcal{T}(0.2, 0.3, 0.4)$
	ϱ_{3j}	Interest rate weights: inflation	$\mathcal{T}(0.2, 0.3, 0.4)$
	ϱ_{4j}	Interest rate weights: productivity.	$\mathcal{T}(0.2, 0.3, 0.4)$
	a	VaR quantile.	0.99
	ρ_j	Propensity to distribute profits.	0.6
	K	Number of firms.	50
	λ	Response to excess demand.	0.7
	χ	Share of profits spent in R& D.	0.2
	η	Share of innovation in R& D.	0.3
	ζ	Relative productivity of labor.	0.5
	ν	Share of observed firms to be imitated.	1
	μ	Mark-up responsiveness to market share.	0.15
	ι_1	Bernoulli innovation parameter.	0.02
K	ι_2	Bernoulli imitation parameter.	0.02
	ω_1	Competitiveness weight on price.	0.5
	ω_2	Competitiveness weight on excess demand.	0.5
	σ	Market share responsiveness to competitiveness.	0.4
	ρ_k	Firm propensity to divide profits.	0.95
	λ_k	Switching: propensity to switch partner.	0.0
	δ	Depreciation rate	0.2
	τ	Income tax rate.	0.2
	H	Number of households.	1000
	ψ_1	Wage weights: unemployment.	0.25
	ψ_2	Wage weights: inflation.	0.25
	ψ_3	Wage weights: productivity.	0.25
H	ψ_4	Wage weights: profits.	0.25
	σ_{hk}	Firm ownership shares.	$1/H$
	λ_h	Switching: propensity to switch partner.	0.9
	$\bar{w}$	Share of reservation-to-market wage.	0.2

Table 8: Symbol, synthetic description and values of the parameters of the model.

Table 9: Correlation structure for selected variables and lags with respect to the business cycle. Monte-Carlo standard errors in parentheses.

Variable (BP-f)	**Correlation coefficient: $\mathbf{Corr(Y, X_{t-k})}$**								
	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
Output	0.734862	0.225444	0.760451	0.440937	1.000000e+00	0.440937	0.760451	0.225444	0.734862
	(0.213331)	(0.470167)	(0.192861)	(0.437971)	(0.63000)	(0.437971)	(0.192861)	(0.470167)	(0.213331)
Consumption	0.763854	0.544393	0.796200	0.579460	8.004218e-01	0.594740	0.862107	0.633160	0.772939
	(0.164144)	(0.349595)	(0.143121)	(0.341057)	(1.423500e-01)	(0.328941)	(0.084684)	(0.324417)	(0.152080)
Investment	0.305695	-0.301984	0.306994	-0.009096	6.793878e-01	-0.018314	0.242654	-0.395792	0.293887
	(0.234077)	(0.281689)	(0.234639)	(0.314632)	(1.420363e-01)	(0.313839)	(0.257458)	(0.286212)	(0.238286)
Credit	0.283310	-0.197544	0.249546	0.134151	7.006619e-01	0.178162	0.260132	-0.234072	0.267808
	(0.196110)	(0.267975)	(0.215850)	(0.265873)	(1.357438e-01)	(0.271186)	(0.212724)	(0.273854)	(0.205177)
Price	0.614289	0.469534	0.527939	0.543665	7.481014e-01	0.684563	0.637300	0.457530	0.540730
	(0.239527)	(0.338466)	(0.271001)	(0.277291)	(1.446160e-01)	(0.222092)	(0.201411)	(0.329708)	(0.271466)
Wage	0.667421	0.533779	0.646873	0.657101	8.190312e-01	0.701153	0.697627	0.556355	0.657917
	(0.188051)	(0.309483)	(0.186411)	(0.237605)	(9.809943e-02)	(0.224719)	(0.152055)	(0.289020)	(0.188987)
Price Inflation	0.043605	-0.263549	0.067551	0.066813	3.963845e-01	-0.080181	-0.132259	-0.328389	0.151344
	(0.161050)	(0.136213)	(0.154169)	(0.183586)	(1.445832e-01)	(0.146091)	(0.203295)	(0.139500)	(0.135751)
Wage Inflation	0.035453	-0.330533	0.253392	0.118377	4.310304e-01	-0.265067	-0.080024	-0.326832	0.263031
	(0.220511)	(0.171377)	(0.184838)	(0.261853)	(1.605204e-01)	(0.181950)	(0.256030)	(0.187103)	(0.165195)
Unemployment	-0.374515	-0.025147	-0.301129	-0.372912	-7.862996e-01	-0.466176	-0.406971	-0.075505	-0.361600
	(0.198038)	(0.317018)	(0.214845)	(0.234145)	(5.594372e-02)	(0.240223)	(0.173017)	(0.295316)	(0.201144)
Public Expenditure	0.663187	0.583366	0.663000	0.696229	7.964262e-01	0.711831	0.683172	0.598207	0.666601
	(0.192312)	(0.273826)	(0.181734)	(0.205188)	(1.144188e-01)	(0.203474)	(0.168104)	(0.260806)	(0.186961)
Bank Price	0.610906	0.765356	0.632722	0.746359	5.870946e-01	0.731256	0.615275	0.760875	0.614071
	(0.237591)	(0.119878)	(0.235843)	(0.120519)	(2.351651e-01)	(0.130162)	(0.242581)	(0.125727)	(0.236867)
Bank Dividend	0.777069	0.480737	0.721933	0.391851	7.171020e-01	0.470535	0.771050	0.451820	0.720747
	(0.080958)	(0.295717)	(0.098142)	(0.288042)	(1.115925e-01)	(0.303358)	(0.089173)	(0.290850)	(0.094462)
Leverage	-0.141856	-0.360436	-0.288054	-0.272442	3.749452e-01	0.562216	0.224828	0.039366	-0.163351
	(0.196982)	(0.171106)	(0.213483)	(0.148953)	(1.908565e-01)	(0.199413)	(0.228982)	(0.195243)	(0.207560)
Interest rate	0.404271	0.307898	0.313304	-0.042527	6.124336e-02	0.111202	0.431488	0.279905	0.285480
	(0.181234)	(0.249085)	(0.161121)	(0.216844)	(2.208146e-01)	(0.254860)	(0.194200)	(0.241874)	(0.168322)
Productivity	0.731281	0.734427	0.736538	0.734823	7.336548e-01	0.729775	0.727533	0.725384	0.723410
	(0.185984)	(0.186150)	(0.187075)	(0.186707)	(1.862947e-01)	(0.185974)	(0.186010)	(0.185473)	(0.186334)

B - Run the Code

Replication codes are freely available at github.com/dcoluccia/SantAnna-Thesis-2019.

All codes are written in `Python3`. The `NumPy`, `SciPy`, `statsmodels`, `time`, `pandas`, `matplotlib`, `csv` and `sklearn` are needed to succesfully run the programs. All files need be placed in the same directory. All libraries are contained in the latest `Anaconda` or `Conda` distributions.

- To replicate the figures of the sample simulation, run `make_figures.py`;
- To generate the Monte Carlo time series, run `panel.py`;
- To perform the analysis of said series, run `MCanalysis.py`. Tables are exported in `.tex` format. Only run after `panel.py` is terminated.

Single realization time series are obtained by setting `T=700` and cutting the first 550 iterations as transient (line `120` of `main.py`). This is done in lines `141-168` of `main.py`. Monte Carlo time series are obtained by setting `T = 750` and cutting the first 450 iterations. This is done respectively in lines `120` and `141-168` in `main.py`.

Runtime of the sample simulation around 20". Runtime of the Monte Carlo simulation around 1h.

References

Abraham, K. G. and Haltiwanger, J. C. (1995). "Real wages and the business cycle", *Journal of Economic Literature*, **33**(3): 1215-1264.

Adrian, T. and Shin, H. S. (2010). "Liquidity and leverage", *Journal of Financial Intermediation*, **19**(3): 418-437.

Adrian, T. and Shin, H. S. (2013). "Procyclical leverage and value-at-risk", *Review of Financial Studies*, **27**(2): 373-403.

Ascari, G., Fagiolo, G. and Roventini, A. (2015). "Fat-tail distributions and business-cycle models", *Macroeconomic Dynamics*, **19**(2): 465-476.

Aymanns, C. and Farmer, J. D. (2015). "The dynamics of the leverage cycle", *Journal of Economic Dynamics and Control*, **50**: 155-179.

Aymanns, C., Caccioli, F., Farmer, J. D. and Tan, V. W. (2016). "Taming the Basel leverage cycle", *Journal of Financial Stability*, **27**: 263-277.

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999). "The financial accelerator in a quantitative business cycle framework", *Handbook of macroeconomics*, **1**: 1341-1393.

Bottazzi, G. and Secchi, A. (2003). "Why are distributions of firm growth rates tent-shaped?", *Economics Letters*, **80**(3): 415-420.

Bottazzi, G. and Secchi, A. (2006). "Explaining the distribution of firm growth rates", *The RAND Journal of Economics*, **37**(2): 235-256.

Bottazzi, G., Secchi, A. and Tamagni, F. (2014). "Financial constraints and firm dynamics", *Small Business Economics*, **42**(1): 99-116.

Braun-Munzinger, K., Liu, Z. and Turrell, A. (2018). "An agent-based model of dynamics in corporate bond trading", *Quantitative Finance*: Forthcoming.

Breeden, D. T. and Litzenberger, R. H. (1978). "Prices of State-Contingent Claims Implicit in Option Prices", *Journal of Business*, **51**(4): 621-651.

Brock, W. A. and Hommes, C. H. (1997). "A rational route to randomness", *Econometrica*, **65**(5): 1059-1095.

Brunnermeier, M. K. (2009). "Deciphering the liquidity and credit crunch 2007-2008", *Journal of Economic Perspectives*, **23**(1), 77-100.

Caballero, R. J. (2010). "Macroeconomics after the crisis: time to deal with the pretense-of-knowledge syndrome", *Journal of Economic Perspectives*, **24**(4): 85-102.

Caiani, A., Godin, A., Caverzasi, E., Gallegati, M., Kinsella, S. and Stiglitz, J. E. (2016). "Agent based-stock flow consistent macroeconomics: Towards a benchmark model", *Journal of Economic Dynamics and Control*, **69**: 375-408.

Christiano, L. J. and Fitzgerald, T. J. (2003). "The band pass filter", *International economic review*, **44**(2): 435-465.

Claessens, S., Kose, M. A. and Terrones, M. E. (2009). "What happens during recessions, crunches and busts?", *Economic Policy*, **24**(60): 653-700.

Dass, N. and Massa, M. (2011). "The impact of a strong bank-firm relationship on the borrowing firm", *Review of Financial Studies*, **24**(4): 1204-1260.

Delli Gatti, D., Di Guilmi, C., Gaffeo, E., Giulioni, G., Gallegati, M. and Palestrini, A. (2005). "A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility", *Journal of Economic Behavior & Organization*, **56**(4): 489-512.

Delli Gatti, D., Gallegati, M., Greenwald, B., Russo, A. and Stiglitz, J. E. (2010). "The financial accelerator in an evolving credit network", *Journal of Economic Dynamics and Control*, **34**(9): 1627-1650.

Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P. and Gallegati, M. (2011). *Macroeconomics from the bottom-up*, Berlin (GER): Springer.

Dosi, G. and Nelson, R. R. (2010). "Technical change and industrial dynamics as evolutionary processes", in *Handbook of the Economics of Innovation*, Amsterdam (NETH): North Holland, **1**, 51-127.

Dosi, G. (2006). "Statistical regularities in the evolution of industries: A guide through some evidence and challenges for the theory", *LEM Working Paper Series*, No. 2006/17.

Dosi, G., Fagiolo, G. and Roventini, A. (2010). "Schumpeter meeting Keynes: A policy-friendly model of endogenous growth and business cycles", *Journal of Economic Dynamics and Control*, **34**(9): 1748-1767.

Dosi, G., Marsili, O., Orsenigo, L., and Salvatore, R. (1995). "Learning, market selection and the evolution of industrial structures", *Small Business Economics*, **7**(6), 411-436.

Dunne, T., Roberts, M. J. and Samuelson, L. (1988). "Patterns of firm entry and exit in US manufacturing industries", *The RAND Journal of Economics*, **19**(4): 495-515.

Eggertsson, G. B. and Krugman, P. (2012). "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach", *Quarterly Journal of Economics*, **127**(3): 1469-1513.

Fagiolo, G. and Roventini, A. (2016). "Macroeconomic Policy in DSGE and Agent-Based Models Redux: New Developments and Challenges Ahead", *Journal of Artificial Societies and Social Simulation*, **20**(1).

Fagiolo, G., Napoletano, M. and Roventini, A. (2008). "Are output growth-rate distributions fat-tailed? some evidence from OECD countries", *Journal of Applied Econometrics*, **23**(5): 639-669.

Fazzari, S. M., Hubbard, R. G., Petersen, B. C., Blinder, A. S. and Poterba, J. M. (1988). "Financing constraints and corporate investment", *Brookings Papers on Economic Activity*, **1**: 141-206.

Fujita, M., Krugman, P. R. and Venables, A. J. (2001). "The spatial economy: Cities, regions, and international trade", Cambridge (MA): MIT press.

Geanakoplos, J. (2010). "The leverage cycle", *NBER macroeconomics annual*, **24**(1): 1-66.

Gibrat, R. (1931). *Les inégalités économiques*, Paris (FR): Librairie du Recueil Sirey.

Hellerstein, J. K., Neumark, D. and Troske, K. R. (1999). "Wages, productivity, and worker characteristics: Evidence from plant-level production functions and wage equations", *Journal of Labor Economics*, **17**(3): 409-446.

Im, K. S., Pesaran, M. H. and Shin, Y. (2003). "Testing for unit roots in heterogeneous panels", *Journal of Econometrics*, **115**(1): 53-74.

Jordà, Ò., Schularick, M. and Taylor, A. M. (2013). "When credit bites back", *Journal of Money, Credit and Banking*, **45**(s2): 3-28.

Jordà, Ò., Schularick, M., and Taylor, A. M. (2015). "Leveraged bubbles", *Journal of Monetary Economics*, **76**(S): 1-20.

Jordá, Ó., Schularick, M. and Taylor, A. M. (2017). "Macrofinancial history and the new business cycle facts", *NBER Macroeconomics Annual*, **31**(1): 213-263.

Kirman, A. (1989). "The intrinsic limits of modern economic theory: the emperor has no clothes", *Economic Journal*, **99**(395): 126-39.

Kiyotaki, N. and Moore, J. (1997). "Credit cycles", *Journal of Political Economy*, **105**(2): 211-248.

López-Salido, D., Stein, J. C. and Zakrajšek, E. 2017. "Credit-market sentiments and the business cycle", *Quarterly Journal of Economics*, **132**(3): 1373-1426.

Lavoie, M. (2014). *Post-Keynesian economics: new foundations*, London (UK): Edward Elgar Publishing.

LeBaron, B. (2006). "Agent-Based Computational Finance", in Tesfation, L. and Judd, K. L. (eds.), *Handbook of Computational Economics*, Amsterdam (Neth): North Holland.

Levin, A., Lin, C. F. and Chu, C. S. J. (2002). "Unit root tests in panel data: asymptotic and finite-sample properties", *Journal of Econometrics*, **108**(1): 1-24.

Mincer, J. and Jovanovic, B. (1981). "Labor Mobility and Wages", in Rosen, S. (ed.): *Studies in Labor Markets*, Chicago (IL): University of Chicago Press: 21-63.

Nelson, R. R. and Winter, S. G. (1982). *An evolutionary theory of economic change*, Cambridge (MA): Harvard University Press.

Ng, V. and Wright, J. H. (2013). "Facts and Challenges from the Great Recession for Forecasting and Macroeconomic Modeling", *Journal of Economic Literature*, **51**(4): 1120-1154.

PHILLIPS, A. W. (1958). “The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957”, *Economica*, **25**(100): 283-299.

POLEDNA, S., THURNER, S., FARMER, J. D. and GEANAKOPLOS, J. (2014). “Leverage-induced systemic risk under Basle II and other credit risk policies”, *Journal of Banking & Finance*, **42**: 199-212.

QUANDT, R. E. (1966). “On the size distribution of firms”, *American Economic Review*, **56**(3): 416-432.

REINHART, C. M. and ROGOFF, K. S. (2008). “Is the 2007 US sub-prime financial crisis so different? An international historical comparison”, *American Economic Review*, **98**(2): 339-44.

REINHART, C. M. and ROGOFF, K. S. (2013). “Banking crises: an equal opportunity menace”, *Journal of Banking & Finance*, **37**(11): 4557-4573.

RICCETTI, L., RUSSO, A. and GALLEGATI, M. (2013). “Leveraged network-based financial accelerator”, *Journal of Economic Dynamics and Control*, **37**(8): 1626-1640.

RICCETTI, L., RUSSO, A. and GALLEGATI, M. (2015). “An agent based decentralized matching macroeconomic model”, *Journal of Economic Interaction and Coordination*, **10**(2): 305-332.

ROMER, P. M. (1990). “Endogenous technological change”, *Journal of Political Economy*, **98**(5): 71-102.

SCHULARICK, M. and TAYLOR, A. M. (2012). “Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008”, *American Economic Review*, **102**(2): 1029-1061.

SHEK, J. SHIM, I. and SHIN, H. S. (2015). “Investor redemptions and fund manager sales of emerging market bonds: how are they related?”, *BIS Working Papers, 509, 201*.

SHEK, J., SHIM, I. and SHIN, H. S. (2017). “Investor redemptions and fund manager sales of emerging market bonds: how are they related?”, *Review of Finance*, **22**(1): 207-241.

SILVERMAN, B. W. (1981). “Using kernel density estimates to investigate multimodality”, *Journal of the Royal Statistical Society. Series B*, **43**(1): 97-99.

SOLOW, R. M. (1956). “A contribution to the theory of economic growth”, *Quarterly Journal of Economics*, **70**(1): 65-94.

STIGLITZ, J. E. and WEISS, A. (1981). “Credit rationing in markets with imperfect information”. *American Economic Review*, **71**(3): 393-410.

STOCK, J. H. and WATSON, M. W. (1999). “Business cycle fluctuations in US macroeconomic time series”, *Handbook of macroeconomics*, **1**: 3-64.

WINDRUM, P., FAGIOLO, G. and MONETA, A. (2007). “Empirical validation of agent-based models: Alternatives and prospects”, *Journal of Artificial Societies and Social Simulation*, **10**(2): 8.