

## Bi-criteria single facility location problem with limited distances

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### Abstract

This paper deals with a bi-criteria model for locating single new facility on the plane where the effective service distance becomes a constant when the actual distance attains or exceeds a certain value. The criteria that are used in this bi-criteria model are minisum, that minimizes the sum of all the transportation costs and minimax, that minimizes the maximum distances from the new facility to the demand points. By using a mixed integer programming formulation, the model is transformed to an easier form, and then the technique of fuzzy programming has been applied to solve it. A numerical example is also presented to illustrate the procedure and then it has been solved by LINGO software.

**Keywords:** Facility Location; Limited Distances; Multi-criteria Decision Making; Minisum & Minimax Problems; Fuzzy Programming.

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### 1. Introduction

In facility location literature, there are two important types of problems that called minisum and minimax location problems (see [1]). For single facility location, the objective of the minisum problem is to locate a new facility on the plane so that a weighted sum of distances from the new facility to the demand points is minimized and the objective of the minimax problem is to minimize the maximum among the weighted distances from the new facility to the demand points (see [2]).

Problems of minisum type arise in locating economic facilities such as plants and warehouses, service facilities such as stores and postal stations, and design problems such as communications

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network design and those found in piping and wiring networks (see [3] & [4]). Problems of minimax type arise in locating emergency service facilities such as fire stations, ambulance centers, police stations, and civil defense, and also in the location of detection stations (see [5] & [6]).

In most researches only one of these criteria are used, for example, some of single-criterion location problems from type of minisum and minimax could be seen in researches of Drezner & Wesolowsky [7] and Rosen & Xue [8]. Sometimes a realistic situation arises where it is not possible to consider only one criterion and should consider many criteria. Therefore, some conflicting objectives need to be considered simultaneously in a multi-criteria formulation.

Multi-criteria formulations of facility location problems in the plane have not been extensively studied in the literature. Among those who have studied multi-criteria aspects of facility location problems are, McGinnis and White [9], Lee et al [10], Melachrinoudis [11] and Brimberg and Juel [12], Ohsawa [13].

McGinnis and White deal with the problem of locating a facility in the plane relative to several demand points using, simultaneously, a minisum and a minimax criterion under rectilinear distances.

Lee et al presented the application of integer goal programming to the facility location and products allocation problem with multiple, competing objectives. Multiple objectives including several qualitative factors of location analysis and cost-related objectives have been considered in their model.

Melachrinoudis developed a maximin-minisum bi-criteria location model with rectilinear distances. He decomposed the model into a series of linear bi-criteria problems which are solved by an adaptation of the Fourier-Motzkin Elimination Method to generate the nondominated and efficient sets of the sub problems.

Brimberg and Juel considered the problem of locating a facility in the plane. In their model two criteria were used: One is the well-known minisum criterion; in the other they minimize the weighted sum of Euclidean distances raised to a negative power and also in the bicriteria model they minimize the weighted sum of the two criteria, with weights adding to one.

Yoshiaki Ohsawa was concerned with a single facility bicriteria location model associated with maximin and minimax criteria in the Euclidean plane. He presented a polynomial-time algorithm for generating the analytical expressions of the efficient set and the tradeoff curve between the conflicting goals. He characterized the set and the curve with the aid of nearest- and farthest-point Voronoi diagrams. Indeed the efficient set and the tradeoff curve will enable planners to reduce alternatives based on two criteria and, consequently, compare these remaining alternatives graphically in terms of these two criteria.

More studies about multi-criteria can be seen in [14] and [15] and specially bi - criteria location including minisum and minimax [16], [17], [18], [19].

This paper deals with a bi-criteria facility location problem with limited distances (BCFPLD) on the plane that is bounded by a convex polygon under minisum and minimax criteria where the actual distances are rectilinear that is based on introduced model by Drezner et al [20] in 1991 in which they considered the location of a new facility in the plane in order to minimize the sum and maximum of "effective service distances" to a given set of users separately, which the effective service distance is proportional to  $l_p$  distance up to a predetermined given value and remains constant thereafter. In fact in this paper both minisum and minimax models with limited distances that had discussed in the paper [20] are considered in the form of a bi-criteria model.

Facility location problem with limited distances (FLPLD) from type minisum, was studied by Chen et al [21] later, they used the D.-C. Program technique to solve that problem where the actual distances between new facility and existing facilities was considered as Euclidean.

For solving our proposed model, first we used a mixed integer programming formulation to convert it to an easier form and then fuzzy programming approach has been applied where the fuzzy multi-objective programming approach was first developed by Zimmermann [22]. Applications of fuzzy programming approach for solving the multi-objective location problems can be seen in [23] and [24], also Uno and Katagiri [26] extended defensive location problem to a multi-objective model that to find a satisfying solution proposed an interactive fuzzy satisfying method.

This paper is organized as follows: Section 2 presents the problem formulation, some notation and preliminaries. In section 3 we introduce fuzzy programming technique for the multi-objective problems and the solution procedure is presented in section 4. Section 5 illustrates a numerical example and finally Section 6 concludes the study.

## 2. Problem Formulation

The Bi-criteria (minisum – minimax) location problem with limited distances is stated as follows: Given a two-dimensional region  $S$  and a set of existing facility points  $P_i = (a_i, b_i)$  for  $i = 1, \dots, n$  located in  $S$ , find a point  $X = (x, y) \in S$  that minimizes the minimum limited distance from  $(x, y)$  to the  $n$ -points and minimizes the total transportation cost between  $(x, y)$  and the  $n$ -points that they are equivalent with

M1:

$$\begin{aligned} \min_{(x,y) \in S} \{Z^1 = \max_i \{d_i(x, y)\}\} \\ \min_{(x,y) \in S} \left\{ Z^2 = \sum_{i=1}^n w_i d_i(x, y) \right\} \\ \text{Subject to} \\ c_{j1}x + c_{j2}y \leq c_{j3} \quad j = 1, 2, \dots, m \end{aligned}$$

Where

- $Z^1$  is the minimax objective function
- $Z^2$  is the minisum objective function
- $d_i(x, y) = \min\{r_i(X, P_i), D_i\}$  is the maximum constrained distance
- $r_i(X, P_i) = [|x - a_i|^p + |y - b_i|^p]^{\frac{1}{p}}$  is the actual distance of  $l_p$  norm between  $X$  and  $P_i$  that  $p$  is the distance parameter, for example, for  $p = 1$  we have rectilinear distances and for  $p = 2$  have Euclidean distances
- $w_i$  is the transportation cost per unit distance between the new facility and existing facility  $i$
- $D_i$  is the “maximum distance” associated with existing facility  $i$
- $c_{j1}, c_{j2}$  and  $c_{j3}$  For  $j = 1, 2, \dots, m$  are constants associated with a given convex polygon.

The assumptions of the bi-criteria facilities location problem with limited distances (BCFLPLD) are:

1. There are  $n$  users or demand points located at given points  $P_i$  for  $i = 1, \dots, n$  and each user is associated with a given value  $D_i$ , the “maximum effective service distance”.
2. The cost of a facility at location  $X$  in a plane to a user is proportional to the weighted distance from his location to this facility this distance is noted  $r_i(X, P_i)$  for  $i = 1, \dots, n$
3. One facility is to be located at a point  $X$  within a convex region  $S$ , such that the two objective function is minimized
4. The constraint of M1 model can be considered with the  $m$  number of linear constraints that make the solution space (see [23] and [24]).

Then, M1 can be rewritten as follows:

M2:

$$\begin{aligned} & \text{Minimize } \{Z^1 = \max_i \{\min\{r_i(X, P_i), D_i\}\}\} \\ & \text{Minimize } \left\{Z^2 = \sum_{i=1}^n w_i \min\{r_i(X, P_i), D_i\}\right\} \\ & \text{Subject to} \\ & c_{j1}x + c_{j2}y \leq c_{j3} \quad j = 1, \dots, m \\ & r_i(X, P_i) = [|x - a_i|^p + |y - b_i|^p]^{\frac{1}{p}} \quad i = 1, \dots, n \end{aligned}$$

### 3. Fuzzy programming technique for the multi-objective problems

Consider the multi-objective programming problem

$$\begin{aligned} & \text{Minimize } Z(x) = [Z^1(x), Z^2(x), \dots, Z^k(x)]^T \\ & \text{Subject to} \\ & x \in S \end{aligned}$$

Where  $x \in R^n$  is an  $n$ -dimensional decision variable and  $S$  is the set of feasible solutions (feasible solution space). One of the approaches that are commonly used for solving the above problem is fuzzy programming.

Looking to history of fuzzy programming literature, specifying that when Bellman and Zadeh [25] introduced the basic concepts of fuzzy goals( $G$ ), fuzzy constraints( $C$ ) and fuzzy decisions( $D$ ), fuzzy sets theory implemented in mathematical programming. Based on these concepts, the fuzzy decision is defined as follows

$$D = G \cap C$$

This problem is characterized by the following membership function:

$$\mu_D(x) = \min (\mu_G(x), \mu_C(x))$$

To be more specific, let us describe the fuzzy goals of a multi-objective problem with the following membership function:

$$\mu_k(Z^k(x)) = \begin{cases} 1 & \text{if } Z^k(x) \leq L^k \\ \frac{U^k - Z^k(x)}{U^k - L^k} & \text{if } L^k \leq Z^k(x) \leq U^k \\ 0 & \text{if } Z^k(x) \geq U^k \end{cases}$$

Where  $U^k$  is the worst upper bound and  $L^k$  is the best lower bound of the objective function  $k$ , respectively. They are calculated as follows

$$U^k = \max_{1 \leq j \leq K} \{Z^k(X^j)\} \quad , \quad L^k = Z^k(X^k) \quad k = 1, \dots, K$$

By using the given membership function and following the fuzzy decision of Bellman and Zadeh [25], the multi-objective problem can be written as follows:

$$\begin{aligned} & \max \min_{k=1, \dots, K} \{\mu_k(Z^k(x))\} \\ & \text{subject to} \end{aligned}$$

$$x \in S$$

By introducing an auxiliary variable  $\lambda$  above problem can be transformed into the following model:

$$\begin{aligned} \max \quad & \lambda \\ \text{subject to} \quad & \lambda \leq \mu_k(Z^k(x)) \quad k = 1, \dots, K \\ & x \in S \end{aligned}$$

#### 4. Solution Procedure

Bi-objective problem  $M2$  simply can be formulated as a mixed integer programming formulation.

$M3$ :

$$\begin{aligned} \text{Minimize } Z^1 &= \max_i \{v_i D_i + (1 - v_i) r_i(X, P_i)\} \\ \text{Minimize } Z^2 &= \sum_{i=1}^n v_i w_i D_i + (1 - v_i) w_i r_i(X, P_i) \\ \text{Subject to} \quad & c_{j1}x + c_{j2}y \leq c_{j3} \quad j = 1, \dots, m \\ & r_i(X, P_i) = [|x - a_i|^p + |y - b_i|^p]^{\frac{1}{p}} \quad i = 1, \dots, n \\ & v_i \in \{0, 1\} \quad i = 1, \dots, n \end{aligned}$$

Where

$$v_i = \begin{cases} 0, & \text{if } r_i(X, P_i) \leq D_i \\ 1, & \text{if } r_i(X, P_i) > D_i \end{cases}$$

In this paper we use rectilinear distance ( $p = 1$ ). For solving bi-objective problem  $M3$ , we use fuzzy programming method, in this way that, at first we solve the bi-objective problem  $M3$  without considering the second objective and again we solve it without considering the first one so we divide it to two sub problems

Consider the first sub problem

$M4$ :

$$\begin{aligned} \text{Minimize } Z^1 &= \max_{i=1, \dots, n} \{v_i D_i + (1 - v_i) r_i(X, P_i)\} \\ \text{Subject to} \quad & c_{j1}x + c_{j2}y \leq c_{j3} \quad j = 1, \dots, m \\ & r_i(X, P_i) = |x - a_i| + |y - b_i| \quad i = 1, \dots, n \\ & v_i \in \{0, 1\} \quad i = 1, \dots, n \end{aligned}$$

The above problem can be written as

$M5$ :

$$\begin{aligned} \text{Minimize } Z^1 & \\ \text{Subject to} \quad & Z^1 \geq v_i D_i + (1 - v_i) r_i(X, P_i) \quad i = 1, \dots, n \end{aligned}$$

$$\begin{aligned} c_{j1}x + c_{j2}y &\leq c_{j3} & j = 1, \dots, m \\ r_i(X, P_i) &= |x - a_i| + |y - b_i| & i = 1, \dots, n \\ v_i &\in \{0, 1\} & i = 1, \dots, n \end{aligned}$$

Denote  $X^1$  as the optimum solution of first sub problem.

Consider the second sub problem

M6:

$$\begin{aligned} &\text{Minimize } Z^2 \\ &\text{Subject to} \\ &Z^2 = \sum_{i=1}^n v_i w_i D_i + (1 - v_i) w_i r_i(X, P_i) \\ &c_{j1}x + c_{j2}y \leq c_{j3} & j = 1, \dots, m \\ &r_i(X, P_i) = |x - a_i| + |y - b_i| & i = 1, \dots, n \\ &v_i \in \{0, 1\} & i = 1, \dots, n \end{aligned}$$

Denote  $X^2$  as the optimal solution of second sub problem.

Combination of  $M5$  and  $M6$  can be written as follows

M7:

$$\begin{aligned} &\text{Minimize } \{Z^1, Z^2\} \\ &\text{Subject to} \\ &Z^1 \geq v D_i + (1 - v_i) r_i(X, P_i) & i = 1, \dots, n \\ &Z^2 = \sum_{i=1}^n v_i w_i D_i + (1 - v_i) w_i r_i(X, P_i) \\ &c_{j1}x + c_{j2}y \leq c_{j3} & j = 1, \dots, m \\ &r_i(X, P_i) = |x - a_i| + |y - b_i| & i = 1, \dots, n \\ &v_i \in \{0, 1\} & i = 1, \dots, n \end{aligned}$$

Each of the above problems are nonlinear programming model that can be solved by any efficient programming code, and the result will be considered as an optimum solution for the two objective issue.

We offer the following paces for achieving the solution of BOFLPLD:

*Step 1:* Develop the BCFLPLD as described in  $M7$ .

*Step 2:* From step 1, solve the first objective function as a single criterion location problem without considering the second constraint of  $M7$ , that is  $M5$  then continue this process again for second objective function without considering the first constraint of  $M7$ , that is  $M6$  and the obtained solutions are represented by  $X^1$  and  $X^2$  respectively.

*Step 3:* calculate the values of  $Z^1$  and  $Z^2$  at both  $X^1$  and  $X^2$  optimal solutions and then put these values in to the payoff table.

Table 1: Payoff table of BCFLPLD

solutions \ objectives	$X^1$	$X^2$
$Z^1$	$Z^1(X^1)$	$Z^1(X^2)$
$Z^2$	$Z^2(X^1)$	$Z^2(X^2)$

Step 4: According to step 3, the worst lower bound and the best upper bound for any of two objective functions are as follows:

$$L^1 = Z^1(X^1) \quad , \quad U^1 = \max \{Z^1(X^1), Z^1(X^2)\}$$

Step 5: Define membership functions of each two objective functions with considering step 4 and following Zimmerman definitions [22]:

$$\mu_1(Z^1) = \begin{cases} 1 & \text{if } Z^1 \leq L^1 \\ \frac{U^1 - Z^1}{U^1 - L^1} & \text{if } L^1 \leq Z^1 \leq U^1 \\ 0 & \text{if } Z^1 \geq U^1 \end{cases} \quad , \quad \mu_2(Z^2) = \begin{cases} 1 & \text{if } Z^2 \leq L^2 \\ \frac{U^2 - Z^2}{U^2 - L^2} & \text{if } L^2 \leq Z^2 \leq U^2 \\ 0 & \text{if } Z^2 \geq U^2 \end{cases}$$

Step 6: By using the membership functions that were specified in step 5, solve the below model:

M8:

Max  $\lambda$

subject to

$$\begin{aligned} \lambda &\leq \mu_1(Z^1) \\ \lambda &\leq \mu_2(Z^2) \\ Z^1 &\geq v_i D_i + (1 - v_i) r_i(X, P_i) & i = 1, \dots, n \\ Z^2 &= \sum_{i=1}^n v_i w_i D_i + (1 - v_i) w_i r_i(X, P_i) \\ c_{j1}x + c_{j2}y &\leq c_{j3} & j = 1, \dots, m \\ r_i(X, P_i) &= |x - a_i| + |y - b_i| & i = 1, \dots, n \\ v_i &\in \{0, 1\} & i = 1, \dots, n \end{aligned}$$

## 5. Numerical example

Suppose there are six existing facilities with locations, weights and maximum distances as given in table 2, also consider the boundary of the given feasible region in table 3

Table 2: Data of numerical example

i	$a_i$	$b_i$	$w_i$	$D_i$
1	5	15	1	5
2	2	10	2	10
3	5	5	3	15
4	0	0	2	4
5	12	4	3	4
6	9	2	1	9

Table 3

j	$c_{j1}$	$c_{j2}$	$c_{j3}$
1	1	0	10
2	0	1	10
3	-1	0	0
4	0	-1	0

According to step 1, BCFLPLD is formulated as follows:

Minimize  $\{Z^1, Z^2\}$

Subject to

$$\begin{aligned} Z^1 &\geq 5v_1 + r_1(1 - v_1) & Z^1 &\geq 10v_2 + r_2(1 - v_2) & Z^1 &\geq 15v_3 + r_3(1 - v_3) \\ Z^1 &\geq 4v_4 + r_4(1 - v_4) & Z^1 &\geq 4v_5 + r_5(1 - v_5) & Z^1 &\geq 9v_6 + r_6(1 - v_6) \\ Z^2 &= 5v_1 + r_1(1 - v_1) + 20v_2 + 2r_2(1 - v_2) + 45v_3 + 3r_3(1 - v_3) + 8v_4 + \end{aligned}$$

$$2r_4(1 - v_4) + 12v_5 + r_5(1 - v_5) + 9v_6 + r_6(1 - v_6)$$

$$\begin{array}{llll} x \leq 10 & y \leq 10 & x \geq 0 & y \geq 0 \\ r_1 = |x - 5| + |y - 15| & r_2 = |x - 2| + |y - 10| & r_3 = |x - 5| + |y - 5| \\ r_4 = |x - 0| + |y - 0| & r_5 = |x - 12| + |y - 4| & r_6 = |x - 9| + |y - 2| \\ v_i \in \{0,1\} & i = 1, \dots, 5 \end{array}$$

According to step 2, for this example we have:

The first sub problem (minimax criterion) by using step 2 can be written as follows

Minimize  $Z^1$

Subject to

$$\begin{array}{lll} Z^1 \geq 5v_1 + r_1(1 - v_1) & Z^1 \geq 10v_2 + r_2(1 - v_2) & Z^1 \geq 15v_3 + r_3(1 - v_3) \\ Z^1 \geq 4v_4 + r_4(1 - v_4) & Z^1 \geq 4v_5 + r_5(1 - v_5) & Z^1 \geq 9v_6 + r_6(1 - v_6) \end{array}$$

$$\begin{array}{llll} x \leq 10 & y \leq 10 & x \geq 0 & y \geq 0 \\ r_1 = |x - 5| + |y - 15| & r_2 = |x - 2| + |y - 10| & r_3 = |x - 5| + |y - 5| \\ r_4 = |x - 0| + |y - 0| & r_5 = |x - 12| + |y - 4| & r_6 = |x - 9| + |y - 2| \\ v_i \in \{0,1\} & i = 1, \dots, 5 \end{array}$$

by solving the above problem we will have

$$X^1 = (x, y) = (3.41392, 3.91392)$$

The second sub problem (minisum criterion) by using step 2 can be written as follows:

Minimize  $Z^2$

Subject to

$$Z^2 = 5v_1 + r_1(1 - v_1) + 20v_2 + 2r_2(1 - v_2) + 45v_3 + 3r_3(1 - v_3) + 8v_4 + 2r_4(1 - v_4) + 12v_5 + r_5(1 - v_5) + 9v_6 + r_6(1 - v_6)$$

$$\begin{array}{llll} x \leq 10 & y \leq 10 & x \geq 0 & y \geq 0 \\ r_1 = |x - 5| + |y - 15| & r_2 = |x - 2| + |y - 10| & r_3 = |x - 5| + |y - 5| \\ r_4 = |x - 0| + |y - 0| & r_5 = |x - 12| + |y - 4| & r_6 = |x - 9| + |y - 2| \\ v_i \in \{0,1\} & i = 1, \dots, 5 \end{array}$$

With solving the above problem:

$$X^2 = (x, y) = (5, 5)$$

By considering step 3, we have:

Table 1: Payoff table of BCFLPLD

Solutions	$X^1 = (3.41392, 3.91392)$	$X^2 = (5, 5)$
objectives		
$Z^1$	$Z^1(X^1) = 7.5$	$Z^1(X^2) = 8$
$Z^2$	$Z^2(X^1) = 55.51648$	$Z^2(X^2) = 48$

$$\begin{array}{ll} L^1 = 7.5 & , \quad U^1 = \max\{7.5, 8\} = 8 \\ L^2 = Z^2(X^2) = 48 & , \quad U^2 = \max\{55.51648, 48\} = 55.51648 \end{array}$$



According to step 5, construct the membership function for the first and second objectives as

$$\mu_1(Z^1) = \begin{cases} 1 & Z^1 \leq 7.5 \\ \frac{8 - Z^1}{8 - 7.5} & 7.5 \leq Z^1 \leq 8 \\ 0 & Z^1 \geq 8 \end{cases}, \quad \mu_2(Z^2) = \begin{cases} 1 & Z^2 \leq 48 \\ \frac{55.51648 - Z^2}{55.51648 - 48} & 48 \leq Z^2 \leq 55.51648 \\ 0 & Z^2 \geq 55.51648 \end{cases}$$

Finally according to step 6 we have:

Maximize  $\lambda$

Subject to

$$\begin{aligned} \lambda &\leq \frac{8 - Z^1}{8 - 7.5} \\ \lambda &\leq \frac{55.51648 - Z^2}{55.51648 - 48} \\ Z^1 &\geq 5v_1 + r_1(1 - v_1) & Z^1 &\geq 10v_2 + r_2(1 - v_2) & Z^1 &\geq 15v_3 + r_3(1 - v_3) \\ Z^1 &\geq 4v_4 + r_4(1 - v_4) & Z^1 &\geq 4v_5 + r_5(1 - v_5) & Z^1 &\geq 9v_6 + r_6(1 - v_6) \\ Z^2 &= 5v_1 + r_1(1 - v_1) + 20v_2 + 2r_2(1 - v_2) + 45v_3 + 3r_3(1 - v_3) + 8v_4 + \\ &\quad 2r_4(1 - v_4) + 12v_5 + r_5(1 - v_5) + 9v_6 + r_6(1 - v_6) \\ x &\leq 10 & y &\leq 10 & x &\geq 0 & y &\geq 0 \\ r_1 &= |x - 5| + |y - 15| & r_2 &= |x - 2| + |y - 10| & r_3 &= |x - 5| + |y - 5| \\ r_4 &= |x - 0| + |y - 0| & r_5 &= |x - 12| + |y - 4| & r_6 &= |x - 9| + |y - 2| \\ v_i &\in \{0,1\} & i &= 1, \dots, 5 \end{aligned}$$

The solution obtained for this problem by using the software lingo is as follows:

$$\lambda = 0.882, \quad x = 4.979, \quad y = 5.42, \quad Z^1 = 7.558, \quad Z^2 = 48.882$$

## 6. Conclusions

We have considered a bi-criteria single facility location problem on the plane with limited distances, where the effective service distance becomes a constant when the actual distance attains or exceeds a certain value. The main aim of this paper is to find a suitable location for single new facility under two criteria of minisum and minimax regarding to predetermine constant values. The problem has transformed to an easier form by using mixed integer programming formulation, and then the technique of fuzzy programming has been applied to solve it.

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