# Effective Inversely Proportional Hypermutations for Unimodal and Multimodal Optimisation

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***Abstract*—Artificial Immune Systems (AIS) employing hypermutations with linear static mutation potential have recently been shown to be very effective at escaping local optima of combinatorial optimisation problems at the expense of being slower during the exploitation phase compared to standard evolutionary algorithms. In this paper, we prove that considerable speed-ups in the exploitation phase may be achieved with dynamic inversely proportional hypermutations (IPH) with mutation potentials and argue that the potential should decrease inversely to the distance to the optimum rather than to the difference in fitness as in the literature. Afterwards, we define a simple (1+1) Opt-IA that uses IPH and ageing for realistic applications where optimal solutions are unknown. The aim of this AIS is to approximate the ideal behaviour of the IPH better and better as the search space is explored. We prove that such desired behaviour and related speed-ups occur for unimodal functions as well as for well-studied bimodal benchmark functions which are hard to optimise for traditional evolutionary algorithms and for AIS using static mutation potentials.**

***Index Terms*—Artificial immune systems, heavy-tailed mutations, inversely proportional hypermutations, theory, runtime analysis**

## I. Introduction

Three recent hot topics in the theory of randomised search heuristics for optimisation in general and in artificial intelligence in particular have been the benefits of mutation operators with higher mutation rates than traditionally employed and recommended [1]–[8], the automatic adaptation of the mutation rate [9]–[13] and the analysis of when non-elitist search heuristics are beneficial compared to elitist ones [14]–[20].

Artificial Immune Systems (AIS) are a class of bio-inspired algorithms which naturally use both high mutation rates via so-called hypermutation operators, often automatically adapt the mutation rate, and use non-elitism implicitly through the use of ageing operators, characteristics that are typical in the immune system of vertebrates from which AIS draw inspiration. In this paper we analyse the performance of an adaptive mutation operator called Inversely Proportional Hypermutation (IPH) that aims to increase the mutation rate with (an estimate of) the distance from the optimum, and propose a considerably improved one for efficient unimodal and multimodal optimisation.

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AIS for optimisation [21], [22] are generally inspired by the clonal selection principle [23]. For this reason they are also often referred to as *clonal selection* algorithms [21]. In the literature two key features of clonal selection algorithms have been identified [24]:

1) The proliferation rate of each immune cell is proportional to its affinity with the selective antigen: the higher the affinity, the higher the number of offspring generated (clonal selection and expansion).
2) The mutation suffered by each immune cell during reproduction is inversely proportional to the affinity of the cell receptor with the antigen: the higher the affinity, the smaller the mutation (affinity maturation).

Indeed well-known clonal selection algorithms employ mutation operators applied to immune cells (i.e., candidate solutions) with a rate that decreases with their similarity to the antigen (i.e., global optima) during affinity maturation. Often such operators are referred to as *inversely proportional hypermutations* (IPH). Popular examples of such clonal selection algorithms are Clonalg [25] and Opt-IA [26].

The ideal behaviour of the IPH operator is that the mutation rate is minimal in proximity of the global optimum and increases as the difference between the fitness of the global optimum and that of the candidate solution increases. However, achieving such behaviour in practice may be problematic because the fitness of the global optimum is usually unknown. As a result, in practical applications information about the problem is used to identify bounds on (or estimates of) the value of the global optimum[1]. Thus, the closer is the estimate to the actual value of the global optimum, the closer should the behaviour of the IPH operator be to the desired one. On the other hand, if the bound is much higher (e.g., for a maximisation problem) than the true value, then there is a risk that the mutation rate is too high in proximity of the global optimum i.e., the algorithm will struggle to identify the optimum.

Previous theoretical analyses, though, have highlighted various problems with IPH operators from the literature even when the fitness value of the global optimum is known. Zarges analysed the effects of mutating candidate solutions with a rate that is inversely proportional to their fitness for the OneMax problem [27]. She considered two different rates for the decrease of the mutation rate as the fitness increases: a linear decay (i.e., each bit flips with probability $\frac{\text{OneMax}(x)}{\text{Opt}}$ where Opt is the optimum value) and an exponential decay

[1] Alternatively, the fitness of the best candidate solution is sometimes used and the mutation rate of the rest of the population is inversely proportional to the best.