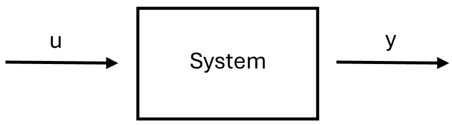
Exercise 1 - Introduction

1 Modeling

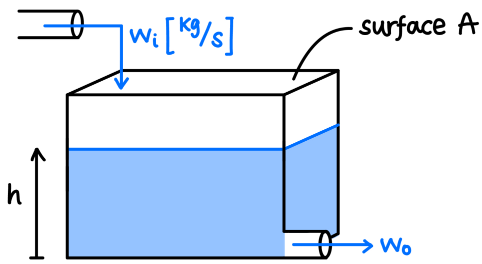
We want to learn how to describe a dynamic system mathematically. The main concepts will be explained in detail in the course Mechanics 3.
For now we can just think of a system as a black box: it gets an input u and gives back an output y.



We want to set up equations that describe the output in terms of the input and some internal parameters. We can do this by applying physical laws, such as Newton's laws, Kirchhoff's laws, etc.
Such systems are usually described by ordinary differential equations (ODEs).
In our case, as you will also see in Thermodynamics I, we often study how some stored quantity changes in time as a function of the flow or some other parameters.

$$\frac{d}{dt}\text{storage} = \sum\text{inflow - }\sum\text{outflow}$$

Example 1: Tank system:



We have a tank with an inlet and an outlet. The inlet flow is $w_i(t)$ and the outlet flow is $w_o(t)$. The height of the water in the tank is $h(t)$.

We want to find an equation that describes how the height $h(t)$ changes in time as a function of the inlet and outlet flows.
We can apply the conservation of mass principle:

$$\tfrac{d}{dt}m(t) = w_i(t) - w_o(t)$$

And substituing $m(t)$ with ρA, where ρ is the density of the water and A is the cross-sectional area of the tank, we get:

$$\rho A \frac{d}{dt}h(t) = w_i(t) - w_o(t)$$
$$y(t) = h(t)$$

In this case $h(t)$ summarizes everything that happens inside the tank and it is called the **state of the system**. States are usually indicated with the letter x.