

The following is pseudo code created for a recursive solution to the z-transformation. The overall logic works; however, extra work will be required to get it working in Python. For example, the initialization that occurs in `z_transformation` has been left out. Please note that in Python, indexing of a two-dimensional array can be accomplished through the following: `array[:,i]` where column `i` and all of the rows of the array will be returned. Lastly, the pseudo code begins indexing at 0.

**z\_helper**( $X, Z, i, \text{degrees}, l_{i-1}, B_{i-1}$ )

**Input:** The original input  $X$ , the  $Z$  array in progress, the bucket index  $i$ , the degrees calculated, the lexicographic array of the previous bucket,  $l_{i-1}$ , and the previous bucket  $B_{i-1}$ . Additionally, let  $d$  be the number of features in  $X$ .

**Output:** The resulting  $Z$  matrix of the concatenated buckets.

```

1: if  $i \geq \text{degrees}$  then //Base Case
2:     return  $Z$ 
3: end if
4:  $l_X = [0 \dots d - 1]$  //Assigning the  $l$  array for X. Can be a parameter instead.
5:  $q = 0$  // Index for the new column in  $B_i$ .
6: for  $j = 0 \dots |l_{i-1}|$  do //For each element in the previous bucket
7:     for  $k = 0 \dots \text{length of } l_X$  do //For each element in original input
8:         if  $l_{i-1}[j] \leq l_X[k]$  then //If the lexicographic value of the bucket is less than or
            equal to the X value...
9:              $l[q] = l_X[k]$  //Update the current  $l$  value with the respective lexicographic
            value in X.
10:             $\text{temp} = B_{i-1}[:, j] * X[:, k]$  //Create the column vector from the element-wise
            multiplication of the respective elements in  $B_{i-1}$  and  $X$ 
11:             $B.\text{append}(\text{temp}, \text{axis} = 1)$  //Append the result to  $Z$ .
12:             $q = q + 1$  //Increment the index of  $B_i$ .
13:        end if
14:    end for
15: end for
16:  $Z.\text{append}(B, \text{axis} = 1)$  //Append the bucket to the final result.
17: return z_helper( $X, Z, i + 1, \text{degrees}, l, B$ ) //Recursively calculate the next bucket(s).

```