`python` Intermediate

Numerical Analysis with `python`

Firsts

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We are almost finished with our course!

So now, we are ready to apply all the tools we have developed so far.

This week, we will see how can use code to solve mathematical problems such as,

- Derivate functions.
- Integrate functions (Definited integrals).
- Root searching.

We will use the libraries available on `python` we have worked on last weeks (NumPy (http://www.numpy.org/) and Matplotlib (https://matplotlib.org/)).

Theory

Searching roots of a function is one of the most common problems solved with computers on science. So we are going to learn two of the most widely used methods,

- Bisection
- Newton-Raphson

Bisection

This method is the simplest when looking for a root[1] of a function f on a given interval $[a, b]$.

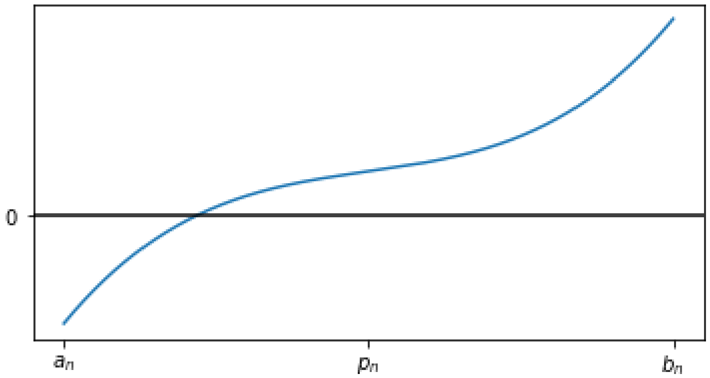
---

1. (Value of x such that $f(x) = 0$).↩

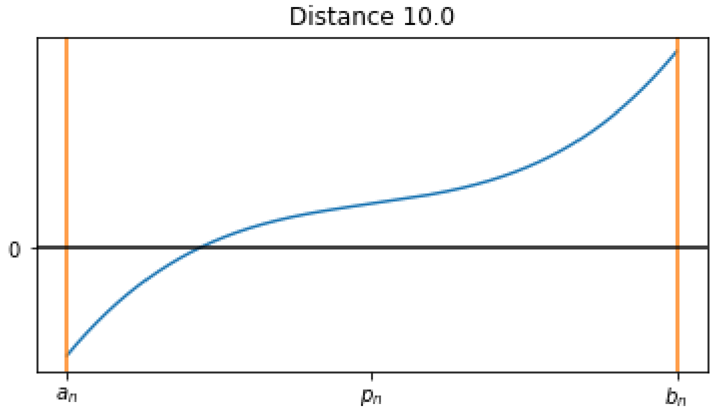
The method work as follows.

1. We select the two points of the interval (the limits).
2. Calculate the middle point.
3. Calculate the function on the midpont.
4. Define the convergence
   A. The interval is small enough.
   B. The value of the function on the midpoint is small enough.
5. If the criteria are not satisfied, check the signs,
   A. If $f(a_n) > 0$ and $f(p_n) > 0$ or $f(a_n) < 0$ and $f(p_n) < 0$, then the root is now on $(p_n, b_n]$.
   B. If $f(a_n) < 0$ and $f(p_n) > 0$ or If $f(a_n) > 0$ and $f(p_n) < 0$, then the root is now on $(a_n, p_n]$.
6. Repeat for the new limits.

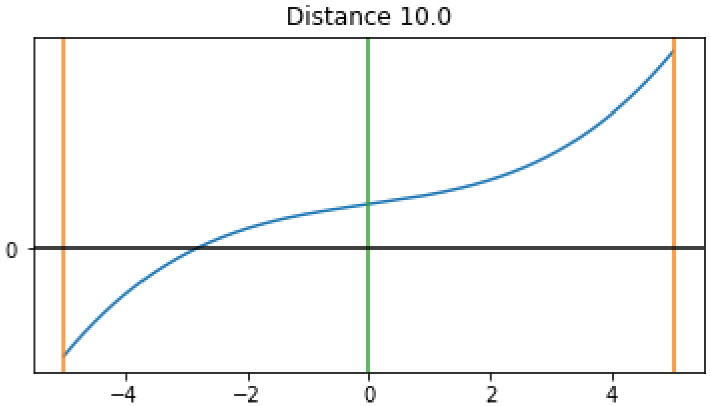
Function



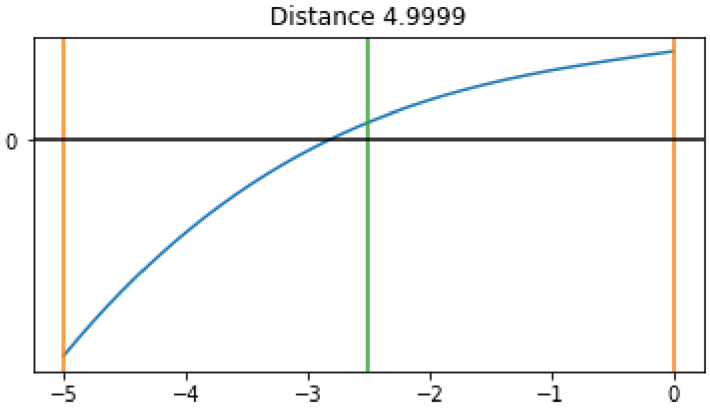
Initial Interval



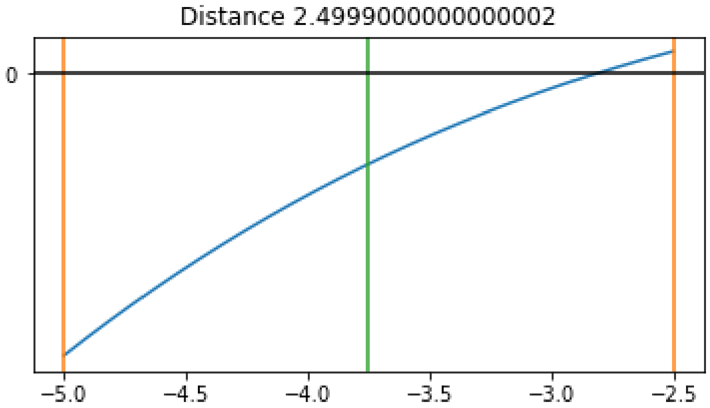
Midpoint



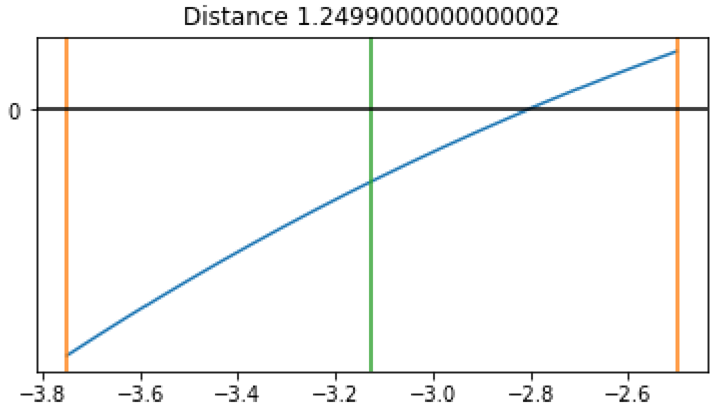
Resize the Interval step 1



Resize the Interval step 2



Resize the Interval step 3



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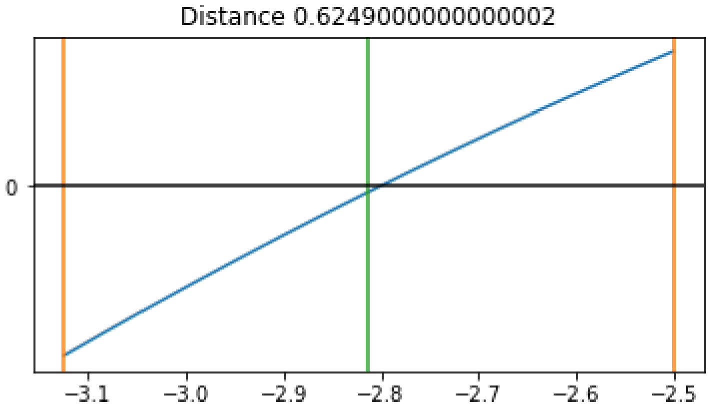

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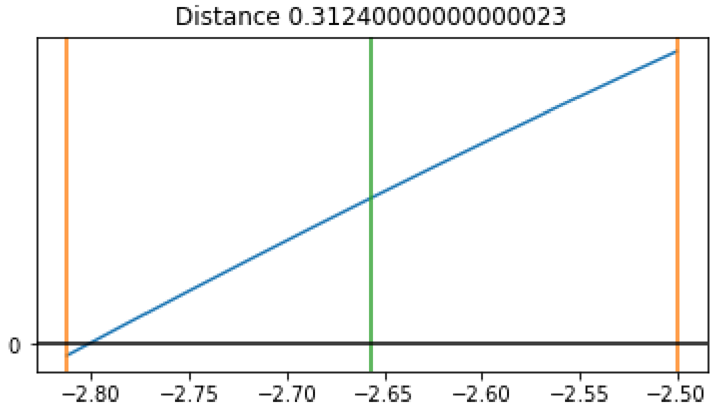
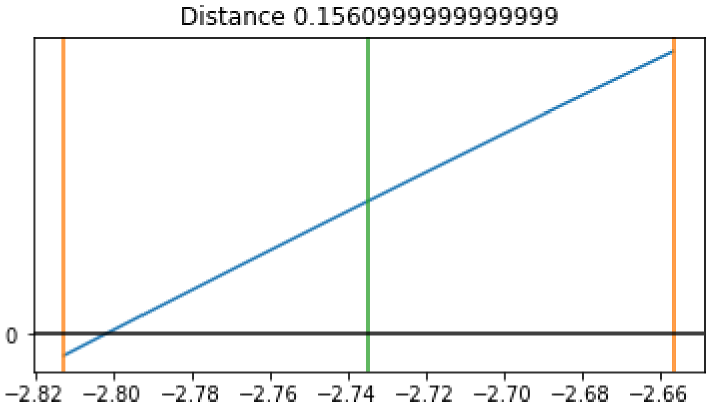
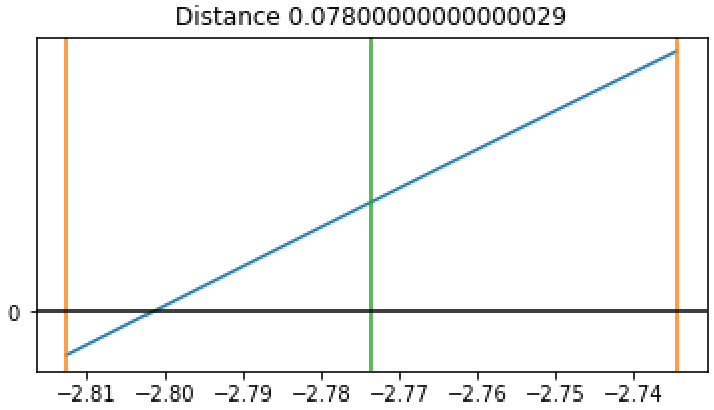


Resize the Interval step 6



Resize the Interval step 7



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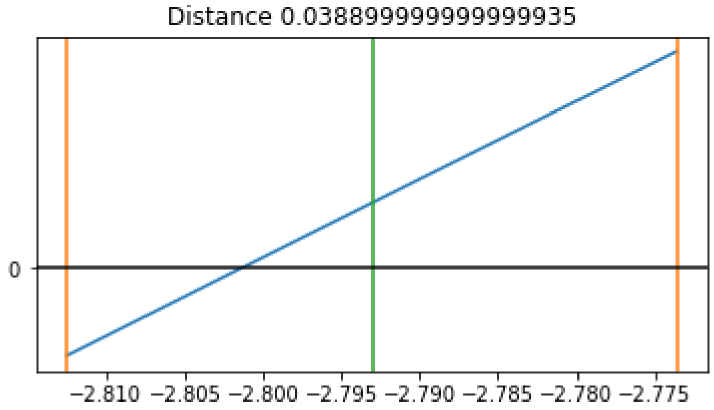
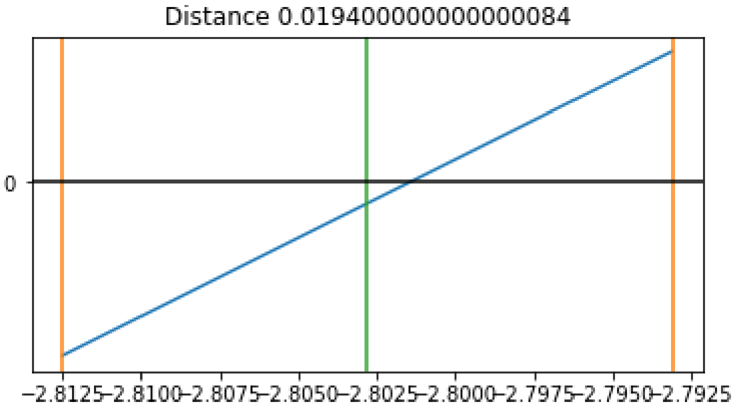
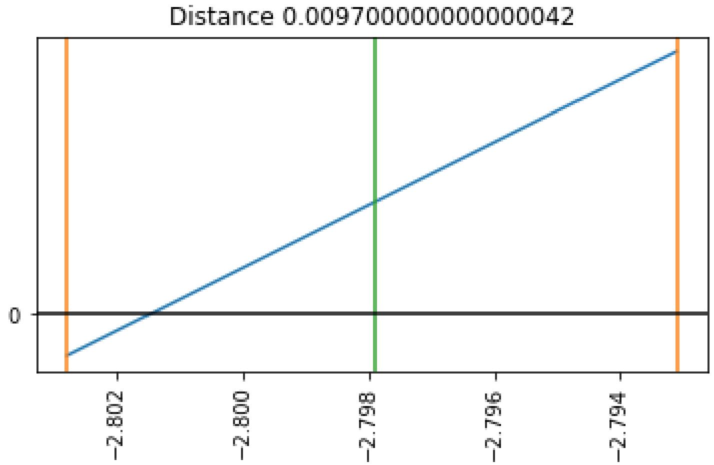


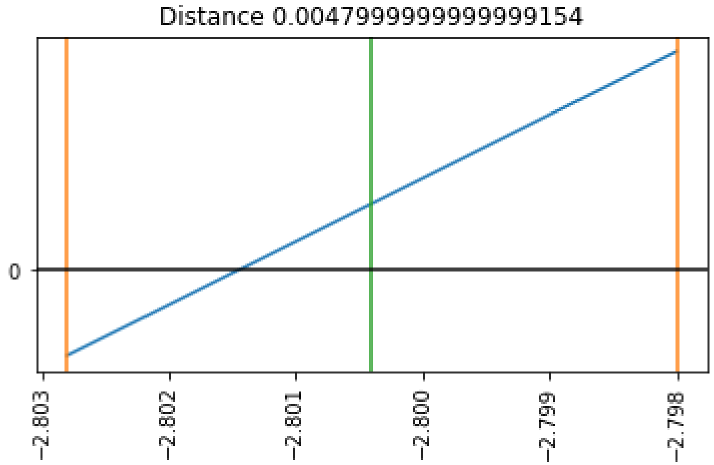
Resize the Interval step 9



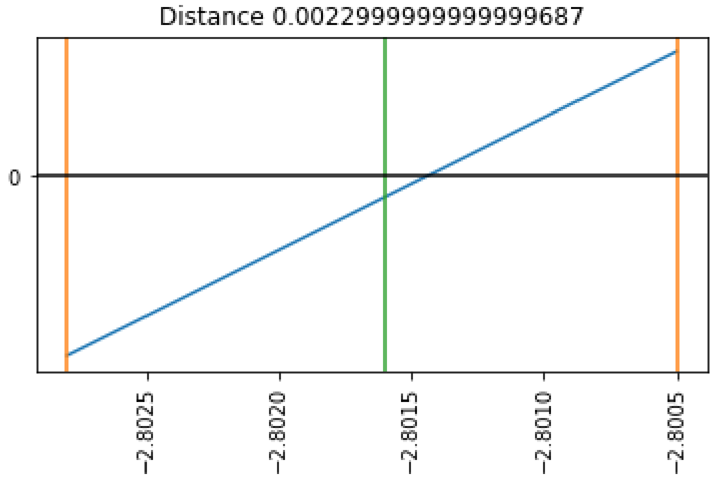
Resize the Interval



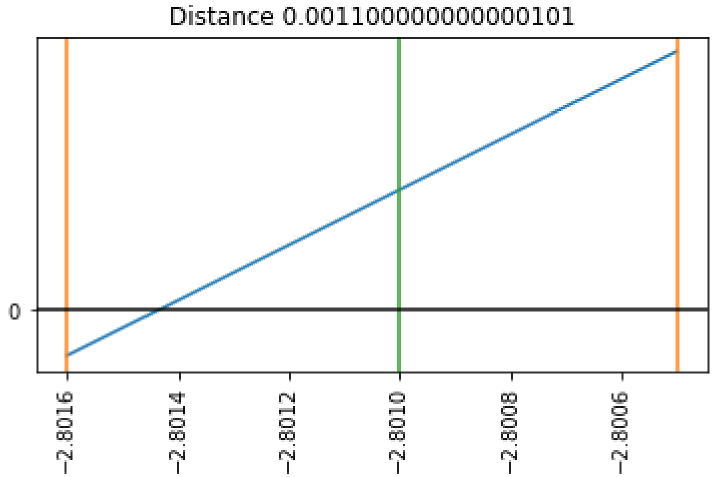
Resize the Interval step 10



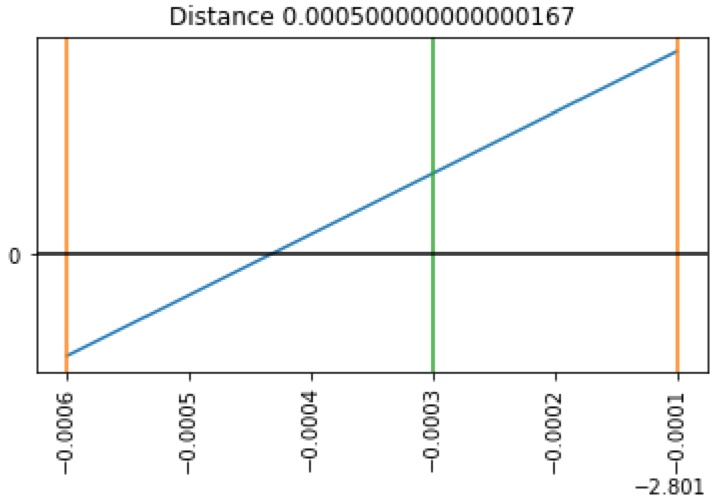
Resize the Interval step 11



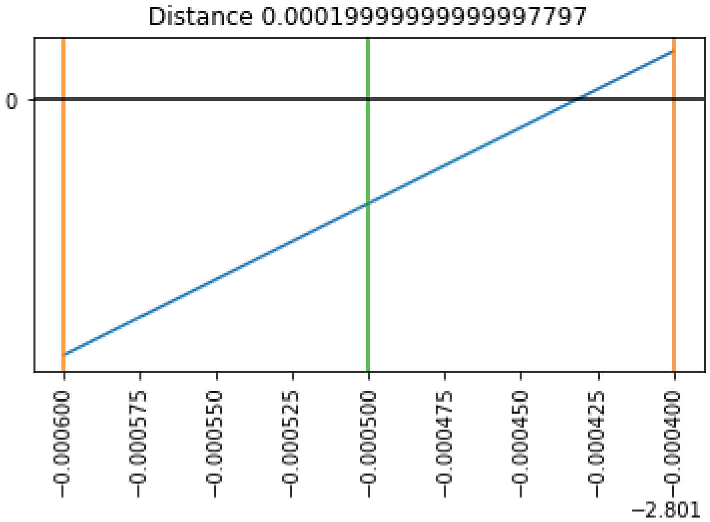
Resize the Interval step 12



Resize the Interval step 13



Resize the Interval step 14



The first problem we will solve, is finding the value of x such that $f(x) = 0$ with

$$f(x) = \left(\tanh\left(\frac{x}{4}\right) + \frac{\cos(x)\sin(x)}{x}\right)^3$$

In [1]:
```
import numpy as np
import matplotlib.pylab as plt
```

In [2]:
```
def f(x):
    return (np.tanh(0.25*x)+(np.cos(x)*np.sin(x))/x)**3
```

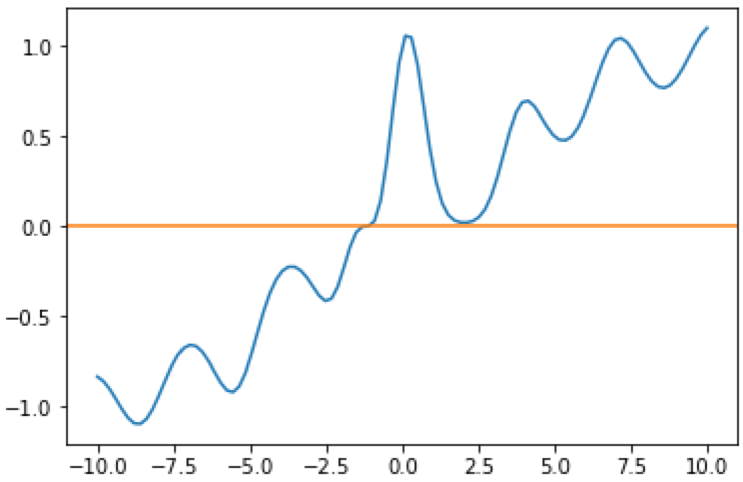
In [3]:
```
x=np.linspace(-10,10,100)
y=f(x)
```

In [4]:

```
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(x,y)
ax.axhline(0,color='C1')
```

Out[4]:

```
<matplotlib.lines.Line2D at 0x11921e128>
```



In [5]:

```
def bisec(f,an,bn):
    eps=1e-10
    while (np.abs(an-bn)>eps):
        pm=(an+bn)/2.
        if (f(an)*f(pm)>0):
            an=pm
        else:
            bn=pm
    return pm
```

In [6]:

```
print(bisec(f,-10,10.1),f(bisec(f,-10,10.1)))
```

```
-1.1908552421034986 1.0627357754887731e-33
```

The first problem we will solve, is finding the value of x such that $f(x) = 0$ with

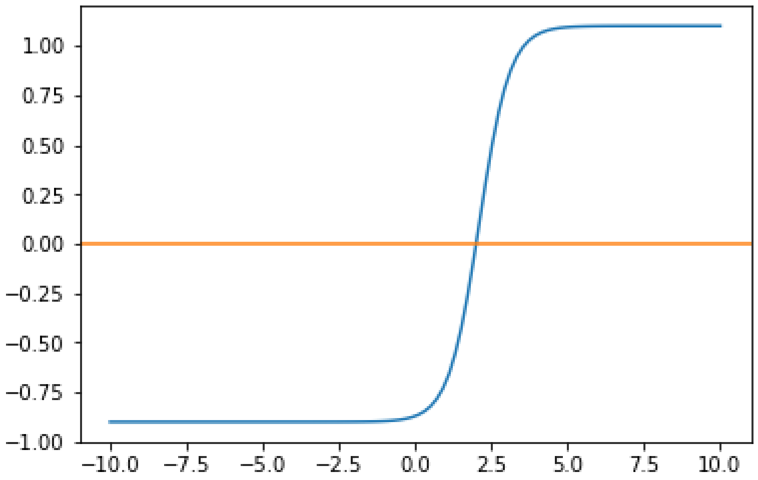
$$f(x) = \tanh(x - 2.1) + 0.1$$

In [7]:
```
def f(x0):
    return np.tanh(x0-2.1)+0.1
```

In [8]:
```
x=np.linspace(-10,10,100)
y=f(x)
```

In [9]:
```
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(x,y)
ax.axhline(0,color='C1')
```

Out[9]:
```
<matplotlib.lines.Line2D at 0x1192c7ef0>
```



In [10]:
```
print(bisec(f,-10,10.1),f(bisec(f,-10,10.1)))
```

```
1.99966465226571 -3.182357155733565e-12
```

Newton

This method is more complex but precise than the bisection. This is also known as the tangent method, because it starts from a guess and moves as the tangent line.

The method work as follows.

We select an initial guess p_0, then the following point is found by taking the zero of the derivative on that point. The equation of that line is

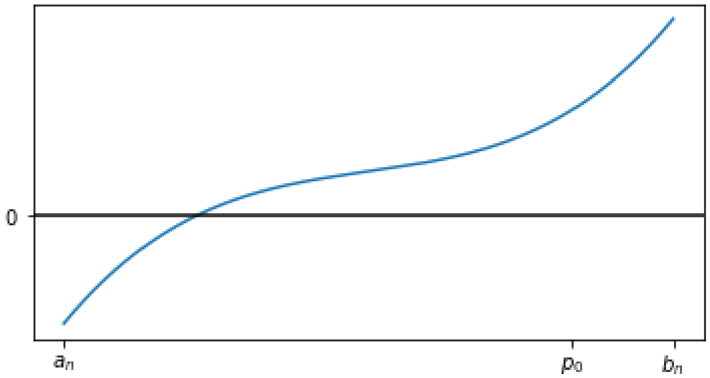
$$y = f'(p_0)(x_1 - x_0) + f(x_0).$$

So, in order to find the value of $x_1, y = 0$, that leads to

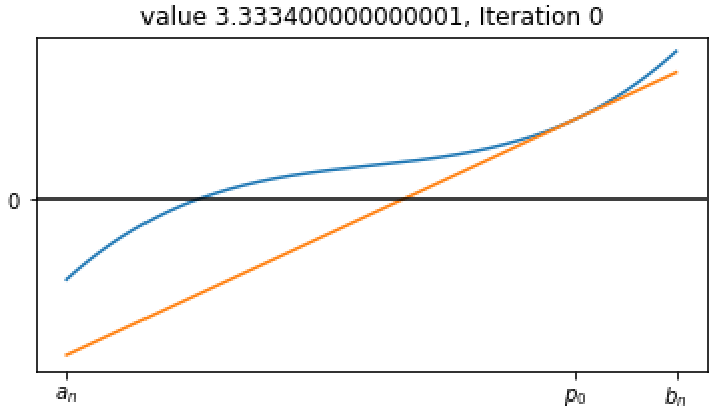
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)},$$

and it is repeated until a tolerance. (Or $f(x_i) == 0$).

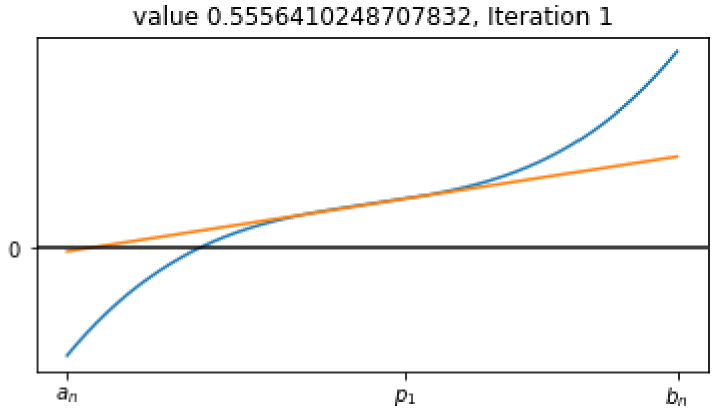
Function



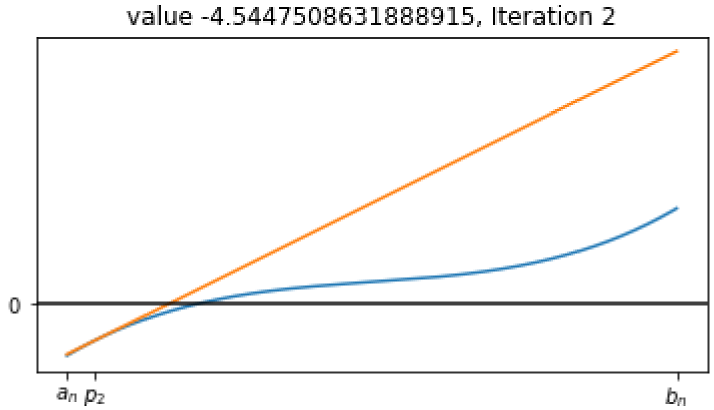
Initial Interval



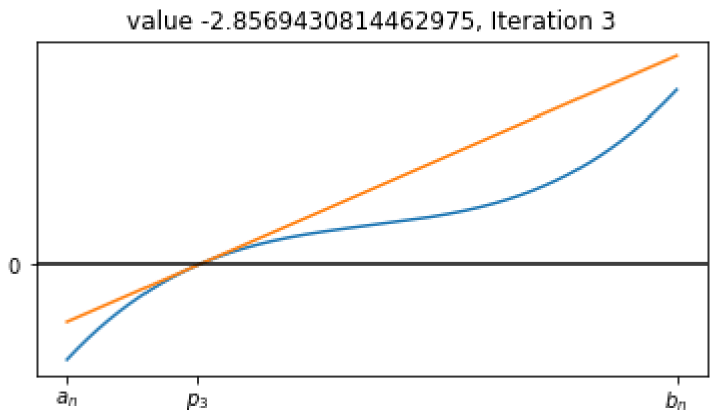
Midpoint



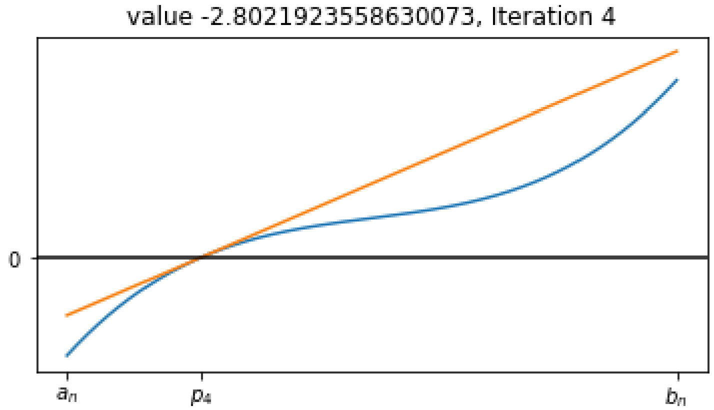
Resize the Interval step 1



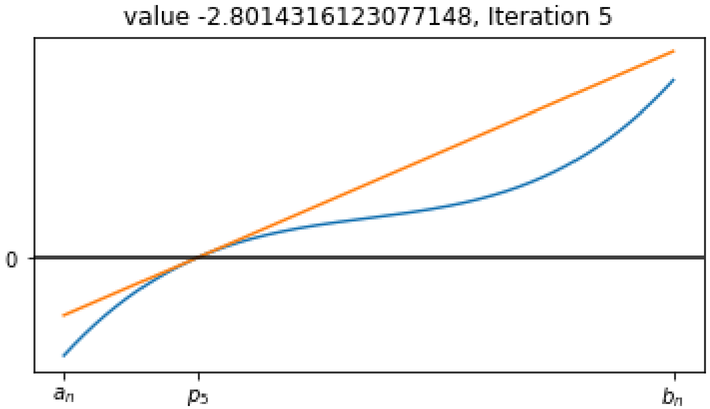
Resize the Interval step 2



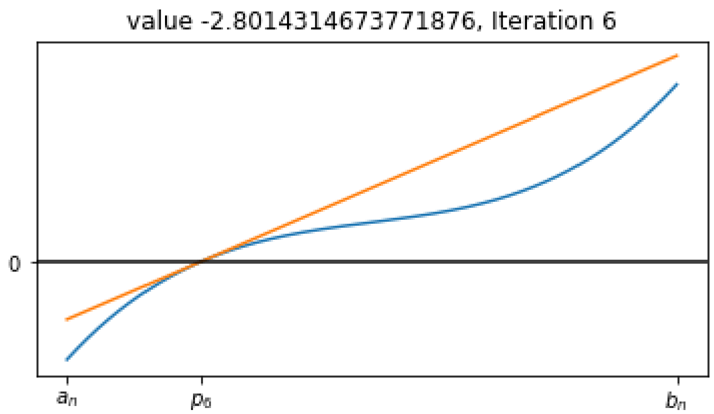
Resize the Interval step 3



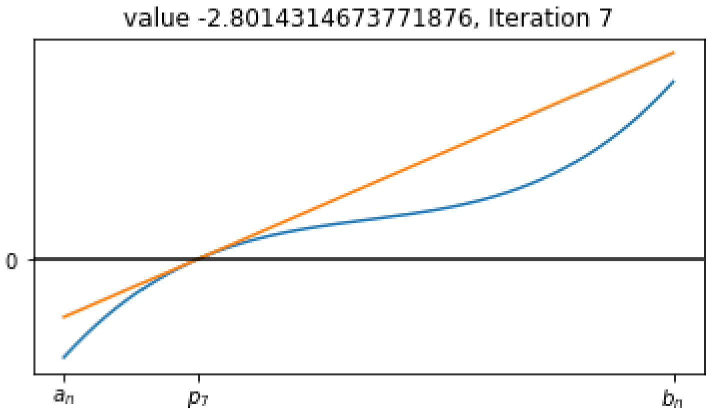
Resize the Interval step 4



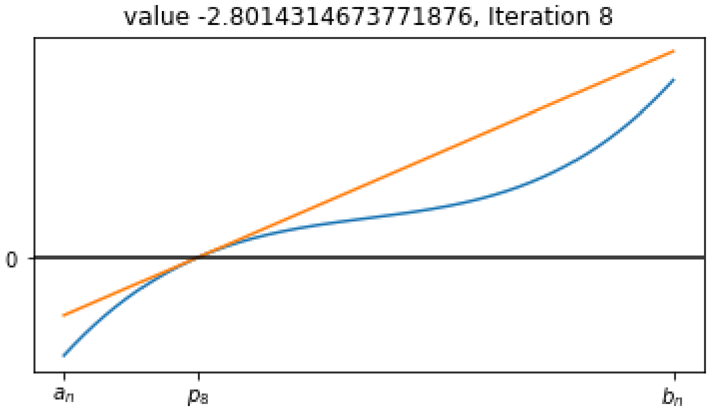
Resize the Interval step 5



Resize the Interval step 6



Resize the Interval step 7



In [11]:

```
def Newton(f0,df0,xi):
    eps=1e-10
    while np.abs(f0(xi))>eps:
        xi-=f0(xi)/df0(xi)
    return xi
```

Again the first function we are testing

$$f(x) = \left(\tanh\left(\frac{x}{4}\right) + \frac{\cos(x)\sin(x)}{x}\right)^3$$

In [12]:
```
def f(x):
    return (np.tanh(0.25*x)+(np.cos(x)*np.sin(x))/x)**3
```

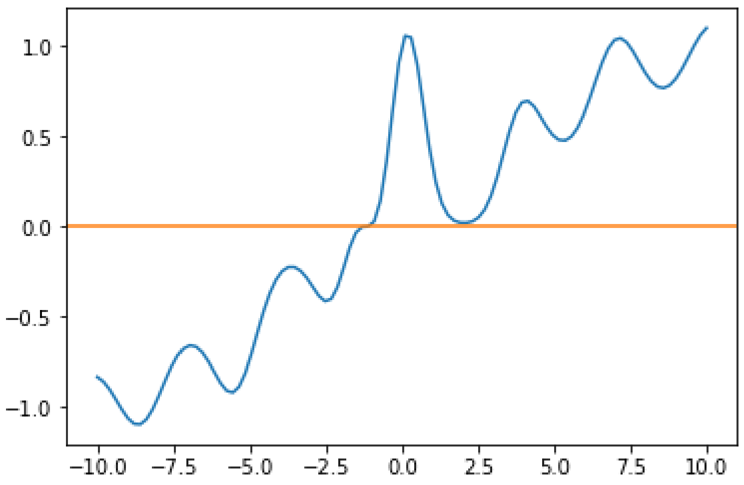
In [13]:
```
x=np.linspace(-10,10,100)
y=f(x)
```

In [14]:

```
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(x,y)
ax.axhline(0,color='C1')
```

Out[14]:

```
<matplotlib.lines.Line2D at 0x1193a4b70>
```



In [15]:
```
def df(x):
    return (0.75/x**4)*((x*np.tanh(0.25*x)+
                         np.sin(x)*np.cos(x))**2.)*(x*x*(np.cosh(0.25*x))**-2.
                                                             -4*x*np.sin(
x)**2+4*x*np.cos(x)**2.-4*np.cos(x)*np.sin(x))
```

In [16]:
```
print(bisec(f,-10,10.1),Newton(f,df,0.5))
```

```
-1.1908552421034986 -1.1911507242488286
```

In [17]:

```
def f1(x0):
    return np.tanh(x0-2.1)+0.1
def df1(x0):
    return 1./np.cosh(x0-2.1)**2.
```

In [18]:

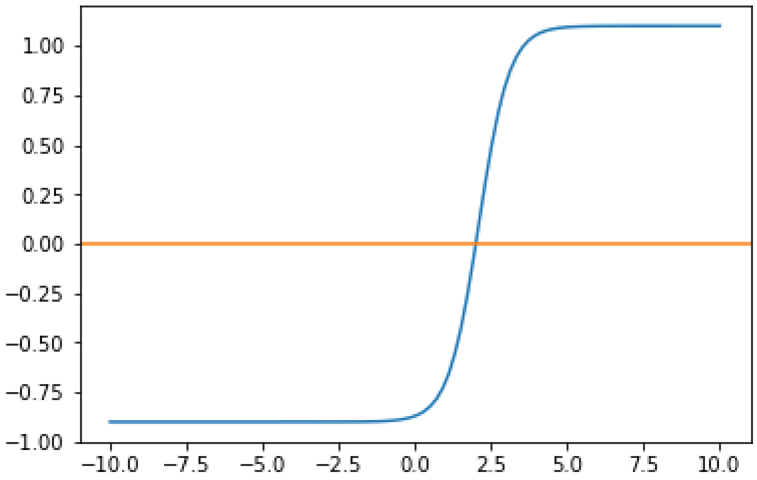
```
x=np.linspace(-10,10,100)
y=f1(x)
```

In [19]:

```
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(x,y)
ax.axhline(0,color='C1')
```

Out[19]:

```
<matplotlib.lines.Line2D at 0x119409630>
```



In [20]:

```
print(bisec(f1,-10,10.1),Newton(f1,df1,1.5))
```

```
1.99966465226571 1.9996646522689245
```

But, what if we consider a different initial condition?

In [21]:
```
print(bisec(f1,-10,10.1),Newton(f1,df1,10.0))
```

```
1.99966465226571 nan

/usr/local/lib/python3.7/site-packages/ipykernel_launcher.py:4: RuntimeWarnin
g: overflow encountered in cosh
  after removing the cwd from sys.path.
/usr/local/lib/python3.7/site-packages/ipykernel_launcher.py:4: RuntimeWarnin
g: divide by zero encountered in double_scalars
  after removing the cwd from sys.path.
/usr/local/lib/python3.7/site-packages/ipykernel_launcher.py:4: RuntimeWarnin
g: invalid value encountered in double_scalars
  after removing the cwd from sys.path.
```