Herramientas Computacionales para Ciencias
Homework 9

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Rules

Note Read carefully the complete homework before starting, so you will know what is the results you have to get!.

This week we are going to concentrate on root finding applications.

On this assignment you will have to construct some functions and use the matplotlib + NumPy structure to plot the results. There is an additional point that must be developed on class before sending the assignment. This part must be saved on a jupyter Notebook named as your UniAndes username.

Introduction

There are a certain families of functions which are defined by the differential equation they are solution to. On this assignment we are going to explore one of them!, and on class we'll see some more.

Bessel Functions! [3.5/5.0]

On my personal experience, the first time I saw the Bessel functions on a physical context, was when solving the wave equation on polar coordinates (radial), in other words, *looking for the possible shapes a circular drum can vibrate* without angular dependence. Strings look like sine and cosine functions meanwhile drums look like Bessel functions.

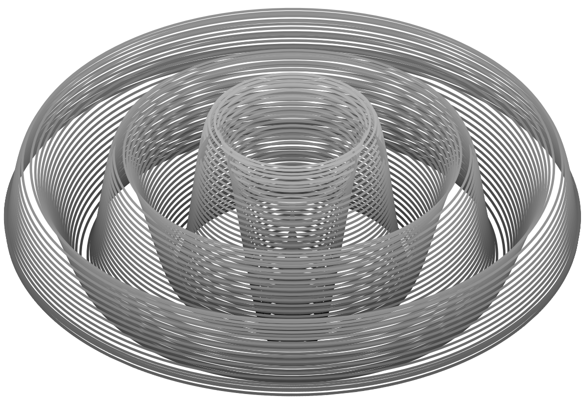

Figure 1: Representation of a circular drum (Using a Bessel function!)

As the membrane drum is fixed at some points, it is necessary to look for the values which the function is zero! (Finding the roots!!).

Do not worry, you won't have to calculate those functions. (At least on this course)

- Use scipy to calculate 10 points of the zeroth first kind Bessel function as follows

```
from scipy import special
x=np.linspace(0,10,10)
y=special.j0(x)
```

- Plot them using matplotlib (I would recommend to use points instead of lines).

  Note: Remember you MUST use labels and titles.

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- From the plots, try to estimate which values x_i are the roots, so that $j_0(x_i) = 0$.
- As we want to use the bisection method to find the zero, now we need to select the intervals such that only one zero is enclosed on each interval!.
  - Choose the three intervals enclosing only one zero each one.
  - Use the bisection method (You can see the slides), BUT!, do only 10 iterations saving the value p_m at each step
  - Use the Newton-Raphson method using as the intial point one of the limits of the interval, BUT!, do only 10 iterations saving the value x_i at each step.

    Note: You will need the derivative of j_0, it satisfies that

    $$\frac{dj_0(x)}{dx} = -j_1(x) \tag{1}$$

    so, define the derivative with

    ```
    def derivative(x):
            return -special.j1(x)
    ```

- Plot the saved values of p_m and x_i. (We must have three plot with two plots each).
- Discuss the last plot.[1]

If you want to test your solutions, you can use

```
print(special.jn_zeros(0,3))
```

The function `jn_zeros(n,m)` will create an array with m zeros of the first kind nth Bessel function.

[1]This is a very important part, you have to make an interpretation of the plot and point out the things you consider important. That is one of the hardest part of doing science and research so it is a good practice to make interpretations of simple plots like this one.