Herramientas Computacionales para Ciencias
Homework 8

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Rules

Note Read carefully the complete homework before starting, so you will know what is the results you have to get!.
This week we are going to concentrate on `matplotlib` (plotting). On this assignment you will have to construct some functions and use the `matplotlib` + `NumPy` structure to plot the results.There is an additional point that must be developed on class before sending the assignment. This part must be saved on a jupyter Notebook named as your UniAndes username.

[0.5/4.0] Simple Pendulum!

The free pendulum equation can be written as,

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0, \tag{1}$$

with g the action of the gravity, ℓ the length of the pendulum. It is usually approximated by using the small angles approximation such that $\sin(\theta) \approx \theta$.

To see the regime in which this approximation makes sense, use the figure structure to make two axes with the following plots, (Use the interval $\theta = [0, \pi/2]$ (it is, 0-90 deg)).

- Axes 1: A plot of $\sin(\theta)$ and θ as a function of θ. (You will have two plots in the first axes!).
- Axes 2: A plot of $\sin(\theta) - \theta$ vs θ.

Using these two plots, try to guess an approximate limit value where the approximation becomes meaningless.

Note: The plots must have legend, labels and title!!.
But, what happen if we have $\theta > \theta_{\text{Limit}}$? We will see some examples.

[1.0/4.0] Dumped Pendulum!

A more realistic model takes into account the friction (Can be the air effect).

$$\frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \frac{g}{\ell}\sin\theta = 0, \tag{2}$$

Load the data from the repository by using this path,
`https://raw.githubusercontent.com/jmsevillam/Herramientas-Computacionales-UniAndes/master/Data/dumped.dat`
There you can find 5 columns t time, θ_1, ω_1, θ_2, ω_2 where the index 1 and 2 mean that the solution is done for two pendulums with slightly different initial conditions.

The parameters used are

g	9.8
ℓ	9.8
γ	0.2
Δt	0.04
Time Steps	2500
θ_1	0.2
$\theta_2 - \theta_1 = \Delta\theta$	0.0001
$\omega 1 = \omega 2$	0.2

Table 1: Parameters of the simulation

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Plot the following relations,

- time vs θ_1
- time vs θ_2
- time vs ω_1
- time vs ω_2
- θ_1 vs ω_1
- θ_2 vs ω_2
- Discuss the previous plots, did you expect these results?
- The two plots of θ_i looks the same, so plot the difference time vs $|\theta_1 - \theta_2|$ (**Hint:** Use `np.abs`)
- Plot again the difference time vs $|\theta_1 - \theta_2|$ but use logarithmic scale on y(**Hint:** Use `ax.set_yscale('log')`). As the relationship looks exponential, the logarithmic scale transform it into a straight line. (I am talking about the maximums, because it looks like some mountains).

[2.0/4.0] Driven Pendulum!

$$\frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \frac{g}{\ell}\sin\theta = F_0\sin(\Omega t), \tag{3}$$

we took $F_0 = 1.2$ so that, the pendulum can reach the top and pass it (Non small angles), and $\Omega = 2/3$.

Repeat the same plots of the previous case but now, with the data https://raw.githubusercontent.com/jmsevillam/Herramientas-Computacionales-UniAndes/master/Data/Driven.dat
Your results are going to be completely different!, try to interpret them.

Do a comparison among the last plot of this point, and the last one of the previous one.

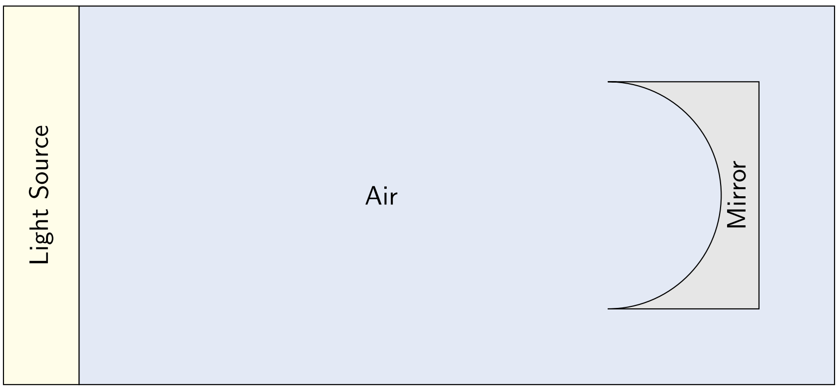
Note

This procedure is done when you are looking for the *chaos* in a system, so all of it will have a huge interpretation that we will discuss at some point!, don't worry if some of these things look new or strange for you.

[0.5/4.0] Light on a Mirror

On this last part, we are going to do a single plot result of a 2D Lattice-Boltzmann simulation for waves.

The system is a light source considering that it is a source of plane waves, travelling from left to right until it gets to the *spherical* mirror where it reflects getting concentrated in the focus.



The data can be found at
https://raw.githubusercontent.com/jmsevillam/Herramientas-Computacionales-UniAndes/master/Data/LBWaves.dat

there, you have a *Matrix* of electric field, so plot it by using `matplotlib.pylab.imshow` or another similar. Try to guess the position of the focus of the mirror.