Problem: How many six-character license plates with letters or numbers in any position and with *at least* one ‘W’ are possible? Start by counting the license plates for which there are *exactly* 1, 2, 3, 4, 5, & 6 ‘Ws’ and add them up.

License plates with exactly 1 W:

$$\left.\begin{array}{cccccc} W & \_ & \_ & \_ & \_ & \_ \\ \_ & W & \_ & \_ & \_ & \_ \\ \_ & \_ & W & \_ & \_ & \_ \\ \_ & \_ & \_ & W & \_ & \_ \\ \_ & \_ & \_ & \_ & W & \_ \\ \_ & \_ & \_ & \_ & \_ & W \end{array}\right\} \qquad \binom{6}{1}\left(35^5\right) = 6 \times 52,521,875$$

License plates with exactly 2 Ws:

$$\left.\begin{array}{cccccc} W & W & \_ & \_ & \_ & \_ \\ W & \_ & W & \_ & \_ & \_ \\ W & \_ & \_ & W & \_ & \_ \\ W & \_ & \_ & \_ & W & \_ \\ W & \_ & \_ & \_ & \_ & W \\ \_ & W & W & \_ & \_ & \_ \\ \_ & W & \_ & W & \_ & \_ \\ \_ & W & \_ & \_ & W & \_ \\ \_ & W & \_ & \_ & \_ & W \\ \_ & \_ & W & W & \_ & \_ \\ \_ & \_ & W & \_ & W & \_ \\ \_ & \_ & W & \_ & \_ & W \\ \_ & \_ & \_ & W & W & \_ \\ \_ & \_ & \_ & W & \_ & W \\ \_ & \_ & \_ & \_ & W & W \end{array}\right\} \qquad \binom{6}{2}\left(35^4\right) = 15 \times 1,500,625$$

License plates with exactly 3 *Ws*:

$$\left.\begin{array}{cccccc}
W & W & W & _ & _ & _ \\
W & W & _ & W & _ & _ \\
W & W & _ & _ & W & _ \\
W & W & _ & _ & _ & W \\
W & _ & W & W & _ & _ \\
W & _ & W & _ & W & _ \\
W & _ & W & _ & _ & W \\
W & _ & _ & W & W & _ \\
W & _ & _ & W & _ & W \\
W & _ & _ & _ & W & W \\
_ & W & W & W & _ & _ \\
_ & W & W & _ & W & _ \\
_ & W & W & _ & _ & W \\
_ & W & _ & W & W & _ \\
_ & W & _ & W & _ & W \\
_ & W & _ & _ & W & W \\
_ & _ & W & W & W & _ \\
_ & _ & W & W & _ & W \\
_ & _ & W & _ & W & W \\
_ & _ & _ & W & W & W
\end{array}\right\} \qquad \binom{6}{3}\left(35^3\right) = 20 \times 42,875$$

License plates with exactly 4 *Ws*:

$$\left.\begin{array}{cccccc}
W & W & W & W & _ & _ \\
W & W & W & _ & W & _ \\
W & W & W & _ & _ & W \\
W & W & _ & W & W & _ \\
W & W & _ & W & _ & W \\
W & W & _ & _ & W & W \\
W & _ & W & W & W & _ \\
W & _ & W & W & _ & W \\
W & _ & W & _ & W & W \\
W & _ & _ & W & W & W \\
_ & W & W & W & W & _ \\
_ & W & W & W & _ & W \\
_ & W & W & _ & W & W \\
_ & W & _ & W & W & W \\
_ & _ & W & W & W & W
\end{array}\right\} \qquad \binom{6}{4}\left(35^2\right) = 15 \times 1,225$$

License plates with exactly 5 *Ws*:

$$\left.\begin{array}{cccccc} W & W & W & W & W & \_ \\ W & W & W & W & \_ & W \\ W & W & W & \_ & W & W \\ W & W & \_ & W & W & W \\ W & \_ & W & W & W & W \\ \_ & W & W & W & W & W \end{array}\right\} \qquad \binom{6}{5}\left(35^1\right) = 6 \times 35$$

License plates with exactly 6 *Ws*:

$$\left.\begin{array}{cccccc} W & W & W & W & W & W \end{array}\right\} \qquad \binom{6}{6}\left(35^0\right) = 1 \times 1$$

So, the total number of license plates with at least one '*W*' is:

$$\begin{aligned} N &= \binom{6}{1}\left(35^5\right) + \binom{6}{2}\left(35^4\right) + \binom{6}{3}\left(35^3\right) + \binom{6}{4}\left(35^2\right) + \binom{6}{5}\left(35^1\right) + \binom{6}{6}\left(35^0\right) \\ &= (6 \times 52,521,875) + (15 \times 1,500,625) + (20 \times 42,875) + (15 \times 1,225) + (6 \times 35) + (1 \times 1) \\ &= 315,131,250 + 22,509,375 + 857,500 + 18,375 + 210 + 1 \\ &= 338,516,711 \end{aligned}$$

On the other hand, consider counting all the possible license plates and subtracting those that have *no Ws*!

$$\begin{aligned} N &= \left(36^6\right) - \left(35^6\right) \\ &= 2,176,782,336 - 1,838,265,625 \\ &= 338,516,711 \end{aligned}$$

What do you notice?