Given...

$\int_z^q (x + axi + bi)e^{-2\cdot\pi\cdot i\cdot n\cdot x}\,dx$, *with a, b, n, q, z constant:*

$$f(x,a,b,n,q,z) = \int_z^q (x + axi + bi)\,e^{-2n\pi ix}\,dx \tag{1}$$

$$= \int_z^q (Ax + B)\,e^{Cx}\,dx \qquad \text{where:} \tag{2}$$

$$A = 1 + ai \tag{3}$$

$$B = bi \tag{4}$$

$$C = -2n\pi i \tag{5}$$

$$\text{and:} \tag{6}$$

$$C^2 = -4n^2\pi^2 \tag{7}$$

$$= \int_z^q Axe^{Cx}\,dx + \int_z^q Be^{Cx}\,dx \tag{8}$$

From Dwight:

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} \qquad 565.1 \tag{9}$$

$$\int xe^{ax}\,dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right] \qquad 567.1 \tag{10}$$

Therefore:

$$A\int xe^{Cx}\,dx + B\int e^{Cx}\,dx = Ae^{Cx}\left[\frac{x}{C} - \frac{1}{C^2}\right] + B\frac{1}{C}e^{Cx} + K \tag{11}$$

$$= \left[\frac{Ax + B}{C} - \frac{A}{C^2}\right]e^{Cx} + K \tag{12}$$

$$= \left[\frac{ACx + BC - A}{C^2}\right]e^{Cx} + K \tag{13}$$

$$= \left[\frac{(1 + ai)(-2n\pi i)\,x + (bi)(-2n\pi i) - (1 + ai)}{(-4n^2\pi^2)}\right]e^{(-2n\pi i)x} + K \tag{14}$$

$$= \left[\frac{(-2n\pi)\,xi - (-2n\pi)\,ax - (-2n\pi)\,b - 1 - ai}{(-4n^2\pi^2)}\right]e^{i(-2n\pi)x} + K \tag{15}$$

$$= \left[\left(\frac{(-2n\pi)\,x - a}{(-4n^2\pi^2)}\right)i - \left(\frac{(-2n\pi)(ax + b) + 1}{(-4n^2\pi^2)}\right)\right]\left[\cos(-2n\pi)\,x + i\sin(-2n\pi)\,x\right] + K \tag{16}$$

$$= \left[\left(\frac{(-2n\pi)\,x - a}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,x - \left(\frac{(-2n\pi)(ax + b) + 1}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,x\right]i \tag{17}$$

$$-\left[\left(\frac{(-2n\pi)\,x - a}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,x + \left(\frac{(-2n\pi)(ax + b) + 1}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,x\right] + K \tag{18}$$

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And the definite integral evaluates to:

$$
\begin{aligned}
A\int_z^q e^{Cx}\,dx + B\int_z^q xe^{Cx}\,dx = &+\left[\left(\frac{(-2n\pi)\,q-a}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,q - \left(\frac{(-2n\pi)\,(aq+b)+1}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,q\right]i && (19)\\
&-\left[\left(\frac{(-2n\pi)\,q-a}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,q + \left(\frac{(-2n\pi)\,(aq+b)+1}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,q\right] && (20)\\
&-\left[\left(\frac{(-2n\pi)\,z-a}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,z - \left(\frac{(-2n\pi)\,(az+b)+1}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,z\right]i && (21)\\
&+\left[\left(\frac{(-2n\pi)\,z-a}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,z + \left(\frac{(-2n\pi)\,(az+b)+1}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,z\right] && (22)
\end{aligned}
$$

So, for real and imaginary parts of $f(x,a,b,n,q,z)$:

$$
\begin{aligned}
\mathrm{Re}(f(x,a,b,n,q,z)) = &-\left[\left(\frac{(-2n\pi)\,q-a}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,q + \left(\frac{(-2n\pi)\,(aq+b)+1}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,q\right] && (23)\\
&+\left[\left(\frac{(-2n\pi)\,z-a}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,z + \left(\frac{(-2n\pi)\,(az+b)+1}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,z\right] && (24)\\
\mathrm{Im}(f(x,a,b,n,q,z)) = &+\left[\left(\frac{(-2n\pi)\,q-a}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,q - \left(\frac{(-2n\pi)\,(aq+b)+1}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,q\right] && (25)\\
&-\left[\left(\frac{(-2n\pi)\,z-a}{(-4n^2\pi^2)}\right)\cos(-2n\pi)\,z - \left(\frac{(-2n\pi)\,(az+b)+1}{(-4n^2\pi^2)}\right)\sin(-2n\pi)\,z\right] && (26)
\end{aligned}
$$

Is it really this complicated?! Where did I go wrong?