

417 • Integrated Analysis

Problem: How many six-character license plates with letters or numbers in any position and with *at least* one ‘W’ are possible? Start by counting the license plates for which there are *exactly* 1, 2, 3, 4, 5, & 6 ‘Ws’ and add them up.

License plates with exactly 1 *W*:

$$\left. \begin{array}{cccccc} W & - & - & - & - & - \\ - & W & - & - & - & - \\ - & - & W & - & - & - \\ - & - & - & W & - & - \\ - & - & - & - & W & - \\ - & - & - & - & - & W \end{array} \right\} \binom{6}{1} (35^5) = 6 \times 52,521,875$$

License plates with exactly 2 *Ws*:

$$\left. \begin{array}{cccccc} W & W & - & - & - & - \\ W & - & W & - & - & - \\ W & - & - & W & - & - \\ W & - & - & - & W & - \\ W & - & - & - & - & W \\ - & W & W & - & - & - \\ - & W & - & W & - & - \\ - & W & - & - & W & - \\ - & W & - & - & - & W \\ - & - & W & W & - & - \\ - & - & W & - & W & - \\ - & - & W & - & - & W \\ - & - & - & W & W & - \\ - & - & - & W & - & W \\ - & - & - & - & W & W \end{array} \right\} \binom{6}{2} (35^4) = 15 \times 1,500,625$$

417 • Integrated Analysis

License plates with exactly 3 W s:

$$\left. \begin{array}{cccccc}
 W & W & W & - & - & - \\
 W & W & - & W & - & - \\
 W & W & - & - & W & - \\
 W & W & - & - & - & W \\
 W & - & W & W & - & - \\
 W & - & W & - & W & - \\
 W & - & W & - & - & W \\
 W & - & - & W & W & - \\
 W & - & - & W & - & W \\
 W & - & - & - & W & W \\
 - & W & W & W & - & - \\
 - & W & W & - & W & - \\
 - & W & W & - & - & W \\
 - & W & - & W & W & - \\
 - & W & - & W & - & W \\
 - & W & - & - & W & W \\
 - & - & W & W & W & - \\
 - & - & W & W & - & W \\
 - & - & W & - & W & W \\
 - & - & - & W & W & W
 \end{array} \right\} \binom{6}{3} (35^3) = 20 \times 42,875$$

License plates with exactly 4 W s:

$$\left. \begin{array}{cccccc}
 W & W & W & W & - & - \\
 W & W & W & - & W & - \\
 W & W & W & - & - & W \\
 W & W & - & W & W & - \\
 W & W & - & W & - & W \\
 W & W & - & - & W & W \\
 W & - & W & W & W & - \\
 W & - & W & W & - & W \\
 W & - & W & - & W & W \\
 W & - & - & W & W & W \\
 - & W & W & W & W & - \\
 - & W & W & W & - & W \\
 - & W & W & - & W & W \\
 - & W & - & W & W & W \\
 - & - & W & W & W & W
 \end{array} \right\} \binom{6}{4} (35^2) = 15 \times 1,225$$

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License plates with exactly 5 W s:

$$\left. \begin{array}{cccccc} W & W & W & W & W & - \\ W & W & W & W & - & W \\ W & W & W & - & W & W \\ W & W & - & W & W & W \\ W & - & W & W & W & W \\ - & W & W & W & W & W \end{array} \right\} \binom{6}{5} (35^1) = 6 \times 35$$

License plates with exactly 6 W s:

$$W \ W \ W \ W \ W \ W \} \binom{6}{6} (35^0) = 1 \times 1$$

So, the total number of license plates with at least one ‘ W ’ is:

$$\begin{aligned} N &= \binom{6}{1} (35^5) + \binom{6}{2} (35^4) + \binom{6}{3} (35^3) + \binom{6}{4} (35^2) + \binom{6}{5} (35^1) + \binom{6}{6} (35^0) \\ &= (6 \times 52,521,875) + (15 \times 1,500,625) + (20 \times 42,875) + (15 \times 1,225) + (6 \times 35) + (1 \times 1) \\ &= 315,131,250 + 22,509,375 + 857,500 + 18,375 + 210 + 1 \\ &= 338,516,711 \end{aligned}$$

On the other hand, consider counting all the possible license plates and subtracting those that have *no* W s!

$$\begin{aligned} N &= (36^6) - (35^6) \\ &= 2,176,782,336 - 1,838,265,625 \\ &= 338,516,711 \end{aligned}$$

What do you notice?