

原初扰动：原初黑洞与标量 诱导引力波

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主要内容

- 背景介绍
- 原初黑洞(PBHs)与标量诱导引力波(SIGWs)
- 非正则暴胀模型
- Chern-Simons 引力中的标量诱导引力波
- 总结

从引力波的发现说起

- 打开研究引力与宇宙的新窗口

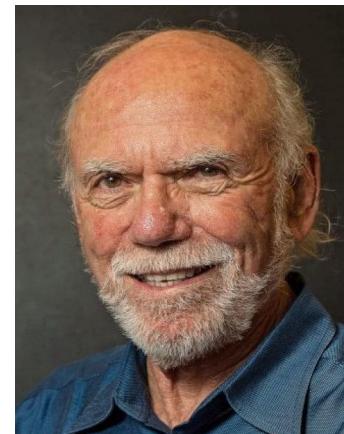


进入多信使天文

Barry C. Barish

Kip S. Thorne

Rainer Weiss



- 2017年诺贝尔物理学奖

引力波事件

	引力波事件	M_{BH}	都有可能是PBHs
LIGO探测到的由 双黑洞并合产生的 的引力波事件	GW150914	$\sim 30M_{\odot}, \sim 30M_{\odot}$	<i>Phys. Rev. Lett., 2016, 116(20):201301</i>
	GW190412	$\sim 30M_{\odot}, \sim 8M_{\odot}$	<i>Phys. Rev. Lett., 2016, 117(6):061101</i>
	GW190521	并合后 $\sim 150M_{\odot}$	<i>Phys. Rev. X 11 (2021) 021053</i>
	GW190814	$\sim 23M_{\odot}, \sim 2.6M_{\odot}$	<i>Phys. Rev. Lett., 2021, 126(7):071101</i>
PTA信号	观测到的信号可以用伴随PBHs形成而产生的随机引力波解释		<i>Phys. Rev. Lett., 2021, 126(5):051303</i>

原初黑洞的概念

- 在宇宙极早期形成的黑洞称为原初黑洞
- 上个世纪 60 年代，Zel'dovich 和 Novikov 指出在早期宇宙中黑洞可以通过对它们周围辐射物质的吸积而显著生长
- 霍金提出早期宇宙的过密区域会直接通过引力塌缩而形成黑洞
- 原初黑洞作为暗物质

Zel'dovich et al *Soviet Astron.* 1967, 10:602

Hawking S et al *Mon. Not. Roy. Astron. Soc.*, 1971, 152:75

Chapline G F et al. *Nature*, 1975, 253(5489):251 – 252

PBHs形成

➤ 形成机制

- 密度不均匀导致的引力塌缩

*S. W. Hawking, Mon. Not. Roy. Astron. Soc., 1971, 152:75.
Carr BJ & S. W. Hawking Mon. Not. Roy. Astron. Soc.,
1974, 168:399 – 415.*

➤ 物理图像

- 暴胀结束后的背景时空：空间平坦的FRW度规

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

- Friedmann 方程

$$H^2 = \frac{8\pi G}{3} \bar{\rho}$$

PBHs形成

◆ 假设存在一个小的球形扰动区域

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - \mathcal{K}(r)r^2} + r^2 d\Omega^2 \right] = -dt^2 + a(t)^2 e^{2\zeta(\hat{r})} \delta_{ij} dx^i dx^j$$

$$r = \hat{r} e^{\zeta(\hat{r})}, \mathcal{K}(r) = -\frac{\zeta'(\hat{r})}{\hat{r}} \frac{2 + \hat{r}\zeta'(\hat{r})}{e^{2\zeta(\hat{r})}}$$

Kehagias A et al JCAP, 2019, 12:029

PBHs形成

忽略 \mathcal{K} 的空间导数

$$H^2 + \frac{\mathcal{K}(r)}{a^2} = \frac{8\pi G}{3}\rho$$

密度扰动 (density contrast):

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} = \frac{\mathcal{K}}{a^2 H^2}$$

$$\delta(\mathbf{x}) = \frac{4(1+w)}{5+3w} \frac{1}{a^2 H^2} e^{-5\zeta(\mathbf{x})/2} \nabla^2 e^{\zeta(\mathbf{x})/2}$$

Musco I Phys. Rev. D, 2019, 100(12):123524

线性阶

$$\delta(\mathbf{x}) = \frac{2(1+w)}{5+3w} \frac{1}{a^2 H^2} \nabla^2 \zeta(\mathbf{x})$$

暴胀时期产生的共动曲率扰动

PBHs暗物质丰度

- 观测限制量
$$Y_{\text{PBH}} = \frac{\Omega_{\text{PBH}0}}{\Omega_{\text{DM}0}}$$
- Λ CDM 模型
$$Y_{\text{PBH}} = \frac{\beta}{3.94 \times 10^{-9}} \left(\frac{0.2}{\gamma} \right)^{\frac{1}{2}} \left(\frac{g_*}{10.75} \right)^{-\frac{1}{4}} \left(\frac{0.12}{\Omega_{\text{DM}} h^2} \right) \left(\frac{M}{M_\odot} \right)^{-\frac{1}{2}}$$
- $\beta = \frac{\rho_{\text{PBH}_i}}{\rho_{\text{tot}_i}}$

β 的计算：Press-Schechter (P-S) 理论

$$\beta = \int_{\delta_c} P(\delta_R) d\delta_R$$

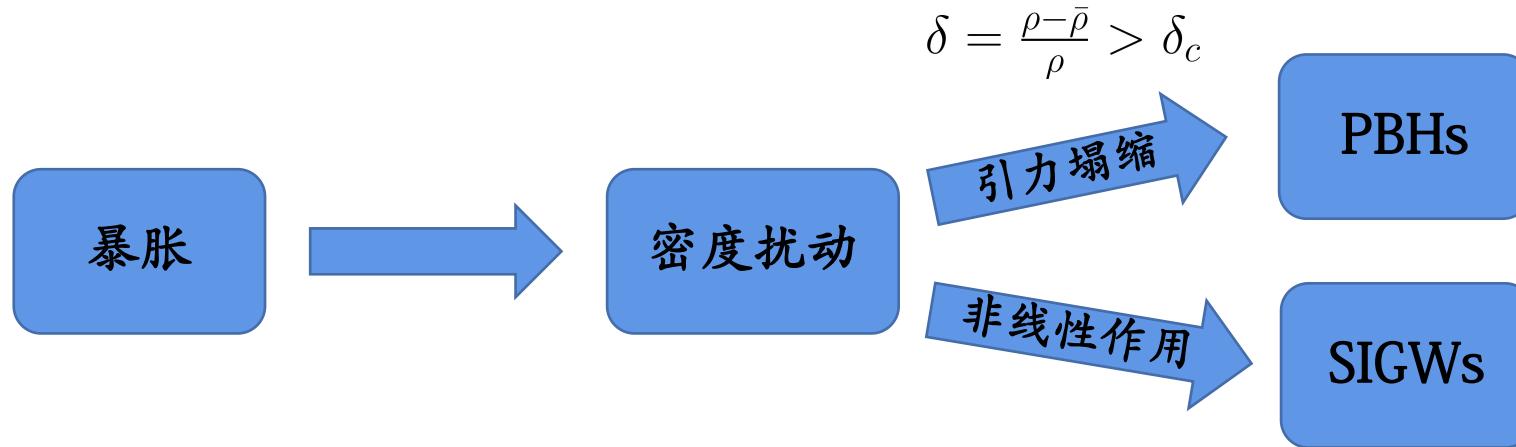
$$\delta_R(\mathbf{x}) = \int d^3x' W(|\mathbf{x} - \mathbf{x}'|, R) \delta(\mathbf{x}') \quad \text{在视界尺度 } R = aH \text{ 上的 smoothing 密度扰动}$$

$$\beta = \operatorname{Erfc} \left(\frac{\delta_c}{\sqrt{2}\sigma_R} \right) \approx \sqrt{\frac{2}{\pi\nu^2}} e^{-\frac{\nu^2}{2}} \quad \nu = \frac{\delta_c}{\sigma_R} \gg 1$$

$$\sigma_R^2 = \langle \delta_R \delta_R \rangle = \int \frac{dk}{k} \tilde{W}(kR)^2 \mathcal{P}_\delta(k)$$

$$\mathcal{P}_\delta(k) = \left(\frac{2(1+w)}{5+3w} \right)^2 (kR)^4 \mathcal{P}_\zeta(k) \quad \mathcal{P}_\zeta \text{ 是共动曲率扰动功率谱}$$

PBHs and SIGWs



广义相对论中的标量诱导引力波

GR:

$$S = \int d^4x \left[\frac{R}{2} + \mathcal{L}_m \right]$$

扰动的度规: $ds^2 = a^2[-(1+2\Phi)d\tau^2 + (1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}^{(2)}dx^j dx^j]$

运动方程:

$$h_{ij}^{(2)''} + 2\mathcal{H}h_{ij}^{(2)' -} - \nabla^2 h_{ij}^{(2)} = -4S_{ij}$$

$$\begin{aligned} S_{ij} = & 2\Phi\Phi_{,ij} + 2\Psi\Psi_{,ij} + \Phi_{,i}\Phi_{,j} + 3\Psi_{,i}\Psi_{,j} - \Psi_{,i}\Phi_{,j} - \Phi_{,i}\Psi_{,j} \\ & - \frac{4}{3(1+w)\mathcal{H}^2}(\Psi' + \mathcal{H}\Phi)_{,i}(\Psi' + \mathcal{H}\Phi)_{,j} \end{aligned}$$

SIGWs

SIGWs的
功率谱

$$\left\langle h_{\mathbf{k}_1}^{(2)s_1}(\tau) h_{\mathbf{k}_2}^{(2)s_2}(\tau) \right\rangle = \frac{2\pi^2}{k_1^3} \delta_{s_1 s_2} \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \mathcal{P}_h(k_1, \tau), \quad s_i = +, \times$$

$$\mathcal{P}_h(k, x) = 4 \int_0^\infty du \int_{|1-u|}^{1+u} dv \left[\frac{4u^2 - (1+u^2-v^2)}{4uv} \right]^2 I^2(u, v, x) \mathcal{P}_\zeta(uk) \mathcal{P}_\zeta(vk)$$



$$\mathcal{P}_h \sim \mathcal{P}_\zeta^2$$

原初曲率扰动功率谱

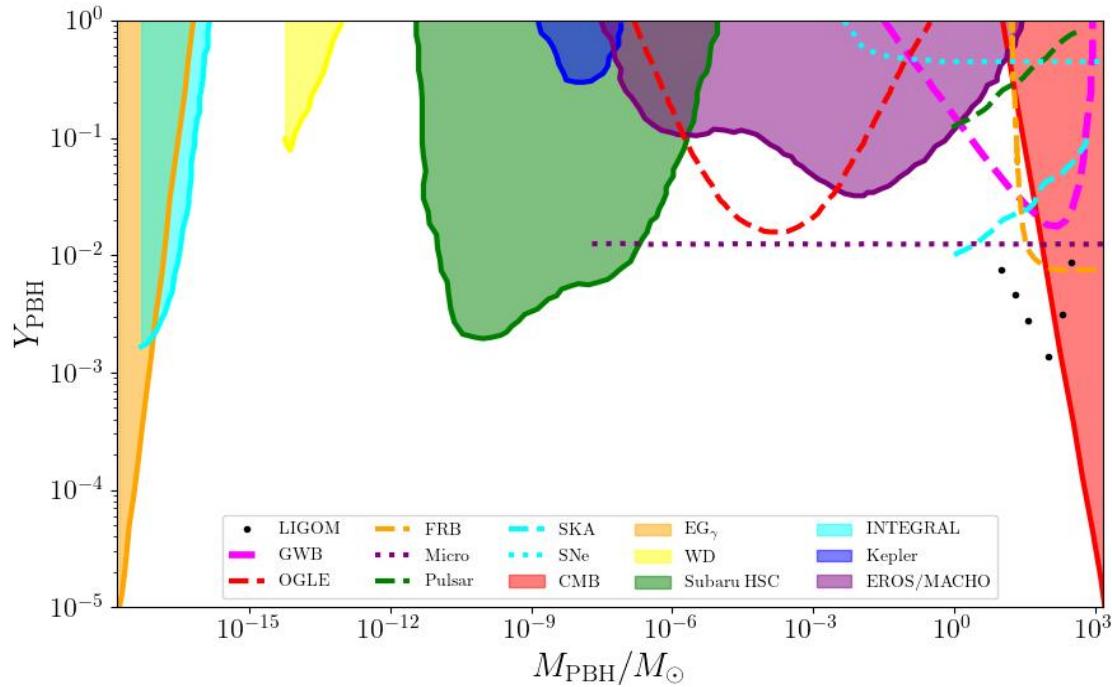
SIGWs的
能量密度
比重参数

$$\Omega_{\text{GW}}(k, \tau) = \frac{1}{24} \left(\frac{k}{\mathcal{H}} \right)^2 \overline{\mathcal{P}_h(k, x)} \quad x = k\tau$$

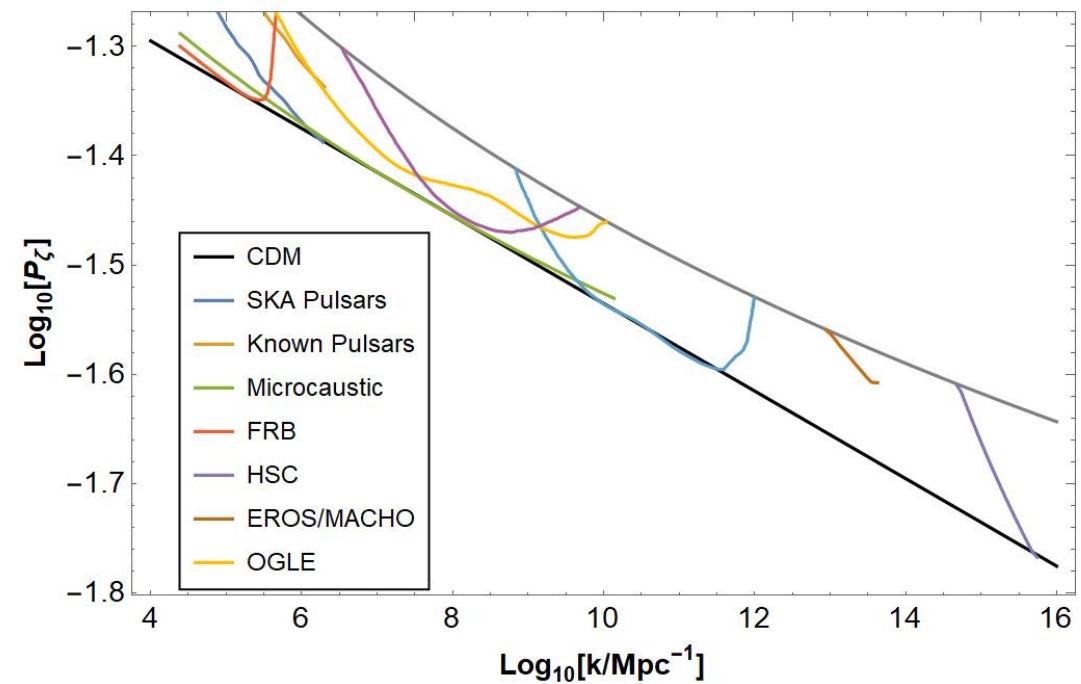
PBHs观测限制

$Y_{\text{PBH}} \rightarrow \mathcal{P}_\zeta$ 功率谱

Sato-Polito et al. Phys. Rev. D, 2019,

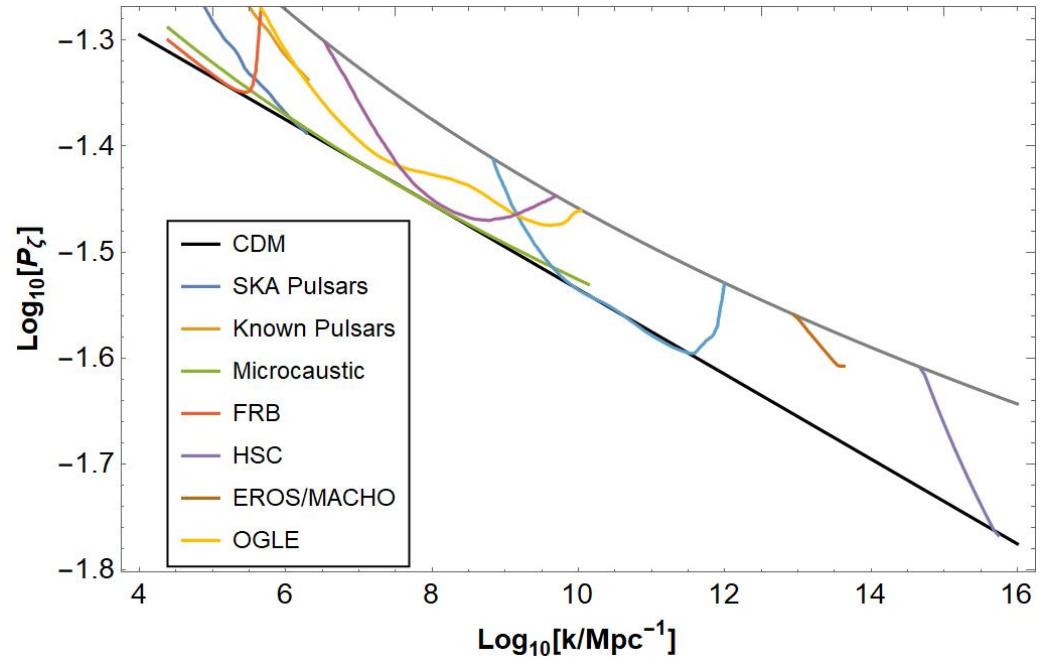


Lu, Gong, Yi, Zhang. JCAP 12 (2019) 031

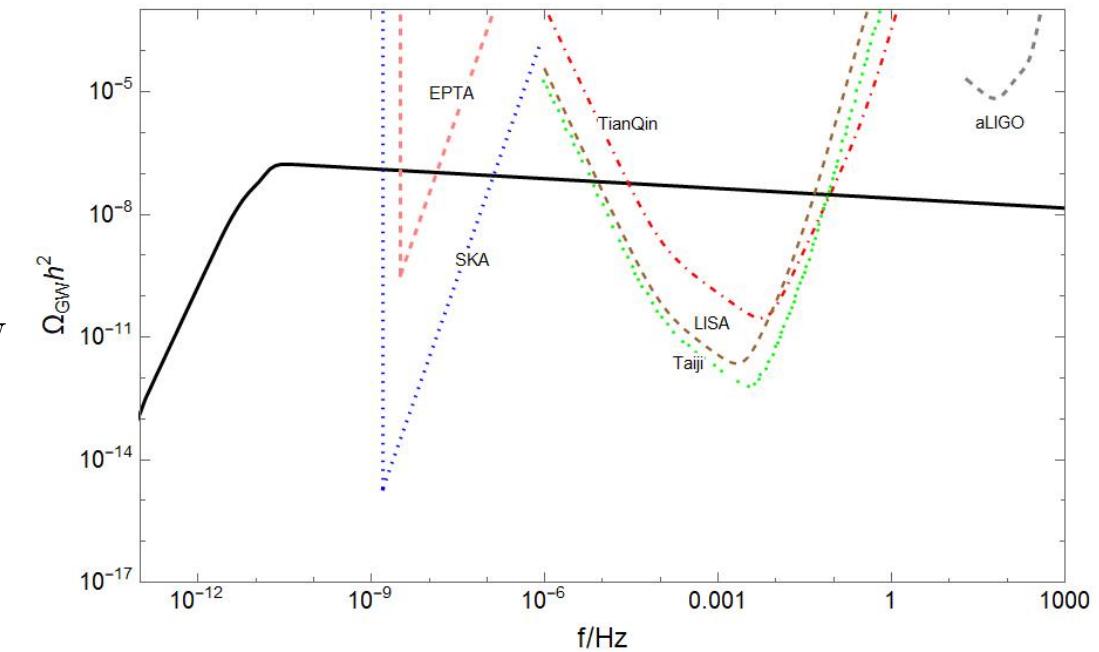


小尺度上: $\mathcal{A}_\zeta \lesssim 0.05$

SIGWs的可探测性

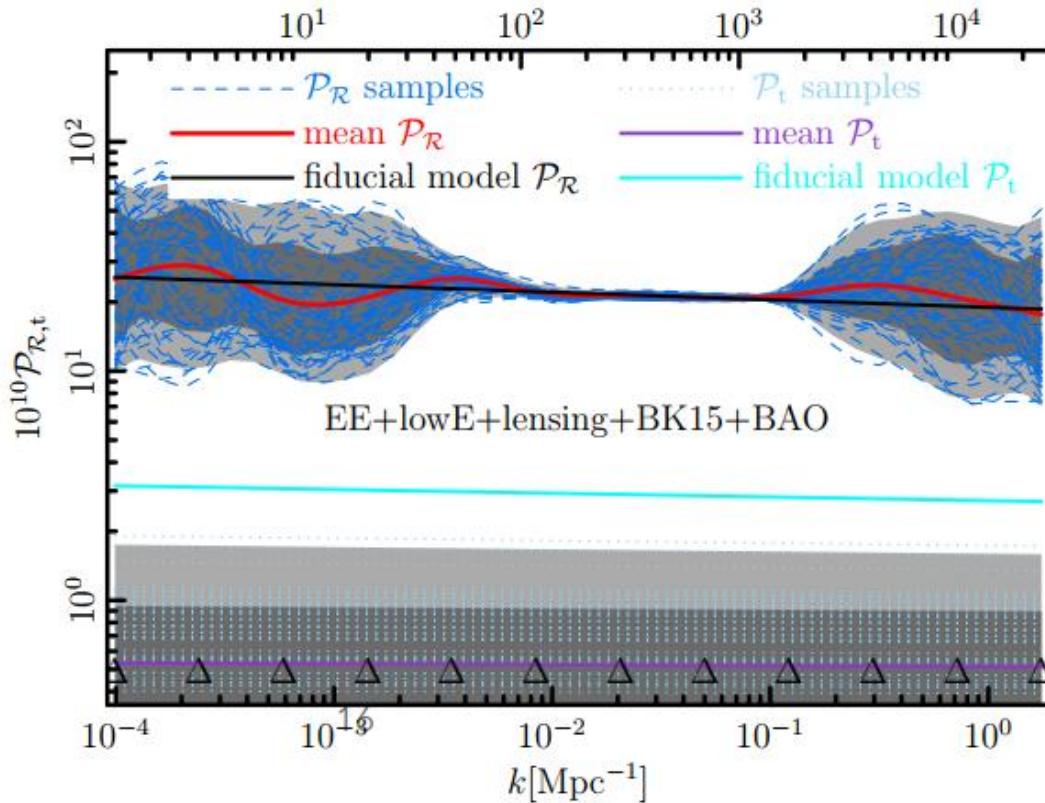


$$\mathcal{P}_\zeta \rightarrow \Omega_{\text{GW}}$$



Lu, Gong, Yi, Zhang. JCAP 12 (2019)
031

CMB尺度的观测限制

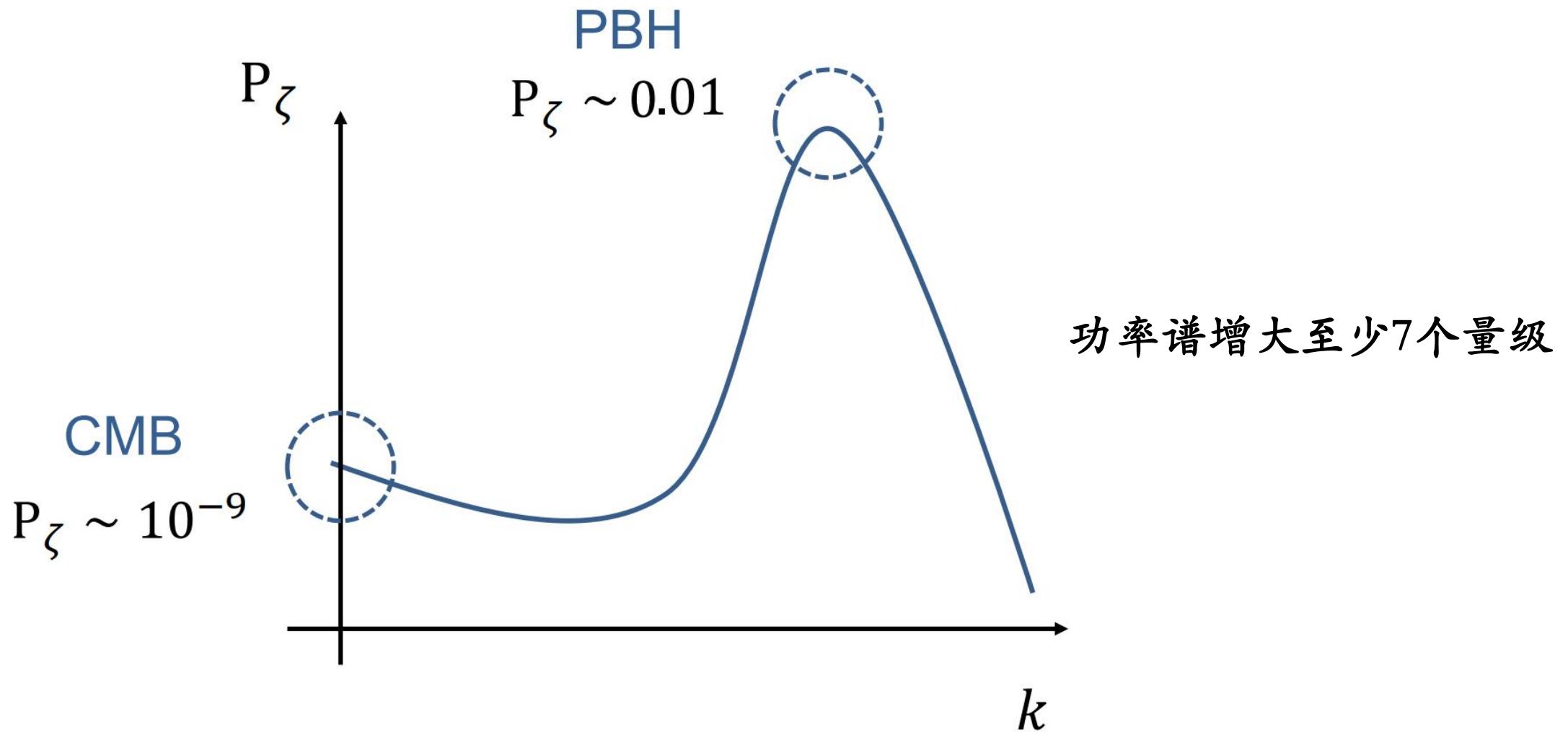


$$\mathcal{P}_\zeta(k) = \mathcal{A}_\zeta \left(\frac{k}{k_*} \right)^{(n_s - 1)}$$

$$\mathcal{P}_T(k) = \mathcal{A}_T \left(\frac{k}{k_*} \right)^{n_T}$$

$$\begin{aligned}\mathcal{A}_\zeta &\simeq 2.1 \times 10^{-9}, n_s = 0.9649 \pm 0.0042, \\ r_{0.05} &= \frac{\mathcal{A}_T}{\mathcal{A}_\zeta} < 0.06 (95\% \text{C.L.}), k_* = 0.05 \text{Mpc}. \end{aligned}$$

Planck 2018



对暴胀模型的要求

- 功率谱在小尺度上达到: $A_\zeta \sim \mathcal{O}(0.01)$
- 功率谱满足CMB限制: $A_\zeta \simeq 2.1 \times 10^{-9}$, $n_s = 0.9649 \pm 0.0042$,
 $r_{0.05} = \frac{A_T}{A_\zeta} < 0.06$ (95% C.L.)
- $N_* \sim 60$
- 单场正则慢滚模型 
- ✓ 考虑慢滚破坏的模型

非正则模型： k/G 暴胀模型

- 作用量: $S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + [1 + G(\phi)]X - V(\phi) \right]$
- 慢滚近似下的功率谱: $\mathcal{P}_\zeta \simeq \frac{V^3}{12\pi^2 V_\phi^2} (1 + G)$
- **$G(\phi)$ 的选择:** $w(\phi) = 1/\phi \rightarrow G(\phi) = \frac{d}{1+|\phi-\phi_p|/c} = \frac{dc}{c+|\phi-\phi_p|}$
- 参数选择: $d \sim \mathcal{O}(10^8)$
- 在大尺度上: $G(\phi_*) \simeq dc \sim \mathcal{O}(1)$ $c \sim \mathcal{O}(10^{-10})$

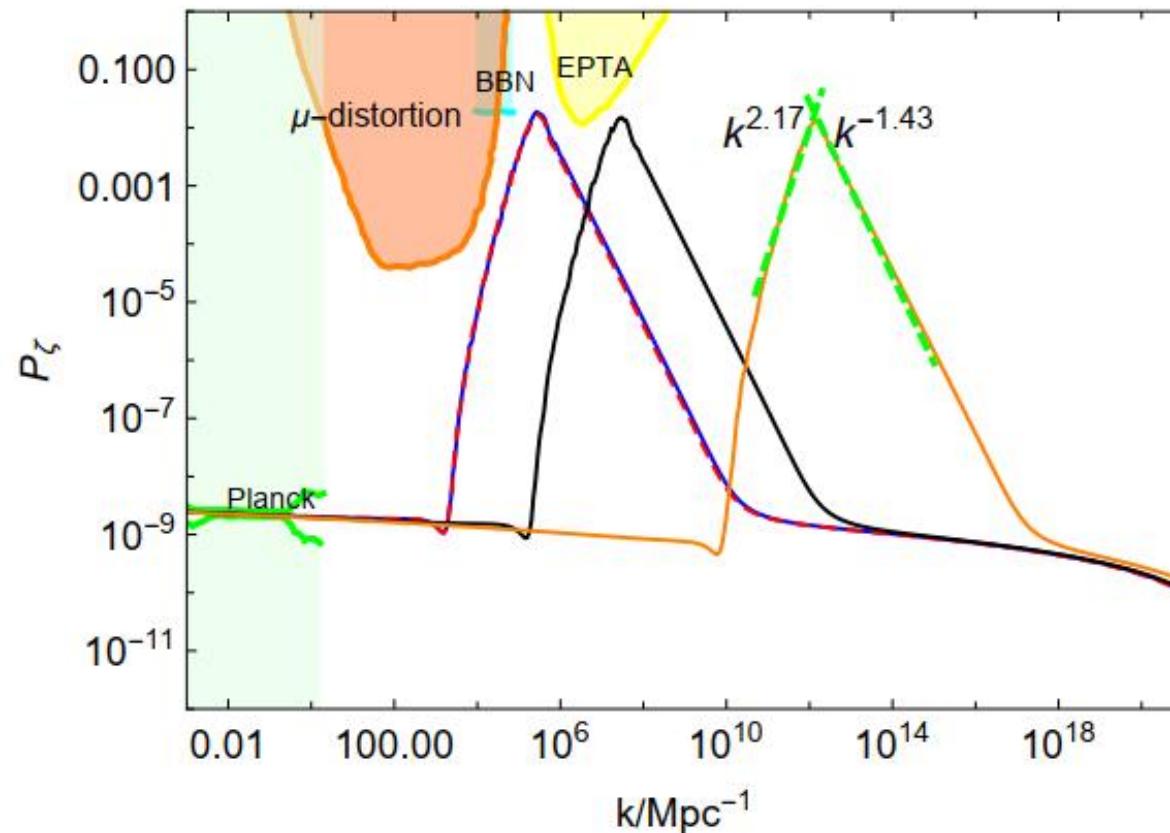
*Lin, Gao, Gong, Lu, Zhang, Zhang.
Phys. Rev. D, 2020,
101(10):103515*

具体模型：

$$V(\phi) = \lambda\phi^{2/5} \quad G(\phi) = \frac{d}{1+|\phi-\phi_p|/c}$$

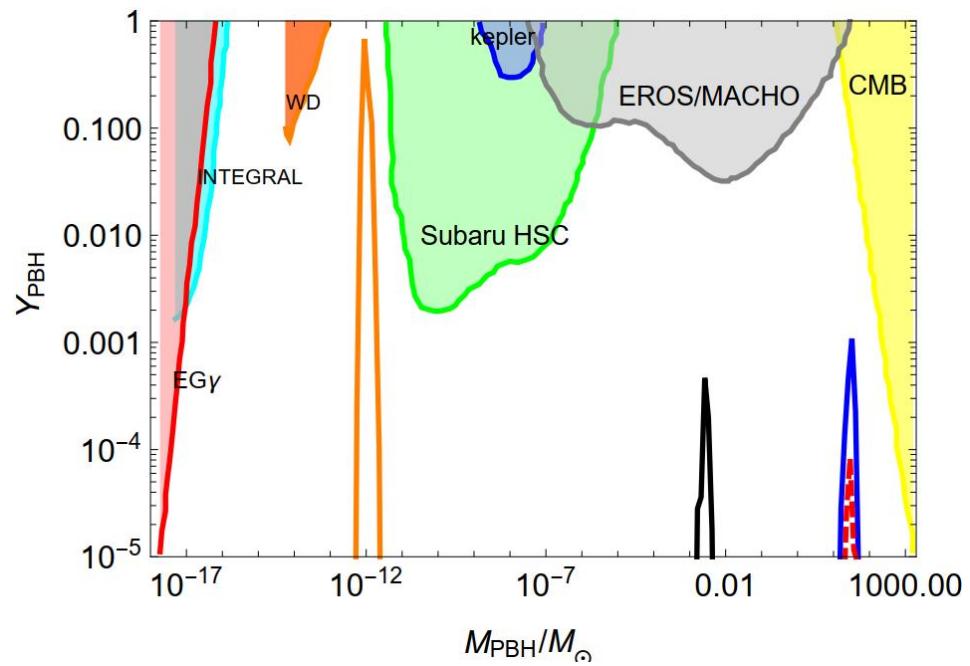
Sets	ϕ_r	c	n_s	$k_{\text{peak}}/\text{Mpc}^{-1}$	\mathcal{P}_ζ	M_{PBH}/M_\odot	Y_{PBH}
A	4.5	9.54×10^{-11}	0.9736	2.86×10^5	1.66×10^{-2}	28.9	7.7×10^{-5}
B	4.5	9.568×10^{-11}	0.9737	2.7×10^5	1.86×10^{-2}	32.5	0.001
C	4.1	1.05×10^{-10}	0.969	3×10^7	1.49×10^{-2}	0.0026	4.7×10^{-4}
D	2.97	1.472×10^{-10}	0.967	1.63×10^{12}	1.32×10^{-2}	9×10^{-13}	0.73

$$N_* \simeq 54$$



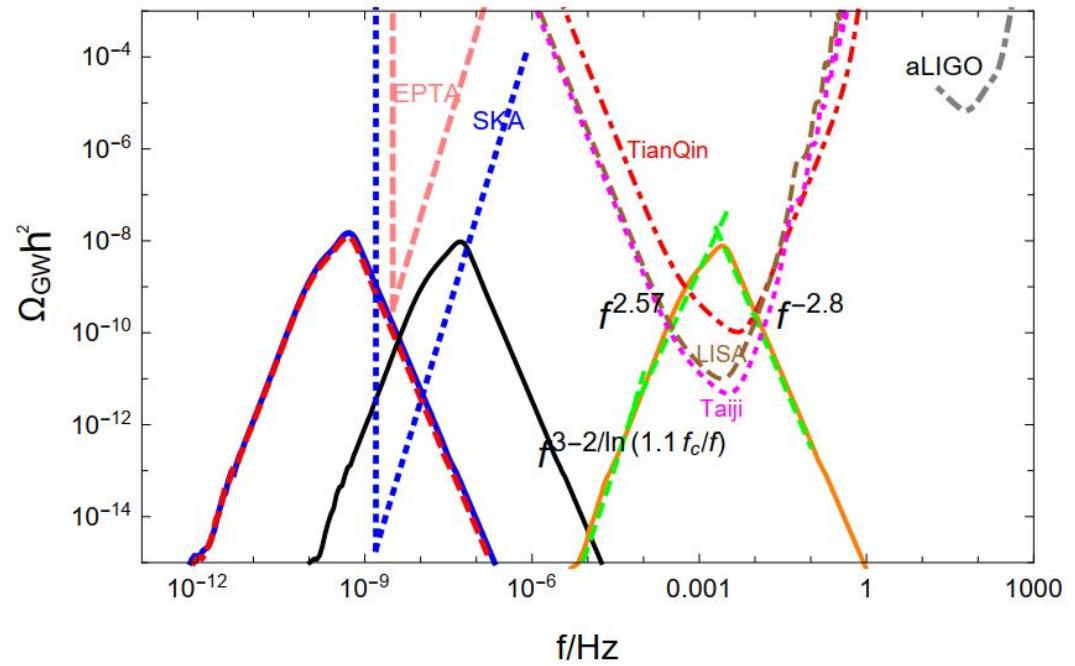
Lin, Gao, Gong, Lu, Zhang, Zhang.
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PBHs 与 SIGWs



$$M_{\text{PBH}} \sim 10^{-12} M_\odot, Y_{\text{PBH}} \sim 0.73$$

$M_{\text{PBH}} \sim 30 M_\odot$ 解释 LIGO 探测到的引力波事件



Lin, Gao, Gong, Lu, Zhang, Zhang.
Phys. Rev. D, 2020,
101(10):103515

宇称破缺引力中的SIGWs

➤ 弱相互作用  引力相互作用

Phys.Rev. 104(1956)245-258, Phys.Rev. 105 (1957) 1413-1414

➤ 量子引力（超弦理论等）的预言

➤ 观测上宇称破缺的迹象：BOSS星系三谱、CMB上E/B谱交叉关联等

Phys.Rev.D 106 (2022)063501, Mon.Not.Roy.Astron.Soc. 522 (2023) 4

Phys.Rev.Lett. 125 (2020)221301, Phys.Rev.D 106 (2022)063503

引力波的偏振

➤ GR:
$$h_k^{A''} + 2\mathcal{H}h_k^{A'} + k^2 h_k^A = 4S_k^A, \quad A = +, \times$$

➤ PV 引力:
$$h_k^{A''} + (2 + \nu_A) \mathcal{H}h_k^{A'} + (1 + \mu_A)k^2 h_k^A = 4\mathcal{S}_k^A, \quad A = R, L$$

➤ 圆偏振度:
$$\Pi = \frac{\mathcal{P}^R - \mathcal{P}^L}{\mathcal{P}^R + \mathcal{P}^L}$$

Chern-Simons (C-S) 引力中的SIGWs

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \frac{1}{4}\vartheta(\varphi)^* RR] - \int d^4x \sqrt{-g} (\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + V(\varphi))$$

$${}^*RR = {}^*R^{\mu\nu\rho\gamma}R_{\mu\nu\rho\gamma} \quad {}^*R^{\mu\nu\rho\gamma} = \frac{1}{2}\varepsilon^{\rho\gamma\alpha\beta}R^{\mu\nu}_{\alpha\beta}$$

C-S 引力

➤ 背景运动方程：

$$\begin{aligned}3\mathcal{H}^2 &= \kappa^2 a^2 \bar{\rho}, \\-\left(\mathcal{H}^2 + 2\mathcal{H}'\right) &= \kappa^2 a^2 \bar{P}, \\\varphi'' + 2\mathcal{H}\varphi' + a^2 V_\varphi(\varphi) &= 0\end{aligned}$$

➤ 线性扰动方程：

$$\begin{aligned}\nabla^2\psi - 3\mathcal{H}(\psi' + \mathcal{H}\psi) &= \frac{1}{2}\kappa^2(\varphi'\delta\varphi' - \varphi'^2\psi + a^2 V_\varphi\delta\varphi) \\\psi' + \mathcal{H}\psi &= \frac{1}{2}\kappa^2\varphi'\delta\varphi \\\psi'' + 3\mathcal{H}\psi' + \mathcal{H}^2\psi + 2\mathcal{H}'\psi &= \frac{1}{2}\kappa^2\left(\varphi'\delta\varphi' - \varphi'^2\psi - a^2 V_\varphi\delta\varphi\right) \\\delta\varphi'' + 2\mathcal{H}\delta\varphi' - \nabla^2\delta\varphi - 4\psi'\varphi' &= -a^2\delta\varphi V_{\varphi\varphi} - 2\psi V_\varphi\end{aligned}$$

SIGWs

➤ SIGWs的运动方程： $u_{\mathbf{k}}^{A''}(\eta) + \left(k^2 - \frac{B^{A''}}{B^A} \right) u_{\mathbf{k}}^A(\eta) = 4 \frac{a S_{\mathbf{k}}^A(\eta)}{\sqrt{z^A}},$

$$u_{\mathbf{k}}^A = B^A h_{\mathbf{k}}^A \quad B^A(k, \eta) = a(\eta) \sqrt{z^A(k, \eta)},$$

$$z^A(k, \eta) = 1 - \frac{k \lambda^A \vartheta(\varphi)'}{a^2(\eta)} \quad S_{\mathbf{k}}^A = S_{\mathbf{k}}^{A(\text{scalar})} + S_{\mathbf{k}}^{A(\text{PV})}$$

SIGWs

➤ 功率谱 : $\langle h_{\mathbf{k}}^A h_{\mathbf{k}'}^C \rangle = \frac{2\pi^2}{k^3} \delta^3(\mathbf{k} + \mathbf{k}') \delta^{AC} \mathcal{P}_h^A(k)$

$$\mathcal{P}_h^A(k, x) = 4 \int_0^\infty du \int_{|1-u|}^{1+u} dv \mathcal{J}(u, v) \frac{I^A(k, u, v, x)^2}{(z^A(k))^2} \mathcal{P}_\zeta(uk) \mathcal{P}_\zeta(vk)$$

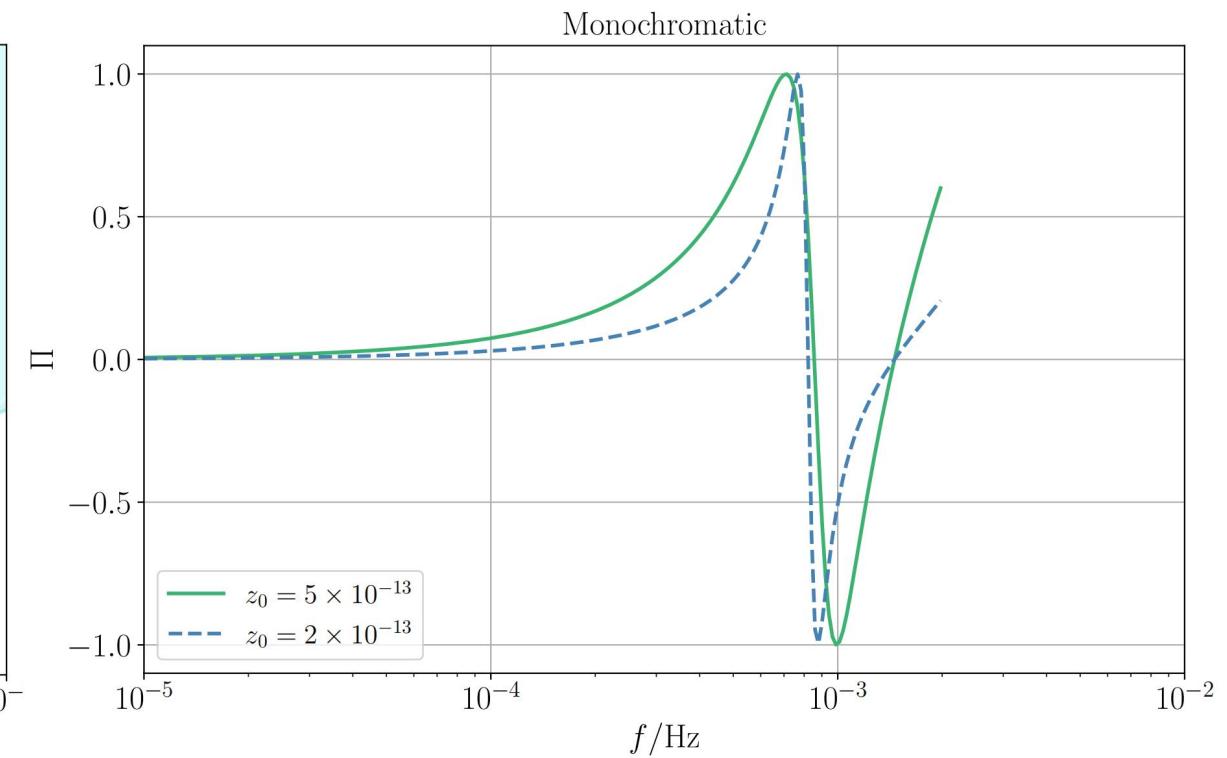
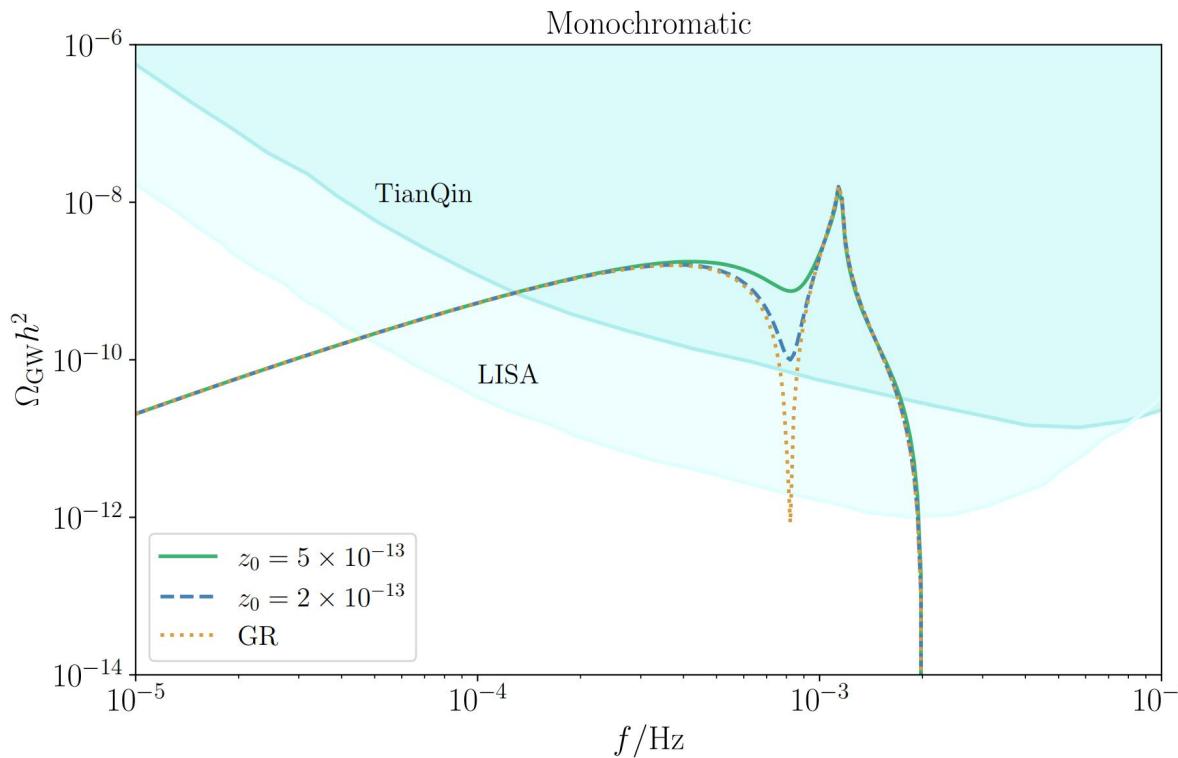
➤ 能量密度 : $\Omega_{\text{GW}}(k, x) = \frac{1}{48} \left(\frac{k}{\mathcal{H}} \right)^2 \sum_{A=R,L} \overline{\mathcal{P}_h^A(k, x)}$

举例说明

➤ 单色的功率谱 :

$$\mathcal{P}_\zeta(k) = \mathcal{A}_\zeta \delta(\ln(k/k_p))$$

$$\Pi = \frac{\mathcal{P}^R - \mathcal{P}^L}{\mathcal{P}^R + \mathcal{P}^L}$$



总结

- 原初扰动在宇宙的演化过程中发挥了重要作用， 我们可以通过对PBHs以及 SIGWs的研究来探索早期宇宙；
- 不同引力理论中的SIGWs具有不同的特点， 研究SIGWs， 有助于我们检验 引力理论。

谢谢！