MF 796 Homework 1
Due: Wed, Jan 29th, 8:00 a.m.

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1. Trading, portfolio management and derivative pricing.

2. Enhance financial modeling ability and get myself familiar with quant's tasks.

3. Python: familiar
   SQL: proficient
   C++: proficient
   Matlab: familiar
   R: familiar

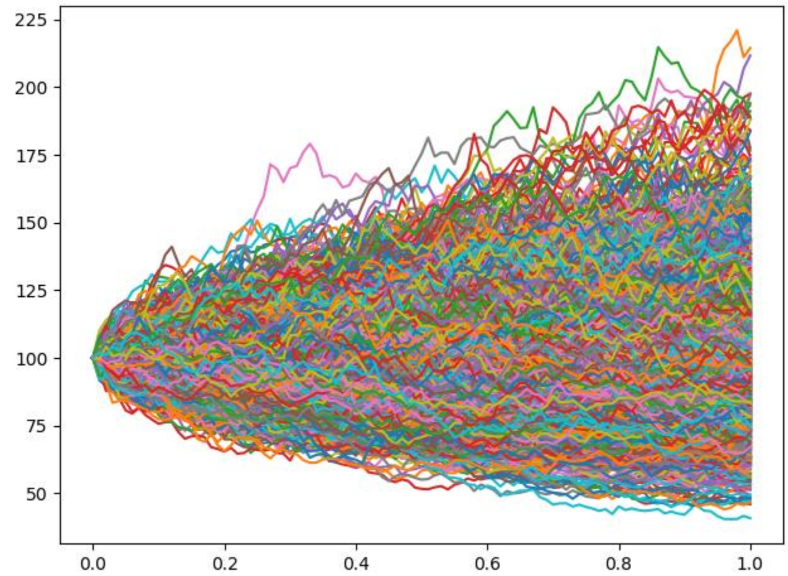
4. According to Black-Sholes PDE,

$$\frac{\partial c_t}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 c_t}{\partial S_t^2} + rS_t \frac{\partial c_t}{\partial S_t} - rc_t = 0$$

   Gamma is $\frac{\partial^2 c_t}{\partial S_t^2}$ measures the sensitivity of the delta to changes in the underlying price of the asset. Theta is $\frac{\partial c_t}{\partial t}$ measures the option's sensitivity to the passage of time prior to expiration. When buying options, theta will always be a negative number for long positions while the opposite is true for gamma. Toward the expiration, theta decreases to zero since option price will be equal to the intrinsic value by the expiration date but delta changes rapidly as option approaches expiration so gamma will increase as expiration approaches. In that sense, there appears to be an inverse correlation between the two variables so as to say we get gamma at the expense of theta.

5. (a)
   S_0 is the initial value of the underlying asset of the option
   r is the risk-free rate of the market
   β is the elasticity of the local volatility function
   σ is the volatility in diffusion coefficient of the price process

   (b)
   via Monte Carlo simulation, the simulation path is shown below,



The simulated price of at-the-money one year European call option is **9.909313007806439**.

(c)

The price of the same European call option via the Black-Scholes formula is **9.94764496602258**.

This price is not the same as the price obtained via simulation, due to unavoidable errors when simulating. However, the two prices should be the same theoretically since the Black-Scholes formula is the analytic solution to the process: $dS_t = rS_t d_t + \sigma S_t dW$. As a result, the difference between the two prices is relatively small.

(d)

The delta of an at-the-money European call option with one year to expiry is **0.5497382248301129**

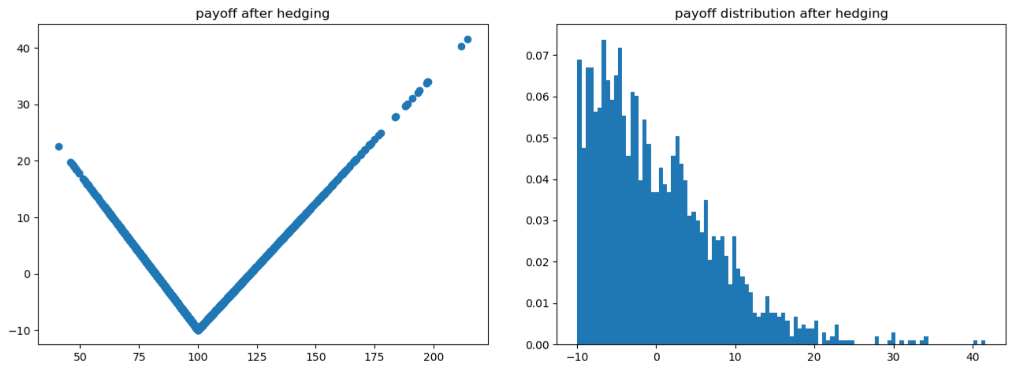
(e)

If we only construct a delta neutral portfolio once at the beginning and keep the position all the time, according to the delta calculated in (c), we need **-0.5497382248301129** shares of stocks to construct a delta neutral portfolio that is long one unit of the call option. In other words, we need to short **0.5497382248301129** shares of stocks.

(f)

The average payoff is **9.956503134360467**.

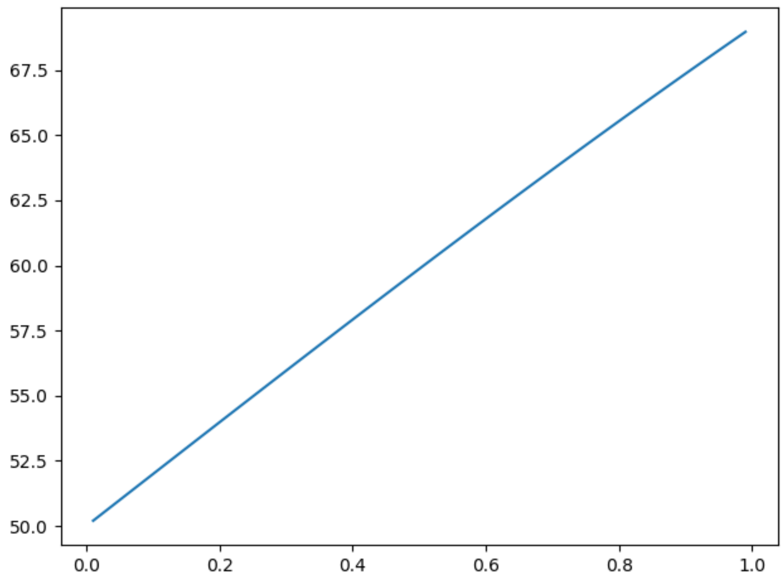
Figures of the payoff of the delta neutral portfolio (the Black-Scholes model price of the option has been subtracted) are shown below.



So compared to the Black-Scholes model price of the option, if the price of the underlying asset falls into the interval $[S_0 - \frac{c_0}{\Delta}, S_0 + \frac{c_0}{1-\Delta}]$, which is [81.9059, 122.0930] in this case, this portfolio will lose money. Otherwise, the portfolio will make money.

(g)
The average payoff now is **0.9918053341165491**, which is lower since given the parameter $\beta = 0.5$, the volatility of the price of underlying asset is smaller so with smaller possibility to jump out of the interval where the portfolio will lose money under the same volatility σ.

(f)
The average payoff now is **15.933393567465432**, which is higher since given the parameter $\sigma = 0.5$, the volatility of the price of underlying asset is higher.
Trying different volatilities, we notice the relationship between the volatility and the delta-neutral portfolio are shown below.



The higher σ is ,the higher payoff the delta-neutral portfolio has. Since higher volatility implies higher possibility the price of underlying asset will jump out of the interval where the portfolio will lose money (even under such situation that the option price will raise correspondingly)