MF 803 Homework 1

Due: Wednesday, September 18th, 6:30 p.m.

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1.

(a) Historical price data of each ETFs on yahoo finance has been downloaded through the API – yfinance. After examining historical price data of each ETFs, no anomalies has been found. So the historical price data is clean and there is no need for extra cleaning.

(b) The annualized return and standard deviation of each ETF are listed below. The annualized return is calculated as $r = ln(\frac{Pt}{P0}) * \frac{T}{t}$. In this situation, T is assumed to be 252 (annual trading days).

Code	Annualized_Return	Standard_Deviation
SPY	**10.09%**	**56.245964**
XLB	**5.71%**	**8.697813**
XLE	**0.39%**	**10.015172**
XLF	**8.99%**	**5.704337**
XLI	**10.64%**	**15.388153**
XLK	**12.90%**	**16.727176**
XLP	**8.55%**	**10.171024**
XLU	**7.28%**	**8.717034**
XLV	**10.97%**	**20.910602**
XLY	**14.60%**	**26.182718**

(c) The table of covariance matrix of daily and monthly returns, the table of correlation matrix of daily and monthly returns are listed below. Especially, monthly returns are calculated by Pt/P0, t =21.

The covariance matrix of Daily returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	8.79368E-05	0.000100564	0.000102578	0.000105612	9.61911E-05	9.31659E-05	5.50193E-05	4.46138E-05	7.58837E-05	9.00707E-05
XLB	0.000100564	0.0001489	0.00013349	0.00012263	0.000117412	0.000100841	5.86133E-05	4.71197E-05	8.10117E-05	0.000100896
XLE	0.000102578	0.00013349	0.000187311	0.000121905	0.000114157	9.83133E-05	5.62788E-05	4.87078E-05	7.92341E-05	9.80954E-05
XLF	0.000105612	0.00012263	0.000121905	0.000157551	0.000117965	0.000102403	6.0205E-05	4.59328E-05	8.57444E-05	0.00010534
XLI	9.61911E-05	0.000117412	0.000114157	0.000117965	0.000120718	9.78146E-05	5.77282E-05	4.57087E-05	7.89375E-05	9.86038E-05
XLK	9.31659E-05	0.000100841	9.83133E-05	0.000102403	9.78146E-05	0.000116822	5.35841E-05	4.0497E-05	7.68319E-05	9.61783E-05
XLP	5.50193E-05	5.86133E-05	5.62788E-05	6.0205E-05	5.77282E-05	5.35841E-05	5.68811E-05	4.34405E-05	5.01455E-05	5.53387E-05
XLU	4.46138E-05	4.71197E-05	4.87078E-05	4.59328E-05	4.57087E-05	4.0497E-05	4.34405E-05	7.82858E-05	3.95826E-05	4.13612E-05
XLV	7.58837E-05	8.10117E-05	7.92341E-05	8.57444E-05	7.89375E-05	7.68319E-05	5.01455E-05	3.95826E-05	8.90775E-05	7.51703E-05
XLY	9.00707E-05	0.000100896	9.80954E-05	0.00010534	9.86038E-05	9.61783E-05	5.53387E-05	4.13612E-05	7.51703E-05	0.000107312

The covariance matrix of monthly returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	0.001294456	0.00156167	0.001604453	0.001600378	0.001538674	0.001378544	0.000726896	0.000394345	0.001019631	0.001371851
XLB	0.00156167	0.002406554	0.002196403	0.001964622	0.001975482	0.0015963	0.000737087	0.000329275	0.001143664	0.001620238
XLE	0.001604453	0.002196403	0.003279444	0.001911776	0.001995976	0.001566683	0.000682569	0.000321593	0.00099501	0.001610699
XLF	0.001600378	0.001964622	0.001911776	0.002493166	0.001976195	0.001554683	0.000741868	0.000242045	0.00119256	0.001665164
XLI	0.001538674	0.001975482	0.001995976	0.001976195	0.002090017	0.001543886	0.000793993	0.000382758	0.001143281	0.001622602
XLK	0.001378544	0.0015963	0.001566683	0.001554683	0.001543886	0.001771609	0.000699042	0.000364205	0.000963708	0.001457637
XLP	0.000726896	0.000737087	0.000682569	0.000741868	0.000793993	0.000699042	0.000857564	0.000599786	0.000672373	0.000767172
XLU	0.000394345	0.000329275	0.000321593	0.000242045	0.000382758	0.000364205	0.000599786	0.001164737	0.000400974	0.000283035
XLV	0.001019631	0.001143664	0.00099501	0.00119256	0.001143281	0.000963708	0.000672373	0.000400974	0.001274248	0.001038809
XLY	0.001371851	0.001620238	0.001610699	0.001665164	0.001622602	0.001457637	0.000767172	0.000283035	0.001038809	0.001695944

The correlation matrix of daily returns:

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	1	0.878836301	0.799258289	0.897260086	0.933606187	0.919198308	0.777939215	0.537703082	0.857390956	0.927199817
XLB	0.878836301	1	0.799318609	0.800638454	0.875746883	0.764583762	0.636889318	0.436429122	0.703422432	0.798182535
XLE	0.799258289	0.799318609	1	0.70962514	0.75915986	0.66461206	0.545228953	0.402231134	0.613403556	0.691897746
XLF	0.897260086	0.800638454	0.70962514	1	0.855376186	0.754813548	0.63597141	0.413590437	0.723787138	0.810132319
XLI	0.933606187	0.875746883	0.75915986	0.855376186	1	0.823674465	0.696655834	0.470188418	0.761226124	0.866329219
XLK	0.919198308	0.764583762	0.66461206	0.754813548	0.823674465	1	0.657338828	0.423466738	0.753174463	0.858993529
XLP	0.777939215	0.636889318	0.545228953	0.63597141	0.696655834	0.657338828	1	0.65098201	0.70447289	0.708304026
XLU	0.537703082	0.436429122	0.402231134	0.413590437	0.470188418	0.423466738	0.65098201	1	0.474000935	0.451259874
XLV	0.857390956	0.703422432	0.613403556	0.723787138	0.761226124	0.753174463	0.70447289	0.474000935	1	0.768843042
XLY	0.927199817	0.798182535	0.691897746	0.810132319	0.866329219	0.858993529	0.708304026	0.451259874	0.768843042	1

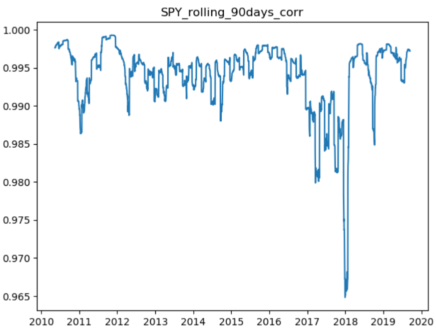
The correlation matrix of monthly returns:

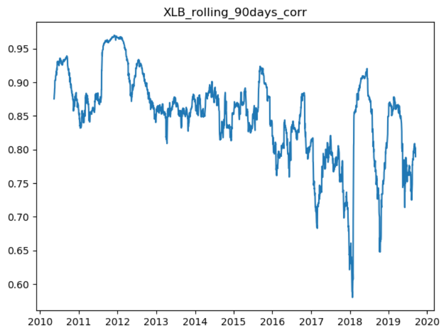
	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
SPY	1	0.884805341	0.778723501	0.890847265	0.93546597	0.910317615	0.689914559	0.32115823	0.793912228	0.925885791
XLB	0.884805341	1	0.78183309	0.802057527	0.880846771	0.773094582	0.513083212	0.196673834	0.653091083	0.802001772
XLE	0.778723501	0.78183309	1	0.668591976	0.762395489	0.64997566	0.407017905	0.164547705	0.486743671	0.68298074
XLF	0.890847265	0.802057527	0.668591976	1	0.865723586	0.739745426	0.507361752	0.142038725	0.66907967	0.809795933
XLI	0.93546597	0.880846771	0.762395489	0.865723586	1	0.802335882	0.59307378	0.245321292	0.700569317	0.861848954
XLK	0.910317615	0.773094582	0.64997566	0.739745426	0.802335882	1	0.567134554	0.253540855	0.641408526	0.840929313
XLP	0.689914559	0.513083212	0.407017905	0.507361752	0.59307378	0.567134554	1	0.600135571	0.643206068	0.636141719
XLU	0.32115823	0.196673834	0.164547705	0.142038725	0.245321292	0.253540855	0.600135571	1	0.329136268	0.201381792
XLV	0.793912228	0.653091083	0.486743671	0.66907967	0.700569317	0.641408526	0.643206068	0.329136268	1	0.706647587
XLY	0.925885791	0.802001772	0.68298074	0.809795933	0.861848954	0.840929313	0.636141719	0.201381792	0.706647587	1

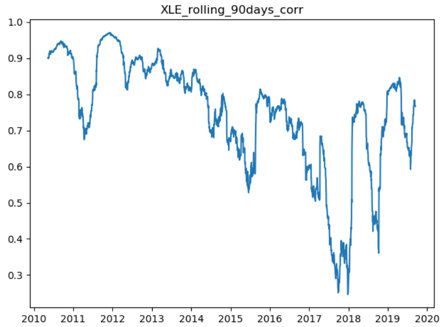
From the two correlation matrix above, we observe that the correlation between different ETF at different frequencies seem to be similar. So, in the long term, the relation between the return of two ETF tend to be stable.

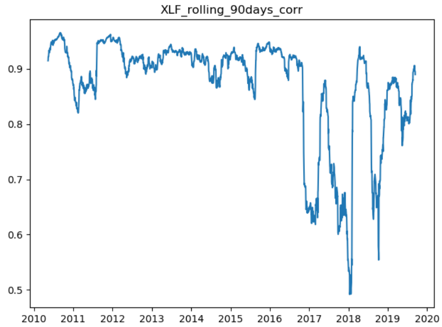
(d) The graph of a rolling 90-day correlation of each sector ETF with the S&P index are listed below. As seen from the figure, the correlations do not appear to be stable over time.

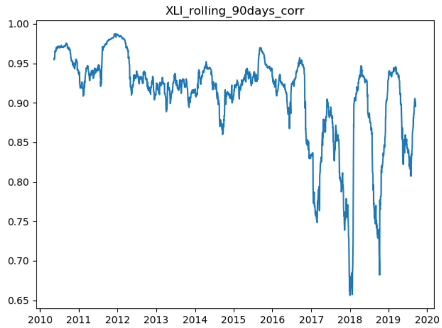
Since each ETF only track stocks of particular industry in S&P index, while the S&P index includes all stocks. Stocks of particular industry has their own lifecycle different from the whole market, plus the ETF confront the risk of holding stocks of only one industry. So the return of each ETF may be further different from the S&P Index.

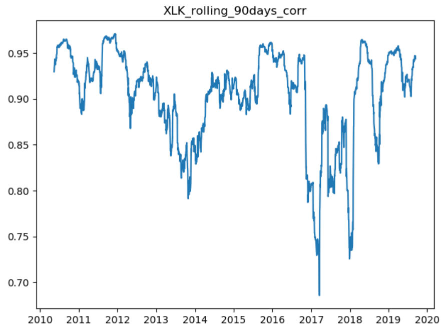


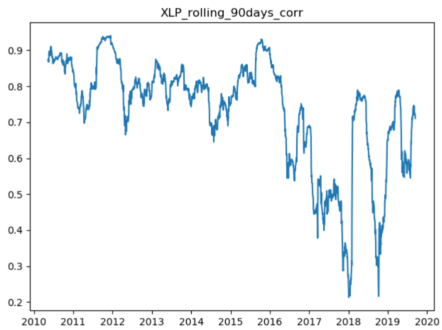


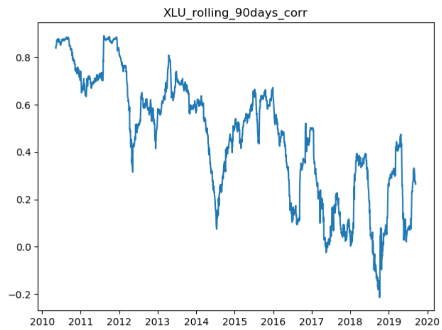


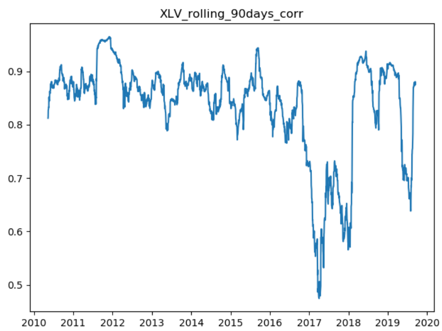


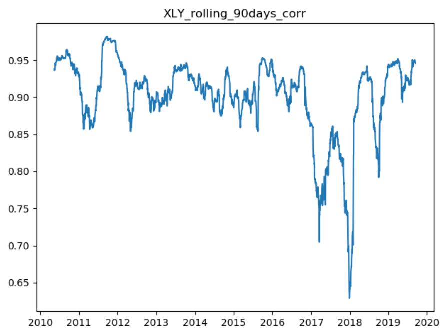








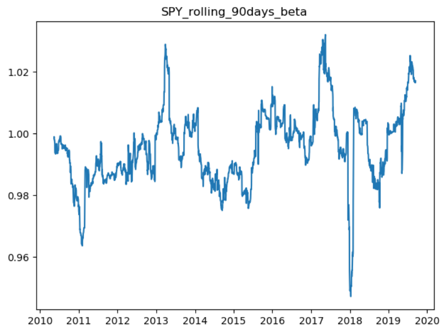


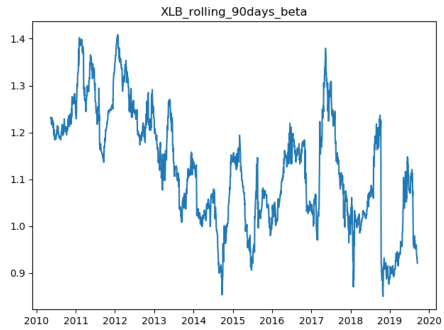


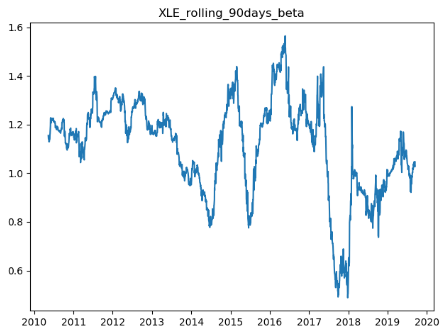
(e) The beta for the entire historical period for each ETF are listed below:

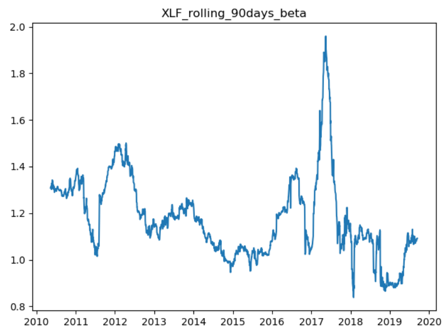
Code	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
Beta	0.99491	1.13989	1.16616	1.1988	1.09105	1.05589	0.61927	0.50133	0.86019	1.02094

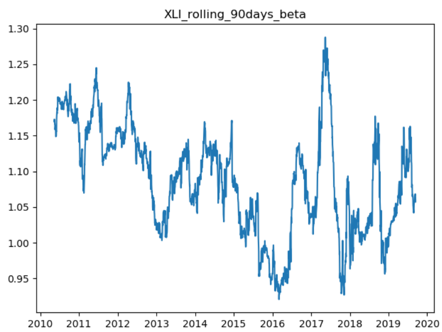
The graph of a rolling 90-day beta of each sector ETF with to the market are listed below. As seen from the figure, the correlations do not appear to be consistent over time.

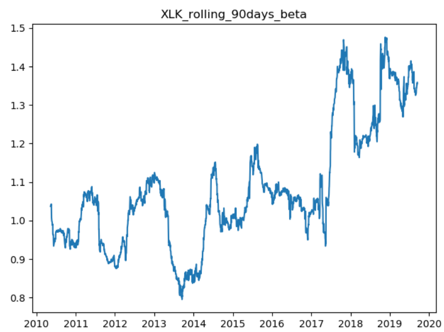

SPY_rolling_90days_beta
1.02
1.00
0.98
0.96
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020

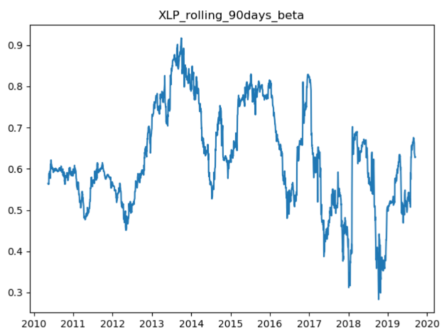

XLB_rolling_90days_beta
1.4
1.3
1.2
1.1
1.0
0.9
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020

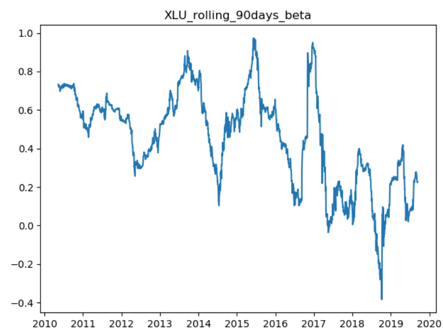

XLE_rolling_90days_beta
1.6
1.4
1.2
1.0
0.8
0.6
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020

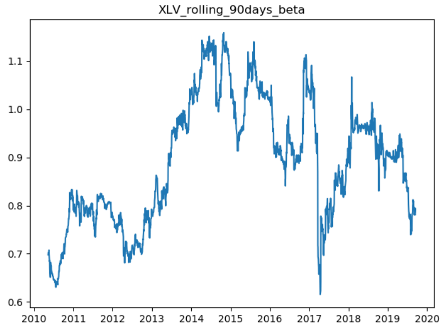

XLF_rolling_90days_beta
2.0
1.8
1.6
1.4
1.2
1.0
0.8
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020

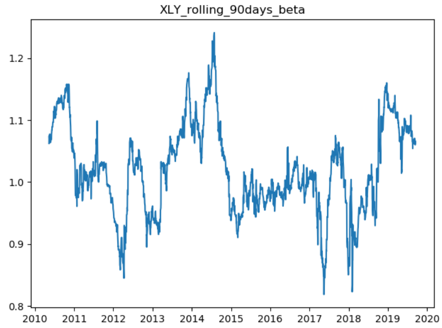

XLI_rolling_90days_beta
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020


XLK_rolling_90days_beta
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020


XLP_rolling_90days_beta
0.9
0.8
0.7
0.6
0.5
0.4
0.3
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020


XLU_rolling_90days_beta
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020





According to the formula listed below, Beta is somewhat related to correlation.

$$r=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}$$

$$\beta=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

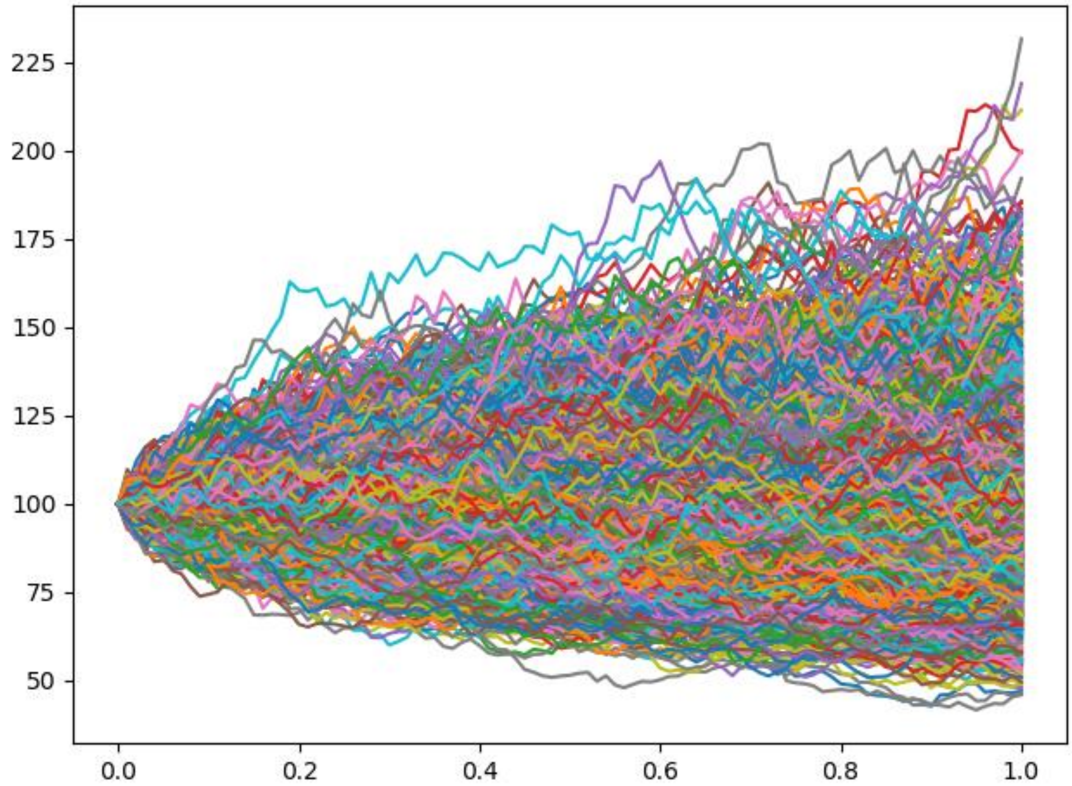
So if correlation is not stable across time, Beta is not consistent, either. And the absolute change of Beta may be higher than correlation. Since correlation only ranges from -1 to 1, while the range of Beta could be wider.

(f) The coefficient of the auto-correlation of each ETF are listed below, along with its corresponding p value. Then, obviously, we can conclude that there is no evidence of auto-correlation in the ETF universe.

Code	Alpha	P_Value
SPY	-1.21E-08	0.96432
XLB	-2.09E-08	0.95252
XLE	-3.57E-07	0.36323
XLF	3.59E-08	0.92063
XLI	-1.24E-07	0.69347
XLK	1.97E-07	0.52623
XLP	-1.17E-07	0.58888
XLU	9.18E-08	0.71784
XLV	-8.84E-08	0.74437
XLY	-1.25E-07	0.67454

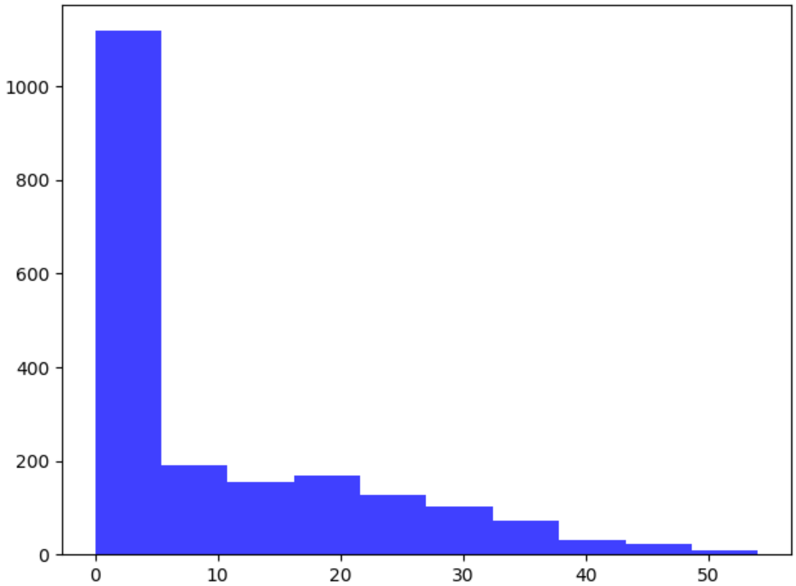
2. In my case, the setting of simulation are such that, the total time is 1, with steps = 100. So dt = Δt = 1/100. And simulation times = 10k.

(a)



Mean of the terminal value = **101.01545961521992**,
variance of the terminal value = **653.4027761174302**.

(b) The histogram of the payoffs for the European option with strike 100 is listed below,



Mean of the payoffs = **9.465678313295951,**
standard deviation of the payoffs = **12.416901995275815**

(c) By taking the average discounted payoffs across all paths, the price of European put option obtained via simulation is **9.465678313295951**.

(d) The price of European put option obtained via simulation is **9.465678313295951**.

The price obtained using the Black-Scholes formula is **9.94764496602258**

The difference is **0.4819666527266282**.

The difference between the two prices becomes small as the simulation times increases. As the simulation times is large enough, the simulated price will be very close to the formulaic price, the difference can be ignored.

(e) The average of the payoff of a fixed strike lookback put option with strike 100 along all simulated path is **16.866794406316064**

Since r = 0 in this case, so the simulation price for the lookback option by averaging the discounted payoffs is the same as the average of the payoff, **16.866794406316064**

(f) In this case, by calculating the difference between the lookback option and the price of European option, the premium that the buyer is charged for the extra optionality embedded in the lookback = **7.4011160930201125**.

According to the definition,

$$premium = e^{-r*t} * max(K - min(S_t), 0) - e^{-r*t} * max(K - S_T, 0)$$

If min(St) = 0, which means the price of underlying assets drops to zero before terminal time, the premium would be highest.

If ST = min(St), which means the price of underlying assets of any time t is higher than the price of terminal time, the premium would be lowest.

But the premium would never be negative, because min(St) can never be higher than ST, else ST would be new minimal price, which make the premium equal to zero.

(g) Using different sigma ranging from 0 to 1, we shall calculate different price of European option and lookback option, as well as the premium between them. From table below, we observe that, as sigma increases, each price and the premium increase corresponding. We can conclude, as the volatility becomes large, investors expect that the price of underlying asset may be more likely to change toward their beneficial side, so the price of option increases, and more premium of lookback option are requested.

Sigma	Euro_option_price	Lookback_option_price	Premium
0	0	0	0
0.05	1.891516	3.563861	1.672345
0.1	3.785067	7.031037	3.24597
0.15	5.678653	10.403156	4.724503
0.2	7.571507	13.680765	6.109258
0.25	9.465678	16.866794	7.401116
0.3	11.359743	19.962191	8.602449
0.35	13.251371	22.96732	9.715949
0.4	15.136814	25.883315	10.746502
0.45	17.017844	28.713015	11.695171
0.5	18.896251	31.459238	12.562988
0.55	20.765051	34.126371	13.36132
0.6	22.621444	36.713138	14.091694
0.65	24.468823	39.220006	14.751184
0.7	26.306546	41.6506	15.344054
0.75	28.134063	44.006411	15.872348
0.8	29.947247	46.284294	16.337047
0.85	31.737077	48.489581	16.752505
0.9	33.498741	50.624216	17.125475
0.95	35.241477	52.690223	17.448746