MF 803 Team Projet

RBF with Adapted Input in Portfolio Selection[1][2][3]

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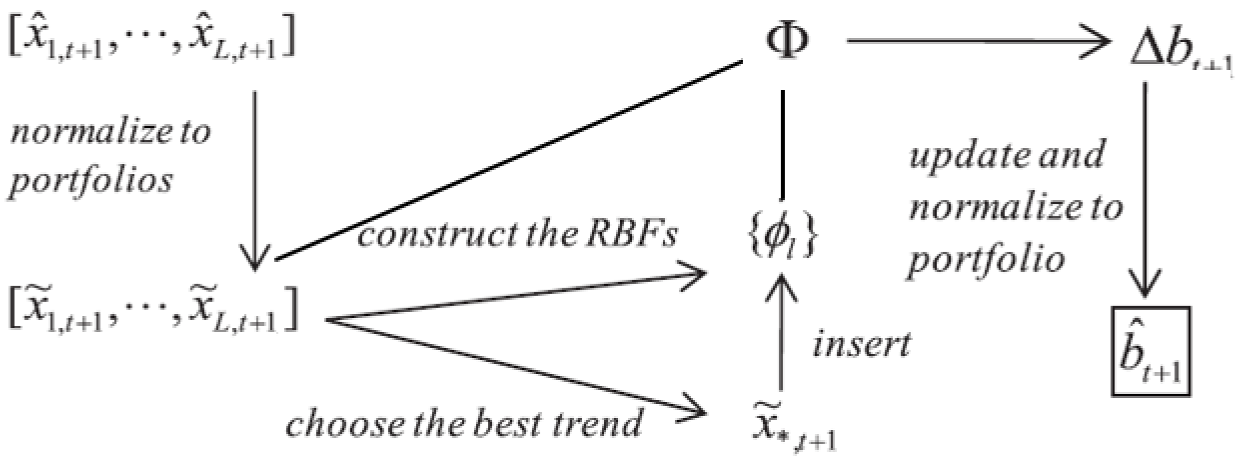
Contents

1. Introduction

Starting from daily price of SP 500 index, we use the price to construct several trend representation. Then we use projection method to normalize each trend representation to eligible portfolio, or say, weight of each composite stock in the light of each trend representation.

However, we have more than one trend representation, so are the eligible portfolios. There, we apply methods to integrate those portfolios and finally get one composite portfolio, the one we used to backtest the performance.

Figure 1.1: Framework of the Project



2. Data

As for the data, we download the data for the year from 2008 to the year 2018. In our case, we used the data downloaded from Yahoo Finance. Since our strategy picked stocks from the SP500 components, there are two types of data: one is the composition data which tells us for each specific month, which specific stocks were in the SP500 index; the other one is historical price data where we extracted the daily close price for further analysis.

As the fact that the SP500 components changes from time to time, we have to make some adjustment to it. In our case, we selected the overlapping component stocks over the last two months. For example, if today is March 15th, 2008, then our strategy will pick out the intersection part of the component stocks over January and February, 2008 as our targeted stocks.

The next step is to compute the trading representations for these targeted stocks. We chose four trading representations, namely EMA, SMA, PP, and LL.

1. For EMA, we set the initial values for all stocks as 1 and then used the updating rule:

$$\mathrm{EMA}_t = \alpha P_t + (1-\alpha) * EMA_{t-1}$$

2. For SMA, it is simple moving average of the past prices.

$$\mathrm{SMA}_t = \sum_{i=0}^{n-1} P_i/n$$

3. For PP, it is the highest price over the past window size days.

$$\mathrm{PP}_t = max_{t-n+1 \leq i \leq t} P_i$$

4. For LL, it is the lowest price over the past window size days.

$$\mathrm{LL}_t = min_{t-n+1 \leq i \leq t} P_i$$

3. Methods

Given parameters $\omega, \{\sigma_l^2\}_{l=1}^L$, compute the trend representations and and the actual price relatives in the recent time window$\{\hat{X}_{l,t-k},_{t-k}\}_{k=0}^{\omega-1}$, and the current portfolio $\hat{b}_t$.
First, Let $X_t \triangleq \frac{p_t}{p_{t-1}}, X_t \in \mathbb{R}_+^d, t = 1, 2, 3, \ldots$, then our methods are constructed in the following ways:

1. Compute the corresponding trend portfolios $\{\tilde{X}_{l,t-k}\}_{k=0}^{\omega-1}$ by

$$\tilde{X}_{l,t-k} = argmin_{x \in \Delta_d} ||X - \hat{X}_{l,t+1}||^2 \quad k = 1, \ldots, L$$

$$\mathrm{b}_t \in \Delta_d := \{b \in \mathbb{R}_+^d : \textstyle\sum_{i=1}^d b^{(i)} = 1\}$$

   Here we project these trend representations onto the constrained domain of x to get eligible portfolios approximating the trend representation. Here we use the Karush Kuhn Tucker condition to find the optimal solution of nonlinear programming.

2. We can use these portfolios in a recent time window to backtest the recent investing performance of each trend. Compute the recent increasing factors $\{R_{l,t-k'}\}_{k=0}^{\omega-1}$ of different trend representations by

$$\mathrm{R}_{l,t-k} = \tilde{X}_{l,t-k}^T X_{t-k} \quad k = 0, \ldots, \omega - 1$$

   to evaluate their recent investing performance.

3. Choose the common adaptive input $\tilde{X}_{*,t+1}$ by

$$* \triangleq argmax_{1 \le l \le L} min_{0 \le k \le \omega-1} R_{l,t-k}$$

   We first find the smallest increasing factor in the time window for each trend portfolio and choose the one with the largest increasing factor among different trend portfolios. By this way, we obtain the best performing trend representation in the worst financial environment, which improves the robustness of the whole system.

4. The $\tilde{X}_{*,t+1}$ is taken as adaptive input for the Radial Basis functions while the previous four portfolios are fixed as centers. Compute the radial basis functions $\{\phi_l\}_{l=1}^{L}$ by

$$\phi_l(\tilde{X}_{*,t+1}) = e^{\frac{-||\tilde{X}_{*,t+1} - \hat{X}_{l,t+1}||^2}{2\sigma_l^2}}$$

5. By using Radial Basis Functions, we measure the similarity between $\tilde{X}_{*,t+1}$ and each portfolio, producing a weight for each portfolio and then get the weighted sum as the output.

$$\hat{b}_{t+1} = \tilde{X}_{t+1}\phi(\tilde{X}_{*,t+1})$$

where the $\tilde{X}_{t+1} = [\tilde{X}_{1,t+1}, \ldots, \tilde{X}_{L,t+1}] \in \mathbb{R}^{d \times L}, \Phi = [\phi_1, \phi_2, \ldots, \phi_L]^T$

4. Empirical Results

From the previous construction, we can see that we do not have much parameters to tune. Since the algorithms are simple and intuitive, the only “selection" we made is the choices of trend representatives. Since our four trend representatives are simple and representing difference aspects of the trend, we do not have a training set to do the selection among trend representatives.

From the previous explanation, we have the period from 2008-2018 be the test set to do back test. It is well-known that the market changed quite a bit after the financial crisis, so the characteristics of the market after 2009 might be quite different from that of 2008-2009. Inspired by this idea, we separate the test set to 2 periods, first of which is 2008-2009, and the second of which is 2010-2018.

Figure 4.1: Results from 2008-2009

First we have different types of strategy. The “S” represents for our strategy, and the “EMA”, “SMA”, “PP”, “LL” represent for the single-factor special case of our strategy respectively, while the “bmk” stands for the equal weighted SP500 components strategy.

Figure 4.2: Results from 2010-2018

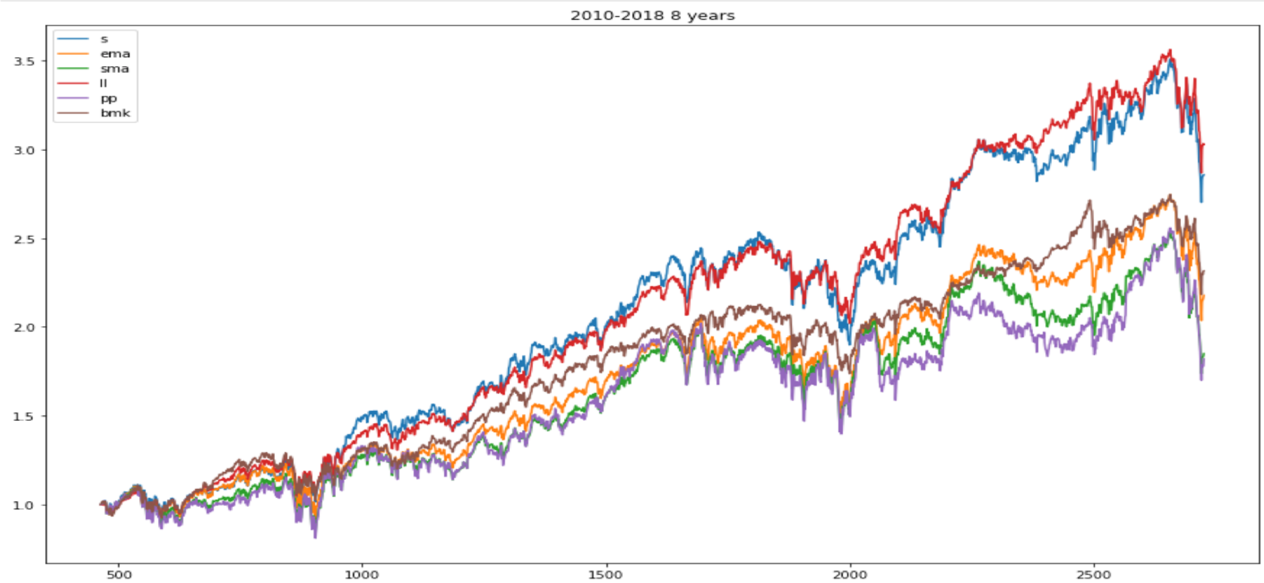


Table 4.1: A simple table

	Mean 2008-2009	IR 2008-2009	Mean 2010-2018	IR 2010-2018
S	14.7%	0.41	13.3%	0.75
EMA	7.3%	0.18	10.5%	0.55
SMA	5.3%	0.12	9.1%	0.43
LL	20.1%	0.60	13.6%	0.86
PP	12.6%	0.26	9.5%	0.40
bmk	7.7%	0.22	10.6%	0.67

From figure 4.1, when there is a turbulence in the market, our strategy outperforms the benchmark, and most of the single-factor models. This is also the case for bull-market period 2010-2018, which is shown in figure 4.2. It is concluded that our model is able to get adapted to the change of market, and stand a good performance comparing with the benchmark.

The result is also concluded in statistical form. Our model have a mean return of 14.7% in 2008-2009 and a mean return of 13.3% after 2009, while the benchmark have a mean return of 7.7% in 2008-2009 and a 10.6% after that. Our model is also standing out when it comes to risk return measure. Our model owns a 0.41 information ratio in 2008-2009, and a higher information ratio as 0.75 in 2010-2018, comparing to the 0.22 and 0.67 of the benchmark.

5. Conclusion

The advantage of our models is self-evident, it can absorb different trend representation, which represent different pattern we expect to find in the price. With the min-max method, we design a scheme to automatically choose an adaptive input with the best investing performance for the RBFs.

There is also some disadvantages. First, the input, that is, a good trend representation is hard to find. As you can see, the trend representation we use are not ideal, since most of time, their pattern are very similar. Besides, we have calculated our daily turnover broadly, which is 1.06. And under an assumption that transaction cost is 0.03%, the performance may be worse than our benchmark. But this drawback may be solved by adding restriction of turnover when we use projection method.

And the efficiency is also an unavoidable problem since everyday we need to identify the overlapping composite stocks. The two drawback may be mitigated by reducing the frequency to monthly or even longer period, which is worth trying.

Bibliography

[1] John Duchi, Shai Shalev-Shwartz, Yoram Singer, and Tushar Chandra. Efficient projections onto the 1-ball for learning in high dimensions. *Proceedings of the 25th International Conference on Machine Learning*, pages 272–279, 07 2008.

[2] Z. Lai, D. Dai, C. Ren, and K. Huang. A peak price tracking-based learning system for portfolio selection. *IEEE Transactions on Neural Networks and Learning Systems*, 29(7):2823–2832, July 2018.

[3] Z. Lai, D. Dai, C. Ren, and K. Huang. Radial basis functions with adaptive input and composite trend representation for portfolio selection. *IEEE Transactions on Neural Networks and Learning Systems*, 29(12):6214–6226, Dec 2018.