MF 803 Homework 4

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a. The price of a zero coupon bond can be calculated by

$$\mathrm{P} = \frac{principal}{(1 + yield)^{Maturity}}$$

Calculated prices of each zero coupon that pays $100 at maturity bond are listed below.

Maturity(years)	Yield	Price($)
1	0.025	97.561
2	0.026	94.996
3	0.027	92.3185
5	0.03	86.2609
10	0.035	70.8919
30	0.04	30.8319

As shown in the table, the zero coupon bond with maturity 1 and yield 2.5% has the highest price. Since the differences on yield between each zero coupon bond is not as significant as the differences on maturity. According to the pricing formula, the higher the maturity, the lower the bond price. So it is reasonable, the zero coupon bond with maturity 1, the smallest maturity, has the highest price.

b. By using finite differences, the duration of each zero coupon bond can be calculated by

$$\mathrm{Duration} = \frac{P_{\_} - P_{+}}{P_0 * (R_{+} - R_{\_})}$$

where P_—— the bond price when the yield drops by x basic points;

P+ —— the bond price when the yield rises by x basis points;

R_—— the yield minus x base points;

R+ —— the yield plus x base points;

P0 —— the initial price of the bond.

Calculated duration of each zero coupon bond are listed below.

Maturity(years)	Yield	Duration
1	0.025	0.97561
2	0.026	1.94932
3	0.027	2.92113
5	0.03	4.85437
10	0.035	9.66184
30	0.04	28.8462

According to the formula and the result, it's known that there is a negative relationship between bond price and bond yield. As the bond yield rises, the bond price would drop.

c. The price of a coupon bond can be calculated by

$$P = \sum_{1}^{Maturity} \frac{principal * coupon\_rate}{(1 + yield)^i} + \frac{principal}{(1 + yield)^{Maturity}}$$

Calculated prices of coupon bonds that pay $100 at maturity at 3% at annually until maturity are listed below.

Maturity(years)	Yield	Price($)
1	0.025	100.48
2	0.026	100.77
3	0.027	100.854
5	0.03	100
10	0.035	95.8417
30	0.04	82.708

Once we rewrite the pricing formula of a coupon bond, we have

$$P = \text{principal} * \frac{coupon\_rate}{yield} * (1 - \left(\frac{1}{1 + yield}\right)^{Maturity}) + \frac{principal}{(1 + yield)^{Maturity}}$$

Given coupon rate 3% and the principal $100, if the bond yield is less than coupon rate 3%, then the bond price will be above $100, the principal. Otherwise, the bond price will be below $100 once the bond yield is higher than coupon. And the conclusion is consistent with our calculation. Coupon bonds with yield less than 0.03 (Bond with maturity 1, 2, 3 in the table), their price are above $100. Coupon bonds with yield higher than 0.03 (Bond with maturity 10, 30 in the table), their price are below $100. Especially, the coupon bond with maturity 5, whose yield is equal to 0.03, its price is equal to $100.

d. The formula of calculating duration by finite differences has been shown above (see b.), and calculated duration of each coupon bond are listed below

Maturity(years)	Yield	Duration
1	0.025	0.97561
2	0.026	1.92104
3	0.027	2.83726
5	0.03	4.57971
10	0.035	8.45875
30	0.04	18.3688

Except for coupon bond and zero coupon bond whose maturities are 1 year (their duration are the same), other coupon bonds we calculated all have smaller duration than the zero coupon bonds with the same maturity and yield. Because duration can be explained as the weighted average maturity of cash flows, intuitively, coupon bonds with cash flows before maturity will has smaller weighted average maturity of cash flows than zero bonds, who only has cash flow at maturity.

e. By using finite differences, the convexity of each zero coupon bond can be calculated by

$$\text{Convexity} = \frac{P_{\_} + P_{+} - 2P_0}{P_0 * (R_{+} - R_{\_})^2}$$

Calculated duration of each zero-coupon bond and coupon bond are listed below

Maturity(years)	Yield	Zero-Coupon Bond Convexity	Coupon Bond Convexity
1	**0.025**	**1.90363**	**1.90363**
2	**0.026**	**5.69976**	**5.5895**
3	**0.027**	**11.3773**	**10.9423**
5	**0.03**	**28.2779**	**26.1524**
10	**0.035**	**102.686**	**85.8288**
30	**0.04**	**859.838**	**460.91**

As shown in that table, this second derivatives are positive,

f. The initial value of the portfolio is **-$0.112522**, means that we can earn $0.112522 at the beginning by long one unit of the 1 year zero-coupon bond, long one unit of the 3 year zero-coupon bond and short two units of the 2 year zero-coupon bond.

g. The duration of this portfolio is **48.6584**, while the convexity of this portfolio is **-1361.91**.
The duration has bigger quantity, since its value is positive.

h. If rates sell off by 100 basis points (each yield rises by 1%), the value of the portfolio would increase $0.0624134, and the new portfolio value would be **-$0.0501082**.

i. If rates rally by 100 basis points (each yield decrease by 1%), the value of the portfolio would decrease $ 0.047089, and the new portfolio value would be **-$0.159611**.
According to calculation in (h) and (i), the change in portfolio value would follow the change in yield, both change toward the same direction. And the absolute change in portfolio value under the scenario each yield rises by 1% is bigger than the absolute change when each yield decreases by 1%, so I would like to own the portfolio since it can hedge the risk of rising yield.

j. The cash flows of a 5-year amortizing bond that repays 20% of its principal annually and pays a 3% coupon (annually) are listed below.

cash flow	**23**	**22.4**	**21.8**	**21.2**	**20.6**
present value	**22.3301**	**21.1141**	**19.9501**	**18.8359**	**17.7697**

k. The price of the amortizing bond is **$100**, and the duration of the amortizing bond is **2.80195**. Compared to its zero coupon and coupon equivalents, the amortizing bond has smallest duration. And the zero coupon equivalents has biggest duration. As discussed above (see d.), duration measures the weighted average maturity of cash flows. Zero coupon equivalents only has one cash flow at maturity, so it is reasonable it has biggest duration. The amortizing bond and its coupon equivalents all have cash flows annually before maturity, and given repay rate, the principal of the amortizing bond has been paid earlier than its coupon equivalents, so the amortizing bond has smallest duration. And this characteristic is beneficial to bond investors, since buying an amortizing bond means investors may gain payment quickly before maturity than buying a zero coupon or coupon equivalent so that being less exposed to changing yield. However, the market is so fair that we can find the price of the amortizing bond is higher than its zero coupon equivalent. Even with the same price as its coupon bond equivalent, its coupon bond equivalent then promises more absolute value of coupon than the amortizing bond.