MF 803 Homework 5

Due: Friday, November 1st, 12:30 p.m.

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By definition, we have

$$L_{t_i} = \frac{1}{\delta}[e^{\int_{t_{i-1}}^{t_i} f ds} - 1], \mathrm{D}(0,t_i) = e^{-\int_0^{t_i} f ds}$$

So

$$\mathrm{S}(0,\mathrm{T}) = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0,t_i)}$$

Simplified, we have

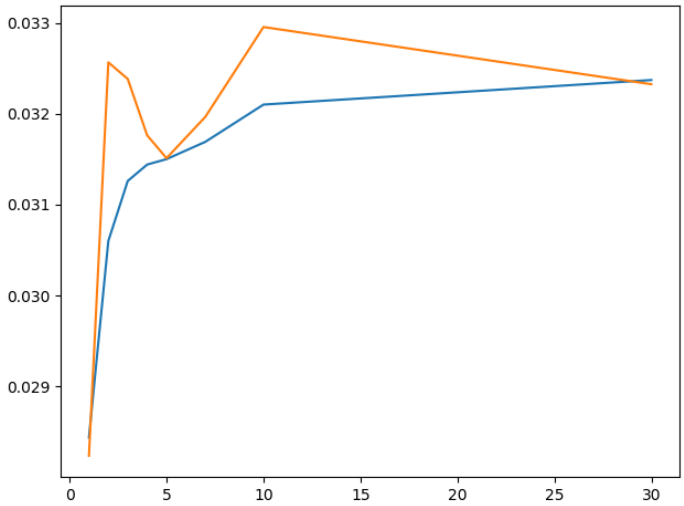
$$\mathrm{S}(0,\mathrm{T}) = \frac{1 - e^{-\int_0^T f ds}}{\delta_{t_i} \sum_{i=1}^{N} e^{-\int_0^{t_i} f ds}}$$

a. According to the formula above, set S(0,1) = 2.8438% as given, we can calculate the forward rate for the first year is **2.8238%**.

b. For $e^{-\int_0^2 f ds} = e^{-\int_0^1 f_1 ds} \cdot e^{-\int_1^2 f_2 ds}$, we set $f_1 = 2.8238\%$. Then modifying the formula above, we can calculate the forward rate from one year to two years is **3.2565%**.

c. Similarly, we continue the process in (b) and extract piecewise constant forward rates for the entire curve. The forward rates and the figure of the forward rates vs. the swap rates are shown below.

Period	Forward Rate
0->1	2.8238
1->2	3.2565
2->3	3.2382
3->4	3.1762
4->5	3.1511
5->7	3.1965
7->10	3.2953
10->30	3.2324



As shown from the figure, the forward rate curve here (the orange one) is above the swap rate curve (the blue one), this is because investors are afraid of rising rate in the future (which is adverse to the bond price), so investors ask for compensation, which reflected in the positive difference of the forward rate to the swap rate.

d. Assume the forward rate from 10 years to 15 years is the same as the forward rate from 10 years to 30 years, we can use the formula above to calculate the swap rate of a 15Y swap, which is **3.2237%**.

e. According to definition,

$$\mathrm{D}(0,t_i) = e^{-\int_0^{t_i} f ds} = D(0,t_1)\cdot D(t_1,t_2)\cdot \ldots \cdot D(t_{i-1},t_i)$$

and

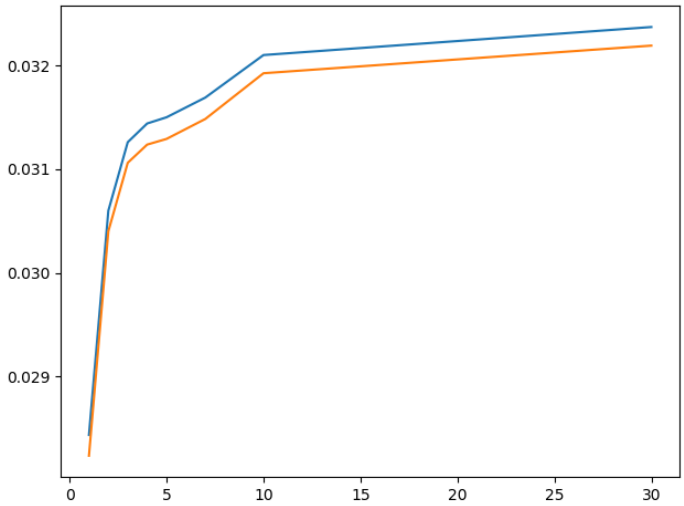
$$\mathrm{D}(0,t_i) = e^{-\int_0^{t_i} r_i ds}$$

while r_i is the zero rate by finding the constant rate that leads to the calibrated discount factors.

The calculated discount factors and zero rates are listed below,

Terms	Discount factor	Zero rate
(0,1)	0.9722	2.8238
(0,2)	0.9410	3.0401
(0,3)	0.9110	3.1061
(0,4)	0.8825	3.1237
(0,5)	0.8552	3.1291
(0,7)	0.8022	3.1484
(0,10)	0.7267	3.1925
(0,30)	0.3807	3.2191

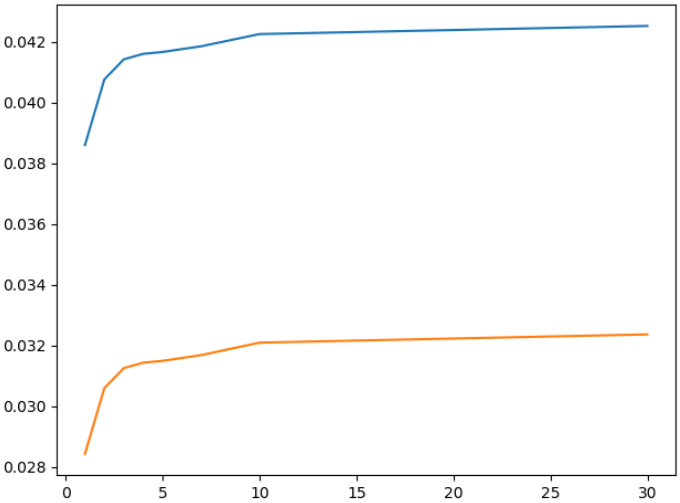
Plot the figure of the zero rates vs the swap rates,



As shown from the figure the zero rate curve is slightly below the swap rate curve. This similarity is reasonable, since the zero rate can be considered as some kinds of weighted average of forward rates.

f. The new swap rates calculated by using shifted forward rates up 100 basis points are listed below,

Terms	Former_swap_rate	New_swap_rate
(0,1)	**2.8438**	**3.8606**
(0,2)	**3.06**	**4.0767**
(0,3)	**3.126**	**4.1427**
(0,4)	**3.144**	**4.1609**
(0,5)	**3.15**	**4.1671**
(0,7)	**3.169**	**4.1860**
(0,10)	**3.21**	**4.2260**
(0,30)	**3.237**	**4.2528**



Obviously, these rates can be viewed as to having shifted the swap rates directly. Since the difference between new swap rates and the former ones is close to 100 basis points.

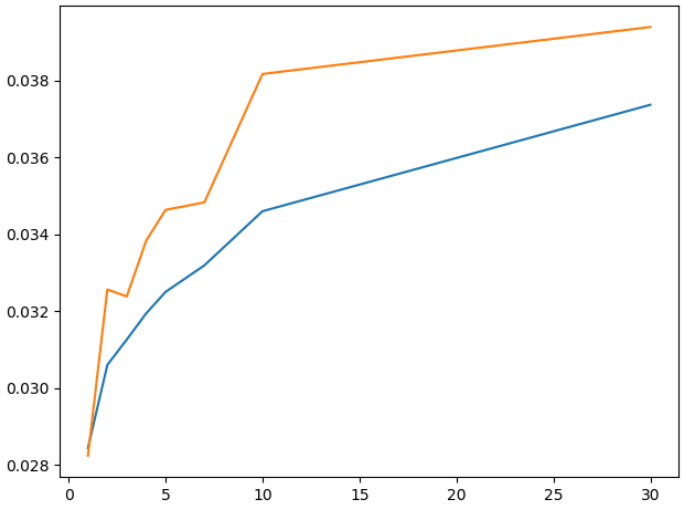
g. Under a bearish steepener, the new swap rates are listed below,

Terms	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 7)	(0, 10)	(0, 30)
swap_rate	2.8438	3.06	3.126	3.194	3.25	3.319	3.46	3.737

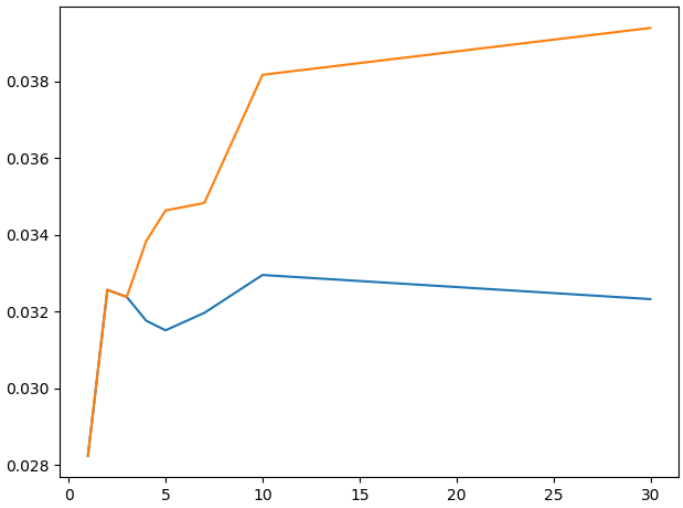
h. Under a bearish steepener, the new forward rates are listed below,

Terms	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 7)	(0, 10)	(0, 30)
forward_rate	2.8238	3.2565	3.2382	3.3831	3.4635	3.4830	3.8171	3.9393

Plot the new forward rates vs the new swap rates,



Plot the new forward rates vs the former forward rates,



As shown from the figure, the new forward rate curve (the orange one) is close to the former forward rate curve (the blue one) at the beginning, then the forward rate curve diverges as the terms become longer.

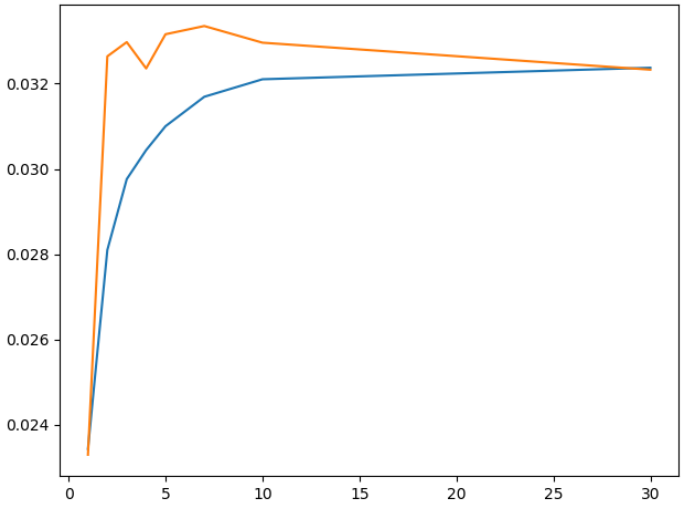
i. Under a bull steepener, the new swap rates are listed below,

Terms	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 7)	(0, 10)	(0, 30)
swap_rate	2.3438	2.8100	2.9760	3.0440	3.1000	3.1690	3.2100	3.2370

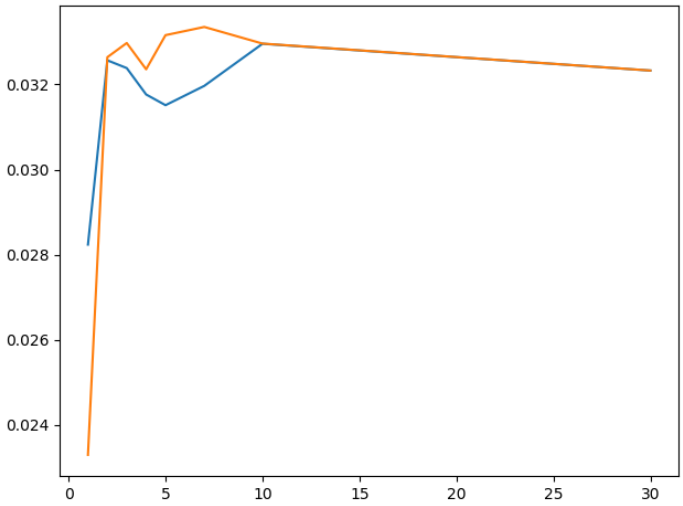
j. Under a bull steepener, the new forward rates are listed below,

Terms	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 7)	(0, 10)	(0, 30)
forward_rate	2.3302	3.2638	3.2970	3.2354	3.3155	3.3347	3.2957	3.2324

Plot the new forward rates vs the new swap rates,



Plot the new forward rates vs the former forward rates,



As shown from the figure, the new forward rate curve (the orange one) diverge from the former forward rate curve (the blue one) at the beginning, then the two curves converge as the terms become longer.