Problem set # 3

Due: Wednesday, February 12, by 8 am

Chang Deng U71725017 dengc@bu.edu

1. **Option Pricing via FFT Techniques** The Heston Model is defined by the following system of stochastic differential equations:

$$
\begin{aligned}
dS_t &= rS_t dt + \sqrt{v_t} S_t dW_t^1 \\
dv_t &= \kappa(\theta - v_t)\, dt + \sigma\sqrt{v_t} dW_t^2 \\
\operatorname{Cov}\left(dW_t^1, dW_t^2\right) &= \rho dt
\end{aligned}
$$

The characteristic function for the Heston Model is known to be:

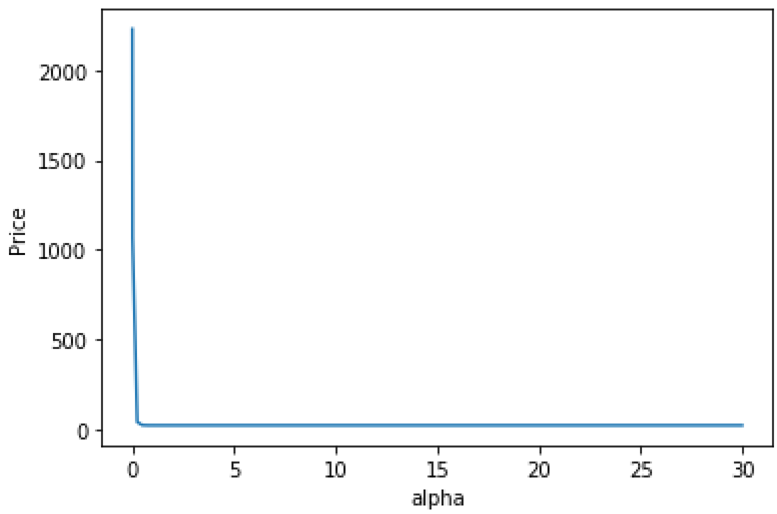
$$
\begin{aligned}
\omega(u) &= \frac{\exp\left(iu\ln S_0 + iu(r-q)t + \frac{\kappa\theta(\kappa - i\rho\sigma u)}{\sigma^2}\right)}{\left(\cosh\frac{\lambda t}{2} + \frac{\kappa - i\rho\sigma u}{\lambda}\sinh\frac{\lambda t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \\
\Phi(u) &= \omega(u)\exp\left(\frac{-\left(u^2+iu\right)v_0}{\lambda\coth\frac{\lambda t}{2} + \kappa - i\rho\sigma u}\right) \\
\lambda &= \sqrt{\sigma^2\left(u^2+iu\right) + (\kappa - i\rho\sigma u)^2}
\end{aligned}
$$

Assume the risk-free rate is 2%, the initial asset price is 250 and that the asset pays no dividends.

(a) **Exploring FFT Technique Parameters** Consider a European Call Option with strike 250 expiring in six months. Additionally, assume you know that the parameters of the Heston Model are:

$$
\begin{aligned}
\sigma &= 0.2 \\
v_0 &= 0.08 \\
\kappa &= 0.7 \\
\rho &= -0.4 \\
\theta &= 0.1
\end{aligned}
$$

i. Calculate the price of the European Call option with many values for the damping factor, α. What values of α seem to lead to the most stable price?



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{0.01: 2229.8687999438275, 0.02: 1066.4147076281254, 0.25: 39.65471098486569,
0.5: 22.447661289815688, 0.8: 21.31589507022012, 1: 21.274373625509067, 1.05:
21.272088364540355, 1.5: 21.26889305392571, 1.75: 21.268868982409007, 10:
21.268867212072806, 30: 21.2688671747742}
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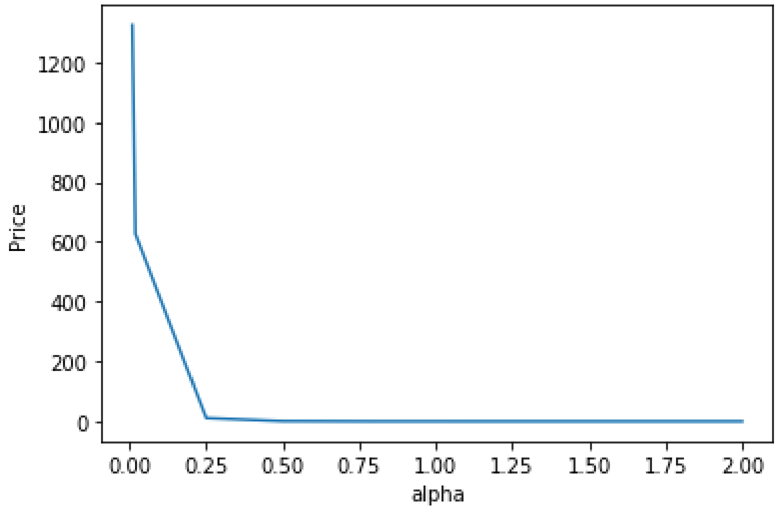
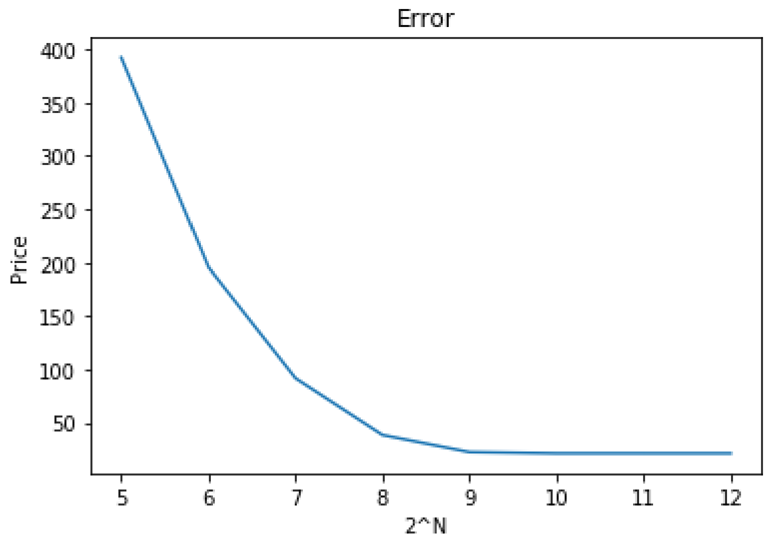
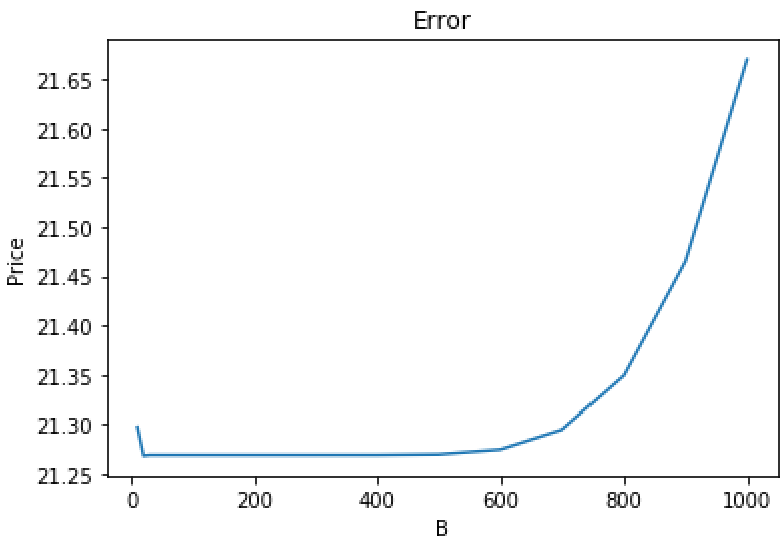


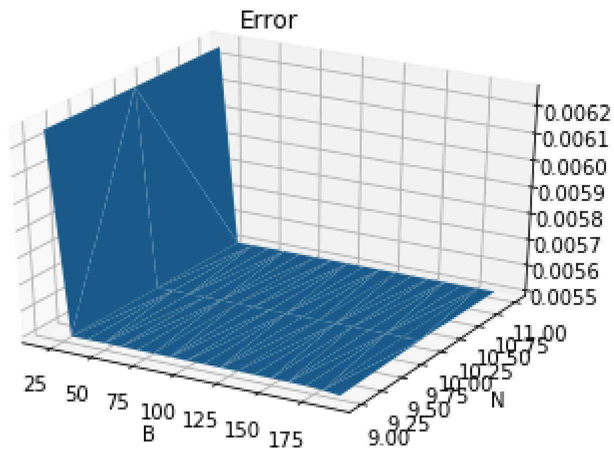
As shown above, α=1 seem to lead to the most stable price.

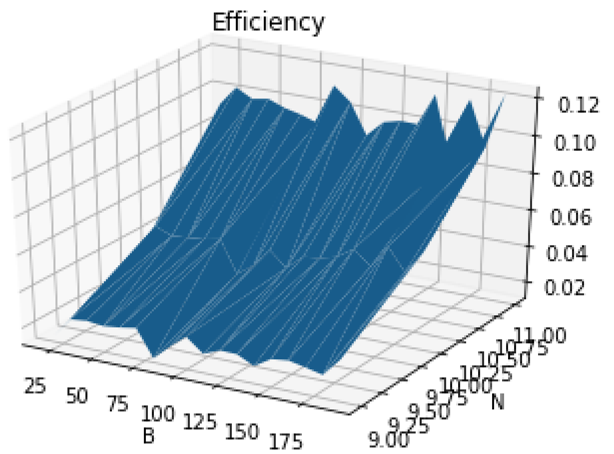
ii. Using the results above, choose a reasonable value of α and calculate the price of the same European Call with various values of N and Δk (or equivalently N and B). Comment on what values seem to lead to the most accurate prices, and the efficiency of each parameterization.

According to 1. i, we choose α=1 which leads to a stable price to calculate the price of the same European Call with various values of N and Δk (or equivalently N and B)



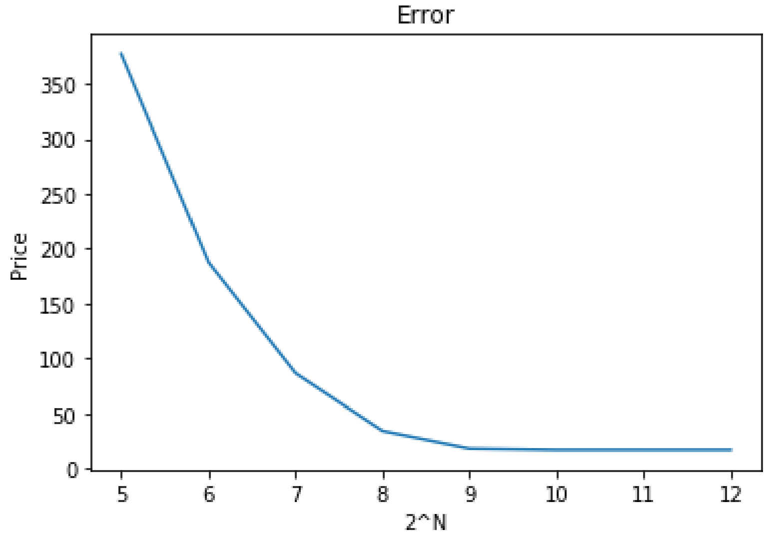


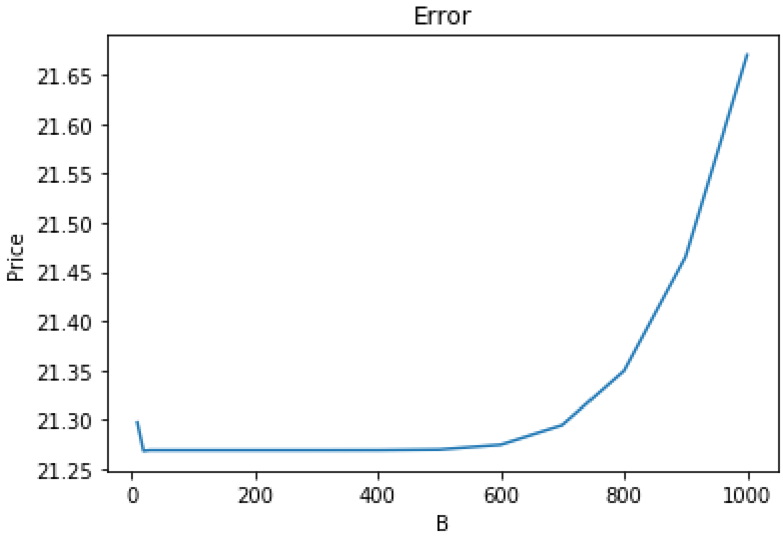


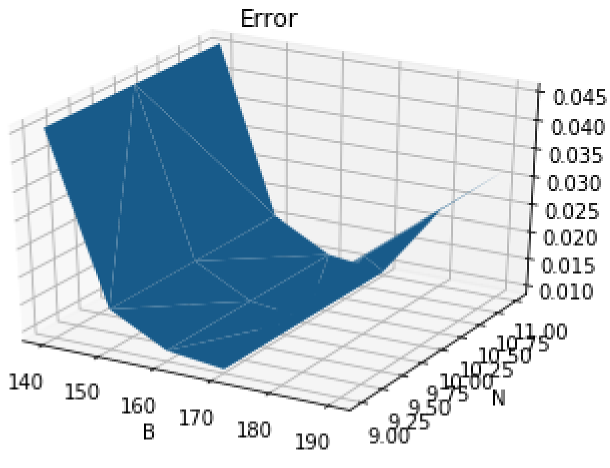


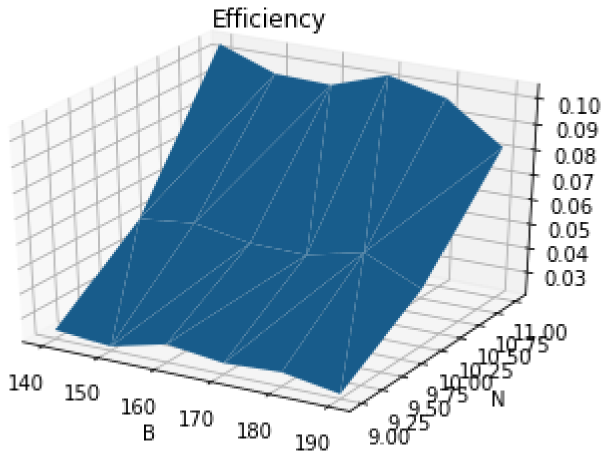
N=2^9,B=80 seem to lead to the most accurate prices under the consideration of the efficiency.

iii. Calculate the price of a European Call with strike 260 using various values of N and Δk (or N and B). Do the same sets of values for N,B and Δk produce the best results? Comment on any differences that arise.


Error
Price
2^N


Error
Price
B


Error
B
N

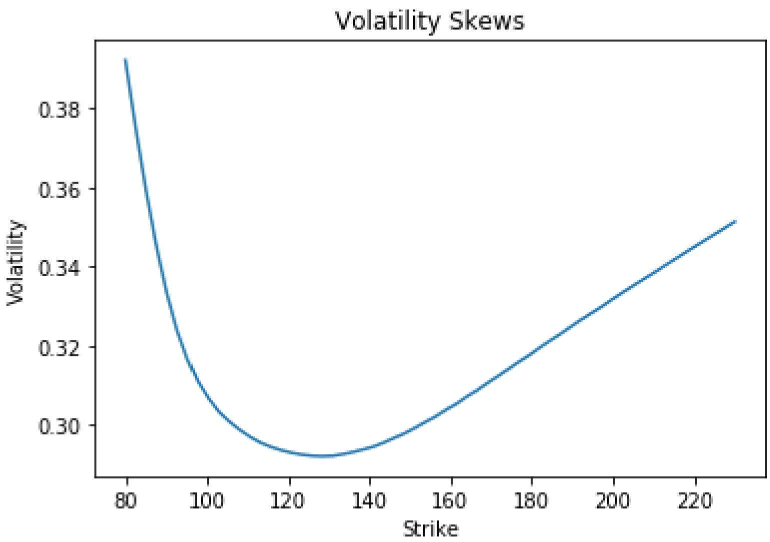


Given a European Call with strike 260, N=2^9,B=170 still seem to lead to the most accurate prices, and the efficiency of each parameterization.

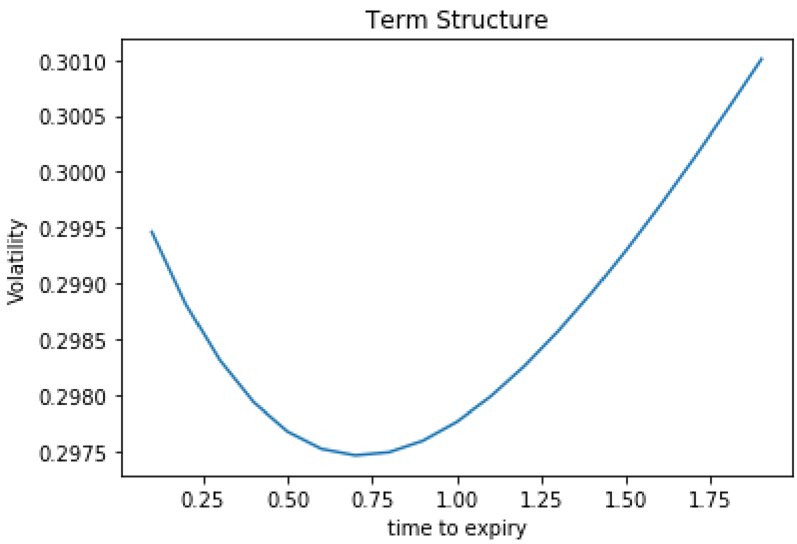
(b) **Exploring Heston Parameters** Assume the risk-free rate is 2.5%, the initial asset price is 150 and that the asset pays no dividends.

$$\begin{aligned} \sigma &= 0.4 \\ v_0 &= 0.09 \\ \kappa &= 0.5 \\ \rho &= 0.25 \\ \theta &= 0.12 \end{aligned}$$

i. Using these parameters, calculate Heston Model prices for three-month options at a range of strikes and extract the implied volatilities for each strike. Plot the implied volatility $\sigma(K)$ as a function of strike.

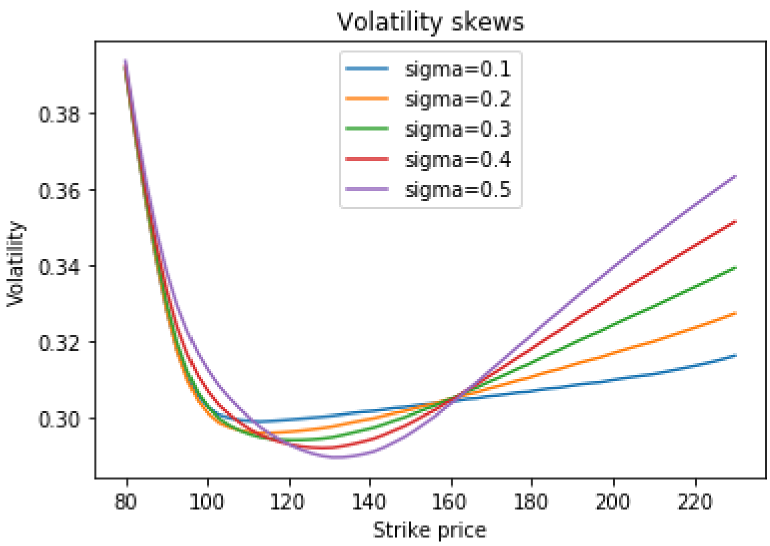


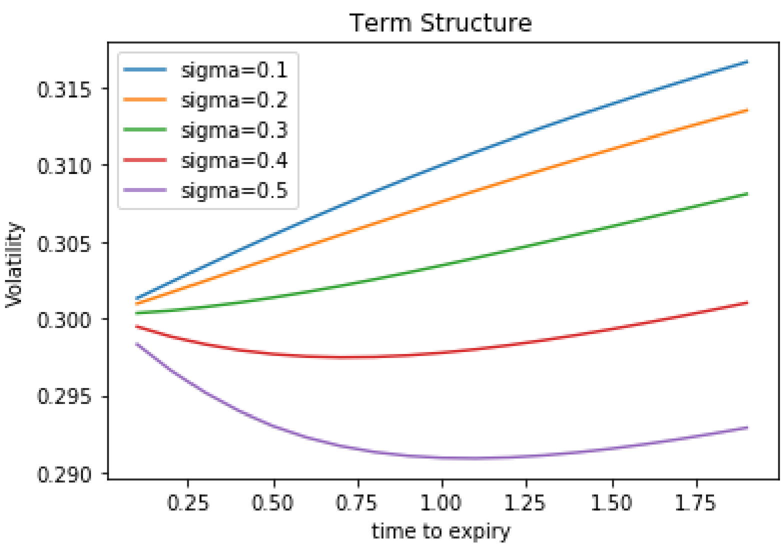
ii. Use the FFT pricing technique to obtain prices of 150 strike calls at many expiries. Extract the implied volatility for each and plot the term structure of volatility by plotting time to expiry on the x-axis and implied volatility on the y-axis.



iii. Holding all other parameters constant, vary each of the model parameters and plot the updated volatility skews and term structures. Comment on the impact that each parameter has on the skew and term structure.

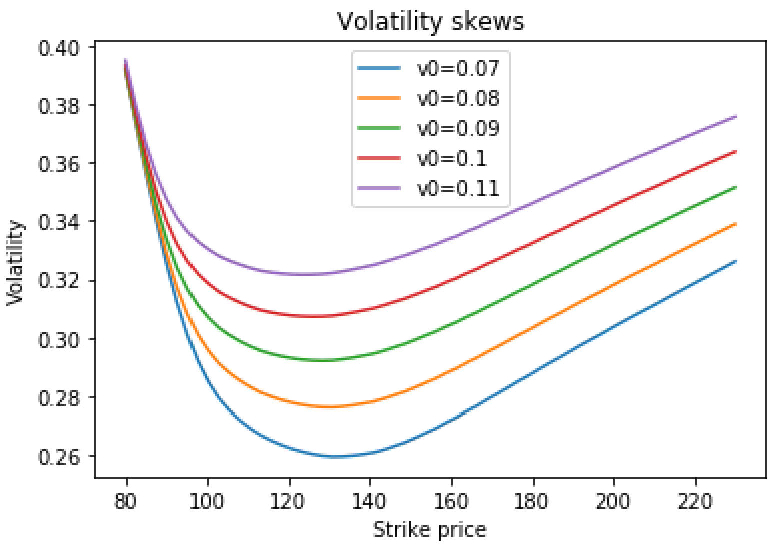
Change σ

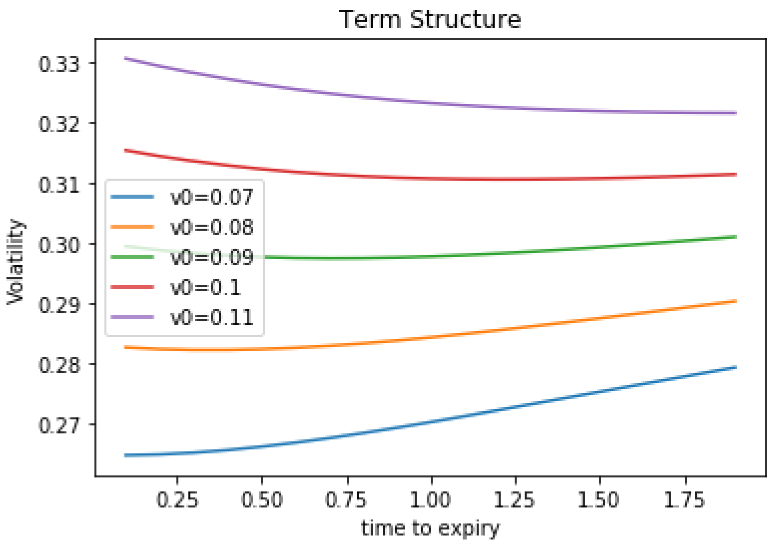




Increase of σ sharpen the volatility skews but may switch the term structures from a increasing function to a decreasing one

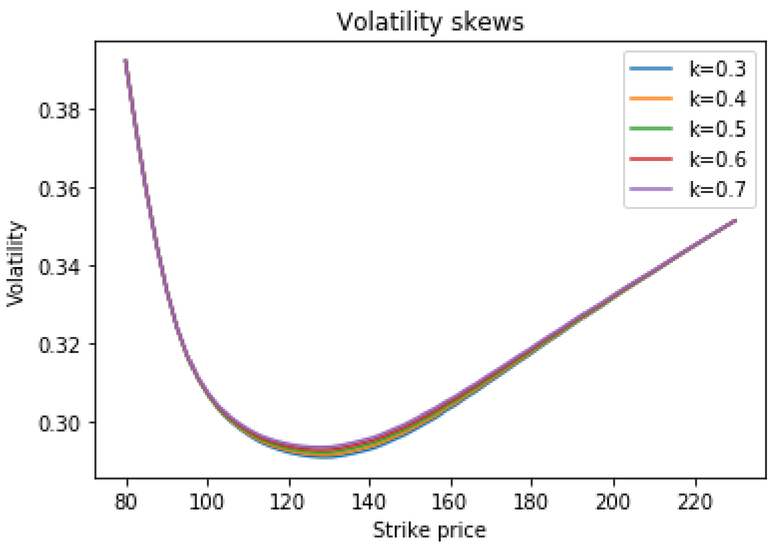
Change $v0$

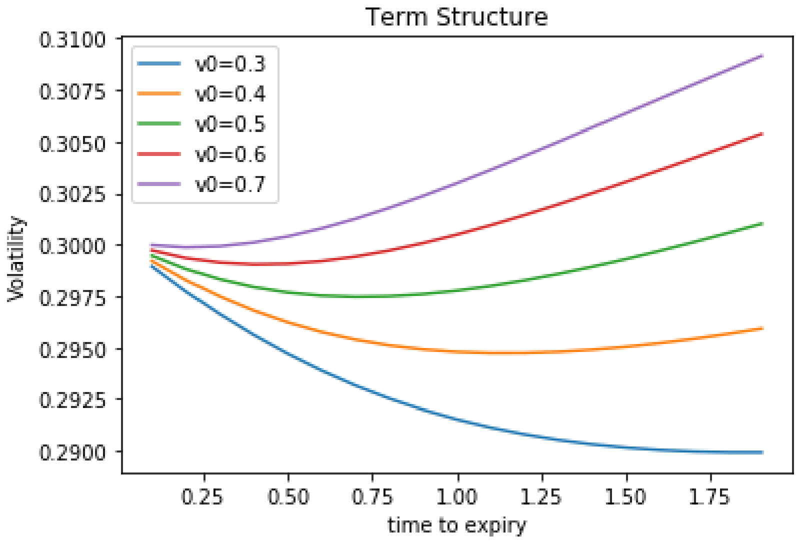




The decrease of $v0$ lead to sharper volatility skews and the term structures seem to switch from decreasing function to a increasing one with a low level of volatility.

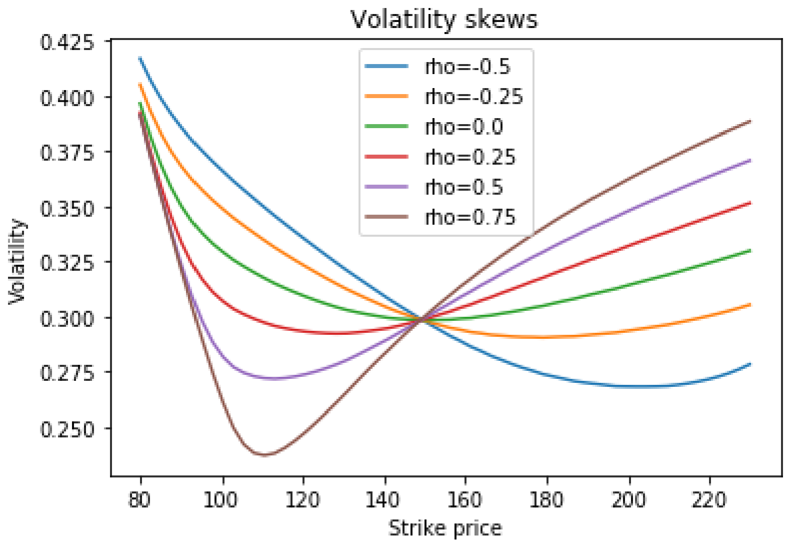
Change k

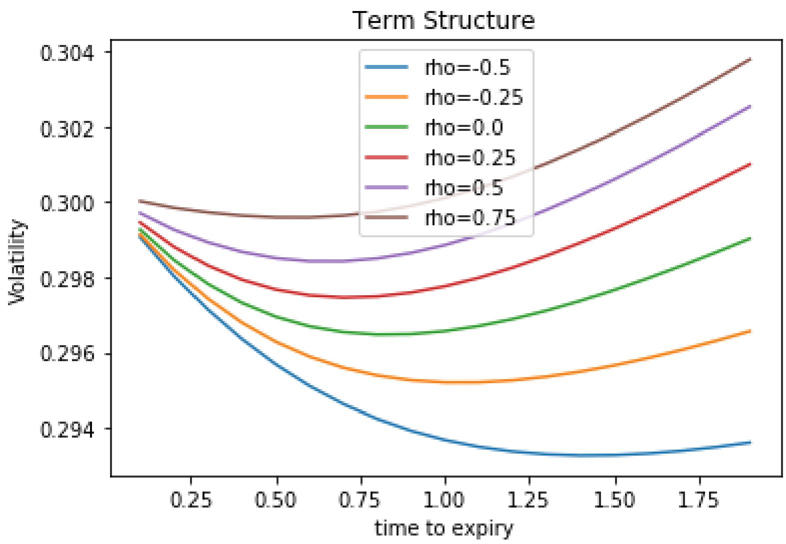




Change of k seems not to affect volatility skews, however increase of k may switch term structures from a decreasing function to a increasing one.

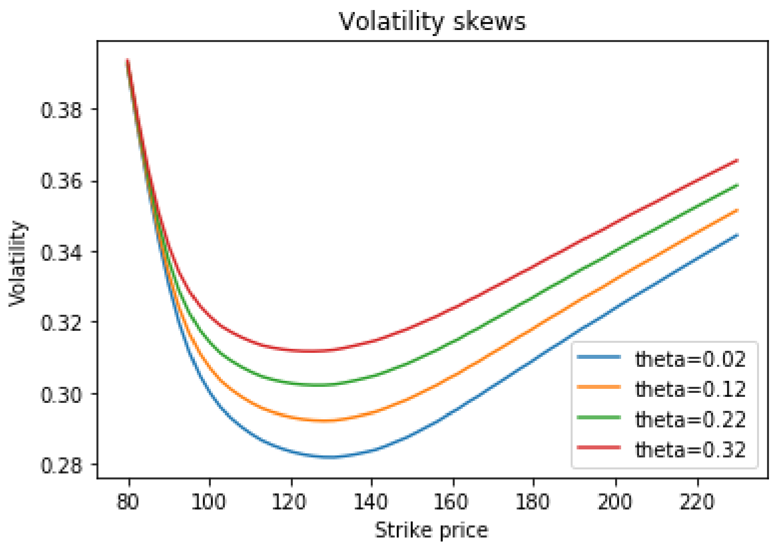
Change ρ

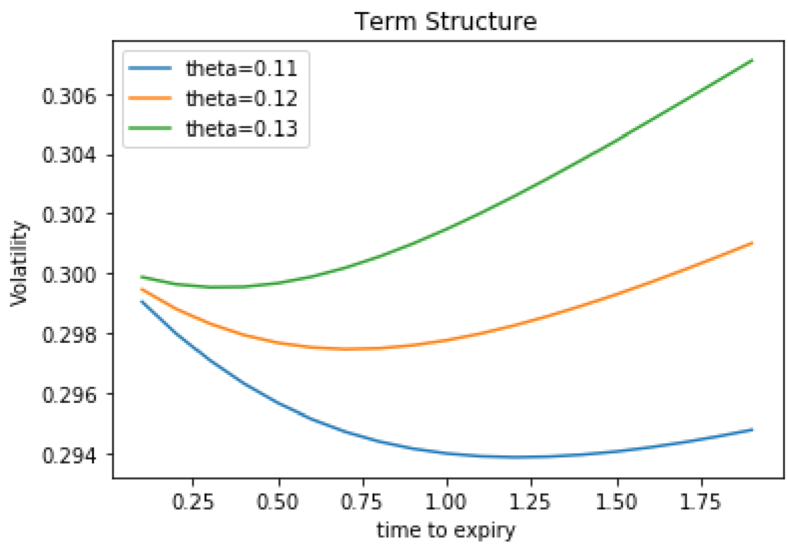




Increase of absolute value of ρ result in sharper volatility skews, increase of ρ may switch term structures from a decreasing function to a increasing one.

Change θ





Decrease of θ result in sharper volatility skews, increase of θ may switch term structures from a decreasing function to a increasing one.