MF 803 Homework 3

Due: Wednesday, October 2nd, 6:30 p.m.

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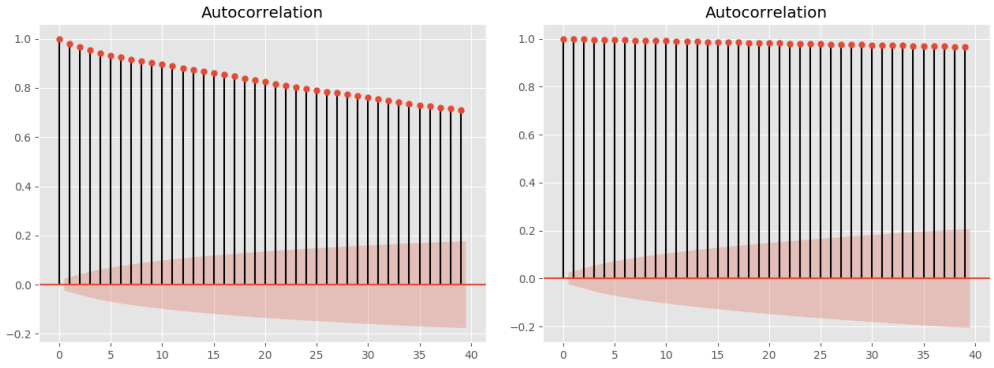
a. Download historical data for the S&P using the SPY ETF and for the VIX index (Date ranges from 1993-01-29 to 2019-09-24), after examining historical data (using the describe module in pandas, the result are shown below), no anomalies has been found.

	Volatility_proxy		Close
count	6712	count	6712
mean	19.237236	mean	135.29867
std	8.007809	std	61.252065
min	9.14	min	43.41
25%	13.39	25%	97.4475
50%	17.255	50%	124.965
75%	22.72	75%	153.575
max	80.86	max	302.01

b. Using the plot_acf module in statsmodels to examine the S&P and the VIX index data for autocorrelation. The left figure is the result of VIX index while the right is for the S&P. Roughly, we shall find there are indications of autocorrelation from the figures.



Using the acf module in statsmodels to calculate the alpha coefficient of S&P and the VIX index data for autocorrelation. Specifically, the lag = 1 in my examination and the results are showed below.

Code	Alpha	P_Value
VIX	0.98091989	0
SPY	0.99902479	0

the p-value of both autocorrelation model are all equal to zero, so the coefficient (alpha) is significant enough to refuse the hypothesis that there is no evidence of autocorrelation. So both series are auto-correlated, and the S&P series has more evidence of auto-correlation. This is consistent with our expectation, since the VIX index is constructed indirectly by the

price of derivatives using mathematical methods, while the S&P series represents the daily price of ETF traded in the market. So the S&P is expected to be more auto-correlated, according to the efficient-market hypothesis, trades expect historical information is contained in present price.

c. Tables of the correlation of the S&P and its implied volatility (using VIX as a proxy) on a daily and monthly basis are shown below.

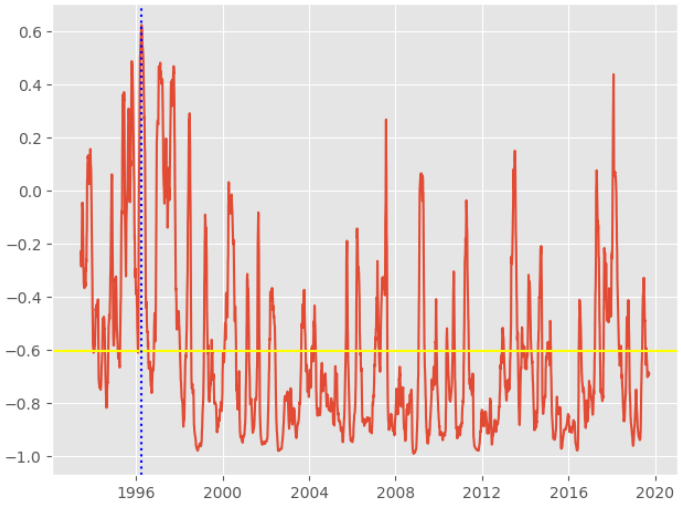
Daily_Cor	SPY	VIX
SPY	1	-0.221244
VIX	-0.221244	1

Monthly_Cor	SPY	VIX
SPY	1	-0.218628
VIX	-0.218628	1

	Correlation	P_Value
Daily	-0.22124355	3.29E-75
Monthly	-0.21862796	7.83E-05

From the table, the correlation of the S&P and its implied volatility (using VIX as a proxy) on a either daily or monthly basis is significant (as the P-value of Pearson Correlation Coefficient implies), which is negative. Which implies once the price of S&P or any assets fluctuates, its implied volatility changes, either. Then for any option pricing model with changing volatilities, such as the Black-Scholes model, the calculated prices will contain premium.

d. The figure of the rolling 90-day correlations of the S&P and its implied volatility is shown below, with the red horizon line indicating its long-run average.



At the early stage (specifically, the Date '1996-04-01'), the rolling correlation deviate the most from its long-run average, for the implied volatility has been deduced from the expected future price before the present price shows fluctuation.

e. The 90-day realized volatility is calculated in the light of the function below.

RealVol Daily Formula

Formula 1

$$Vol = 100 \bullet \sqrt{\frac{252}{n}\sum_{t=1}^{n} R_t^2}$$

Where:

Vol = Realized volatility
252 = a constant representing the approximate number of trading days in a year
t = a counter representing each trading day
n = number of trading days in the measurement time frame
Rt = continuously compounded daily returns as calculated by the formula:

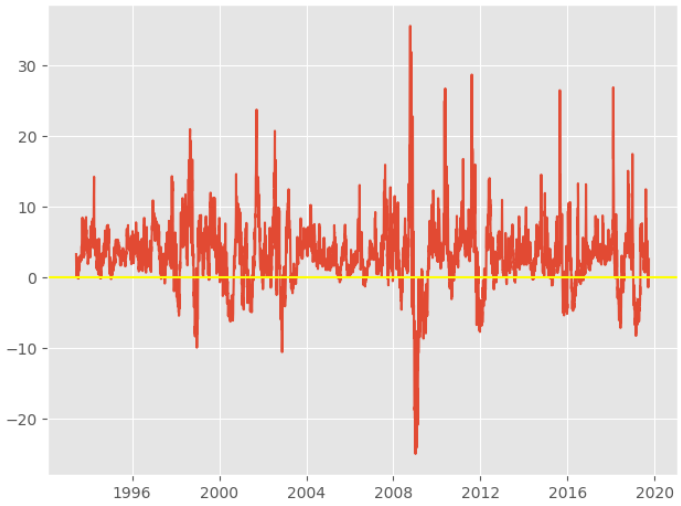
Formula 2

$$R_t = Ln\frac{P_t}{P_{t-1}}$$

Where:

Ln = natural logarithm
Pt = Underlying Reference Price ("closing price") at day t
Pt–1 = Underlying Reference Price at day immediately preceding day t

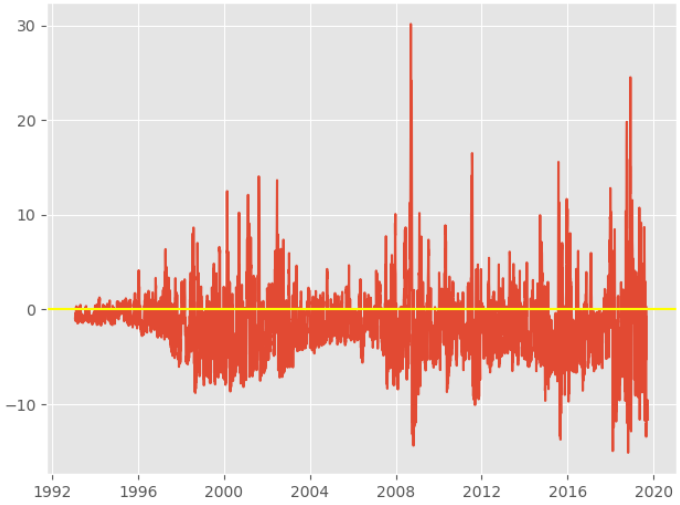
And the figure of the premium of implied volatility over realized volatility is shown below.



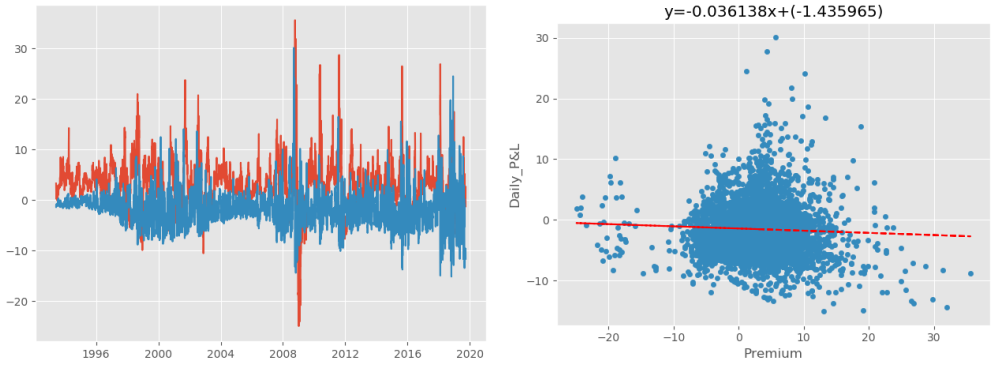
Since the trend line is always above the horizon line y = 0, so the premium is generally positive. The highest occurred during the period around 2008 when the financial market is seemingly booming. And the lowest premium occurred during the period around 2009, when the financial crisis just happened.

f. Construct a portfolio that buys a 1M at-the-money straddle (long an at-the-money call and long an at-the-money put) every day in my historical period. In my case, the historical period ranges from 1993 to 2019. The 1M implies 21 trading days, so the steps between ST and S0 for the option in my data set is 21.

g. In my case, the average P&L is **-1.534813646709434.** The figure of daily P&L is shown below.



h. The figures of daily P&L against the premium between implied and realized volatility are shown below.



From the figures, a strong relationship between the daily P&L and the premium can be observed. There is a negative relationship between the daily P&L and the premium, which is consistent with our expectation. For the straddle (the difference between the initial price and the strike price) is positively related to the realized volatility and the option prices are positively related to the implied volatility. So the P&L (=straddle – option prices) is negatively related to the premium (= implied volatility – realized volatility).