MF 796: Computational Methods of Mathematical Finance

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Problem set # 2

Due: Wednesday, February 5, 2020 by 8 am.

Problem 1: Evaluation of a known integral using various quadratures: In this problem we are going to compute the price of a European call option with 3 month expiry, strike 12, and implied vol 20, Assume the underlying is 10 now and the interest rate is 4%.

1. Use Black-Scholes formula to compute the price of the call analytically.
2. Calculate the price of the call numerically using the following 3 quadrature methods:
   (a) Left Riemann rule
   (b) Midpoint rule
   (c) Gauss nodes of your choice (say explicitly why you made that choice)

   with the number of nodes $N = 5, 10, 50, 100$ and compute the calculation error as a function of N for each of the methods.
3. Estimate the experimental rate of convergence of each method and compare it with the known theoretical estimate.
4. Which method is your favorite and why?

Problem 2: Calculation of Contingent Options: Let S_1 be a random variable that takes on the value of SPY one year from now and let S_2 take on the values of SPY 6 months from now. Assume that they are jointly normally distributed with

$$\begin{aligned} \sigma_1 &= 20 \\ \sigma_2 &= 15 \\ \rho &= 0.95 \end{aligned}$$

By ρ here we mean correlation between S_1 and S_2. Also, assume that interest rate is zero. Please specify where you got the current price of the underlying.

1. Evaluate the price of the one year call on SPY with the strike $K_1 = 370$. This is an example of a vanilla option.
2. Evaluate the price of the one year call on SPY with the strike $K_1 = 370$, contingent on SPY at 6 months being below 365. This is a contingent option.
3. Calculate the contingent option again, but with $\rho = 0.8$, $\rho = 0.5$, and $\rho = 0.2$.
4. Does dependence on ρ make sense?
5. Calculate the contingent option again, but with SPY at 6 months below $360, 350$, and 340.
6. Does the dependence on the 6 month value make sense?
7. Under what conditions do you think the price of the contingent option will equal the price of the vanilla one?