Problem set # 6

Due: Wednesday, March 18th, by 8am

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Problem 1: Numerical PDEs: Suppose that the underlying security SPY evolves according to standard geometric brownian motion. Then its derivatives obey the Black-Scholes equation:'

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial s} - rc = 0$$

Use SPY's closing price on March 4, 2020.

We are going to find the price of a call spread with the right of early exercise. The two strikes of the call spread are K1 = 315 and K2 = 320 and the expiry is September 30, 2020.

1. Explain why this instrument is not the same as being long an American call with strike 315 and short an American call with strike 320, both with expiry September 30, 2020.

   Short an American call offer your opponent the right of early exercise， which implies that the two options may not be early exercised simultaneously as the two options in the call spread with the right of early exercise.

2. For riskless rate r, use the 3-month US Treasury bill at the close of March 4, 2020. Say where you got the rate and why you consider it a reliable source.

   From the https://www.treasury.gov/, I got the 3-month US Treasury bill at the close of March 4, 2020 which is 0.72%. It is the official website of U.S. department of the treasury, so I consider it as a reliable source.

3. Let's assume that we are not able to find σ by calibrating to the European call spread price and must find it by other means. Find a way to pick the σ, explain why you chose this method, and then find the σ.

   Since we are not able to find σ by calibrating, I'd like to acquire the implied volatility traded in the market. From yahoo finance, I find a series of strike price and its implied volatility of SPY's option with expiry September 30, 2020. And I use linear interpolation to calculate the implied volatility of given strikes and finally decide to use the average implied volatility of K1 and K1 for following question, so the σ is 25.95875%.

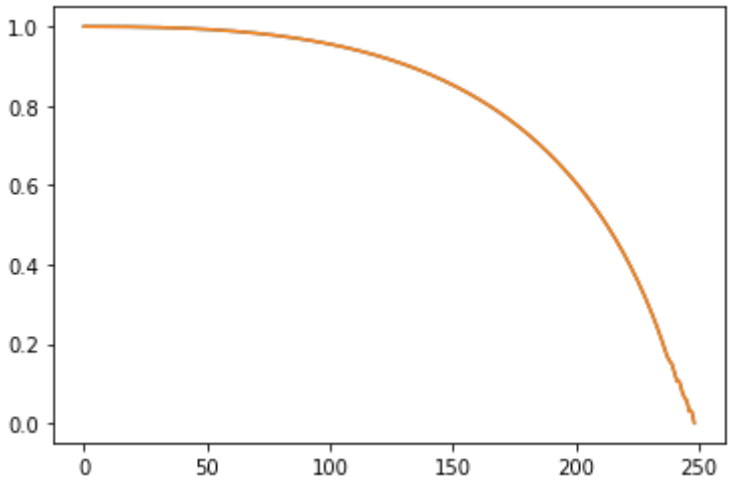
4. Set up an explicit Euler discretization of (1). You will need to make decisions about the choice of smax, hs, ht, etc. Please explain how you arrived at each of these choices.

   From yahoo finance I get the SPY's closing price on March 4, 2020 is 312.86, So I choose Smax to be 500, which is two sigma times above current price so as to be high enough to be the upper bound. And I first set hs =2 to arrive a relative accurate price of option then in order to satisfy the stability condition, which is

   $$h_t = O\left(\frac{1}{\sigma^2 s_{\max}^2}\right) h_s^2$$

   So I set ht = T/4000 where T in this case is about 7/12, seven months from March to September.

5. Let A be the update matrix that you created in the previous step. Find out its eigenvalues and check their absolute values.



The plot of ordered absolute eigenvalues of the update matrix A are shown above, since I have set the parameters to satisfy the stability condition so the eigenvalues should be between -1 and 1.

6. Apply your discretization scheme to find today's price of the call spread without the right of early exercise. The scheme will produce a whole vector of prices at time 0. Explain how you chose the one for today's price.

   After applying the discretization scheme to find today's price of the call spread without the right of early exercise, I got a series of underlying prices from 2 to 498 and the corresponding price of the call spread. Then I use linear interpolation to find today's price of the call spread without the right of early exercise given the SPY's closing price on March 4, 2020 (312.86), which is about 2.19.

7. Modify your code in the previous step to calculate the price of the call spread with the right of early exercise. What is the price?

   Similarly, I modify my code and go through the same procedure then get the price of the call spread with the right of early exercise given the SPY's closing price on March 4, 2020 (312.86), which is about 4.50.

8. Calculate the early exercise premium as the difference between the American and European call spreads. Is it reasonable?

   By calculating the early exercise premium as the difference between the American and European call spreads, the premium is about 2.31, which is reasonable since the American call spread gives the investors the right to exercise early so promises potentially higher payoff, sot the price of American call spread is more expensive.