MF 803: Advanced Programming for Mathematical Finance

Professor Chris Kelliher Fall 2019

Problem set # 4

Due: Wednesday, October 16th, by 6:30pm.

1. **Bond Pricing in C++:**

   Consider the following table of bond yields by maturity:

Maturity(years)	Yield
1	0.025
2	0.026
3	0.027
5	0.03
10	0.035
30	0.04

   Note: these are yield to maturities for each bond can be used for all its underlying cashflows. For example, for a 5-year coupon (or zero-coupon) bond, use a constant yield of 3% for all the bonds cashflows.

   (a) Calculate prices of a zero coupon bond that pays \$100 at maturity for each maturity & yield combination. Which price is the highest? Is this reasonable?

   (b) Calculate the duration of each zero coupon bond, or sensitivity of the bond price to a change in bond yield, using using finite differences. What is the relationship between bond prices and bond yields?

   (c) Calculate prices of coupon bonds that pay \$100 at maturity at 3% annually until maturity. Which prices are below \$100? Which prices are above? Why?

   (d) Calculate the duration of each coupon bond using finite differences. Do zero-coupon bonds or coupon bonds have higher duration? Why?

   (e) Calculate the second derivative of each bond price with respect to yield (commonly known as convexity). Are the second derivatives positive or negative?

   (f) Consider a portfolio that is long one unit of the 1 year zero-coupon bond, long one unit of the 3 year zero-coupon bond and short two units of the 2 year zero-coupon bond. Calculate the initial value of the portfolio.

   (g) Calculate the duration of this portfolio. Calculate the convexity of the portfolio as well. Which quantity is bigger?

(h) Suppose you own this portfolio and rates sell off by 100 basis points (each yield rises by 1%). What happens to the value of your portfolio?

(i) Now suppose you own this portfolio and rates rally by 100 basis points (each yield decreases by 1%). What happens to the value of your portfolio? Is this a portfolio you would want to own?

(j) Print the cashflows of a 5-year amortizing bond that repays 20% of its principal annually and pays a 3% coupon (annually).

(k) Calculate the price and duration of the amortizing bond using finite differences. Comment on the difference between this bond and its zero coupon and coupon equivalents.

NOTE: All code for completing these exercises should be completed in C++ and should be written generically. You may end up using this code on future assignments so I encourage you to code thoughtfully.