

MF 803 Homework 2

Due: Wed, September 25th, 6:30 p.m.

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1.

(a) historical data on the Fama-French factors from Ken French's website has been downloaded as a csv file. Using read_csv module in pandas to parse them into a Python dataframe. After examining historical price data of each ETFs (using describe module in pandas, the result are shown below), no anomalies has been found. So the historical data of Fama-French factors is clean and there is no need for extra cleaning.

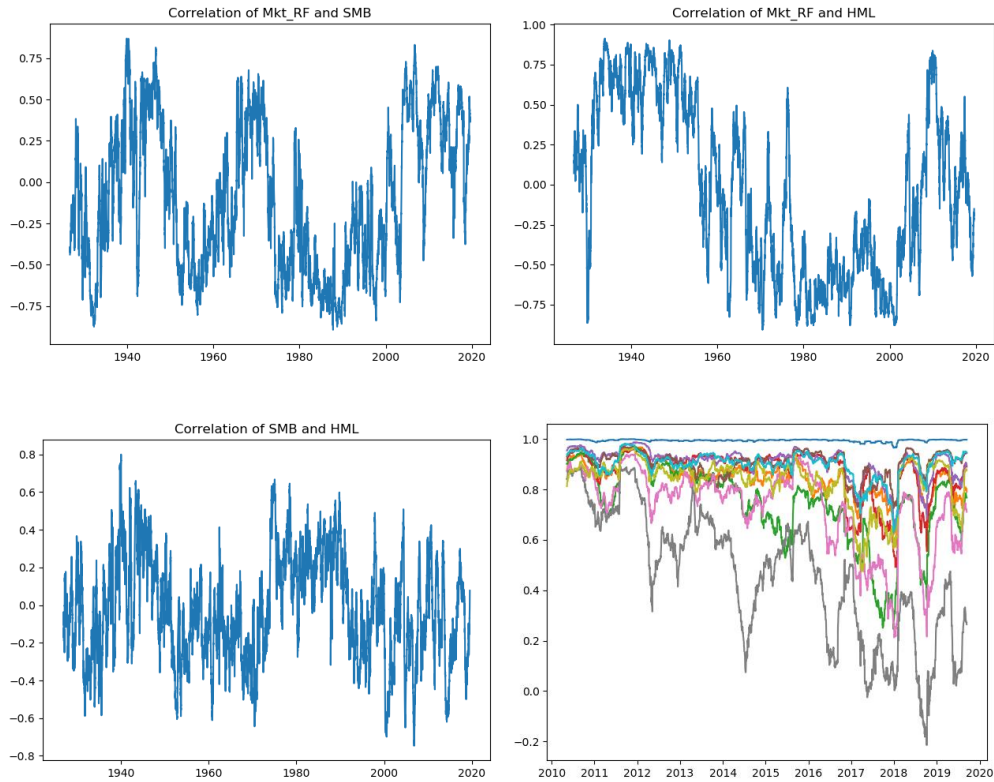
	Mkt_RF	SMB	HML
count	24535	24535	24535
mean	0.02936	0.00473	0.01528
std	1.06196	0.58316	0.5842
min	-17.44	-11.62	-5.98
25%	-0.4	-0.24	-0.24
50%	0.06	0.02	0
75%	0.49	0.27	0.26
max	15.76	8.21	8.43

(b) The daily covariance and correlation matrix of the factor returns over the entire time period are shown below.

	Mkt_RF	SMB	HML		Mkt_RF	SMB	HML
Mkt_RF	1.12777	-0.1077	0.10458	Mkt_RF	1	-0.1739	0.16858
SMB	-0.1077	0.34007	-0.0261	SMB	-0.1739	1	-0.0765
HML	0.10458	-0.0261	0.34129	HML	0.16858	-0.0765	1

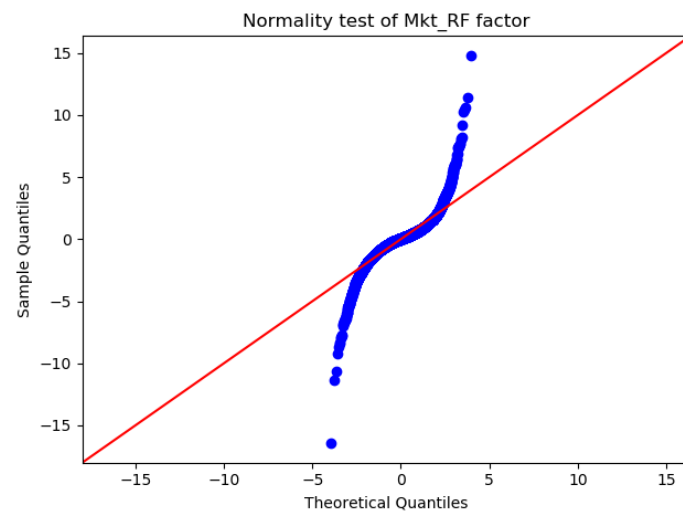
Compared to the correlations of the sector ETFs (shown below), factor returns is less correlated to each other, which is consistent with intuition, as each factor return has been Schmidt orthogonalized.

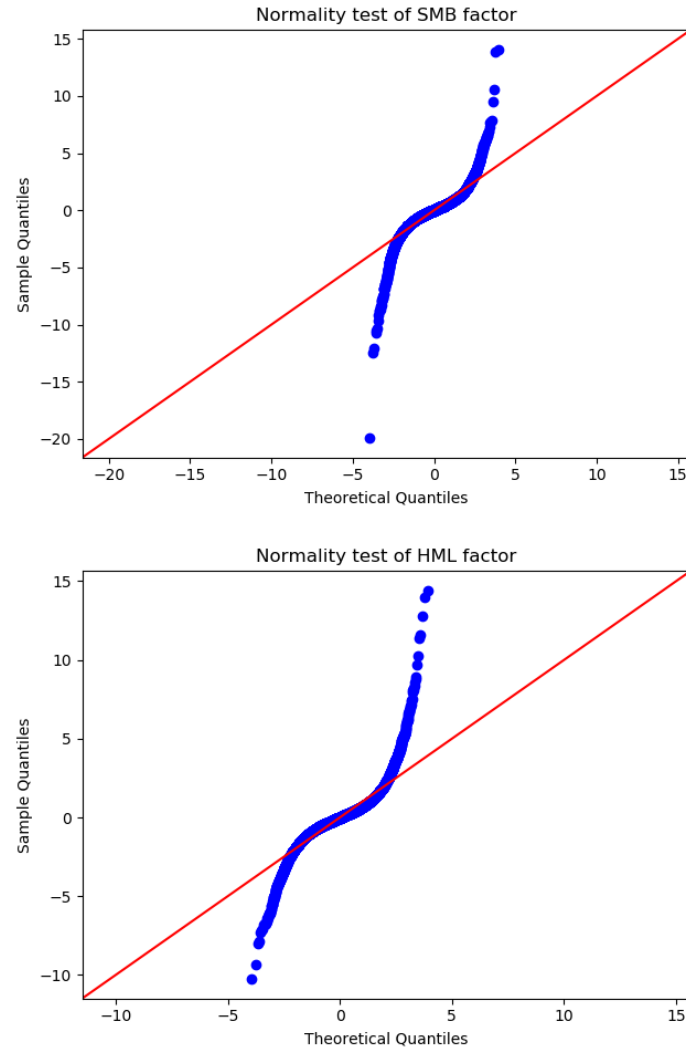
(c) The figures of rolling 90-day correlations for the factor returns, as well as the rolling 90-day correlations of the ETFs to S&P index are shown below.



As seen from the absolute change of correlation, the rolling 90-day correlations for the factor returns is relatively stable over time. Especially compared to the correlations of the ETFs from HW1.

(d) Applying qqplot module in statsmodels to check the factor returns for normality, the result are shown below.



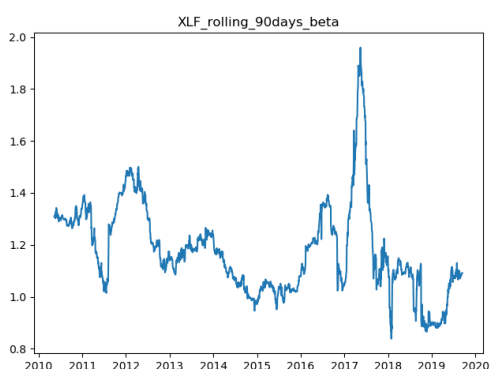
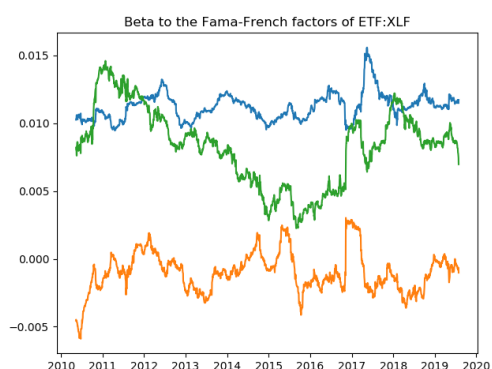
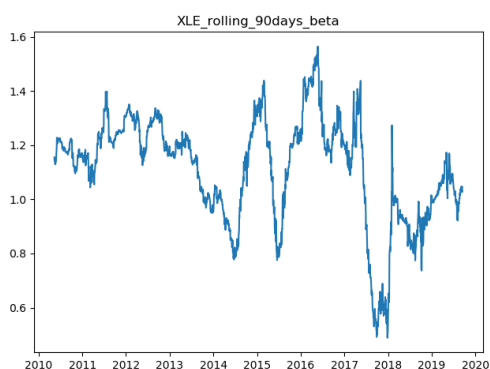
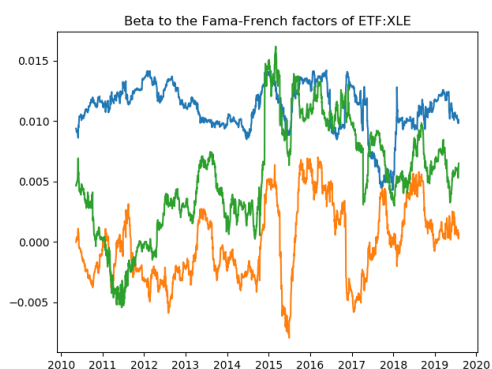
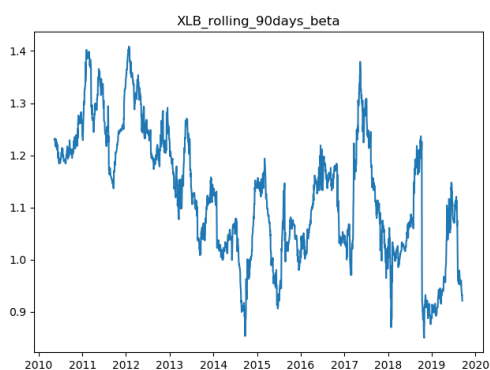
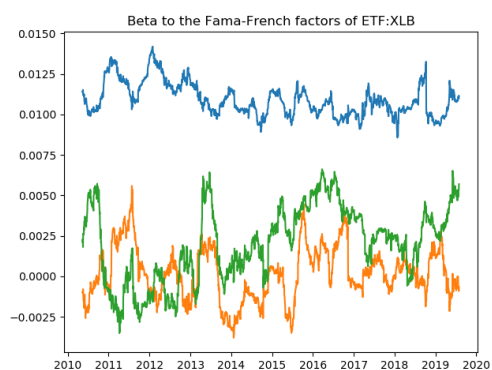
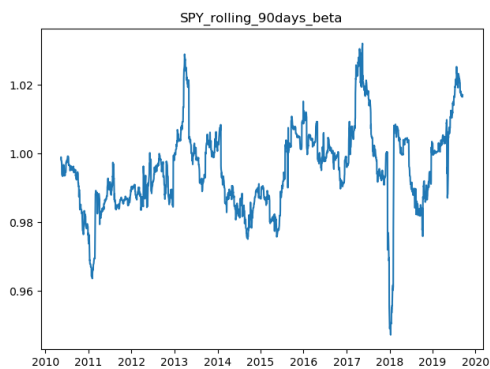
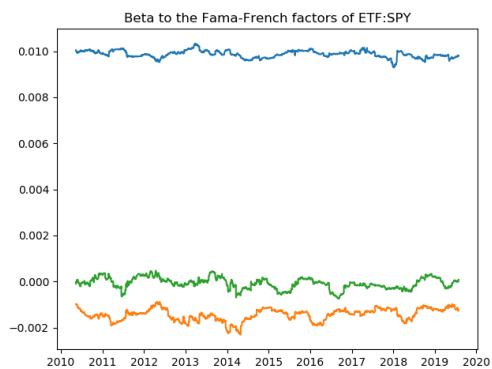


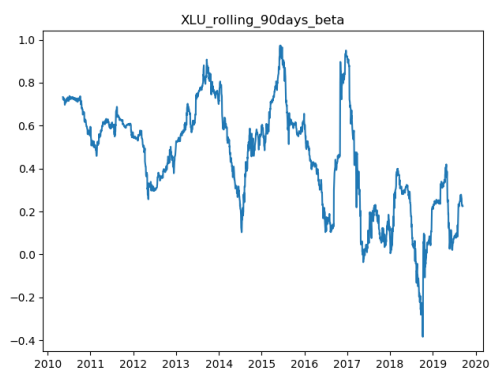
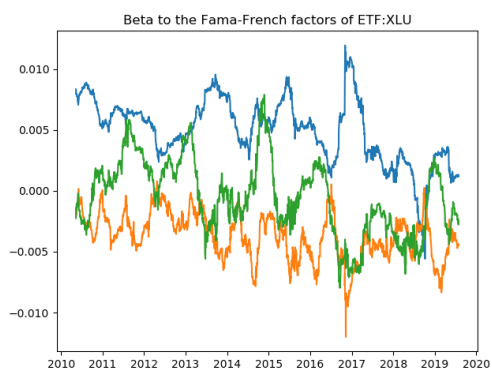
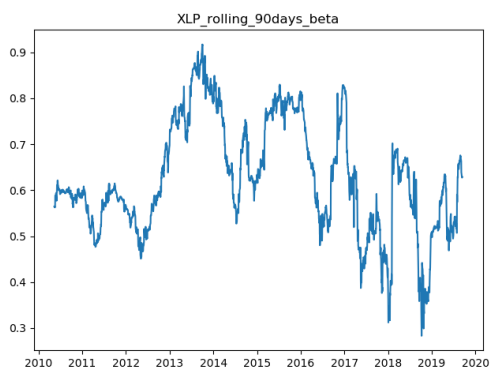
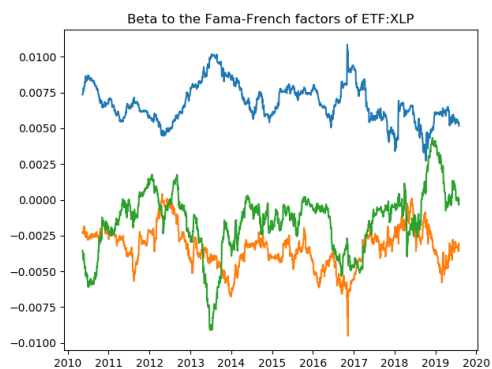
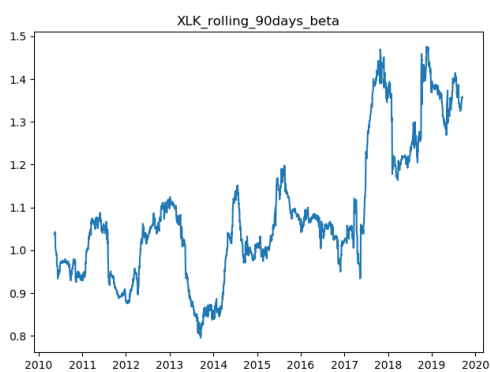
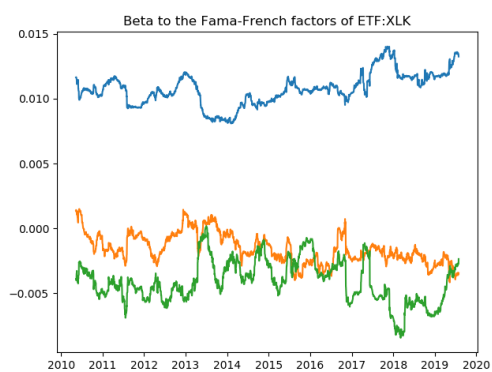
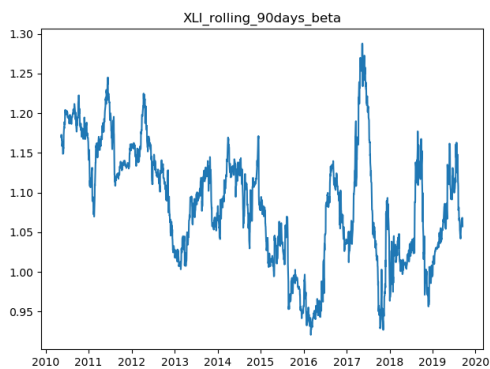
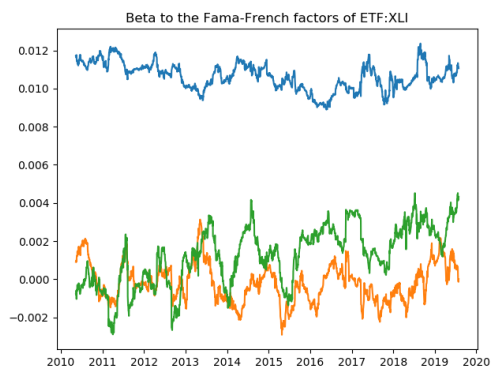
Obviously, the factor returns do not appear normal, or almost every point of factor returns should stick to the slope.

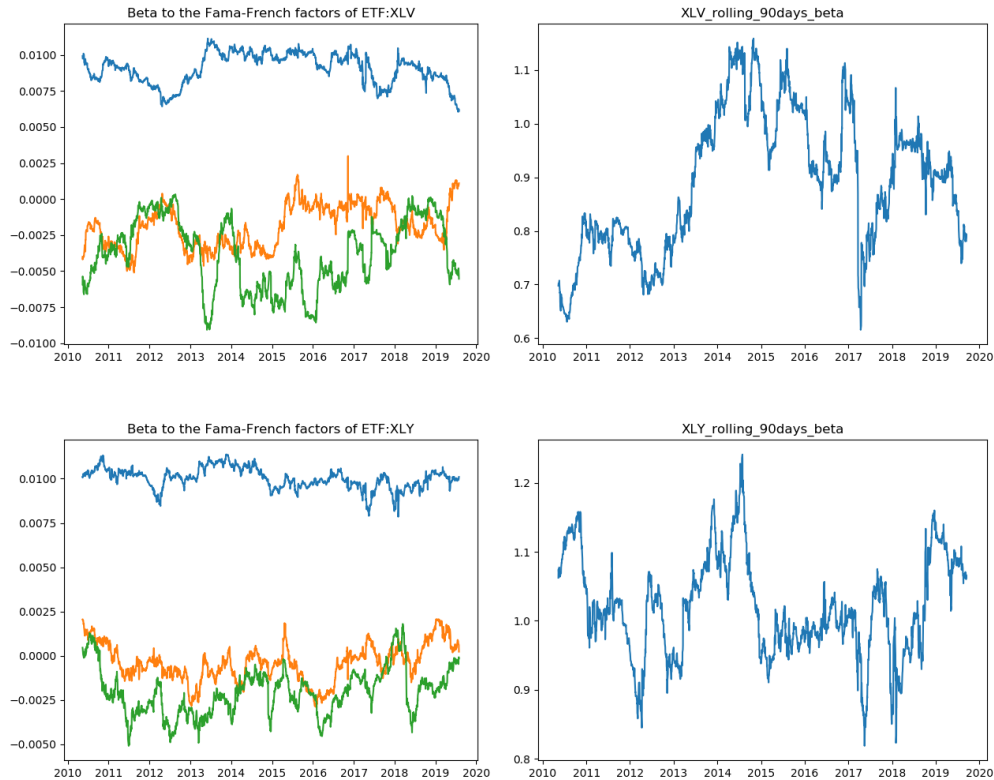
(e) Table below are the β 's of the 3-factor Fama-French model for the sector ETFs.

Code	Beta_Mkt_RF	Beta_SMB	Beta_HML
SPY	0.009877	-0.001417	-0.000007
XLB	0.010937	0.000459	0.00282
XLE	0.011047	-0.000051	0.005681
XLF	0.011358	-0.000679	0.007958
XLI	0.010519	0.000099	0.001668
XLK	0.010747	-0.001854	-0.004315
XLP	0.006475	-0.003351	-0.000771
XLU	0.005281	-0.003432	0.000365
XLV	0.008826	-0.001896	-0.0036
XLY	0.010081	-0.000125	-0.001735

And the figures of the rolling 90-day β 's of the 3-factor Fama-French model for the sector ETFs and the rolling 90-day beta of each sector ETF with to the market are shown below.





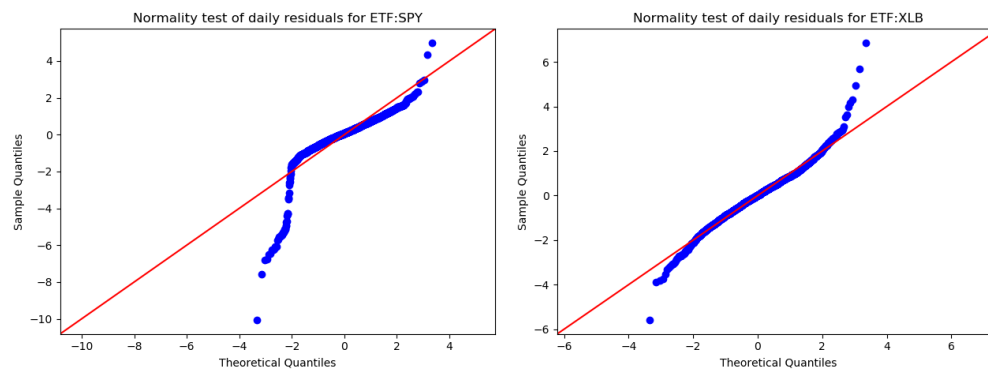


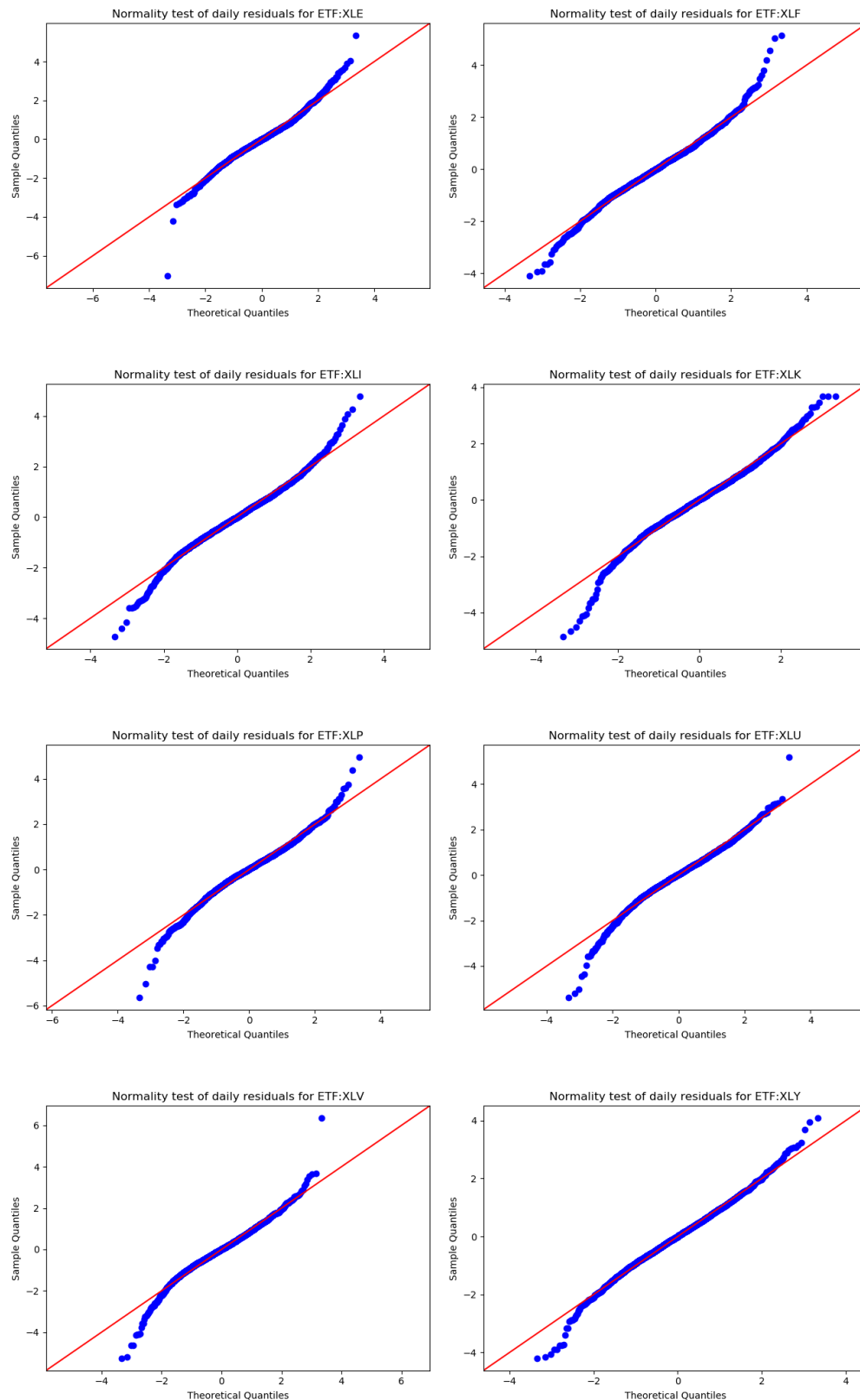
From these graphs, we can see the rolling 90-day β 's of the 3-factor Fama-French model for the sector ETFs are more consistent than the rolling 90-day beta of each sector ETF with to the market.

(f) The mean and variance of the daily residuals for each sector ETF are shown below.

Code	E_Mean	E_std
SPY	8.39E-19	0.000882
XLB	1.13E-18	0.0056
XLE	-6.63E-20	0.007707
XLF	2.37E-18	0.003841
XLI	-9.63E-19	0.003823
XLK	1.76E-19	0.003718
XLP	2.53E-19	0.004688
XLU	3.94E-19	0.007432
XLV	4.11E-19	0.004627
XLY	-5.04E-21	0.003695

Using qqplot to check the daily residuals for normality,





Apparently, the daily residuals fail to be normal. the 3-factor Fama-French model can explain part of the return of each ETF, but can not perfectly attribute the return to factor returns, or the residuals should appear to be normal.

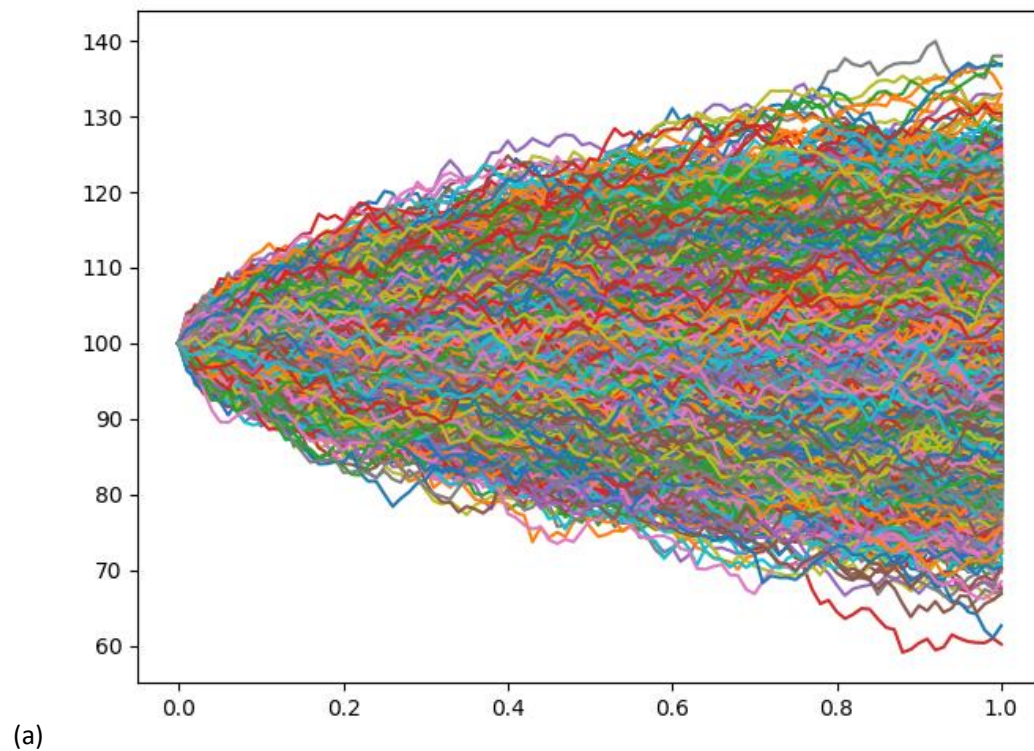
Besides, I use the auto-regression model to test whether the residuals will be auto-correlated.

The result is listed below.

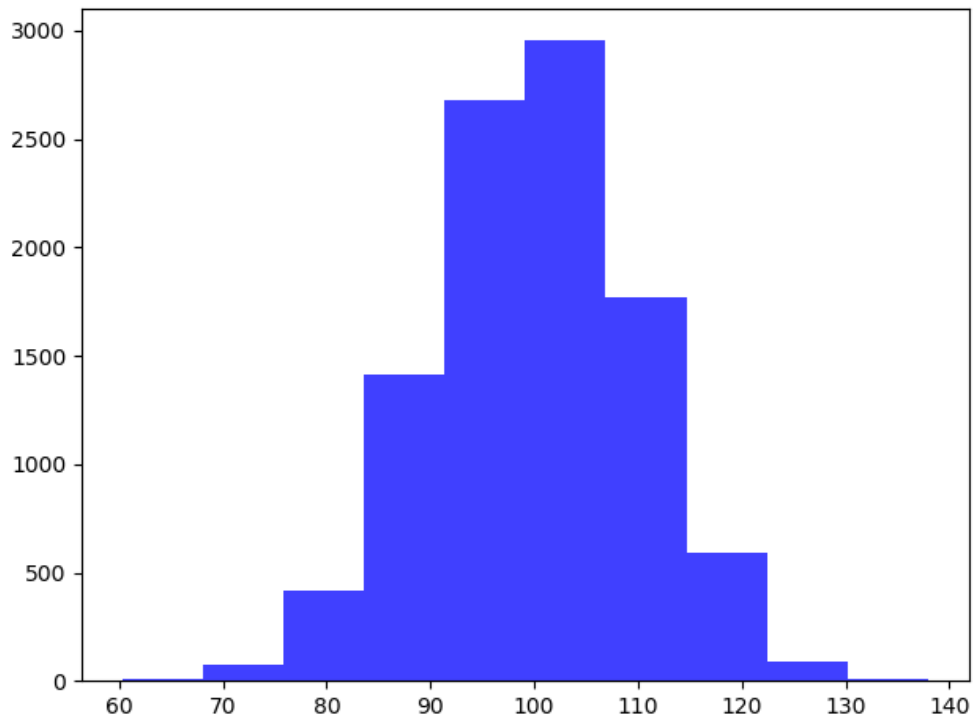
Code	Alpha	P_Value
SPY	-0.1736	9.65E-18
XLB	-0.0391	5.49E-02
XLE	-0.0248	2.24E-01
XLF	-0.0326	1.10E-01
XLI	-0.0509	1.25E-02
XLK	-0.0375	6.60E-02
XLP	-0.0178	3.83E-01
XLU	-0.0084	6.81E-01
XLV	0.01829	3.69E-01
XLY	-0.0388	5.69E-02

The P-Value is too small to conclude that the alpha coefficient of auto-regression is significant, so the residuals are not self-correlated.

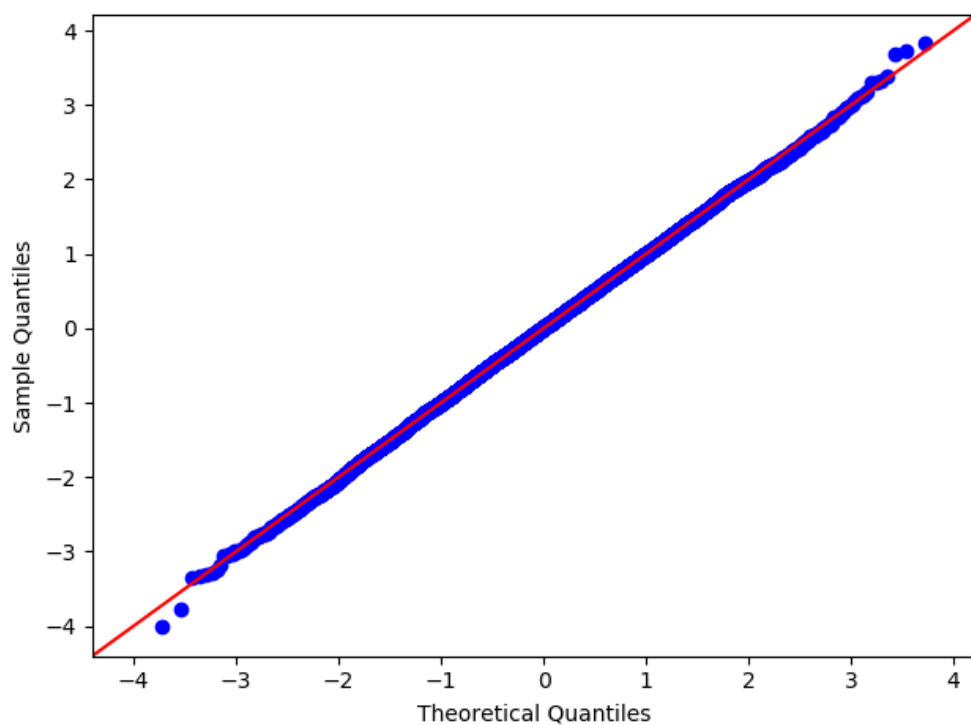
2. In my case, the setting of simulation are such that, the total time is 1, with steps = 100. So $\Delta t = 1/100$. And simulation times = 10k.



(b) The histogram of the ending values of the asset price along the simulated paths. is listed below,

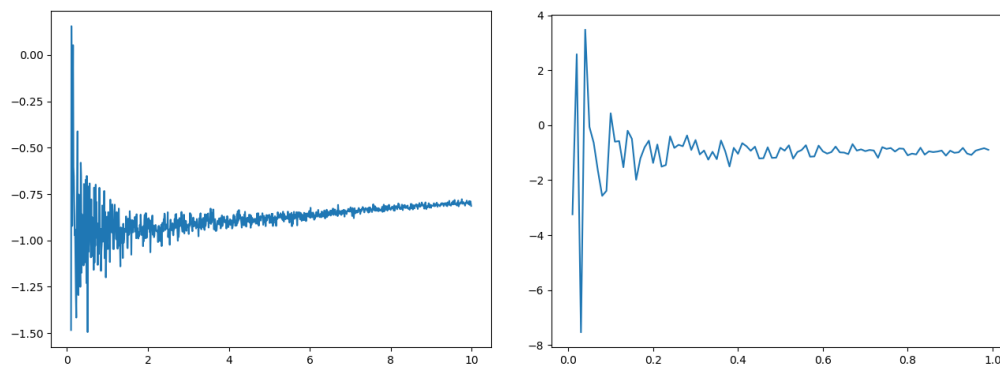


Using qqplot module to check whether the ending values of your simulated paths are normally distributed. The figure below shows that the ending values do are normally distributed.



(c) In my case, the simulated price of a Lookback put option with strike 100 under the Bachelier model is **7.346911090824913**, while the price of the lookback option in the Black-Scholes model price obtained in HW1 is **17.148212989083444**.

(d) The figure of values of Δ under different values of ε are shown below.



As we can learn from the figure, the optimal value of ε maybe 1, which located in the range where has a stable Δ and the value of Δ is also close to its theoretical value, -1. Since Δ measures the sensitivity of option price to the initial price of underlying asset, the smaller the ε and the closer it is to zero, the more inconsistent Δ sticks to its theoretical value. So the value of ε approaches to zero, there will be the largest amounts of error.