CS475 Machine Learning, Fall 2012: Homework 5

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Question 1:

(a)

$$V_c = (2r)^d \tag{1}$$

$$V_s = \frac{r^d \sqrt{\pi}^d}{\gamma(\frac{d}{2}+1)} \tag{2}$$

$$\begin{aligned}
\lim_{d\to\infty} \frac{V_s}{V_c} &= \lim_{d\to\infty} \frac{r^d \sqrt{\pi}^d}{2^d r^d \gamma(\frac{d}{2}+1)} && \text{let } d = 2z && (3)\\
&= \lim_{z\to\infty} \frac{r^2 z \sqrt{\pi}^2 z}{2^2 z r^2 z \Gamma(z+1)} \frac{\frac{1}{\sqrt{2\pi z} e^{-z} z^z}}{\frac{1}{\sqrt{2\pi z} e^{-z} z^z}} && && (4)\\
&= \lim_{z\to\infty} \frac{\frac{\sqrt{\pi}^{2^z}}{\sqrt{2\pi z} e^{-z} z^z 2^{2z}}}{\frac{\Gamma(z+1)}{\sqrt{2\pi z} e^{-z} z^z}} && && (5)\\
&= \lim_{z\to\infty} \frac{\pi^{z-.5} e^z}{2^{2z+.5} z^{z+.5}} && && (6)\\
&= 0 && && (7)
\end{aligned}$$

(b) Let us look at the ϵ ball variant of KNN. We have some n dimensional cube that represents our total feature space. And we pick a radius ϵ for our ball so that it encompasses some small but existent portion of the dataset. In low dimensions this is feasible to do. But in higher dimensions, we see that even if we pick the maximum possible diameter - the length of a side of the cube - then the ratio of the volume of our hypersphere to our total feature space will approach 0 (the limit from part a). This means that in high dimensions the probability of finding examples near our test example to create a label from will be 0, making prediction impossible.

Question 2:

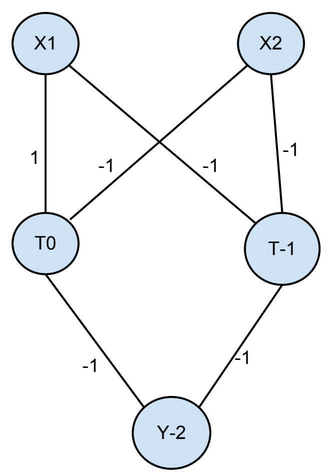
(a) The 1-NN classifier will be $O(nd)$ assuming that the examples are not organized in a K-D tree. The linear SVM in primal form will be $O(d)$.

(b) We should choose K_2. The K_2 classifier will have less variance than K_1 because it is less dependent on the training examples, but still has similar prediction power to the K_1 classifier becuase they achieve the same cross validation error.

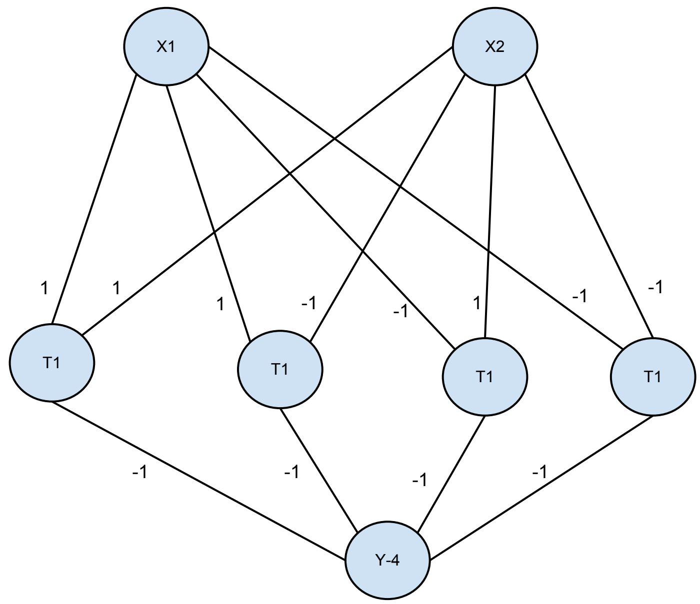
(c) KNN essentially dra

Question 3:

(a) In this case, we make the observation that the condition is satisfied if $x_1 - x_2 \leq 0$ and $1 \leq -x_1 - x_2 <= -1$. T_0 handles the first case and T_{-1} handles the second case. Each output -1 if activated, and so the condition is only satisfied if both cases are satisfied.



(b) In this case we recognize that the shape we are recognizing is a square and so we must check that our point is within the square. We can decompose this into recognizing whether it is on the correct side of each of the four borders of the square. These conditions translate to $x_2 - x_1 \leq 1$, $x_2 + x_1 \leq 1$, $x_1 - x_2 \leq 1$, $-x_2 - x_1 \leq 1$.



(c)

Question 4:

(a)

(b)

Question 5:

(a)

(b)

(c)