Measuring Energy Intake via Energy Balance Principle While Accounting for Measurement Error

Danny Ries

Iowa State University

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The Obesity Epidemic

Over 35% of Americans are obese, and over 75% of men are either overweight or obese. Obesity is linked to many different medical, psychological, emotional, and economic effects such as:

- Type 2 Diabetes
- Coronary Heart Disease
- High Blood Pressure
- Clinical Depression
- Anxiety
- Increased Health Care Costs
- Lost Wages
- Discrimination

The “Fatal Flaw in Obesity Research”

It has been said the “Fatal Flaw in Obesity Research” is our inability to accurately measure how much someone eats (EI) in free living situations

- Current measures of EI; ie. self report, are clouded with (measurement) error
- Garbage in Garbage out
- Tough to understand dietary trends over the years
- Cannot measure adherence to clinically prescribed interventions

This error in measurement extends to EE and body composition, albeit not nearly as severe

2 Remedies to aid in Obesity Research

1. Accurately and efficiently measure Energy Intake (EI)
2. Assess compliance to *2008 Physical Activity Guidelines*

Modeling Energy Balance

Energy Balance

The application of the first law of thermodynamics to nutrition/exercise science:
Change in Energy Stores (ΔES) = Energy Intake (EI) - Energy Expenditure (EE)

where $\Delta \text{ES} = c_1 \frac{\Delta FM}{\Delta T} + c_2 \frac{\Delta FFM}{\Delta T}$

This provides an alternative way to measure EI for an individual

Modeling Energy Balance Cont.

We are now in a situation where we must measure both EE and ΔES in order to calculate EI
$\rightarrow$ But gold standard measures for both exist!

In a world of unlimited resources, researchers needing EI for individuals could use gold standard measures of EE and ΔES and use simple measurement error models

Unfortunately,
DLW $\sim$ \$500/person
DXA $\sim$ \$100/person

Modeling Energy Balance Cont.

There are many other cheaper measures of EE and ΔES, that even when used together to calculate EI, are still more accurate than self-reported EI

Goal: Create a statistical measurement model for gold standard and cheap measurements of both EE and ΔES in order to develop calibration equations for cheap measurements.

This will allow future research to calibrate cheaper measurements (when gold standard measures aren't used) and thus eliminate known biases

Modeling Energy Balance Cont.

- Lots of research has been done for calibrating and evaluating measurement error for EI
- Some research for EE
- Little research for ΔES

To the best of our knowledge, no research has been done in evaluating the measurement error and calibrating measurements jointly via the Energy Balance principle

Energy Balance Study

The Energy Balance Study (EBS) was conducted 2011-2012 at the University of South Carolina

- 430 male and females aged 20-35
- 5 DXA scans, one every 3 months
- Sensewear Armband measuring EE every 3 months (averaged across 10 days)
- Subset of 119 participants receieved DLW at end of 12 months, with additional DXA scan
- Demographic variables

Although these data don't have perfect replicates, it provides a baseline to start our modeling of measurement error in Energy Balance

Checking Normality of Measurement Errors

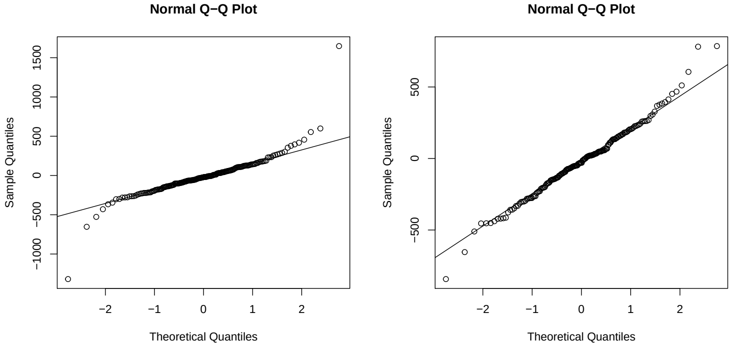


Figure: Differenced DXA ΔES

Checking Normality of Measurement Errors

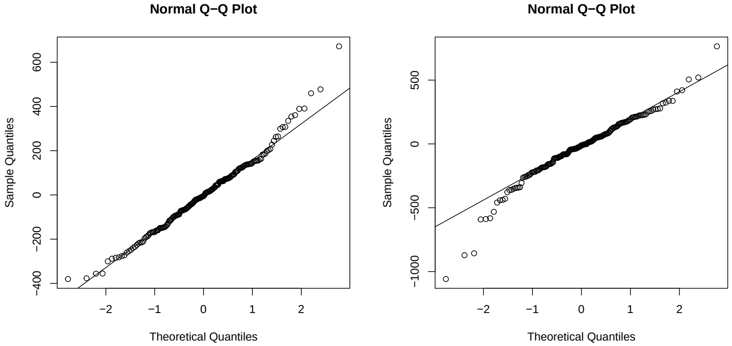


Figure: Differenced SWA EE

Näive Check for Biases

It seems reasonable that cheap measurement tools could be affected by factors other than the *truth*, ie. demographics

$\rightarrow$ Fit the following multiple regression model with the subset data from EBS

SWA EE $= \beta_0 + \beta_1$DLW EE $+ \beta_z$' Demographic Variables $+ \epsilon$

This is not ideal since we would want True EE not DLW EE in the regression, but still provides motivation

Table: Regression of Sensewear Armband EE on DLW EE, Age, BMI, Gender using EBS data. Results show systematic biases could exist in cheap EE measurements.

Coefficient	Estimate	Std Error	P-value
Intercept	878.422	154.113	<0.0001
DLW EE	0.558	0.040	<0.0001
Age	-7.351	3.999	0.0676
Male	305.582	43.258	<0.0001
BMI	14.146	3.988	0.0004

Notation

let i represent individual and j represent replicate number

Observable:

- W_{ij}^{EE} and $W_{ij}^{\Delta ES}$ represent gold standard measures of EE and ΔES
- Y_{ij}^{EE} and $Y_{ij}^{\Delta ES}$ represent cheap measures of EE and ΔES
- Z_i represent a $k \times 1$ vector of error free covariates

Latent:

- X_i^{EE} and $X_i^{\Delta ES}$ represent *usual* EE and ΔES

Independence Assumptions

Given X_i^{EE}, Z_i

- Y_{ij}^{EE} are mutually independent for all i, j
- W_{ij}^{EE} are mutually independent for all i, j
- Y_{ij}^{EE} is independent of W_{ij}^{EE} for all i, j
- Y_{ij}^{EE} is independent of $W_{ij}^{\Delta ES}$ and $Y_{ij}^{\Delta ES}$ for all i, j
- W_{ij}^{EE} is independent of $W_{ij}^{\Delta ES}$ and $Y_{ij}^{\Delta ES}$ for all i, j

Same assumptions hold for reverse case (replace EE with ΔES and ΔES with EE)

Model for Observed Variables

$$
\begin{aligned}
Y_{ij}^{EE} &= m_{ee}(X_i^{EE}, Z_i) + \epsilon_{ij}^{EE} \\
Y_{ij}^{\Delta ES} &= m_{es}(X_i^{\Delta ES}, Z_i) + \epsilon_{ij}^{\Delta ES} \\
W_{ij}^{EE} &= X_i^{EE} + \nu_{ij}^{EE} \\
W_{ij}^{\Delta ES} &= X_i^{\Delta ES} + \nu_{ij}^{\Delta ES}
\end{aligned}
$$

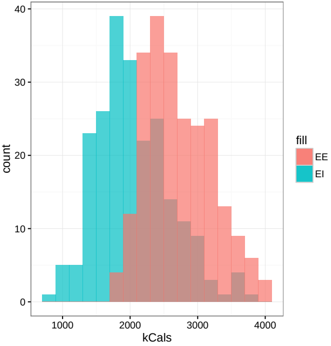
$$E(\epsilon_{ij}^{EE}) = E(\epsilon_{ij}^{\Delta ES}) = E(\nu_{ij}^{EE}) = E(\nu_{ij}^{\Delta ES}) = 0$$

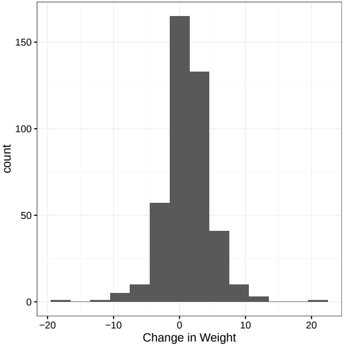
Joint Likelihood

$$
\begin{aligned}
L_i(\boldsymbol{\theta}) &= \prod_{j=1}^{J} f(W_{ij}^{EE}, W_{ij}^{\Delta ES}, Y_{ij}^{EE}, Y_{ij}^{\Delta ES} | Z_i, \boldsymbol{\theta}) \\
&= \int_{\mathcal{X}_{es}} \int_{\mathcal{X}_{ee}} \prod_{j=1}^{J} f(W_{ij}^{EE}, W_{ij}^{\Delta ES}, Y_{ij}^{EE}, Y_{ij}^{\Delta ES}, X_i^{EE}, X_i^{\Delta ES} | Z_i, \boldsymbol{\theta}) dX_i^{EE} dX_i^{\Delta ES} \\
&= \prod_{j=1}^{J} \int_{\mathcal{X}_{es}} \int_{\mathcal{X}_{ee}} f(W_{ij}^{EE} | X_i^{EE}, X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{wee}}) f(W_{ij}^{\Delta ES} | X_i^{EE}, X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{wes}}) \times \\
&\quad f(Y_{ij}^{EE} | X_i^{EE}, X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{yee}}) f(Y_{ij}^{\Delta ES} | X_i^{EE}, X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{yes}}) f(X_i^{EE}, X_i^{\Delta ES} | Z_i, \boldsymbol{\theta_x}) dX_i^{EE} dX_i^{\Delta ES} \\
&= \prod_{j=1}^{J} \int_{\mathcal{X}_{es}} \int_{\mathcal{X}_{ee}} f(W_{ij}^{EE} | X_i^{EE}, Z_i, \boldsymbol{\theta_{wee}}) f(W_{ij}^{\Delta ES} | X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{wes}}) \times \\
&\quad f(Y_{ij}^{EE} | X_i^{EE}, Z_i, \boldsymbol{\theta_{yee}}) f(Y_{ij}^{\Delta ES} | X_i^{\Delta ES}, Z_i, \boldsymbol{\theta_{yes}}) f(X_i^{EE}, X_i^{\Delta ES} | Z_i, \boldsymbol{\theta_x}) dX_i^{EE} dX_i^{\Delta ES}
\end{aligned}
$$

$$
L(\boldsymbol{\theta}) = \prod_{i=1}^{n} L_i(\boldsymbol{\theta})
$$

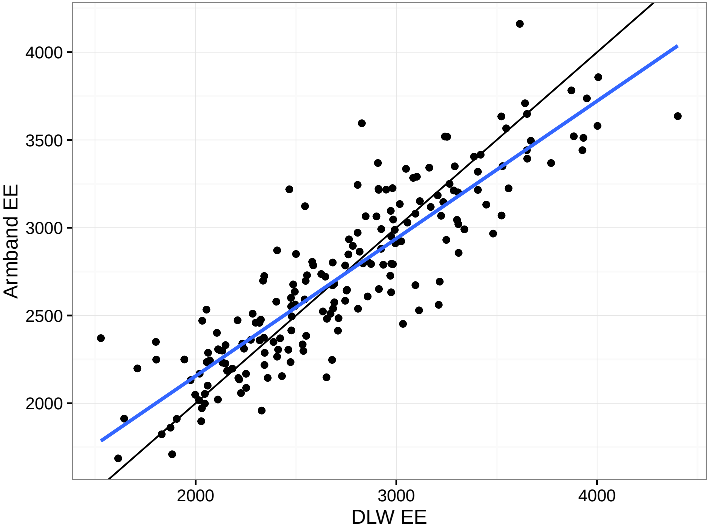
We can create energy!





*Data from Energy Balance Study

Armband vs DLW



Modeling Energy Balance

Measurement Methods

Measurement of EE and ΔES is less noisy than EI and true gold standard measurements exist

- Pedometer
- Consumer grade wearables
- Sensewear Armband
- DLW (gold standard)

- Body Weight
- Calipers
- BodPod
- Bioelectrical impedence
- DXA (gold standard)

Notice how none of these methods require self report?

Research Goal

Research Outcomes

Our overall goal is to provide a cheap, easy, noninvasive *prediction of EI for an individual in free living situations*

To achieve our goal:

- Statistically assess the measurement error in various instruments of both EE and ΔES
- Calibrate cheaper measures so they provide reasonable accuracy without the expense and expertise required for DLW and DXA

Data Required

Because we are assessing the measurement error in these instruments, we need replicate measures of all EE and ΔES measurements on an individual that accounts for all measurement error.

This is a little tricky...

- Replicates measurements of EE for an individual
- Replicate measurements of ΔES for an individual
- Demographic covariates for an individual

Luckily, we have a wealth of data from the Energy Balance Study to help propose a model and empirically check assumptions.

We can further specify forms for each component:

$$Y_{ij}^{EE} = m_{ee}(X_i^{EE}) + \gamma_{ee} Z_i + \epsilon_{ij}^{EE} \tag{1}$$
$$Y_{ij}^{\Delta ES} = m_{es}(X_i^{\Delta ES}) + \gamma_{es} Z_i + \epsilon_{ij}^{\Delta ES} \tag{2}$$
$$W_{ij}^{EE} = X_i^{EE} + \nu_{ij}^{EE} \tag{3}$$
$$W_{ij}^{\Delta ES} = X_i^{\Delta ES} + \nu_{ij}^{\Delta ES} \tag{4}$$

- Because we never can observe $(X_i^{EE}, X_i^{\Delta ES})$, it is difficult to say the functional relationship (m_{ee} and m_{es}) between it and cheap measurements Y^{EE} and $Y^{\Delta ES}$
- Because of this, we choose to model m_{ee} and m_{es} with B-splines
- Must specify number of knots k and knot locations $\zeta_1, ...\zeta_k$

$$m_{ee}(X_i^{EE}) = \sum_{i=1}^{k_{ee}} b_{i,ee}(\zeta_{i,ee})\beta_{i,ee} = B_{ee}(\zeta_{ee})\beta_{i,ee} \quad (5)$$

$$m_{es}(X_i^{\Delta ES}) = \sum_{i=1}^{k_{es}} b_{i,es}(\zeta_{i,es})\beta_{i,es} = B_{es}(\zeta_{es})\beta_{i,es} \quad (6)$$

We let k and $\zeta_1, ...\zeta_k$ vary according to the data

Specifying the Likelihood

$$\epsilon_{ij}^{EE} \overset{iid}{\sim} N(0, \sigma^2_{i,yee}) \tag{7}$$

$$\epsilon_{ij}^{\Delta ES} \overset{iid}{\sim} N(0, \sigma^2_{i,yes}) \tag{8}$$

$$\nu_{ij}^{EE} \overset{iid}{\sim} N(0, \sigma^2_{i,wee}) \tag{9}$$

$$\nu_{ij}^{\Delta ES} \overset{iid}{\sim} N(0, \sigma^2_{i,wes}) \tag{10}$$

Latent Variable Likelihood

The final component to have a joint likelihood specified is the bivariate latent variable. Because we *never ever* observe these values, we must specify its distribution carefully

$$(X_i^{EE}, X_i^{\Delta ES}) \overset{iid}{\sim} \sum_{h=1}^{\infty} \pi_h N(\mu_h, \Sigma_h) \tag{11}$$

$$\pi_h \overset{iid}{\sim} Stick(\alpha) \tag{12}$$

$$\sum_{h=1}^{\infty} \pi_h = 1 \tag{13}$$

$$\pi_h = V_h \prod_{\ell < h} (1 - V_h) \tag{14}$$

$$V_H = 1 \tag{15}$$

$$V_h \sim Beta(1, \alpha), h < H \tag{16}$$

Priors

Need to assign priors for unknown parameters

$$\boldsymbol{\theta} = (\{\sigma_{i,yee}\}_{i=1}^{n}, \{\sigma_{i,yes}\}_{i=1}^{n}, \{\sigma_{i,wee}\}_{i=1}^{n}, \tag{17}$$

$$\{\sigma_{i,wes}\}_{i=1}^{n}, \{\Sigma_i\}_{i=1}^{n}, \{\gamma_{i,yee}\}_{i=1}^{p}, \tag{18}$$

$$\{\gamma_{i,yes}\}_{i=1}^{p}, \{\beta_{i,yee}\}_{i=1}^{k_{ee}}, \{\beta_{i,yes}\}_{i=1}^{k_{kes}}, k_{ee}, k_{es}, \tag{19}$$

$$\{\zeta_{i,ee}\}_{i=1}^{k_{ee}}, \{\zeta_{i,es}\}_{i=1}^{k_{es}}) \tag{20}$$

Assume independent priors (for now)

$$p(\sigma_{i,yee}) \overset{iid}{\sim} C^+(0,1) \quad (21)$$
$$p(\sigma_{i,yes}) \overset{iid}{\sim} C^+(0,1) \quad (22)$$
$$p(\sigma_{i,wee}) \overset{iid}{\sim} C^+(0,1) \quad (23)$$
$$p(\sigma_{i,wes}) \overset{iid}{\sim} C^+(0,1) \quad (24)$$
$$p(\Sigma_i) \overset{iid}{\sim} \text{Inv-Wish}(I_{2x2}, 3) \quad (25)$$

$$p(\gamma_{i,ee}) \overset{iid}{\sim} N(w_{i,ee}, B_{i,ee}) \quad (26)$$
$$p(\gamma_{i,es}) \overset{iid}{\sim} N(w_{i,es}, B_{i,es}) \quad (27)$$
$$p(\beta_{i,ee}) \overset{iid}{\sim} N(v_{i,ee}, C_{i,ee}) \quad (28)$$
$$p(\beta_{i,es}) \overset{iid}{\sim} N(v_{i,es}, C_{i,es}) \quad (29)$$
$$p(k_{ee}) \sim Poi(a_{ee}) \quad (30)$$
$$p(k_{es}) \sim Poi(a_{es}) \quad (31)$$

$$p(\zeta_1, ..., \zeta_{k_{ee}} | k_{ee}) \sim DUnif(x_1^{EE}, ..., x_n^{EE}) \quad (32)$$
$$p(\zeta_1, ..., \zeta_{k_{es}} | k_{es}) \sim DUnif(x_1^{\Delta ES}, ..., x_n^{\Delta ES}) \quad (33)$$

Priors likely to change as we elicit expert information and use of past data

Estimation

Since we are taking a Bayesian approach, the posterior distribution $p(\boldsymbol{\theta}|Y, W, Z)$ gives us everything we need for inference

- Using Bayes Rule:

$$p(\boldsymbol{\theta}|Y, W, Z) = \frac{p(Y, W|\boldsymbol{\theta}, Z)p(\boldsymbol{\theta})}{\int p(Y, W, \boldsymbol{\theta}|Z)d\boldsymbol{\theta}} \tag{34}$$

- Integral is impossible to evaluate analytically, use MCMC to simulate draws from joint posterior
- Will use Gibbs Sampler to update parameters
- Reversible Jump MCMC step necessary for B-splines since number of knots k and knot locations $\zeta_1, ...\zeta_k$ are random variables and therefore dimension of posterior is allowed to change
- Implement in R/C++

Model Assessment and Comparason

We will assess the fit of our model through the use of the posterior predictive distributions and relevant discrepancy measures $D()$

$$p(Y^*|W,Y,Z) = \int\int p(Y^*|\boldsymbol{\theta},X,Z)p(\boldsymbol{\theta},X|Y,W)d\boldsymbol{\theta}dX \tag{35}$$

$$p(W^*|W,Y,Z) = \int\int p(W^*|\boldsymbol{\theta},X,Z)p(\boldsymbol{\theta},X|Y,W)d\boldsymbol{\theta}dX \tag{36}$$

$$p(X^*|W,Y,Z) = \int\int p(X^*|\boldsymbol{\theta})p(\boldsymbol{\theta},X|Y,W)d\boldsymbol{\theta}dX \tag{37}$$

For each simulated replicate data set (for each Y, W, X) calculate $D()$. Compare to $D()$ from true data

Model Comparason

Although I presented only one specific model here, there are simplifying (and more complicating) assumptions we could (and will) make. To compare models we can use:

- DIC
- Bayes Factors
- PMSE of EI — this is not straightforward
- Parsimony and Practical Interpretation