

Measurement Error Modeling of Energy Balance and Physical Activity Data

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March 9, 2017

The Obesity Epidemic

Over 35% of Americans are obese, and over 75% of men are either overweight or obese. Obesity is linked to many different medical, psychological, emotional, and economic effects such as:

- Type 2 Diabetes
- Coronary Heart Disease
- High Blood Pressure
- Clinical Depression
- Anxiety
- Increased Health Care Costs
- Lost Wages
- Discrimination

2 Remedies to aid in Obesity Research

1. Accurately and efficiently measure Energy Intake (EI)
2. Assess compliance to *2008 Physical Activity Guidelines*

To address the difficulty in measuring EI:

Bayesian Semi-Parametric Energy Balance Measurement Error Model

Modeling Energy Balance

Energy Balance

Change in Energy Stores (ΔES) = Energy Intake (EI) - Energy Expenditure (EE)

$$\text{where } \Delta ES = C_{FM} \frac{\Delta FM}{\Delta T} + C_{FFM} \frac{\Delta FFM}{\Delta T}$$

Current measures of EI; ie. self report, are clouded with (measurement) error

Research Goal

Create a joint statistical measurement model for gold standard and inexpensive measurements of both EE and ΔES in order to develop calibration equations for cheap measurements.

Energy Balance Study

The Energy Balance Study (EBS) was conducted 2011-2012 at the University of South Carolina

- 430 male and females aged 20-35
- 5 DXA scans, one every 3 months
- Sensewear Armband measuring EE every 3 months (averaged across 10 days)
- Subset of 119 participants received DLW at end of 12 months, with additional DXA scan

Naïve Check for Biases

$$\text{SWA EE} = \beta_0 + \beta_1 \text{DLW EE} + \beta_z' \text{ Demographic Variables} + \epsilon$$

Coefficient	Estimate	Std Error	P-value
Intercept	878.422	154.113	<0.0001
DLW EE	0.558	0.040	<0.0001
Age	-7.351	3.999	0.0676
Male	305.582	43.258	<0.0001
BMI	14.146	3.988	0.0004

Notation

let i represent individual and j represent replicate number

Observable:

- W_{ij}^{EE} and $W_{ij}^{\Delta ES}$ represent gold standard measures of EE and ΔES
- Y_{ij}^{EE} and $Y_{ij}^{\Delta ES}$ represent inexpensive measures of EE and ΔES
- Z_i represent a $k \times 1$ vector of error free covariates

Latent:

- X_i^{EE} and $X_i^{\Delta ES}$ represent *usual* EE and ΔES

$$E(W_{ij}^{EE} | i) = X_i^{EE}$$
$$E(W_{ij}^{\Delta ES} | i) = X_i^{\Delta ES}$$

Independence Assumptions

We assume individuals are independent

Additionally, given X_i^{EE}, Z_i

- Y_{ij}^{EE} are independent for all j
- W_{ij}^{EE} are independent for all j
- Y_{ij}^{EE} is independent of W_{ij}^{EE} for all j
- Y_{ij}^{EE} is independent of $W_{ij}^{\Delta ES}$ and $Y_{ij}^{\Delta ES}$ for all j
- W_{ij}^{EE} is independent of $W_{ij}^{\Delta ES}$ and $Y_{ij}^{\Delta ES}$ for all j

Same assumptions hold for reverse case (replace EE with ΔES and ΔES with EE)

Naïve Model

The Naïve Model assumes no measurement error in gold standard measurements

$$(Y_{ij}^{EE} | W_{ij}^{EE}, Z_i, \theta_{yee}) \stackrel{ind}{\sim} N(\beta_{0,ee} + \beta_{1,ee} W_{ij}^{EE} + \gamma_{ee}' Z_i, \sigma_{\epsilon}^2{}_{EE})$$

$$(Y_{ij}^{\Delta ES} | W_{ij}^{\Delta ES}, Z_i, \theta_{yes}) \stackrel{ind}{\sim} N(\beta_{0,es} + \beta_{1,es} W_{ij}^{\Delta ES} + \gamma_{es}' Z_i, \sigma_{\epsilon}^2{}_{\Delta ES})$$

Linear Measurement Error Model (LMEM)

Basic modification to the Naïve model when we account for measurement error in a covariate

$$(Y_{ij}^{EE} | X_i^{EE}, Z_i, \theta_{yee}) \stackrel{ind}{\sim} N(\beta_{0,ee} + \beta_{1,ee} X_i^{EE} + \gamma_{ee}' Z_i, \sigma_{\epsilon_{EE}}^2)$$

$$(Y_{ij}^{\Delta ES} | X_i^{\Delta ES}, Z_i, \theta_{yes}) \stackrel{ind}{\sim} N(\beta_{0,es} + \beta_{1,es} X_i^{\Delta ES} + \gamma_{es}' Z_i, \sigma_{\epsilon_{\Delta ES}}^2)$$

$$(W_{ij}^{EE} | X_i^{EE}, Z_i, \theta_{wee}) \stackrel{ind}{\sim} N(X_i^{EE}, \sigma_{\nu_{EE}}^2)$$

$$(W_{ij}^{\Delta ES} | X_i^{\Delta ES}, Z_i, \theta_{wes}) \stackrel{ind}{\sim} N(X_i^{\Delta ES}, \sigma_{\nu_{\Delta ES}}^2)$$

$$(X_i^{EE}, X_i^{\Delta ES} | \theta_X) \stackrel{ind}{\sim} N\left(\begin{bmatrix} \mu_{EE} \\ \mu_{\Delta ES} \end{bmatrix}, \Sigma_X\right)$$

Extending the Linear Model

Relax the assumption that the relationship between an inexpensive measurement and *usual* EE and Δ ES is linear

We propose using cubic free knot splines to model the relationship between inexpensive measurement and *usual*

- Allows for a flexible nonlinear relationship
- No need to specify number or location of knots
- Using Reversible Jump MCMC, incorporates uncertainty in spline selection

Free Knot Spline Model (SMEMN)

$$(Y_{ij}^{EE} | X_i^{EE}, Z_i, \theta_{yee}) \stackrel{ind}{\sim} N(s_{ee}(X_i^{EE}; \beta_{ee}) + \gamma_{ee} Z_i, \sigma_{\epsilon^{EE}}^2)$$

$$(Y_{ij}^{\Delta ES} | X_i^{\Delta ES}, Z_i, \theta_{yes}) \stackrel{ind}{\sim} N(s_{es}(X_i^{\Delta ES}; \beta_{\Delta es}) + \gamma_{es} Z_i, \sigma_{\epsilon^{\Delta ES}}^2)$$

$$s_{ee}(X_i^{EE}; \beta_{ee}) = \sum_{i=1}^{k_{ee}+4} b_{i,ee}(\mathbf{X}^{EE}) \beta_{i,ee} = B_{ee}(\mathbf{X}^{EE}) \beta_{ee}$$

$$s_{es}(X_i^{\Delta ES}; \beta_{\Delta es}) = \sum_{i=1}^{k_{es}+4} b_{i,es}(\mathbf{X}^{\Delta ES}) \beta_{i,es} = B_{es}(\mathbf{X}^{\Delta ES}) \beta_{es}$$

In order to ensure the functions are monotone, constrain

$$\beta_{1,ee} \leq \beta_{2,ee} \leq \dots \leq \beta_{k_{ee}+4,ee}$$

Extending the Normality of Latent Variables

Allow more flexibility in modeling of latent variables, use Dirichlet process mixture prior (SMEMDP):

$$X_i^{EE}, X_i^{\Delta ES} | \zeta_i = h \stackrel{ind}{\sim} N \left(\begin{bmatrix} \mu_{ee,h} \\ \mu_{es,h} \end{bmatrix}, \Sigma_h \right)$$

$$\zeta_i \stackrel{iid}{\sim} \text{Cat}(H, \pi)$$

$$V_h \sim \text{Beta}(1, \alpha)$$

$$V_H = 1$$

$$\pi_h = V_h \prod_{\ell < h} (1 - V_\ell)$$

Gibbs Sampler

We use conjugate priors to help simplify the sampler

For iteration $k=1,\dots,K$, sample from its full conditional:

$$1. \{ \Sigma^{(k)} \} \mid \cdot \stackrel{ind}{\sim}$$

$$Inv - Wish(d + n, \psi + (\mathbf{X}_i^{(k-1)} - \boldsymbol{\mu}^{(k-1)})'(\mathbf{X}_i^{(k-1)} - \boldsymbol{\mu}^{(k-1)}))$$

$$2. \{ (\mu_{EE}^{(k)}, \mu_{\Delta ES}^{(k)}) \} \mid \cdot \stackrel{ind}{\sim} N(M'_\mu, C'_\mu)$$

$$C'_\mu = (C_\mu^{-1} + n\Sigma^{-1(k)})^{-1}$$

$$M'_\mu = C'_\mu (C_\mu^{-1}M + n\Sigma^{-1(k)}\bar{\mathbf{X}}^{(k-1)})$$

$$\bar{\mathbf{X}} = \frac{1}{n} \sum_{i=1}^n \mathbf{X}_i$$

$$3. \text{Update } \mathbf{X}_i \text{ with a random walk}$$

$$\{ \mathbf{X}_i^{(k)} : i = 1, \dots, n \} \mid \cdot \text{ for } i = 1, \dots, n \text{ sample } \mathbf{X}_i^* \text{ from } N(\mathbf{X}_i^{(k)}, C_{X_i})$$

$$\text{Set } \mathbf{X}_i^{(k)} = \mathbf{X}_i^* \text{ with probability } \alpha_{X_i}, \text{ otherwise set } \mathbf{X}_i^{(k)} = \mathbf{X}_i^{(k-1)}$$

$$\alpha_{X_i} = \min \left(1, \frac{f(\mathbf{X}_i^* \mid \cdot)}{f(\mathbf{X}_i^{(k-1)} \mid \cdot)} \right)$$

4. Update $\beta_{ee}, \beta_{es}, k_{ee}, k_{es}, r_{ee}, r_{es}, \gamma_{ee}, \gamma_{es}$ using RJMCMC described next. Calculate $s_{ee}(\mathbf{X}^{EE(k)}; \beta_{ee}^{(k)})$ and $s_{es}(\mathbf{X}^{\Delta ES(k)}; \beta_{es}^{(k)})$
5. $\sigma_{\epsilon^{EE}}^{2(k)} | \cdot \sim IG(a_{yee} + J \times \frac{n}{2}, b_{yee} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^J (Y_{ij}^{EE} - s_{ee}(X_i^{EE(k)}; \beta_{ee}^{(k)}) - \gamma_{ee}^{(k)'} Z_i)^2)$
6. $\sigma_{\epsilon^{\Delta ES}}^{2(k)} | \cdot \sim IG(a_{yes} + J \times \frac{n}{2}, b_{yes} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^J (Y_{ij}^{\Delta ES} - s_{es}(X_i^{\Delta ES(k)}; \beta_{es}^{(k)}) - \gamma_{es}^{(k)'} Z_i)^2)$
7. $\sigma_{\nu^{EE}}^{2(k)} | \cdot \sim IG(a_{wee} + J \times \frac{n}{2}, b_{wee} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^J (W_{ij}^{EE} - X_i^{EE(k)})^2)$
8. $\sigma_{\nu^{\Delta ES}}^{2(k)} | \cdot \sim IG(a_{wes} + J \times \frac{n}{2}, b_{wes} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^J (W_{ij}^{\Delta ES} - X_i^{\Delta ES(k)})^2)$

RJMCMC

Green (1995) introduced RJMCMC as a way to “jump” between different models and dimensions as a means for model selection. Denison (1998) introduced a simple and effective implementation of a Bayesian free-knot spline

We extend method of Denison in three ways:

1. The spline covariate is a latent variable that is also sampled from
2. There are additional linear components to the mean function
3. We make the modification from DiMatteo (2001) so the likelihood ratio is an approximation of BIC

$\mathcal{A} = \{x_i, i = 1, \dots, n : x_i \text{ is not currently a knot or within } \ell + 1 \text{ locations of a current knot}\}$

1. Calculate $b_k = c \times \min\left(1, \frac{p^{(k+1)}}{p^{(k)}}\right)$
 $d_k = c \times \min\left(1, \frac{p^{(k)}}{p^{(k+1)}}\right)$
2. Select birth, death, or move step with probabilities $b_k, d_k, 1 - b_k - d_k$ respectively
3. **Knot Changes**

If birth step:

Select a new knot location at random from the set \mathcal{A} and join with current knots $r^{(k-1)}$ to create the proposed knot locations r^*

If death Step:

Sample one knot location from $r^{(k-1)}$ at random and remove it.

If move step:

Sample one knot location from $r^{(k-1)}$ at random, and change it to a new knot location at random from the set \mathcal{A}

4. Calculate the spline basis matrix $B^*(\mathbf{X}^{(k)})$ using $\mathbf{X}^{(k)}$ and proposed knot locations r^*
5. Calculate proposed spline and linear regression coefficients β^*, γ^* by using OLS to regress $\bar{\mathbf{Y}}$ on $B^*(\mathbf{X}^{(k)}) + \mathbf{Z}$
6. Accept proposed knots and coefficients with probability α
Otherwise set $r^{(k)} = r^{(k-1)}$, $\beta^{(k)} = \beta^{(k-1)}$, $\gamma^{(k)} = \gamma^{(k-1)}$

$$\alpha_{birth} = \min \left(1, \text{Likelihood ratio} \times \frac{n - Z(k)}{n} \right)$$

$$\alpha_{death} = \min \left(1, \text{Likelihood ratio} \times \frac{n}{n - Z(k)} \right)$$

$$\alpha_{move} = \min(1, \text{Likelihood ratio})$$

$$Z(k) = 2(\ell + 1) + k(2\ell + 1)$$

$$k = \text{length}(r^{(k-1)})$$

7. Compute mean function $s(\mathbf{X}^{(k)}; \beta^{(k)})$ using spline basis matrix $B'(\mathbf{X}^{(k)})$ and $r^{(k)}$

Simulation Study

Simulated 200 data sets for both 2 and 4 replicates per individual. Number of individuals was set to be 300. Three different measurement errors were used for each set of replicates:

- Normal
- Skew-Normal
- Bimodal (50-50 mixture of two Normals)

For each simulated data set, we ran the MCMC for 12,000 iterations, using the first 2000 as burn in

Issues with SMEMDP with 2 replicates

LMEM	σ_{yee}	σ_{yes}	σ_{wee}	σ_{wes}	$\gamma_{1,ee}$	$\gamma_{2,ee}$	$\gamma_{3,ee}$	$\gamma_{1,es}$	$\gamma_{2,es}$	$\gamma_{3,es}$
Mean Est	432.67	289.34	254.13	69.67	251.72	13.38	-5.46	-196.76	7.82	-4.96
Std Err	16.70	9.90	11.99	2.63	41.34	3.95	3.35	25.32	2.39	2.25
Bias	27.17	-44.66	4.13	-3.20	-48.28	-0.62	1.54	3.24	-0.18	0.04
Truth	405.50	334.00	250.00	72.86	300.00	14.00	-7.00	-200.00	8.00	-5.00

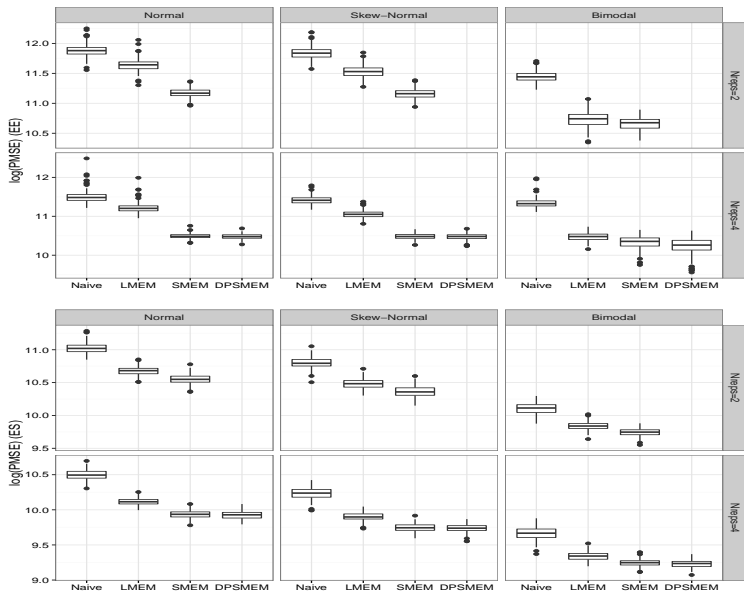
SMEMN	σ_{yee}	σ_{yes}	σ_{wee}	σ_{wes}	$\gamma_{1,ee}$	$\gamma_{2,ee}$	$\gamma_{3,ee}$	$\gamma_{1,es}$	$\gamma_{2,es}$	$\gamma_{3,es}$
Mean Est	393.56	282.68	247.00	68.93	306.58	14.27	-6.95	-197.44	7.76	-4.92
Std Err	13.33	10.96	9.54	2.56	36.09	3.35	2.77	24.41	2.42	2.10
Bias	-11.94	-51.32	-3.00	-3.93	6.58	0.27	0.05	2.56	-0.24	0.08
Truth	405.50	334.00	250.00	72.86	300.00	14.00	-7.00	-200.00	8.00	-5.00

Table: Skewed Errors, 2 replicates per person

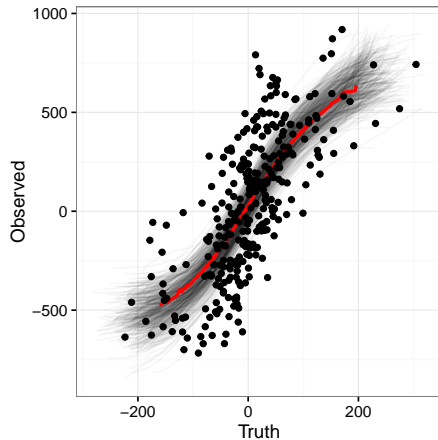
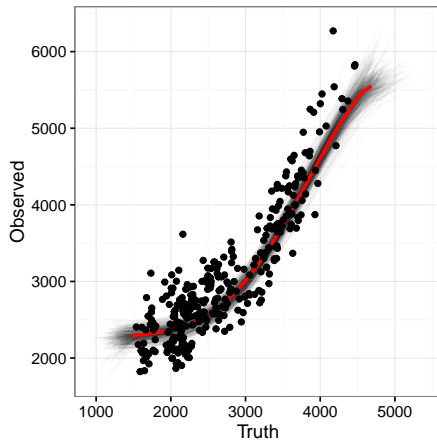
LMEM	σ_{yee}	σ_{yes}	σ_{wee}	σ_{wes}	$\gamma_{1,ee}$	$\gamma_{2,ee}$	$\gamma_{3,ee}$	$\gamma_{1,es}$	$\gamma_{2,es}$	$\gamma_{3,es}$
Mean Est	264.16	220.65	207.41	52.92	216.24	11.50	-5.75	-201.48	7.90	-4.74
Std Err	15.07	7.31	11.92	1.85	32.90	3.20	3.07	17.18	1.66	1.49
Bias	-141.34	-113.35	-42.59	-19.94	-83.76	-2.50	1.25	-1.48	-0.10	0.26
Truth	405.50	334.00	250.00	72.86	300.00	14.00	-7.00	-200.00	8.00	-5.00

SMEMN	σ_{yee}	σ_{yes}	σ_{wee}	σ_{wes}	$\gamma_{1,ee}$	$\gamma_{2,ee}$	$\gamma_{3,ee}$	$\gamma_{1,es}$	$\gamma_{2,es}$	$\gamma_{3,es}$
Mean Est	257.20	217.62	182.30	51.86	220.14	12.25	-5.90	-201.34	7.87	-5.08
Std Err	11.41	7.51	8.22	1.87	30.56	3.24	2.86	17.33	1.60	1.46
Bias	-148.30	-116.38	-67.70	-21.01	-79.86	-1.75	1.10	-1.34	-0.13	-0.08
Truth	405.50	334.00	250.00	72.86	300.00	14.00	-7.00	-200.00	8.00	-5.00

Table: Bimodal Errors, 2 replicates per person



Example of one data set



Calibration

$$X_{calibrated} = s^{-1}(y - \gamma'Z)$$

For $r = 1, \dots, R$

1. Calculate $y_i^* = y_i - \gamma^{(r)'} Z_i$, where Z_i are the covariate values for individual i
2. Use optimize for the function $|s_i(x) - y_i^*|$ to choose the value of x that will minimize the aforementioned criteria, call this $x_{i,calibrated}^{(r)}$. $s_i(x)$ is the predicted value of y_i for the given value x using the MCMC draw for the spline coefficients, latent variables, and knot locations from the r^{th} draw of the chain

3 example calibrations

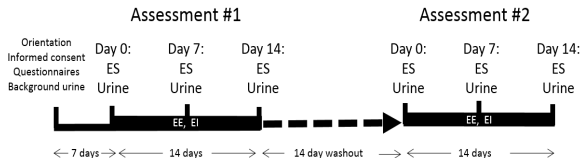
	Gender	BMI	Age
1	M	28.6	20.5
2	F	21.5	30.1
3	M	38.6	22.8

	Lower	Median	Upper	Observed	Truth
EE	2574.18	2666.00	2736.39	3028.89	2199.25
	3452.51	3525.18	3619.08	4119.26	3588.12
	2571.99	2665.46	2744.65	2555.86	2643.14
Δ ES	25.15	42.35	60.57	142.30	64.17
	-104.21	-82.93	-63.90	-405.74	-21.08
	-8.41	3.91	17.83	96.06	-0.48

Table: 95% credible interval for mean calibration estimate for cheap measurements under Skewed Errors

Data Collection

- 30 participants
- Free living, no treatment
- M/F
- Ages 25-35
- BMI 18-30
- Convenience sample



EE: DLW (subcontract with Mass Spectrometry Core at Pennington Biomedical Research Center), Actigraph wGT3X-BT, Actigraph Link, Metalogic Lume

EI: 24-hr dietician administered dietary recalls (subcontract with Cancer Prevention and Control Program in the Arnold School of Public Health at the University of South Carolina)

ES: DXA (administered by Nutrition and Wellness Research Staff), body weight, waist circumference, In-Body bioelectrical impedance, Hologic bioelectrical impedance

To Assess Compliance with *2008 Physical Activity Guidelines*:

A Bayesian two-part model with measurement error: Assessing adult moderate with vigorous physical activity and compliance to 2008 Physical Activity Guidelines

2008 Physical Activity Guidelines

Because of the many benefits linked to physical activity, the Department of Health and Human Services Recommends Adults 18-65 participate each week in at least:

- 150 Minutes of Moderate Physical Activity (3-6 METs), or
- 75 Minutes of Vigorous Physical Activity (6+ METs), or
- 75-150 Minutes during some combination of Moderate and Vigorous Physical Activity (MVPA)
- Resistance training targeting all major muscle groups twice
- Most benefits occur when activity is performed in at least 10 minute *bouts*

What proportion of adults adhere to these guidelines? Are there differences among different demographics?

Physical Activity Measurement Survey (PAMS)

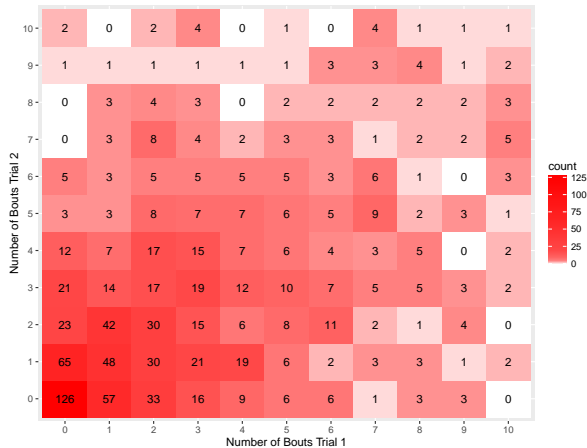
2 year study starting in 2009 collecting 2 days of physical activity data via Sensewear Armband and 24 Hour Recall on adults in four Iowa counties

- Measurements of physical activity are over 24 hours
- Replicate measurements taken 2-3 weeks apart
- Minute by minute data from armband (in MET-minutes)
- Survey weights needed
- Measurement error involved

- Y_{1ij} : Number of bouts individual i during trial j participated in
- Y_{2ij} : Total MET-minutes individual i during trial j accrued in at least 10 minute bouts minus $30 * Y_{1ij}$ (total excess MET-minutes)
- Z_i : Observed demographic variables for individual i
- X_{1i} Individual i 's expected number of bouts per day
- X_{2i} Individual i 's expected total excess MET-minutes per day
- X_{3i} Individual i 's expected total MET-minutes in MVPA per day
($= 30 * X_{1i} + X_{2i}$)

Exchangeability within person for Y_1

Bowker's test of symmetry: p-value = 0.1474



Exchangeability within person for Y_2

Total excess MET-minutes depends on number of bouts, need to account for it as well as eliminate individual effects

$$Y_{2i1} - Y_{2i2} = \beta_0 + \beta_1(Y_{1i1} - Y_{1i2}) + \beta_2(Weekend_{i1} - Weekend_{i2}) + \epsilon_i$$

$$\epsilon_i \stackrel{iid}{\sim} N(0, \sigma^2)$$

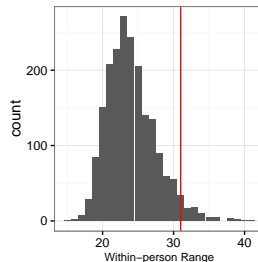
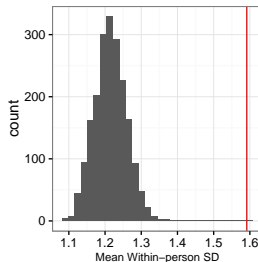
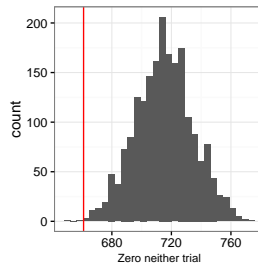
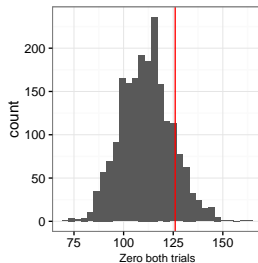
Coefficient	Estimate	Std Error	P-value
β_0	10.129	7.765	0.192
β_1	75.275	2.315	<0.0001
β_2	3.234	12.868	0.802

Modeling Number of Bouts

Model Y_1 as a Poisson-Gamma mixture with mean of Gamma as a regression on covariates \mathbf{Z} (use BMI, age, gender, smoker)

$$\begin{aligned}
 Y_{1ij} | X_{1i} &\overset{\text{ind}}{\sim} \text{Poisson}(X_{1i}) \\
 X_{1i} | \eta, \mu_i &\overset{\text{ind}}{\sim} \text{Gamma}\left(\eta, \frac{\eta}{\mu_i}\right) \\
 \mu_i &= e^{\mathbf{Z}_i' \boldsymbol{\beta}} \\
 p(\eta) &\sim \text{Gamma}(a_1, a_2) \\
 p(\boldsymbol{\beta}) &\sim N\left(\mathbf{0}_k, \frac{1}{v} I_{k \times k}\right)
 \end{aligned}$$

Assessing Overdispersion



Two-Part Model for Y_2

$$Y_{2ij}|X_{2i}, X_{1i} \stackrel{ind}{\sim} (1 - \pi_{ij}(\alpha))\delta_0(Y_{2ij}) + \pi_{ij}(\alpha)LN(\mu_{y,i}(\beta), \sigma_y^2)$$

$$X_{2i}|X_{1i}, Z_i \stackrel{ind}{\sim} LN(\mu_{x,i}(\gamma), \sigma_x^2)$$

$$\pi_{ij}(\alpha) = \Phi(\alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i})$$

$$\mu_{y,i}(\beta) = E(\log Y_{2ij} | Y_{2ij} > 0) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$

$$\mu_{x,i}(\gamma) = E(\log X_{2ij}) = \gamma_0 + \gamma_1 X_{1i} + \gamma_z' Z_i$$

Model Assessment

$$T_1(\mathbf{Y}_2) = \sum_{i=1}^{1057} \sum_{j=1}^2 I(Y_{2ij} = 0)$$

$$T_2(\mathbf{Y}_2) = \sum_{i=1}^{1057} I(Y_{2i1} = Y_{2i2} = 0)$$

$$T_3(\mathbf{Y}_2) = \frac{1}{2114} \sum_{i=1}^{1057} \sum_{j=1}^2 I\left(Y_{2ij} > \frac{450}{7}\right)$$

$$T_4(\mathbf{Y}_1, \mathbf{Y}_2) = \beta_1 \text{ coefficient from regression assessing day effect of for } Y_2$$

Estimating Probability of Compliance on Average

For ℓ from $\ell = 1, 2, \dots, L$ do:

1. Sample $\theta^{(\ell)}$ from the posterior distribution $p(\theta|\mathbf{Y}, \mathbf{Z})$
2. Simulate $X_{1i}^{*(\ell)}$ from $p(X_{1i}^*|\theta^{(\ell)}, Z_i)$ for $i = 1, \dots, n$
3. Simulate $X_{2i}^{*(\ell)}$ from $p(X_{2i}^*|\theta^{(\ell)}, X_{1i}^{*(\ell)}, Z_i)$ for $i = 1, \dots, n$
4. Calculate $X_{3i}^{*(\ell)} = 30X_{1i}^{*(\ell)} + X_{2i}^{*(\ell)}$ for $i = 1, \dots, n$
5. Calculate $p^{(\ell)} = \frac{1}{n} \sum_{i=1}^n I\left(X_{3i}^{*(\ell)} \geq \frac{450}{7}\right)$

Further work

We plan on the third chapter being a problem motivated from the PAMS data, the data we will be collecting, or the OPEN Study data

Like the first two chapters, it will likely be centered around Bayesian measurement error modeling