PHY820/422 HW #2 — Due Monday 9/18/17 @ 5pm
Lagrangians: Formal proofs and some applications

1. José and Saletan, Chapter 2, problem 3

   Prove that if L' is defined by (2.36), then Eq.(2.28) implies Eq.(2.37).

   Hints: You can use the following two facts that we showed in class (if you are not sure, try to prove them again starting from Eq. (2.35) of the textbook):

   (I) d/dt and $\partial/\partial q^\alpha$ commute. In particular, we have (now formulated in primed coordinates)

$$\frac{\partial}{\partial q'^\gamma}\dot{q}^\alpha = \frac{d}{dt}\left(\frac{\partial q^\alpha}{\partial q'^\gamma}\right) \tag{1}$$

   (II) Cancellation of dots in generalized coordinates:

$$\frac{\partial \dot{q}^\alpha}{\partial \dot{q}'^\gamma} = \frac{\partial q^\alpha}{\partial q'^\gamma} \tag{2}$$

2. Goldstein (Ed. 2), Chapter 1, problem 14

   If L is a Lagrangian for a system satisfying Lagrange's equations, show by direct substitution that

$$L' = L + \frac{dF(q^\alpha, t)}{dt}$$

   also satisfies Lagrange's equations where F is any arbitrary, but differentiable, function of its arguments.

   Hint: Use another result that we derived in class. For any function $F = F(q^\alpha, t)$, we have

$$\frac{dF(q^\alpha, t)}{dt} = \frac{\partial F}{\partial q^\alpha}\,\dot{q}^\alpha + \frac{\partial F}{\partial t} \tag{3}$$

   Don't forget that repeated indices mean a summation (in this case, over α).

3. Goldstein (Ed. 2), Chapter 1, problem 16

   A Lagrangian for a particular physical system can be written as

$$L' = \frac{m}{2}(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2) - \frac{K}{2}(ax^2 + 2bxy + cy^2)$$

   where a, b, and c are arbitrary constants but subject to the condition that $b^2 - ac \neq 0$. What is the physical system described by the above Lagrangian?

4. José and Saletan, Chapter 2, problem 9

   A double plane pendulum consists of a simple pendulum (mass m_1, length l_1) with another simple pendulum (mass m_2, length l_2) suspended from m_1, both constrained to move in the same vertical plane.

   (a) Write down the Lagrangian of this system in suitable coordinates.

   (b) Derive the Euler-Lagrange equations.

5. José and Saletan, Chapter 2, problem 10

   Consider a stretchable plane pendulum, that is, a mass m suspended from a spring of spring constant k and unstretched length l, constrained to move in a vertical plane. Define the angle from the vertical axis to be θ and the length stretch to be s. Write down the Lagrangian and obtain the Euler-Lagrange equations. Is energy to conserved? Why or why not? **Bonus point:** Use the method (to be covered on Wednesday) to obtain a constant of motion, and check if it is the same as or different from energy.