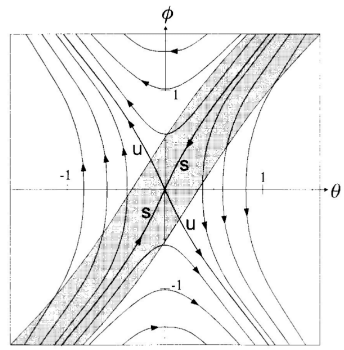
# PHY820/422 HW #6 — Due Monday 10/23/17 @ 5pm
# Scattering and linear oscillations

1. **Scattering off of two disks.** On the following figure identify the points that correspond to trajectories that leave the scattering region above and those that leave it below the two disks. Argue that the curve of trapped orbits is the separatrix between these two sets. Find a way to determine which region of the figure is forbidden to first hits.



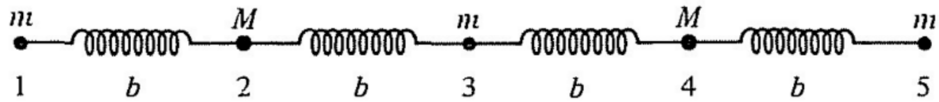

2. **Two vs three disks.** This is a question where you are expected to provide qualitative answers and plots. Consider the scattering off of the two disks that we discussed in class. For simplicity, let's fix $\theta_0 = \pi/4$ and vary $\phi_0$ (see the lecture notes or the textbook for the definition of the angles). Plot the qualitative features of the "dwell time", i.e. the time that the particle spends in the scattering region, as a function of the angle $\phi_0$. How does this compare to the scattering features for three disks in Fig. 4.11?

3. **Cantor sets and the Lyapunov exponent.**

   (a) A Cantor set can be formed by removing some fraction $1/f$ other than 1/3 of the intervals in each step. What is the fractal dimension of such a Cantor set? It will be useful to define the quantity $g = (f-1)/f$.

   (b) In the case of the scattering off of three disks, let's define two trajectories to be *equivalent of order* $n$ if they are the same $n$-string in our representation in terms of $l$ and $r$. What is the probability that two trajectories that are equivalent of the order $n$ would also be equivalent of the order $n+1$? (Assume that the "size" of the set $I_n$ is reduced by a factor $g$ upon the $n+1$-th bounce.) What is the probability that two trajectories will be separated in $m$ collisions? Expressing this probability as $e^{-\lambda m}$ the exponent $\lambda$ is called the Lyapunov exponent. What is the Lypaunov exponent in terms of $g$?

4. **Linear oscillations.** A 5-atom linear molecule is simulated by a configuration of masses and ideal springs that looks like the following diagram:




All force constants are equal. find the eigenfrequencies and normal modes for longitudinal vibration. [Hint: transform the coordinates $x_i$ to $\xi_i$ defined by

$$x_1 = \frac{\xi_1 + \xi_5}{\sqrt{2}}, x_2 = \frac{\xi_2 + \xi_4}{\sqrt{2}}, \quad x_3 = \xi_3, \quad x_4 = \frac{\xi_2 - \xi_4}{\sqrt{2}}, \quad x_5 = \frac{\xi_1 - \xi_5}{\sqrt{2}}$$

The matrix $\Lambda$ takes a simpler form in the new basis.]

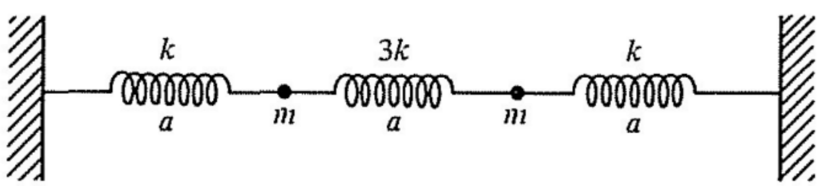
5. **Linear oscillations.**

   (a) Three equal mass points have equilibrium positions at the vertices of an equilateral triangle. They are connected by equal springs that lie along the arcs of the circle circumscribing the triangle. Mass points and springs are constrained to move only on the circle, so that, e.g., the potential energy of a spring is determined by the arc length covered. Determine the eigenfrequencies and normal modes of small oscillations in the plane. Identify physically any zero frequencies.

   (b) Suppose one of the springs has a change in force constant $\delta k$, the others remaining unchanged. To first order in $\delta k$ what are the changes in the eigenfrequencies and normal modes?

   (c) Suppose what is changed is the mass of one of the particles by an amount $\delta m$. Now how do the normal eigenfrequencies and normal modes change?

6. **Linear oscillations.** Two particles move in one dimension at the junction of three springs, as shown in the figure. The springs all have unstretched lengths equal to $a$, and the force constant and masses are shown.




Find the eigenfrequencies and normal modes of the system.