PHY820/422 HW #3 — Due Monday 9/25/17 @ 5pm
Lagrangians: More practice

1. José and Saletan, Chapter 2, problem 11 (simplified version)

   A wire is bent into the shape given by $z = A\rho^2$ and oriented vertically opening upward, in a uniform gravitational field g. Here, z and ρ are defined in cylindrical coordinates. The wire rotates at a constant angular velocity Ω about the vertical (z) axis, and a bead of mass m is free to slide on it without friction.

   (a) Find the equilibrium height of the bead on the wire.

   (b) Find the frequency of small vibrations about the equilibrium position(s).

   You are excepted to tackle part (b) in two different ways. (i) Expand the Euler-Lagrange equation around the equilibrium point(s) (ii) Construct a conserved quantity from the Lagrangian, and expand it around the equilibrium point(s) to find the frequency of small oscillations.

   Bonus point: Solve the same problem for a wire bent into the shaped $z = A\rho^n$ for a positive n.

2. Goldstein (Ed. 2), Chapter 1, Problem 18

   A particle of mass m moves in one dimension such that it has the Lagrangian

   $$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - V(x)^2$$

   where V is some differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation.

3. Jose and Saletan, Chapter 3, Problem 12(a).

   Describe the motion of the Lagrangian $L = \dot{q}_1\dot{q}_2 - \omega^2 q_1 q_2$. Describe the physical motion and write another Lagrangian (L') that produces the same equations of motion. Is it possible to relate the two Lagrangians by a total time derivative, i.e., $L - L' = dF/dt$ for a function $F(q_1, q_2, t)$?

   Hint: Read section 2.2.2 of the textbook.