NAME: Key In Class Jan 30

1. Answer the following questions about the matrices below:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 11 & -5 & 3 \\ 1 & 3 & -1 & 2 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}.$$

(a) Compute BC. What effect does B have on the rows of C? *[1 point]*

$$BC = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$$

switches/exchanges row 1 & 2

B is a permutation or row exchange matrix

(b) Compute ABC. What effect does A have on the rows of BC? *[1 point]*

$$A(BC) = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 0 & -11 & 5 & -3 \end{pmatrix}$$

A is an elimination matrix (E_{41}) eliminates entry in Row 4 Col 1 by subtracting 4 times R1 from R4 of BC

(c) Write the inverse matrix, A^{-1}, which reverses the effect of A on matrix rows. *[1 point]*

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}$$

This matrix adds 4 times R1 to Row 4

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2. Write down the augmented matrix $[\ A \mid \vec{b}\ ]$ for the following system of equations. Use elimination to reduce the system to upper triangular form, and then back substitute for z, y, x. Show all your steps and write down the elimination (row exchange) matrix used in each step. *[4 points]*

$$x + 2y + z = 1$$
$$3x + 7y + 3z = 1$$
$$-2x - 3y - 4z = -1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 3 & 7 & 3 & 1 \\ -2 & -3 & -4 & -1 \end{array}\right]$$ augmented matrix

Elimination

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{21}[A \mid \vec{b}] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ -2 & -3 & -4 & -1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad E_{31}(E_{21}[A \mid \vec{b}]) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$R_3 \to R_3 + 2R_1$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad E_{32}(E_{31}E_{21}[A \mid \vec{b}]) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & 3 \end{array}\right]$$

$R_3 \to R_3 - R_2$

upper triangular $\to$

$x = \frac{13}{2}, y = -2, z = -\frac{3}{2}$

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3. Choose the numbers p, q, r, s in this augmented matrix so that there is (a) no solution (b) infinitely many solutions.

$$\left(A \mid \vec{b}\right) = \left(\begin{array}{ccc|c} 3 & 12 & -6 & p \\ 0 & 1 & 3 & q \\ 0 & 0 & s & r \end{array}\right)$$

Which of the numbers p, q, r or s have no effect on the solvability? *[3 points]*

a) no solution when $s=0$ and $r \neq 0$

then R_3 is $0x+0y+0z = r$, not possible

b) infinitely many solutions when

$s=0$ and $r=0$

then R_3 is $0x+0y+0z=0$, infinitely many!

NOTE: p & q do not affect the solvability