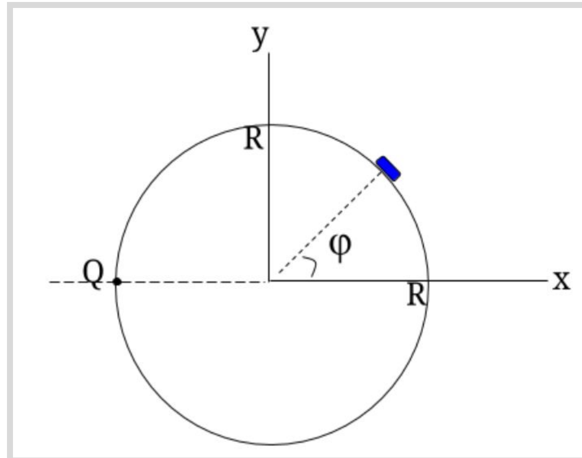
Name_____solution key________________________________

A car drives around a circular track (radius = R) with constantly increasing speed. (See the figure.) The angle ϕ as a function of time t is $\phi(t) = ½\ \alpha\ t^2$ where α is constant.



(A) Write the coordinates x(t) and y(t).

(B) Calculate the velocity and acceleration vectors. Make an ***accurate*** drawing that shows the velocity and acceleration vectors when the car first passes the point Q.

(A)

$x = R \cos\phi = R \cos(\alpha t^2/2)$

$y = R \sin\phi = R \sin(\alpha t^2/2)$

(B)

$\mathbf{v} = \mathbf{e}_x x' + \mathbf{e}_y y' = -R\,\alpha t \sin(\alpha t^2/2)\,\mathbf{e}_x + R\,\alpha t \cos(\alpha t^2/2)\,\mathbf{e}_y$

$\mathbf{a} = \mathbf{e}_x x'' + \mathbf{e}_y y'' = [-R\,\alpha \sin(\alpha t^2/2) - R(\alpha t)^2 \cos(\alpha t^2/2)]\,\mathbf{e}_x$

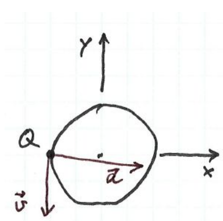
$\qquad + [R\,\alpha \cos(\alpha t^2/2) - R(\alpha t)^2 \sin(\alpha t^2/2)]\,\mathbf{e}_y$

At Q,

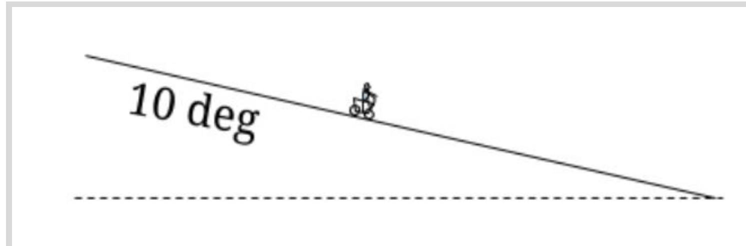
$\phi = \alpha t^2/2 = \pi$; sin = 0 and cos = – 1;

$\mathbf{v} = -R\,\alpha\,t\,\mathbf{e}_y$

and $\mathbf{a} = R(\alpha t)^2\,\mathbf{e}_x - R\,\alpha\,\mathbf{e}_y = R\,\alpha\,\{2\pi\,\mathbf{e}_x - 1\,\mathbf{e}_y\}$



A bicycle rider coasts down a long hill. (See the figure.) The angle of the slope is θ = 10 degrees = 0.174 radians. (A) Using your knowledge about air resistance, derive a formula for the terminal speed of the bicycle. Be sure to define any parameters that you use. (A numerical calculation is not necessary.)



(B) Derive the speed as a function of time, starting from speed $v_0 = 0$ at $t = 0$.

(A) Let v be the velocity pointing down the slope.

$m v' = F = mg \sin(\theta) - c v^2$

At terminal velocity, F = 0 ; thus $v_{ter} = \text{Sqrt}[\, mg \sin(\theta)/c \,]$

(B) Using separation of variables,

$$\frac{dv}{(mg \sin(\theta) - cv^2)} = \frac{dt}{m}$$

Integrate both sides of the equation == > $t/m = (1/c)\, \text{arctanh}(v/v_{ter})$

Result is $v(t) = v_{ter}\, \text{Tanh}[\, ct/m \,]$

A single-stage rocket has these parameters:

mass rate = 100 kg/s ; $=K$

exhaust relative speed = 400 m/s; $=v_{ex}$

the initial mass = 1000 kg ; $= m_0$

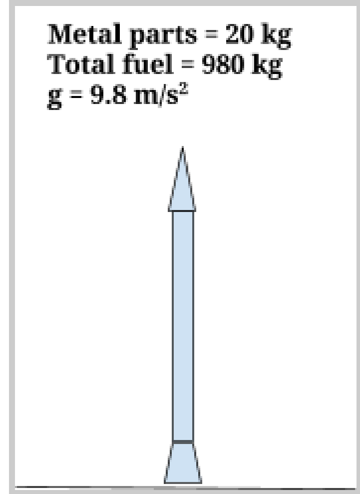
the final mass = 20 kg. $= m_F$

The rocket is fired upward from the surface of the Earth.

(A) Calculate the speed as a function of time.

(B) Sketch an accurate plot of v versus time.

(C) Use the plot to estimate the height when the fuel runs out.



(A) The rocket equation is

$$m\, v' = K\, v_{ex} - m\, g \qquad \text{where } m = m_0 - K\, t$$

Integrate the equation:

$$dv/dt = - g + K\, v_{ex}/(m_0 - K\, t)$$

$$v = - g\, t + K\, v_{ex}\, (- 1/K) \ln (m_0 - Kt') \,\Big|_{t'=0}^{t' = t}$$

$$= - gt + K\, v_{ex} \ln \{ (m_0 /(m_0 - Kt) \}$$

(B) $v(0) = 0$; $v_F = - g\, t_F + K\, v_{ex} \ln \{ m_0 / m_F \}$

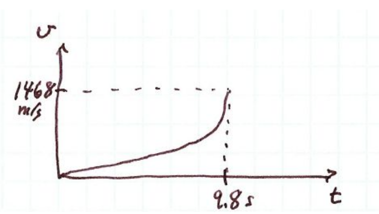
Numerical: At burnout, $t_B = 980 \text{ kg} / (100 \text{ kg/s}) = 9.8 \text{ s}$

(i) $- g\, t_B = - 9.8 * 9.8 \text{ m/s} = - (10-0.2)^2 = - (100 - 4 + 0.04) = - 96 \text{ m/s}$

(ii) other term = $100 * 400 * \ln (1000/20) =$ 1564 m/s

$v_F = 1468$ m/s

(C)



height = area under the curve

≈ 0.5 ½(9.8 s)(1468 m/s)

≈ 3600 m = 3.6 Km