Measuring Trapping Forces in Optical Tweezers

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Introduction

Since the work of Ashkin[1] on radiation pressure, the possibility to use the optical force of optical fields to trap and manipulate microparticles has found a wide range of applications. From atomic and nonlinear physics to biology, optical forces have provided a convenient way to control the dynamics of small particles[1].Optical tweezers, for example, have proved useful not only for trapping particles but also for assembling objects ranging from microspheres to biological cells[2].

In this paper, we trap polystyrene particles of different sizes with a TEM_{00} Gaussian beam and a radially polarized doughnut mode beam of different powers. We then measure the transverse trapping forces with the viscous-drag-force technique.

Trapping Theory

Optical tweezers use gradient forces to trap particles. The gradient force used in optical tweezers arises from oscillating electric dipoles that are induced when light passes through transparent objects, which consequently experience a time-averaged force in the direction of the field gradient. Trapping is stable when the gradient force overcomes the scattering force, which is the force due to light scattering. The scattering force acts in the direction of light propagation, i.e. in axial direction in case of a propagating laser beam.

In the geometrical optics regime, where d >> λ, a simple ray-optic picture can explain the phenomenon (Figure 1). [3]
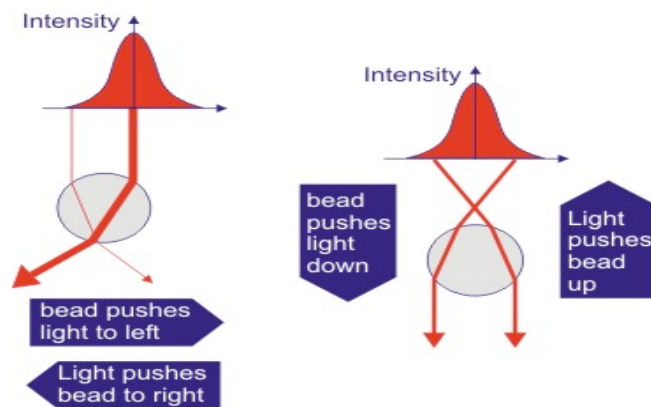

Figure .1 Ray-optic picture of the optical gradient force. From Ref.[4].

Rays of light carry momentum and are bent by refraction when passing through a dielectric sphere with a refractive index, n, greater than the surrounding medium. By conservation of momentum, the rate of change of momentum in the deflected rays conveys an equal and opposite rate of change in momentum to the sphere (Newton's Second Law). When a dielectric sphere is placed in a light gradient, the sum of all rays passing through it generates an imbalance in force, tending to push the sphere towards the brighter region of the light.

In this regime[4], for a single ray $F_{scat} = \dfrac{n_m P}{c} q_s$, $F_{grad} = \dfrac{n_m P}{c} q_g$

where,

$$q_s = 1 + R_{TE} \cos 2\theta_1 - T_{TE}^2 \frac{\cos 2(\theta_1 - \theta_2) + R_{TE} \cos 2\theta_1}{1 + 2R_{TE} \cos 2\theta_2 + R_{TE}^2}$$

$$q_g = R_{TE} \cos 2\theta_1 - T_{TE}^2 \frac{\sin 2(\theta_1 - \theta_2) + R_{TE} \cos 2\theta_1}{1 + 2R_{TE} \cos 2\theta_2 + R_{TE}^2}$$

and

$$R_{TE} = \left(\frac{\cos\theta_1 - \sqrt{n^2 - \sin^2\theta_1}}{\cos\theta_1 + \sqrt{n^2 - \sin^2\theta_1}}\right)^2 , \quad T_{TE} = 1 - R_{TE} , \quad n = {n_p}/{n_m} ,$$

where n_m is the refractive index of the surrounding medium, n_p the refractive index of the particle, P the power of the trapping laser, q_s the scattering efficiency, q_g the gradient trapping efficiency, θ_1 the incident angle, θ_2 the refractive angle, R the reflection coefficient, and T the transmission coefficient. If we add all forces associated with different incident angles θ_1 we obtain the total force F_{scatt} and F_{grad} acting on the particle.

In the Rayleigh regime [6], where d $\ll \lambda$, the particle acts as a dipole and the forces arise from dipole-field interactions [5].

$$F_{scat} = \frac{I_0}{c} \frac{128\pi^5 r^6}{3\lambda^4} \left(\frac{n^2 - 1}{n^2 + 2}\right)^2 n_m$$

$$F_{grad} = -\frac{n_m}{2} \alpha \nabla E^2 = -\frac{n_m^3 r^3}{2} \left(\frac{n^2 - 1}{n^2 - 2}\right) \nabla E^2 , \tag{1}$$

where m is the effective index defined as the ratio of the refractive indices of particle and environment, I_o is the laser beam intensity, $r_o = d/2$ is the radius of the particle, and λ is the wavelength of the laser.

In the case where d is comparable to the wavelength of the trapping light, optical forces need to be calculated using Mie theory [7]. Here, we use a simplified Mie theory [8] in our theoretical calculations. The focused Gaussian laser beam intensity can be approximated as $I = I_o \exp\left(-\dfrac{r^2}{2\omega^2} - \dfrac{z^2}{2\omega^2\varepsilon^2}\right)$, where ε is the axial direction eccentricity of the focal volume, ω is the beam waist radius, and r and z are cylindrical coordinates. Using this intensity profile, the transverse trapping force in the plane z=0 is given by (Ref[8])

$$F_{grad}(r) = \alpha I_o \omega^2 A(\varepsilon) e^{-\left(\frac{r^2}{2\omega^2}\right)} \left[\sinh\left(a_c \frac{r}{\omega}\right)\right] \tag{2}$$

where $\alpha = \dfrac{\varepsilon_p}{\varepsilon_m} - 1$ is the relative difference of the dielectric constants of the

particle ε_p, and the surrounding medium ε_m, $A(\varepsilon) = 4\pi \ \varepsilon \ erf(\dfrac{a_c}{\sqrt{2}})erf(\dfrac{a_c}{\varepsilon\sqrt{2}})e^{-a_c^2/2}$

and $a_c = \dfrac{r_o}{\omega}\left(\dfrac{\pi}{6}\right)^{1/3}$.

Since recently, optical tweezers make increasingly use of laser beams with a doughnut-shaped beam profile. As mentioned in Ashkin's paper [5], although the doughnut mode beam will reduce the transverse trapping efficiency a little bit, it will improve the axial trapping efficiency. If absorbing particles, such as metal particles, are being trapped, then the scattering force will dominate over the gradient force if a TEMoo (Gaussian) beam is employed. Therefore, for absorbing particles, the doughnut beam becomes the only choice for stable trapping. In this paper, we also measured the transverse trapping force of a doughnut mode beam, and compared it with the trapping force of a Gaussian mode trap.

Experimental configuration

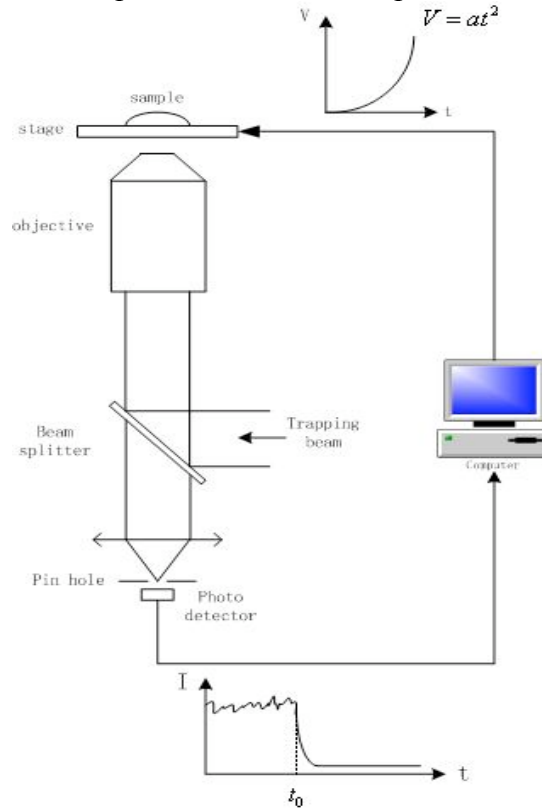Figure 2 shows a schematic representation of our experimental setup.

Figure 2. Experimental setup
The stage control signal and the detector output are also shown

A diode pumped frequency-doubled Nd: YVO$_4$ laser (Coherent Verdi 8W max. at 532 nm) is used as a trapping source. After passing through a single-mode fiber, light from the laser is first collimated and then passed through λ/4 and λ/2 waveplates to obtain linear polarization. The beam is then further expanded and is directed to a custom built inverted microscope. Alternatively, the expanded Gaussian beam can be sent

through a mode converter in order to achieve a radially polarized doughnut mode. The mode converter we are using consists of four half-waveplates, one in each quadrant as shown in figure 3. The optical axis of each segment is oriented such that the field is rotated to point in the radial direction after passing through the converter.
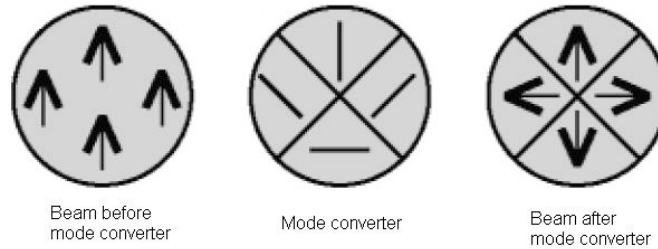

Beam before mode converter   Mode converter   Beam after mode converter

Figure 3 Mode converter

The final beam is reflected by a 90/10 nonpolarizing beam splitter and focused by a high NA oil immersion objective (Nikon 60×, NA=1.4) onto a microscope cover slip, hosted on a piezo driven stage (MadCity Labs, Inc). A droplet with a factory prepared polystyrene bead solution with bead diameters of 350, 500 and 1000 nm (Polyscience, Inc) is placed on the cover slip.

The scattered light from an individual trapped polystyrene bead is collected by the focusing objective and is focused through a 500 micron pinhole to reduce background light. The scattered light is then detected by a PIN photodiode. The signal from the photodiode is acquired by a National Instruments data acquisition card and is analyzed by custom written Labview software. The same card is used to generate an output signal for controlling the piezo stage. The computer generated control signal y(t) is a second-order power function $y = at^2$ with an acceleration a. This function accelerates the stage together with the sample in a given direction. Consequently, the viscous force $F_{vis} = 6\pi\eta rv = 6\pi\eta rat$ acting on the trapped particle increases accordingly. Here, η is the viscous coefficient of water (10^{-3} at room temperature) and r is the radius of trapped particle, and 'v' the relative speed between particle and surrounding medium. When the viscous force equals the trapping force, the trapped particle escapes from the laser focus. This event causes a drop in the detected scattered light signal from the photodetector. The detector signal is recorded and analyzed to calculate the speed v_o of the piezo stage at the time t_0 of the particle's escape. Since the particle's motion in a viscous fluid is overdamped, the optical gradient force must equal the viscous force, and hence the velocity v_o determines the maximum magnitude of the trapping force. The experiment has been repeated several times to reduce measurement errors due to Brownian motion.

Experimental Results

Figure 4 shows the theoretical and experimental curves for the maximum trapping force as a function of the particle size. The theoretical curve is calculated using equation (2). The trapping beam power is 32 mw in both cases. The three points on the experimental curve correspond to 350, 500 nm and 1000 nm polystyrene beads.
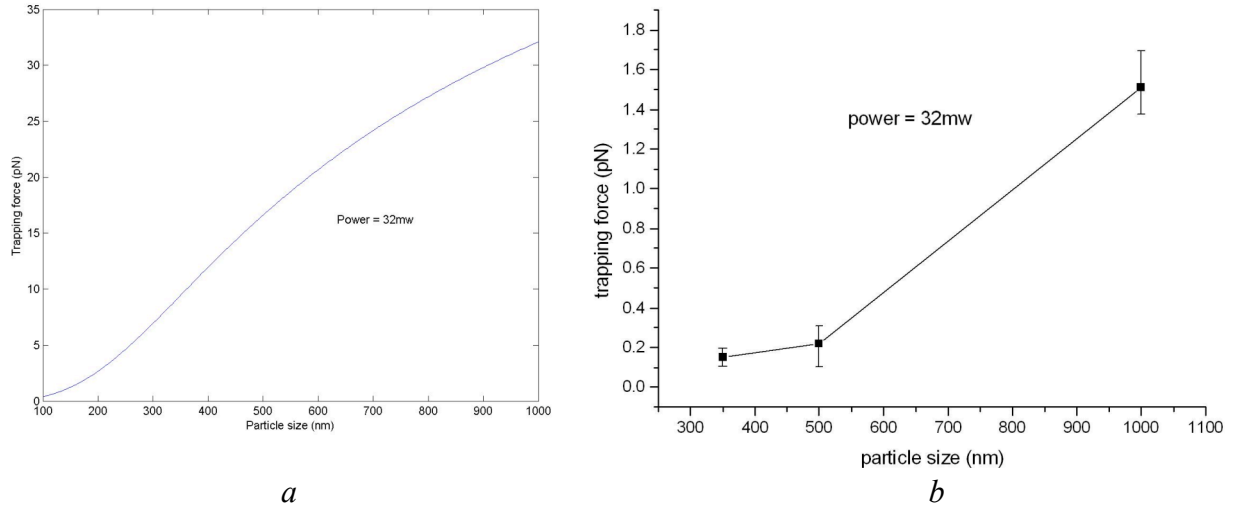
a   *b*

Figure4. Trapping force as a function of particle size
a)   theoretical curve b) measured value

We can see, the slopes of these two curves are different. If we study equation (2) relating the gradient force F_{grad} to the particle size r_0, we find that one possible reason for this inconsistency is the deviation of our focus from the assumed model. Thus, the beam waist ω could be different from what it should be.

Figure 5 shows the theoretical and experimental curves for the maximum trapping force for 1 micrometer beads as a function of trapping laser power. The measured trapping force is less than the force predicted by theoretical calculations which could originate from a) beam aberrations leading to a distorted laser focus, b) absorption in the objective lens and c) other experimental errors.
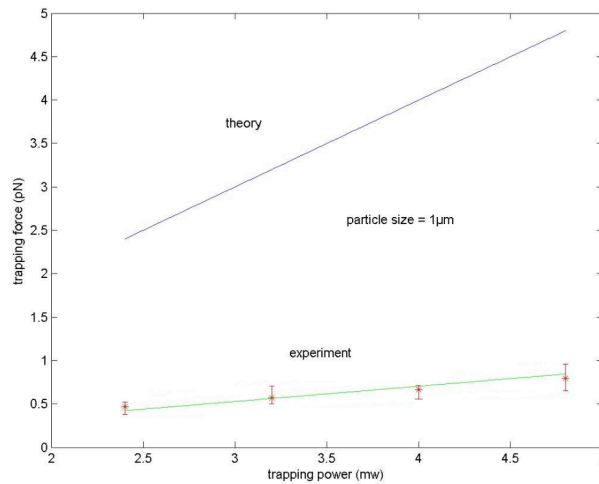

Figure 5: Trapping force as a function of trapping power.
(blue line) theoretical curve, (green line) measured curve.

The slopes of these two lines are also different. This again could originate from an imperfect laser focus.

In figure 6, we compare the trapping force acting on a 1000 nm polystyrene bead interacting with a Gaussian beam and a radially-polarized doughnut beam.
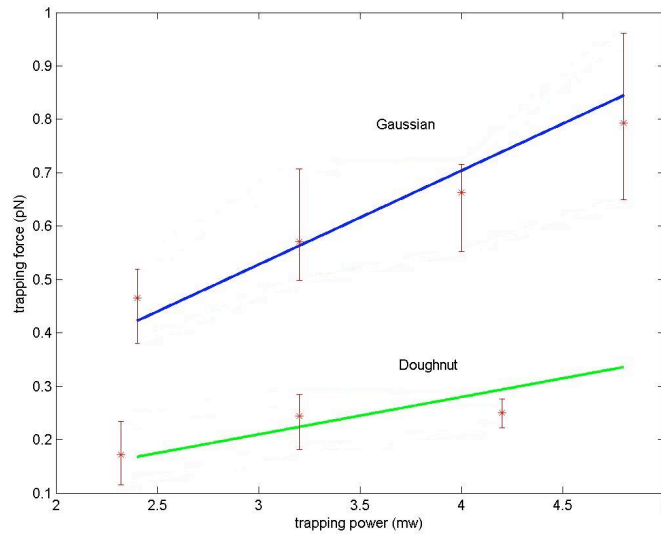

Figure 6. Trapping force as a function of trapping power
for a Gaussian beam and a radially polarized doughnut mode beam.
(blue line) Gaussian beam, (green line) doughnut mode beam.

We observe that the slope of the green line i.e. the transverse trapping efficiency of the doughnut beam is lower than that of the Gaussian beam (blue line) as predicted in Ashkin's work.

Conclusions

We have developed an optical tweezers system and tested its performance by trapping polystyrene beads of different sizes using a regular Gaussian beam and a radially polarized doughnut beam. We have measured the trapping force with the drag force technique and compared the results with theoretical predictions. The measured trapping force has linear dependence on trapping power, and we compared the trapping forces for a Gaussian beam and a doughnut mode beam.

References

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