TAKE OFF FROM EARTH'S SURFACE

We have

$$m\,(dv/dt) = v_{ex} K - mg$$

Approximate

g = constant and v_{ex} = constant.

How to integrate the diff. eq.?

$$m\,dv = v_{ex} K\,dt - mg\,dt$$

but $m = m(t)$!

$$= -\,v_{ex}\,dm - m\,g\,dt$$

$$dv = -\,v_{ex}\,(dm/m) - g\,dt$$

Integrate

$$\int_0^v dv' = -\,v_{ex} \int_{m0}^{m} dm'/m' - g \int_0^t dt'$$

$$v(t) = v_{ex}\,\ln\,[m_0/m(t)] - gt$$

Also, $m(t) = m_0 - Kt$ *(assuming K is constant)*

Graph of v as a function of t

(1) Slope ~ $(v_{ex}K - m_0 g)/m_0$

(2) rocket runs out of fuel

(3) slope = $-\,g$

(4) rocket starts to fall downward

Area under the curve = height