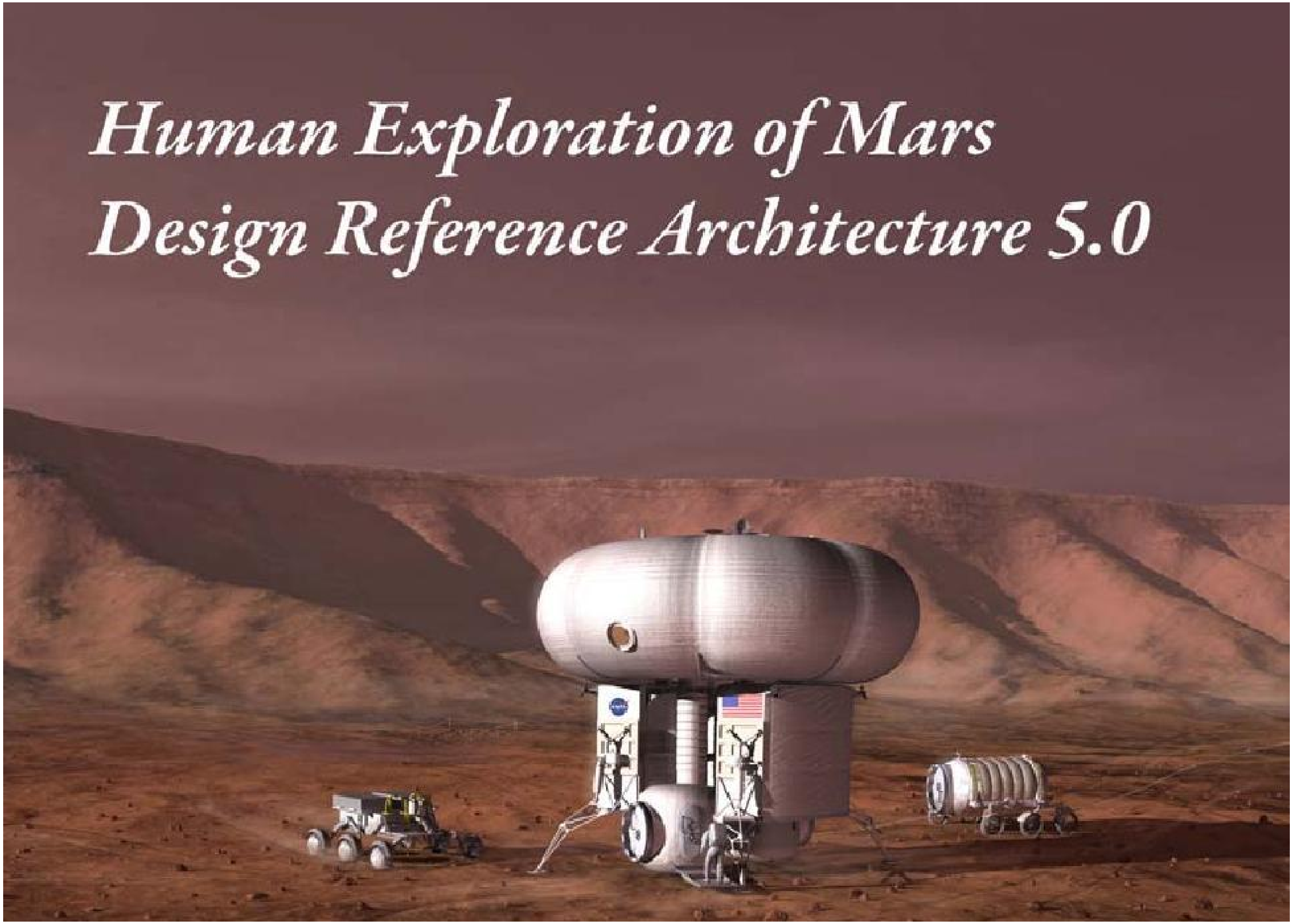
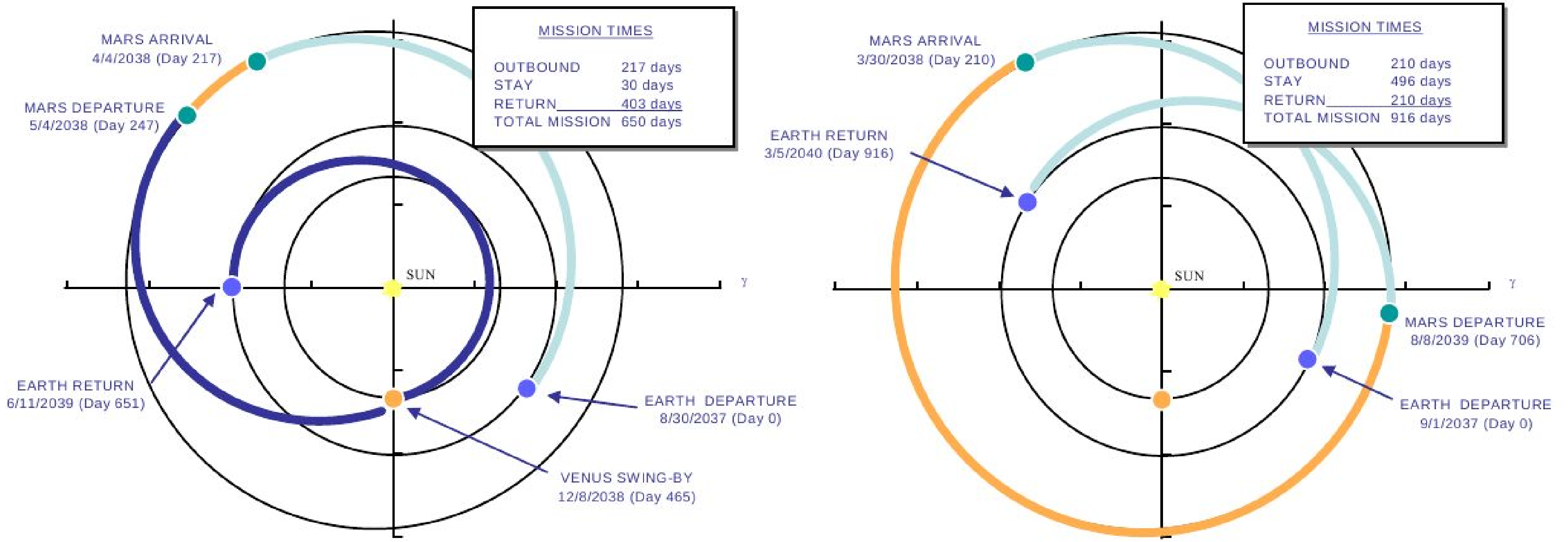
NASA is making plans to send astronauts to Mars (year 2040?)
If you want to read about it, search Google for the report ;
search for "Mars Design Reference Architecture 5.0 - NASA"



6.2 Decision 1: Mission Type

exists only two choices



a) Opposition Class: Short-Stay Mission

b) Conjunction Class: Long-Stay Mission

Figure 6-2. Comparison of (a) Opposition-class and (b) Conjunction-class mission profiles.

"short stay mission"
stay 30 days on Mars

difficult orbital dynamics

"long stay mission"
stay 496 days on Mars

**preferred
but very costly!**

Interesting complexity of the transfer orbit:
they must take into account the eccentricities of the orbits of Mars (0.0934) and Earth (0.0167) . It is not accurate enough to approximate the planetary orbits as circles.

Parametric equations for Keplerian orbits

Kepler's Equation $\quad M = E - \varepsilon \sin(E)$

This equation was published by Kepler (1619)

- M = "mean anomaly"
- E = "eccentric anomaly"
- ε = eccentricity

The coordinates of the planet are

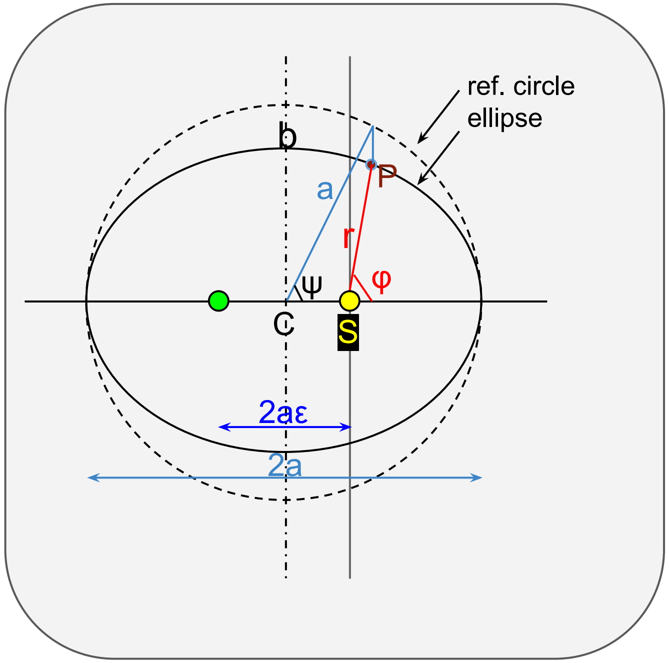
$$x = a\,[\cos(E) - \varepsilon\,]$$
$$y = b \sin(E)$$

where

- a = semimajor axis
- b = seminor axis

Kepler's equation is transcendental. Numerical analysis is necessary to solve for "E" .

I'll use ψ to denote the "eccentric anomaly".



reference circle; radius = a
orbit ellipse; semimajor axis = a
C = center ; S = Sun ; P = planet

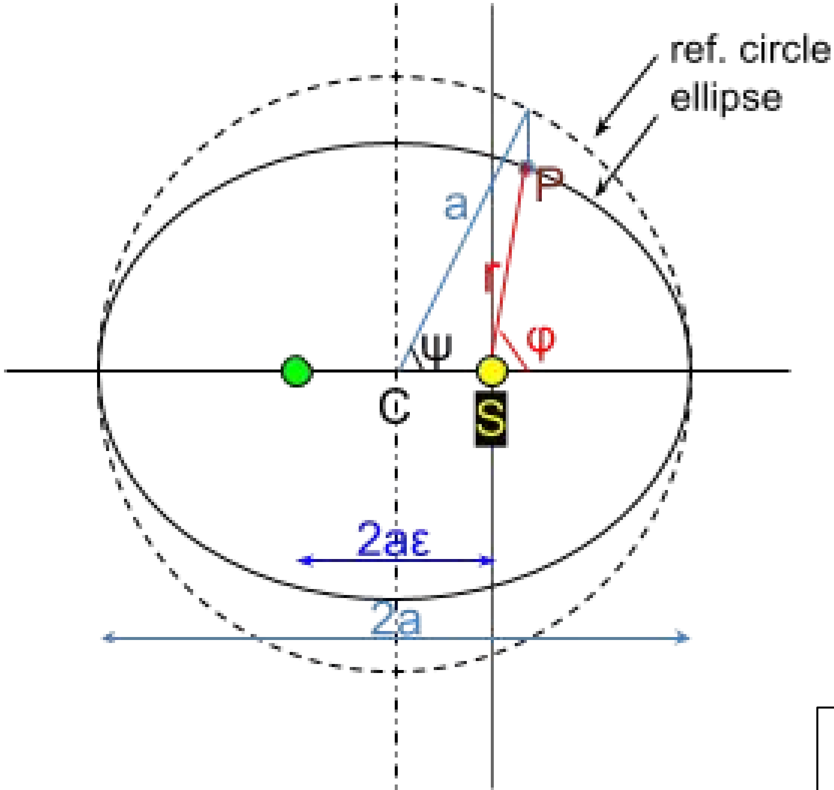
ψ = eccentric anomaly

$x = r \cos \varphi = a \cos \psi - a\varepsilon$

$y = r \sin \varphi = b \sin \psi$ *why? b/c*

$(x+a\varepsilon)^2 /a^2 + y^2 /b^2 = 1$ *(ellipse)*

Max. $x = a\,(1 - \varepsilon)$ at $\psi = 0$; <u>*CHECK*</u>
Max. $y = b$ at $\psi = \pi/2$;
$r = a$ at $\psi = \pi/2$
$(a\varepsilon)^2 + b^2 = a^2$ so $b = a$ SQRT[$1 - \varepsilon^2$]



reference circle; radius = a
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C = center ; S = Sun ; P = planet

ψ = eccentric anomaly

$x = r \cos\varphi = a \cos\psi - a\varepsilon$

$y = r \sin\varphi = b \sin\psi$

$(x+a\varepsilon)^2 / a^2 + y^2 / b^2 = 1$ *(ellipse)*

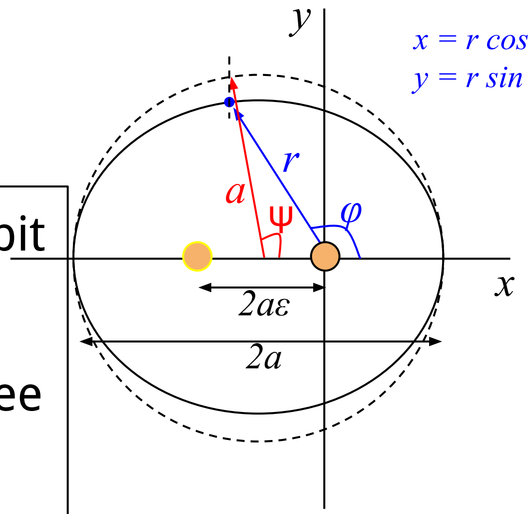
The sun is at the origin and the plane of the orbit has Cartesian coordinates x and y. The ***center*** is at $\{x,y\}=\{-a\varepsilon, 0\}$.

We can write parametric equations for all three variables
(time = t and spatial coordinates = x and y)
in terms of the independent variable ψ :

$$t = T / (2\pi) \ (\psi - \varepsilon \sin\psi) \qquad \textbf{(1)}$$

$$x = a \ (\cos\psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a \ (1 - \varepsilon^2)^{1/2} \ \sin\psi \qquad \textbf{(3)}$$



$x = r \cos\varphi = a\,(\cos\psi - \varepsilon\,)$
$y = r \sin\varphi = b \sin\psi$

The sun is at the origin and the plane of the orbit has Cartesian coordinates x and y.

We can write parametric equations for the three variables

(time = t and spatial coordinates = x and y)

in terms of the independent variable ψ :

$t = T\,/\,(2\pi)\;(\,\psi - \varepsilon \sin\psi\,)$ **(1)**

$x = a\,(\,\cos\psi - \varepsilon\,)$ **(2)**

$y = a\,(\,1 - \varepsilon^2\,)^{1/2}\,\sin\psi$ **(3)**

The parameters T , a and ε are

T = period of revolution; **$\psi \mapsto \psi + 2\pi$**

a = semimajor axis

ε = eccentricity.

In term of Kepler's variables,

$\psi = E$

$t = T/(2\pi)\;\; M$

$M = E - \varepsilon \sin E$

Proof of the parametric equations.

$$t = T/(2\pi)\ (\psi - \varepsilon \sin\psi) \qquad \textbf{(1)}$$

$$x = a(\cos\psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a(1-\varepsilon^2)^{1/2} \sin\psi \qquad \textbf{(3)}$$

$dt/d\psi = T/(2\pi)\ (1 - \varepsilon \cos\psi)$

We must prove that E (energy) and ℓ (ang. momentum) are constants of the motion.

Theorem 1

The angular momentum (ℓ) is a constant of the motion.

Proof

$$\ell = \mu(x\dot{y} - y\dot{x})$$

$$dx/dt = (dx/d\psi)(d\psi/dt) = (-a \sin\psi)(2\pi/T)(1 - \varepsilon\cos\psi)^{-1}$$

$$dy/dt = (dy/d\psi)(d\psi/dt) = (b\cos\psi)\ (2\pi/T)(1-\varepsilon\cos\psi)^{-1}$$

$$= \mu ab\{(\cos\psi - \varepsilon)\cos\psi + \sin^2\psi\}(2\pi/T)(1-\varepsilon\cos\psi)^{-1}$$

$$= \mu ab(2\pi/T) \text{ which is constant} \quad ✓$$

Also note: $T = (\pi ab)(2\mu/\ell)$ which agrees with Kepler's second law ; $dA/dt = \ell/(2\mu)$ (3.17) ➡ $A/T = \ell/(2\mu)$ ✓

$$t = T/(2\pi)\ (\psi - \varepsilon \sin\psi) \qquad \textbf{(1)}$$

$$x = a(\cos\psi - \varepsilon) \qquad \textbf{(2)}$$

$$y = a(1 - \varepsilon^2)^{1/2} \sin\psi \qquad \textbf{(3)}$$

Theorem 2

The energy (E) is a constant of the motion. Proof

$$E = \frac{1}{2}\mu(\dot{x}^2 + \dot{y}^2) - \frac{\gamma}{r}$$

$$= \frac{1}{2}\mu\{a^2 \sin^2\psi + b^2\cos^2\psi\}\left(\frac{2\pi}{T}\right)^2 (1 - \varepsilon\cos\psi)^{-2} - \frac{\gamma}{a}(1 - \varepsilon\cos\psi)^{-1}$$

$$= \frac{1}{2}\mu a^2 (2\pi/T)^2 \{1 - \varepsilon^2\cos^2\psi\}(1 - \varepsilon\cos\psi)^{-2} - \frac{\gamma}{a}(1 - \varepsilon\cos\psi)^{-1}$$

$$= \left\{\frac{1}{2}\mu a^2 \left(\frac{2\pi}{T}\right)^2 (1 + \varepsilon\cos\psi) - \frac{\gamma}{a}\right\}(1 - \varepsilon\cos\psi)^{-1}$$

$$= \{C_1 + C_2\,\varepsilon\cos\psi\}(1 - \varepsilon\cos\psi)^{-1} = E$$

(needs more!)

$$dx/dt = (dx/d\psi)(d\psi/dt)$$
$$= (-a \sin\psi)(2\pi/T)(1 - \varepsilon\cos\psi)^{-1}$$

$$dy/dt = (dy/d\psi)(d\psi/dt)$$
$$= (b\cos\psi)(2\pi/T)(1 - \varepsilon\cos\psi)^{-1}$$

$$r^2 = x^2 + y^2$$
$$= a^2\{(\cos\psi - \varepsilon)^2 + (1 - \varepsilon^2)\sin^2\psi\}$$
$$= a^2\{1 - 2\varepsilon\cos\psi + \varepsilon^2\cos^2\psi\}$$
$$= a^2(1 - \varepsilon\cos\psi)^2$$

$$b^2 = (1 - \varepsilon^2)a^2$$

$$\frac{A^2 - B^2}{(A - B)^2} = \frac{A+B}{A-B}$$

So, we have this …

$$\{ C_1 + C_2\, \varepsilon \cos\psi \} (1 - \varepsilon \cos\psi)^{-1} = E$$

where

$$C_2 = \tfrac{1}{2}\, \mu\, a^2\, (2\pi/T)^2$$

and $C_1 = C_2 - \gamma/a.$

$C_1 (1 + C_2/C_1\, \varepsilon \cos)(1 - \varepsilon \cos)^{-1} = E$

This must be a constant (E).

$C_2 / C_1 = -1$

$C_1 = E$

So, we require $C_2 / C_1 = -1$ and $C_1 = E$.

$C_1 = C_2 - \gamma/a = - C_1 - \gamma/a$

$C_1 = - \gamma / (2a)$

$C_2 = - C_1 = \gamma / (2a)$

$\therefore \gamma / (2a) = \tfrac{1}{2}\, \mu\, a^2\, (2\pi/T)^2$

<u>Result</u>

The theorem is true, and E is given by

$$E = - \frac{\gamma}{2a}. \quad ✓$$

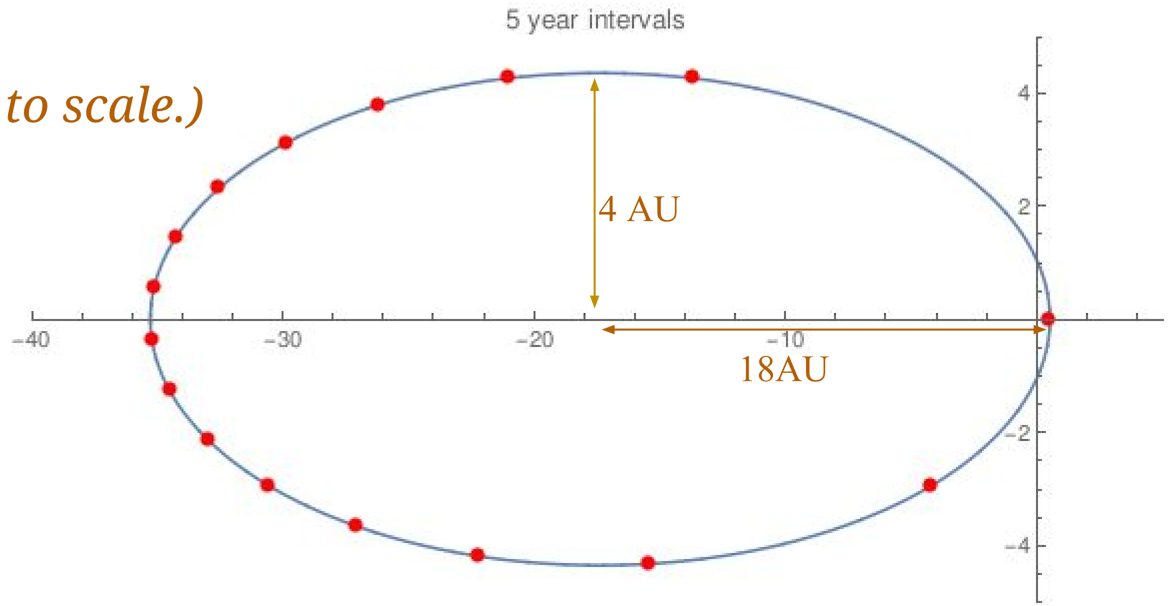
Also, we find $\gamma / (2a) = \tfrac{1}{2}\, \mu\, a^2\, (2\pi/T)^2$;

$$\therefore \quad T^2 = \frac{4\pi^2 a^3}{GM} \qquad \text{because } \gamma / \mu = GM,$$

which is Kepler's third law.

Example A. The orbit parameters of Halley's comet are a = 17.9 AU and ε = 0.97. Plot of the orbit of Halley's comet.

(Not drawn to scale.)



Example B. Calculate the perihelion distance.

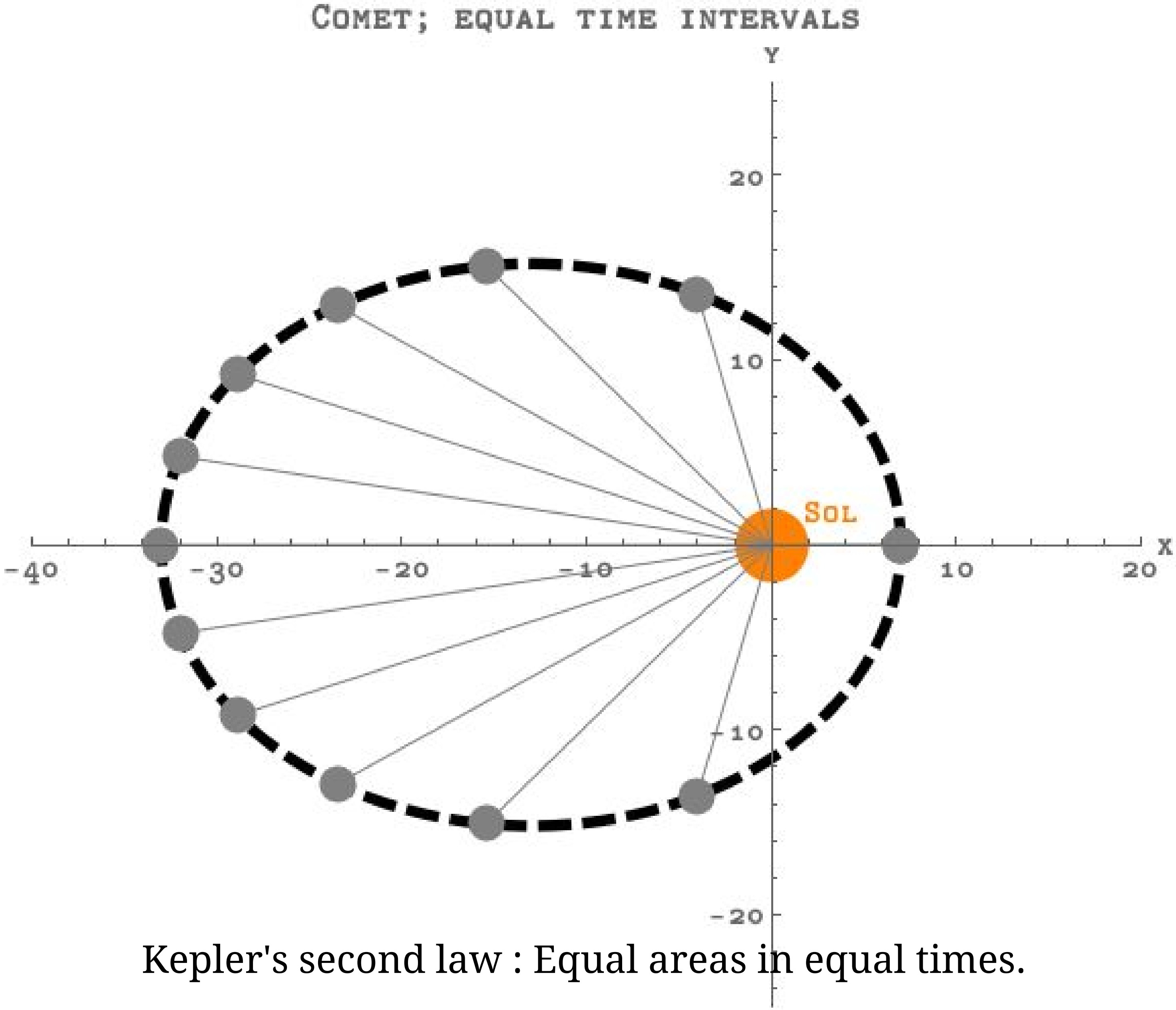
$$r_{min} = a\,(1 - \varepsilon) = 0.537 \text{ AU}$$

Example C. Calculate the aphelion distance.

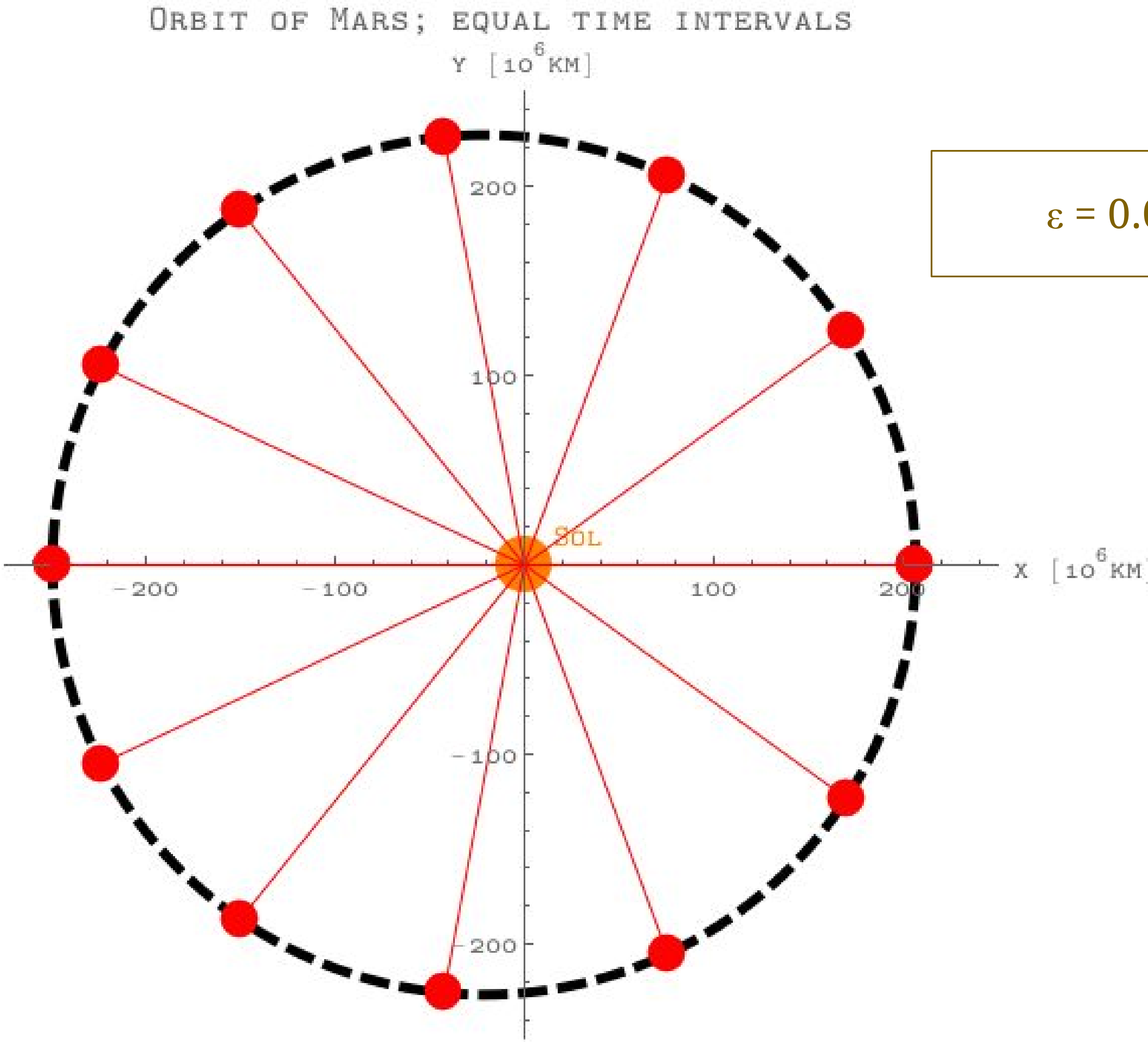
$$r_{max} = a\,(1 + \varepsilon) = 35.3 \text{ AU}$$

Example D. Calculate the period of revolution.

$$T = 2\pi\ \textbf{SQRT}\ (a^3/GM) \quad = 76 \text{ years}$$



Kepler's second law : Equal areas in equal times.


ORBIT OF MARS; EQUAL TIME INTERVALS
Y [10^6 KM]
X [10^6 KM]
SOL
200
100
-100
-200
-200
-100
100
200

ε = 0.0934