Phy321 Spring 2017
HW5.1

1. $F = m\dot{v} = -Kv^3$

$$m\frac{dv}{dt} = -Kv^3$$

$$\int \frac{dv}{v^3} = \int -\frac{K}{m}\,dt \quad \cancel{+\text{const}}$$

$$\frac{v^{-2}}{-2} = \frac{-K}{m}t \quad + \text{const}$$

$$\frac{-1}{2v^2} = \frac{-K}{m}t - \frac{1}{2v_0^2}$$

$$\frac{1}{v^2} = \frac{1}{v_0^2} + \frac{2Kt}{m}$$

$$v^2 = \frac{1}{\frac{1}{v_0^2} + \frac{2Kt}{m}} = \frac{v_0^2}{1 + \frac{2Kt}{m}v_0^2}$$

$$\boxed{v = \frac{v_0}{\sqrt{1 + \frac{2Ktv_0^2}{m}}}}$$

HW 5.2

2. (a) $M = \int dm = \int (r\,dr\,d\phi)\,\sigma$

$$= \int_0^R r\,dr \int_0^{\pi/2} d\phi\ \sigma$$

$$= \int_0^R r\,dr \int_0^{\pi/2} d\phi\ (c\,r\cos\phi)$$

$$= c\,\frac{R^3}{3}\Big[\sin\phi\Big]_0^{\pi/2} = \frac{cR^3}{3}$$

(b) $I = \int dm\, r^2$

$$= \int_0^R r\,dr \int_0^{\pi/2} d\phi\,(c\,r\cos\phi)\,r^2$$

$$= \frac{R^5}{5}\,c\,\Big[\sin\phi\Big]_0^{\pi/2} = \boxed{\frac{cR^5}{5}}$$

(could also write $I = \frac{3}{5}MR^2$)

(c) $M\,X_{cm} = \int dm\ x$

$$= \int_0^R r\,dr \int_0^{\pi/2} d\phi\,(c\,r\cos\phi)\,r\cos\phi$$

$$M x_{cm} = c \int_0^R r^3 dr \int_0^{\pi/2} d\phi \, \cos^2\phi$$

$$= c \frac{R^4}{4} \underbrace{\int_0^{\pi/2} d\phi \left[\frac{1+\cos(2\phi)}{2}\right]}_{\underbrace{\frac{1}{2}\left[\phi + \frac{1}{2}\sin(2\phi)\right]_0^{\pi/2}}_{\pi/4}}$$

$$= c \frac{R^4}{4} \frac{\pi}{4} = \frac{c\pi R^4}{16}$$

$$M y_{cm} = c \int_0^R r^3 dr \int_0^{\pi/2} d\phi \, \cos\phi \sin\phi$$

$$= c \frac{R^4}{4} \left[\frac{(\sin\phi)^2}{2}\right]_0^{\pi/2}$$

$$= \frac{c R^4}{8}$$

$$\therefore \boxed{\begin{aligned} x_{cm} &= \frac{c\pi R^4/16}{c R^3/3} = \boxed{\frac{3\pi}{16} R} \\ y_{cm} &= \frac{c R^4/8}{c R^3/3} = \boxed{\frac{3}{8} R} \end{aligned}}$$