Chapter 4. Energy

Section 4.1 : Kinetic Energy and Work

Section 4.2 : Potential Energy

Read Sections 4.1 and 4.2.

What is energy?

from the Oxford Dictionary of Physics ...

Energy: "A measure of a system's ability to do work."   OK, then, what is "work"?

ODP: "the scalar product of force and displacement vectors".

Momentum $\mathbf{p} = m\,\mathbf{v}$

$d\mathbf{p} = \mathbf{F(r)}\,dt$ eq. of *motion*

Kinetic energy $T = ½\, m\, v^2$

$dT = \mathbf{F(r)} \cdot d\mathbf{r}$ eq. of *motion*

Potential energy $U(\mathbf{r})$

$dU = -\,\mathbf{F(r)} \cdot d\mathbf{r}$ eq. of *position*

4.1. Kinetic Energy and Work

The kinetic energy of a particle (i.e., due to translational motion); notation = T;

$$T = ½\, mv^2$$

Calculate dT/dt

$$\frac{dT}{dt} = \frac{1}{2} m\, 2\vec{v} \cdot \frac{d\vec{v}}{dt}$$

$$= m\,\vec{v} \cdot \vec{a}$$

$$= \vec{v} \cdot \vec{F}$$

Figure 4.1

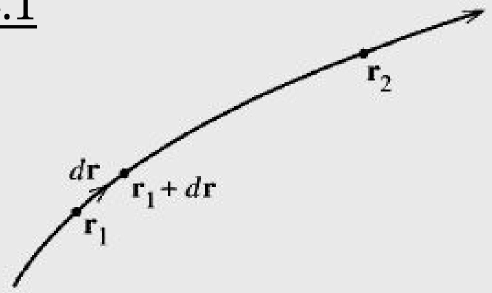


Figure 4.1 Three points on the path of a particle: $\mathbf{r}_1$, $\mathbf{r}_1 + d\mathbf{r}$ (with $d\mathbf{r}$ infinitesimal) and $\mathbf{r}_2$.

$$\frac{dT}{dt} = \vec{v} \cdot \vec{F} = \frac{d\vec{r}}{dt} \cdot \vec{F}$$

$$dT = \vec{F} \cdot d\vec{r}$$

$$\Delta T = T_2 - T_1 = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

along the trajectory of the motion

an example of a "line integral"

The Work–Kinetic Energy Theorem

$dT = \mathbf{F} \cdot d\mathbf{r}$ or, $\Delta T = \int \mathbf{F} \cdot d\mathbf{r}$

or, $dT/dt = \mathbf{F} \cdot \mathbf{v}$

In these equations, d**r** means a displacement along the trajectory of the particle motion.

4.2. Potential Energy

For *<u>conservative</u>* *forces, we may define a potential energy function; notation U(**r**) .*

- Definition of a conservative force:

A force is conservative if (i) $\boldsymbol{F}$ depends only on $\boldsymbol{r}$, ***<u>and</u>*** (ii) the work done by $\boldsymbol{F}$ when the particle moves from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$ is independent of the path from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$.

Work = $\int_\Gamma \boldsymbol{F} \,.\, d\boldsymbol{r}$ ← *a line integral*

Γ is some path in space from r_1 to r_2 ;
but Γ *is not necessarily the "trajectory"!*

For a conservative force the Work depends on the endpoints, but the Work is the same for all paths connecting the endpoints.

Figure 4.3

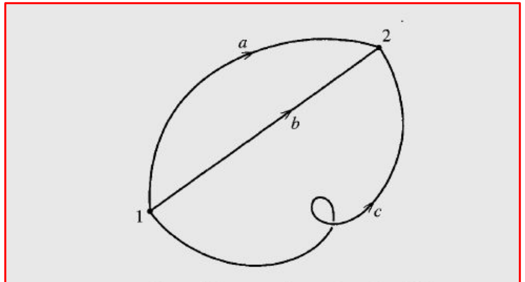


Figure 4.3 Three different paths, a, b, and c, joining the same two points 1 and 2.

- Definition of the potential energy function:

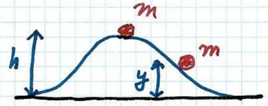
$$\Delta U = -W \quad \textit{(sign is important)}$$

$$U(\boldsymbol{r}_2) - U(\boldsymbol{r}_1) = -\int_\Gamma \mathbf{F} \,.\, d\boldsymbol{r}$$

Or, $U(\boldsymbol{r}+d\boldsymbol{r}) - U(\boldsymbol{r}) = -\mathbf{F} \,.\, d\boldsymbol{r}$

Note that U(**r**) is a scalar.

- Think of a ball on a hill.



$U = mgh$ at top

$U = mgy$ at height y

$$U(h) - U(y) = \int_h^y \vec{F} \cdot d\vec{r} = -mg(y-h)$$

$= mgh - mgy$ ($\vec{F}$ points from high U to low U)

Example 4.1

Three line integrals

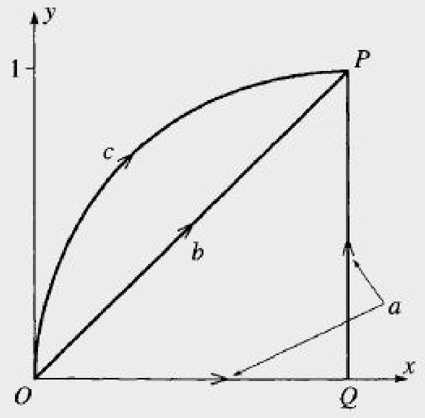


Figure 4.2 Three different paths, a, b, and c, from the origin to the point $P = (1, 1)$.

For a conservative force,

$$W_a = W_b = W_c = -[\,U(P) - U(0)\,]$$

where $W_\alpha = \int_\alpha \boldsymbol{F} \cdot d\boldsymbol{r}$ (def. of work)

For a conservative force,

$$W_{i \to f} = U(i) - U(f)$$

for any path from i to f ;

W = work done by F;

U = potential energy corresponding to F.

Taylor's Example 4.2

The potential energy for a charge q in a static (*time independent*) electric field $\mathbf{E(r)}$.

$$U(\mathbf{r}_1) - U(\mathbf{r}_2) = \int_\Gamma q\,\mathbf{E}\,.d\mathbf{r}$$

Recall from PHY 184: for a static field,

$$\mathbf{E} = -\nabla V; \qquad (V = voltage)$$

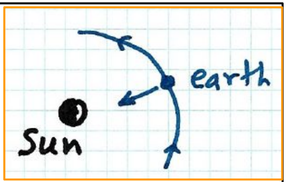
so

$$U(\mathbf{r}_1) - U(\mathbf{r}_2) = -[\,q\,V(\mathbf{r}_2) - q\,V(\mathbf{r}_1)\,]$$

$$U(\mathbf{r}) = q\,V(\mathbf{r})$$

Suppose that q is positive; then the potential energy is high where the voltage is high; the force points from higher potential to lower potential.

Example

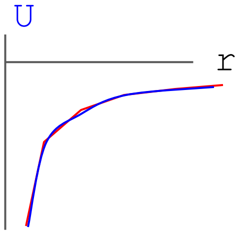


The gravitational potential energy of the Earth, due to the gravitational force exerted by the Sun …

Newton's theory,

$$\boldsymbol{F(r)} = -\frac{GMm}{r^2}\,\boldsymbol{e_r}$$

$$U(\boldsymbol{r}) = -\frac{GMm}{r}$$



Why is $U(\boldsymbol{r})$ negative?

(1) $U(r) - U(\infty) = \int_r^\infty (-GMm)/r'^2\,dr'$

$= GMm/r' \,\big|_r^\infty = -GMm/r$

(2) Or: increasing r is like "going up hill" .

Mechanical energy of a particle

$$E = T + U$$

$$E = \tfrac{1}{2}\, m\, v^2 + U(\mathbf{r})$$

Why is this important?

Theorem:

If the force is conservative then
E is a constant of the motion.

Proof:

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU(\vec{r})}{dt}$$

$$= \vec{F}\cdot\vec{v} + \frac{\partial U}{\partial \vec{r}}\cdot\frac{d\vec{r}}{dt}$$

$$= \vec{F}\cdot\vec{v} - \vec{F}\cdot\vec{v} = 0$$

Several Forces

Suppose $m\dfrac{d\vec{v}}{dt} = F_1 + F_2 + F_3 \dots + F_n$

where all the forces are conservative.

Each force has a corresponding P.E.,

$$\vec{F}_i \Rightarrow U_i(\vec{r})$$

Then $$\frac{dT}{dt} = \sum_{i=1}^{n} \vec{F}_i\cdot\vec{v} = \sum_{i=1}^{n} \vec{F}_i\cdot d\vec{r}/dt$$

$$= \sum_{i=1}^{n} -\frac{dU_i}{dt}$$

$E = T + \sum_{i=1}^{n} U_i$ is constant. $\boxed{\dfrac{dE}{dt} = 0}$

"total mechanical energy"

Note:
$\Delta U = -W$; so $dU = -\mathbf{F}\,.\,d\mathbf{R}$; so $\partial U/\partial \mathbf{r} = -\mathbf{F}$

Nonconservative Forces

Suppose $m\frac{d\vec{v}}{dt} = \vec{F}_{cons} + \vec{F}_{nc}$

Define $E = \frac{1}{2}mv^2 + U_{cons.}$

$$dE = m\vec{v}\cdot d\vec{v} + dU_{cons.}$$

$$= \vec{v}\cdot\vec{F}\,dt - \vec{F}_{cons}\cdot d\vec{r}$$

$$= (\vec{F} - \vec{F}_{cons})\cdot d\vec{r} = \vec{F}_{nc}\cdot d\vec{r}$$

$$\Delta E = \int_\Gamma \vec{F}_{nc}\cdot d\vec{r} = W_{nc}$$

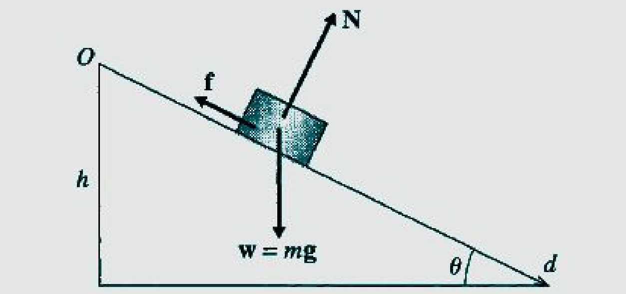


Figure 4.6 A block on an incline of angle θ. The length of the slope is d, and the height is $h = d\sin\theta$.

Example 4.3

a block sliding down an incline

We solved this example before using forces. Now, use *energies* to find the final velocity. Friction is not conservative; *the principle is* : $\Delta(T+U) = W_{nc}$

$$\Delta T = T_{bottom} - T_{top} = \frac{1}{2}mv^2$$

$$\Delta U = U_{bottom} - U_{top} = -mgh$$

$$W_{nc} = \int_0^d \vec{f}\cdot d\vec{r} = \int_0^d (-)\mu mg\cos\theta\, ds$$

do you understand the minus sign?

Result,

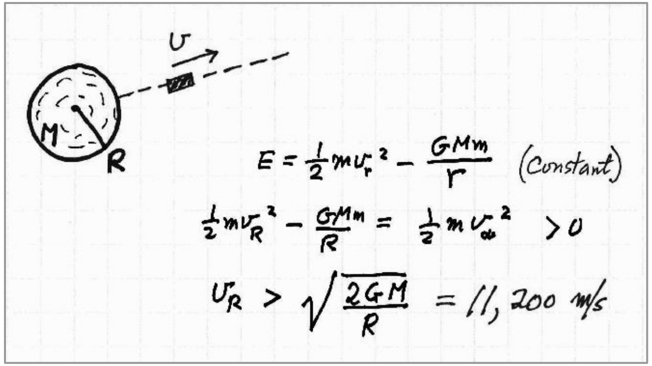
$$\frac{1}{2}mv^2 - mgh = -\mu mg\cos\theta\, d$$

$h = d\sin\theta$

$$v^2_{bottom} = 2gd(\sin\theta - \mu\cos\theta)$$

Example. Calculate the *escape velocity* from the surface of the Earth.

That is, if an object is at the surface of the Earth and moving upward with speed v > v_{escape}, then the object will escape from Earth's gravity.



Test yourself:
Calculate the escape velocity from a distance r from the center of the Earth, for r = $2R_{Earth}$.

Homework Assignment #7
due in class Friday, October 21
[31] Problem 4.3 **
[32] Problem 4.8 **
[33] Problem 4.9 **
[34] Problem 4.10 *
[35] Problem 4.18 **
[36] Problem 4.23 **
Use the cover page.

This is a pretty long assignment,

so start working on it today.