(b)

Name: KEY

Section:

Please work together to solve the problems.

1. Reduce these matrices to their ordinary echelon forms U:

(a)
$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

Which are the free variables and which are the pivots?

a)
$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 4 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} R_2 \rightarrow R_2 - R_1 | b) B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix} R_3 \rightarrow R_3 - 2R_2$$

Pivots: X1, X3 (columns 1 + columns 3 have 1st nonzero entries un Rows 1 + 2)

Free variables & X2, X4, X5

Pivots o X1, X2 free o X3 2. The matrix below is in reduced row echelon form. The reduced row echelon matrix R has 1's as pivots and has zeros above the pivots as well as below.

What are the free variables and what are the pivots? Find a special solution for each free variable (set the free variable to 1 and set the other free variables to 0).

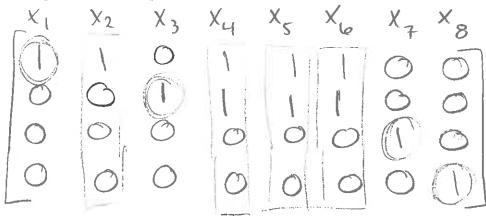
Special Solution #-1

$$X_3=1$$
, $X_5=0$
 $1X_4+4X_5=0 \implies X_4=0$
 $1X_2+2X_3+3X_5=0$
 $1X_2+2X_3+3X_5=0$
 $1X_2+2X_3+3X_5=0$
 $1X_1+1X_3-1X_5=0$
 $1X_1+1X_3-1X_5=0$
 $1X_1+1=0 \implies X_1=1$

linear combinations of
$$3$$
, 4 , 3

Apecial solution #2 $X_{c}=1$, $X_{3}=0$ xy +4x = 0 X4+4=0=x4=-4 1x2 +2x3 +3x==0 X2+3=0=3 X=-3 1x1 + x3 -1x= 0 $x_1 = \frac{1}{3}$ $\hat{S}_2 = \frac{1}{3}$

3. Put as many 1's as possible in a 4 by 8 reduced echelon matrix R so that the free columns are 2, 4, 5, 6.



Free o X2, X4, X5, X6 pivot & XI) X3, X7, X8

4. Construct a matrix A such that its nullspace contains all multiples of (2, -1, 3, 1).

(2,-1,3,1) X4 must be yree, X,, X2, X3 pivot
"special solution" free variable is 1 $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

5. Suppose an m by n matrix has r pivots. Answer the following questions:

Suppose an m by n matrix has r pivots. Answer the following questions:

(a) The number of special solutions is $\underbrace{ \bigcap - \bigcap }_{\text{COLL}} (N-\Gamma)$ free Collemns)

(b) The nullspace contains only $\vec{x} = \vec{0}$ when $r = \underbrace{ \bigcap }_{\text{COLL}} (N-\Gamma)$ (all vows are pivot vows)