Chapter 5. Oscillations

Section 5.1. *Hooke's law*
Section 5.2. *Simple Harmonic Motion*

Read Sections 5.1 and 5.2.

Robert Hooke (1635 – 1703) lived at about the same time as Isaac Newton. (Hooke was a little older.)

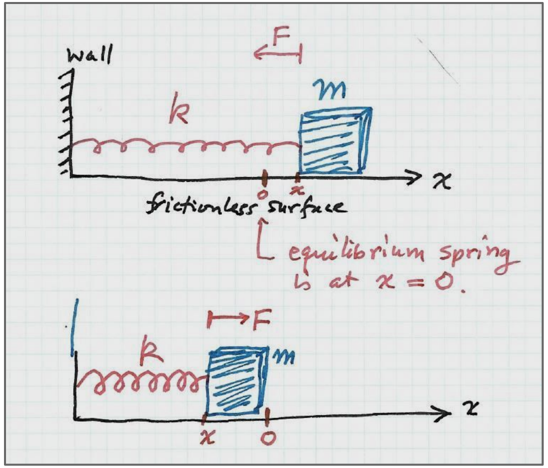
They worked on similar topics in physics [mechanics; optics; microscopes (Hooke) and telescopes (Newton)].

But they were not friends, because each one thought that he was superior to the other guy.

5.1. Hooke's law

Hooke's law states that the force exerted by a spring is
F(x) = –k *x*
(1 dimension)
where *x* = the displacement from equilibrium.

k = Hooke's constant;
(units = N/m)



Essential equations

$F(x) = -\,k\,x$ **"restoring force; F points toward x = 0"**

$U(x) = \frac{1}{2}\,k\,x^2$; $F = -\,dU/dx$

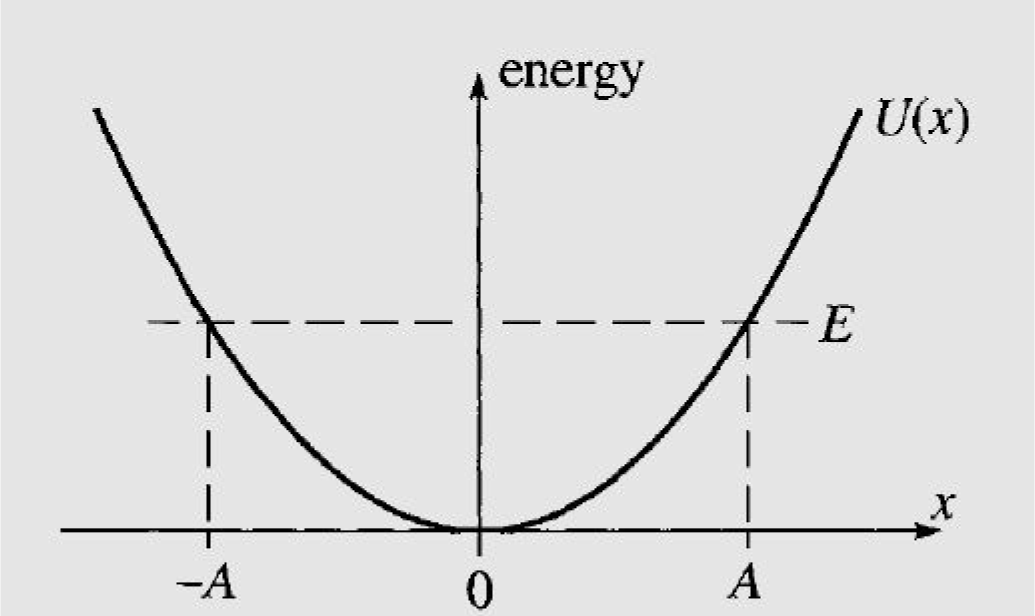
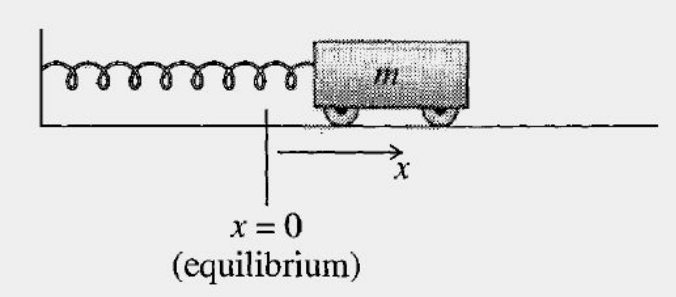


Figure 5.1 A mass m with potential energy $U(x) = \frac{1}{2}kx^2$ and total energy E oscillates between the two turning points at $x = \pm A$, where $U(x) = E$ and the kinetic energy is zero.

5.2. Simple Harmonic Motion



The equation of motion is

$$m\,\ddot{x} = -\,k\,x$$

Or, write

$$\ddot{x} = -\,\omega^2\,x \qquad (1)$$

where $\omega = \sqrt{k/m}$.

Eq. (1) has many solutions …

- sine and cosine solutions;
- *complex* exponential solutions;
- linear combinations of solutions (*the superposition principle);*
- *Initial conditions are necessary to have a unique solution.*

We could write the solution in several ways. We could write

$$\mathbf{x(t) = A\cos(\omega t) + B\sin(\omega t)\ ;}$$

in this form, the initial position is

$$\mathbf{x_0 = x(0) = A}$$

and the initial velocity is

$$\mathbf{v_0 = \dot{x}(0) = \omega B}\ .$$

$$\mathbf{(A = x_0\ \text{and}\ B = v_0/\ \omega)}$$

Figure 5.3

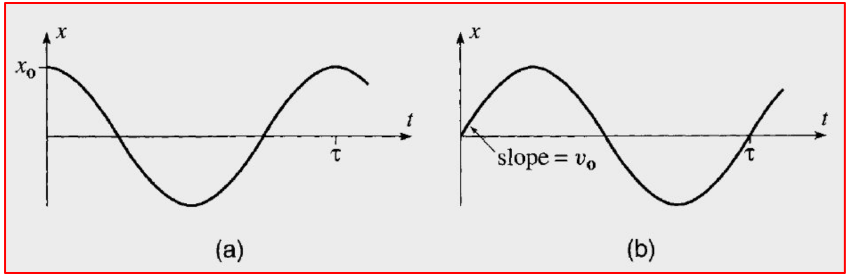


Figure 5.3 **(a)** Oscillations in which the cart is released from x_0 at $t = 0$ follow a cosine curve. **(b)** If the cart is kicked from the origin at $t = 0$, the oscillations follow a sine curve with initial slope v_0. In either case the period of the oscillations is $\tau = 2\pi/\omega = 2\pi\sqrt{m/k}$ and is the same whatever the values of x_0 or v_0.

Example (b) is an example of a *phase-shifted cosine solution*, where the phase shift is 90 degrees.

(a) $x(t) = x_0 \cos(\omega t)$

(b) $x(t) = (v_0/\omega) \sin(\omega t) = (v_0/\omega) \cos(\omega t - \pi/2)$

∗The general phase-shifted cosine solution is

$$x(t) = A \cos(\omega t - \delta) \,.$$

A = amplitude;
δ = phase shift .

This is the same as

$$x(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t) \,,$$

where $B_1 = A \cos\delta$ and $B_2 = A \sin\delta$.

∗The general solution as the real part of a complex exponential is

$$x(t) = C_1 e^{i\omega t} + C_1^* e^{-i\omega t}$$

//second derivative of $e^{i\omega t} = -\omega^2 e^{i\omega t}$ //

Note: $z + z^* = 2 \text{ Re } (z)$

Relations between the three forms of solution

Write $C_1 = |C_1|\, e^{-i\delta}$

(1) $x(t) = B_1 \cos\omega t + B_2 \sin\omega t$

$= A\cos(\omega t - \delta)$

$= A\cos\delta \cos\omega t + A\sin\delta \sin\omega t$

$\therefore$ $B_1 = A\cos\delta$ and $B_2 = A\sin\delta$

$B_1^2 + B_2^2 = A^2$ and $\tan\delta = \frac{B_2}{B_1}$

(2) $x(t) = C_1 e^{i\omega t} + C_1^* e^{-i\omega t}$

$= |C_1|\{e^{i\omega t} e^{-i\delta} + e^{-i\omega t} e^{i\delta}\}$

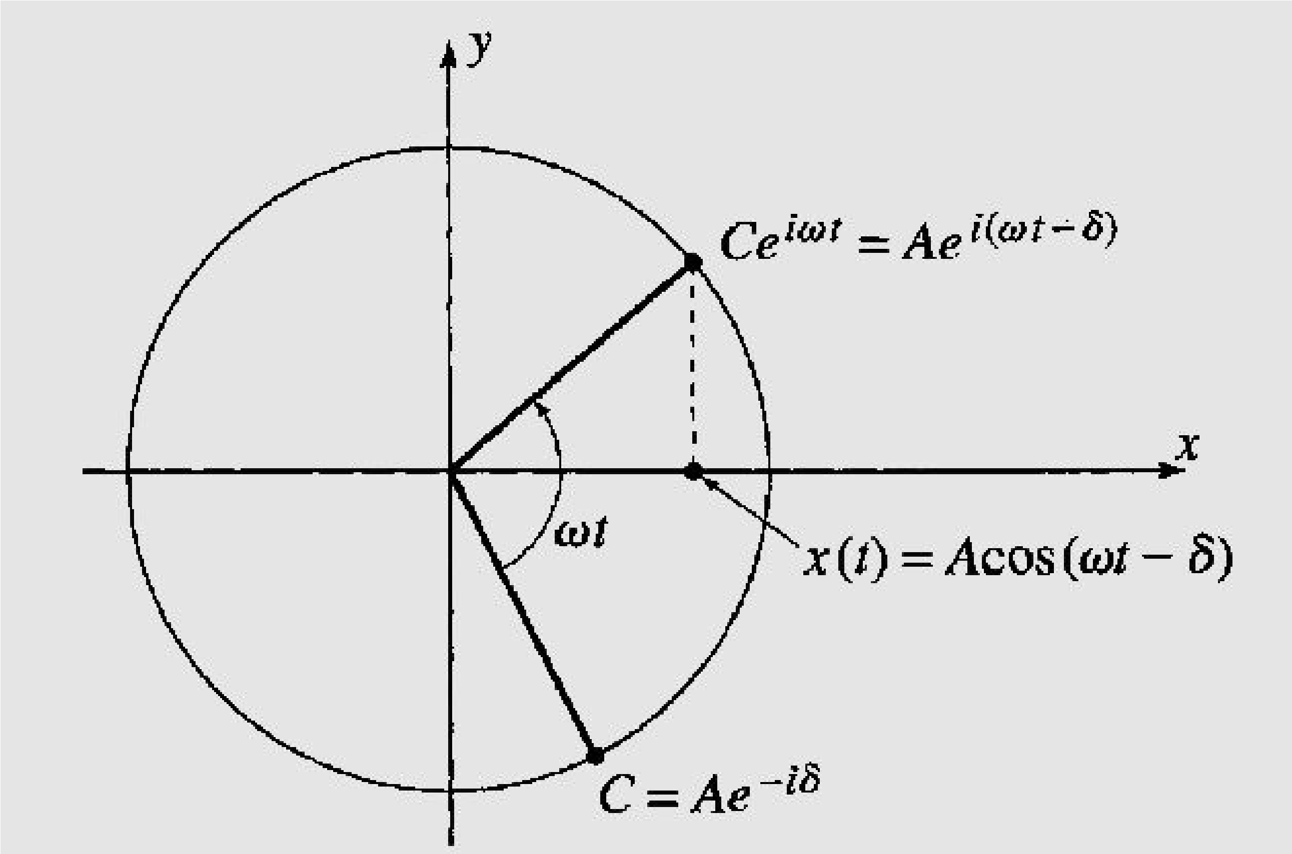
$= |C_1|\, 2\cos(\omega t - \delta)$

$= A\cos(\omega t - \delta)$

$\therefore$ $A = 2|C_1|$ and $C_1 = |C_1| e^{-i\delta}$

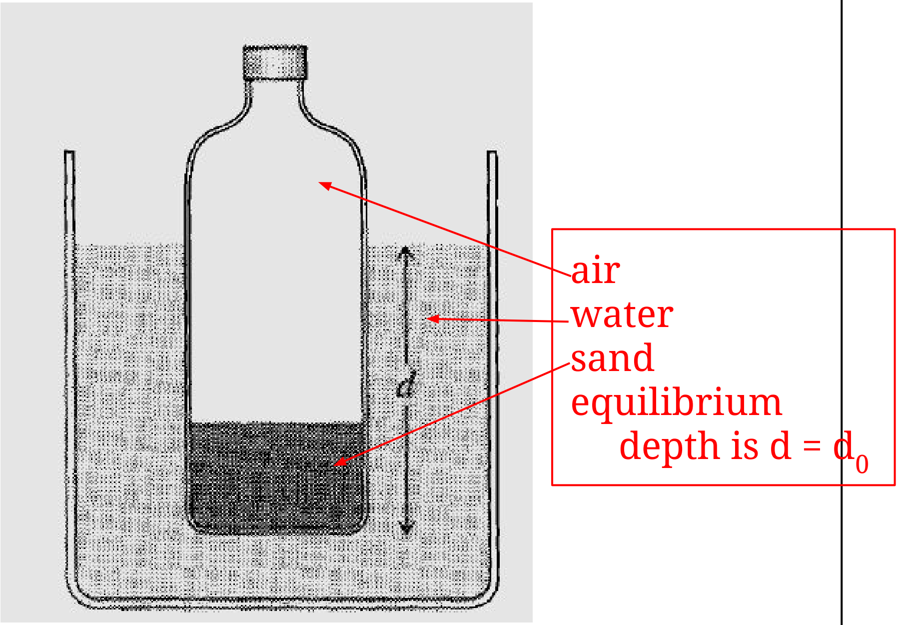
Figure 5.5
Geometrical picture of the complex exponential function

$$C\,e^{i\omega t}$$



Example 5.2

a bottle in a bucket



Show that the bottle undergoes S. H. M.

Let x = displacement downward from equilibrium.

Then $d = d_0 + x$. **Understand the sign.**

Newton's second law,

$$m \ddot{x} = mg - \rho g A (d_0 + x)$$

gravity and buoyancy forces

Equilibrium is at x = 0 , so

$$mg = \rho g A d_0 \ .$$

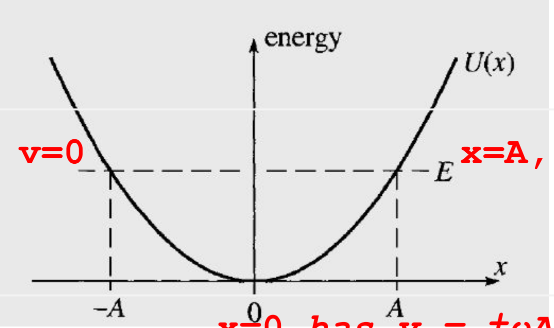
Thus $\ddot{x} = -\omega^2 x \qquad (\bigstar)$

where $\omega^2 = \rho g A / m = g / d_0$.

And (★) is the equation for S. H. M.

Taylor: " Try the experiment yourself. But be aware that the details of the flow of water around the bottle complicate the situation. The calculation here is a very simplified version of the truth."

Energy considerations in S.H.M.



x=−A, v=0

x=A, v=0

x=0 *has* v = ±ωA

$x(t) = A \cos(\omega t - \delta)$

Energy is conserved in SHM.

$T = \frac{1}{2} m (x')^2 = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t - \delta)$

$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t - \delta)$

Recall, $m\omega^2 = k$.

$T + U = \frac{1}{2} k A^2$ **(constant in t)**

or, $T + U = \frac{1}{2} m v(0)^2$ **(same constant)**

Homework Assignment #9

due in class Friday, November 4

[41] Problem 4.41 and Problem 4.43

[42] Problem 5.3 *

[43] Problem 5.5 *

[44] Problem 5.9 *

[45] Problem 5.12 **

[46] Problem 5.18 ***

Use the cover sheet.

Do it now so you will have time to study for …

Second Exam: Friday November 4