1.1 vector addition and linear combinations

Notation

two-dimensional vector

$\vec{v}$ $\vec{v}$ ← w/ handwriting

in a book, vector is denoted by a bold usually lower case letter

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

v_1 & v_2 are components

v_1 ← 1st component

v_2 ← 2nd component

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$ column vector

row vector $\vec{w} = [w_1 \quad w_2]$

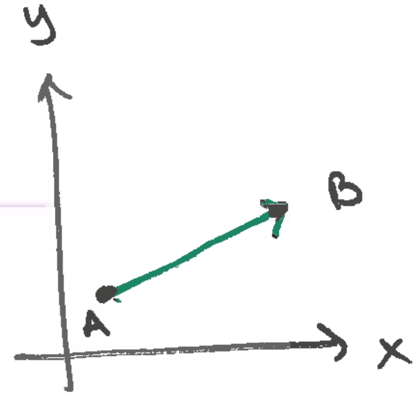
$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \neq \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

vectors in the plane: a directed line segment in the plane ~~that~~ that corresponds to a displacement from point A to point B

(here, usually A is the origin)

vector $\vec{v} = \overrightarrow{AB}$ initial pt. is A

terminal pt. is B



Vector addition

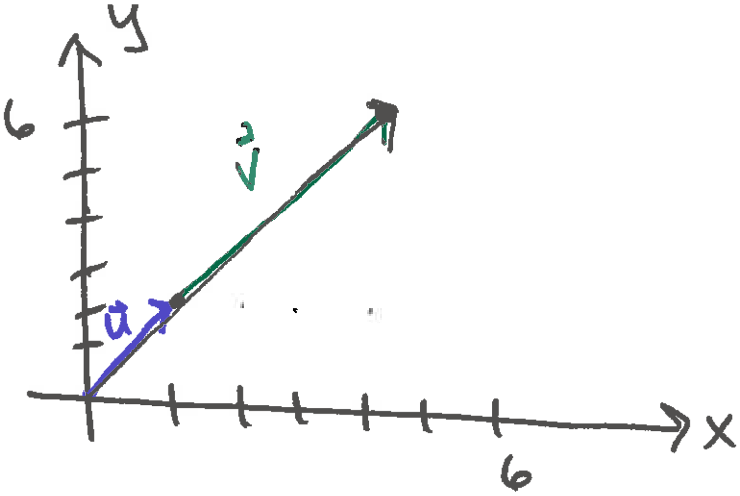
$\vec{u} + \vec{v}$ $\quad \vec{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\vec{u} + \vec{v} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

Ex) $\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$\vec{u} + \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

Picture



Scalar multiplication

$$c\vec{v} = c\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} cv_1 \\ cv_2 \end{bmatrix}$$

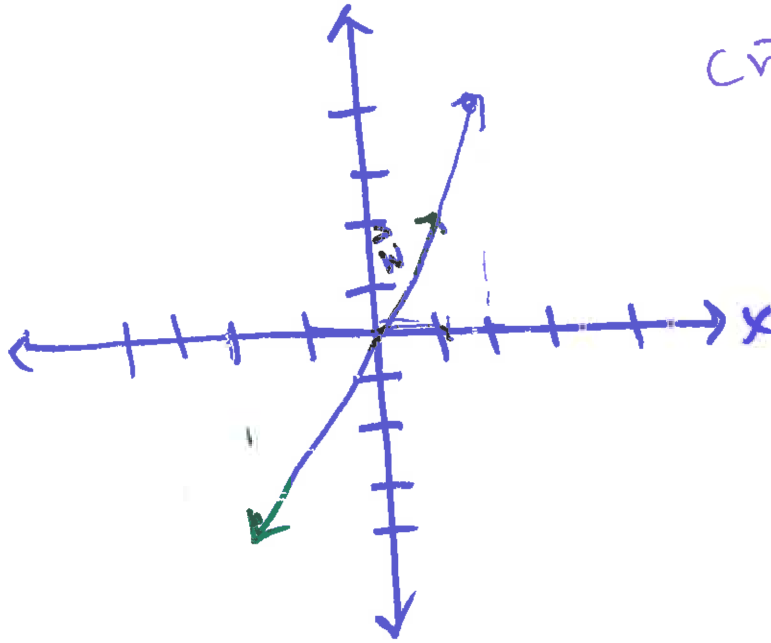
$\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ $c = 4$

$$4\vec{v} = 4\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 20 \end{pmatrix}$$

* note: $c\vec{v}$ has the same direction as $\vec{v}$ if $c>0$ & opposite direction if $c<0$

$\vec{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $c = 2$ vs. $d = -2$

y



$c\vec{w} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$d\vec{w} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

Thm: Algebraic Properties of vectors in $\mathbb{R}_n$

Note: $\mathbb{R} \leftarrow$ real numbers

$\mathbb{R}^2 \leftarrow$ 2 dimensions, x-y plane

$\mathbb{R}^3 \leftarrow$ x, y, z plane, 3-d space

if $\vec{v} \in \mathbb{R}^n$ $\quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$\in$ belongs to

Let $\vec{u}, \vec{w}, \vec{v}$ be vectors in $\mathbb{R}^n$ and c, d are Scalars
$c, d \in \mathbb{R}$

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{u} + \vec{0} = \vec{u}$ $\qquad \vec{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ } n zeros
4. $\vec{u} + (-\vec{u}) = \vec{0}$

5. $c(\vec{u}+\vec{v}) = c\vec{u} + c\vec{v}$

6. $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

7. $c(d\vec{u}) = (cd)\vec{u}$

8. $\vec{1}\vec{u} = \vec{u}$