Homework Assignment #2 due Friday, September 16

Name_____grading key________

Cover sheet: Staple this page in front of your solutions.

Write the requested *answers* to the problems (without calculations) on this page; and write your *solutions* to the problems on your own paper.

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[6] Problem 1.35.* *Answer: The distance where the golf ball hits the ground is* $(2 v_0^2 / g) \sin\theta \cos\theta$

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[7] Problem 1.38.* *Answer: The distance from the puck to O, when the puck returns to floor level, is ...* $2 v_{0x} v_{0y} / (g \sin\theta)$

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[8] Problem 1.39.** *(There is no answer to report here.)* *see the solutions*

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[9] Problem 1.44.* *(There is no answer to report here.)* *see the solutions*

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[10] Problem 1.51.***[computer]

(Refer to the Mathematica sample program to get started.)

Hand in the Mathematica program and the plots.

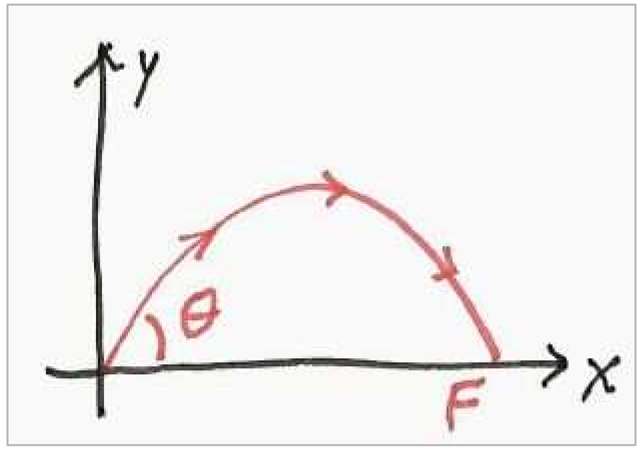
Answer here : Comment on your two graphs

The true period is longer than the harmonic approximation.

Homework Assignment #2

Problem 1.35

A golf ball ...



$x(t) = v_0 \cos\theta \;\; t$

$y(t) = v_0 \sin\theta \;\; t - ½\, g\, t^2$

At the final point F : $y = 0$ so

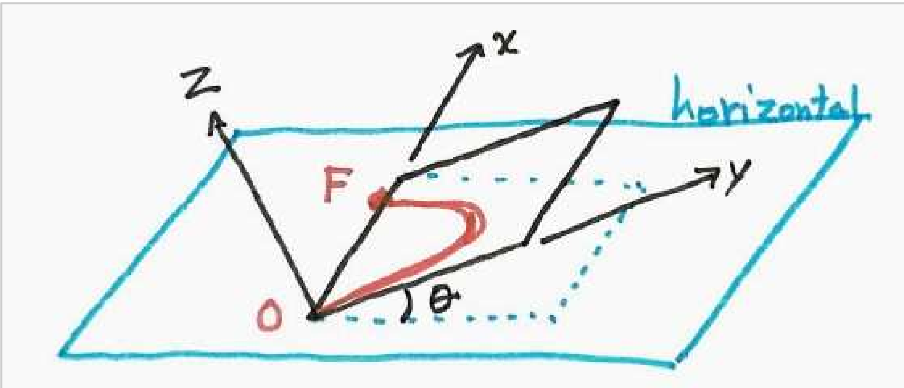
$$t_F = (2v_0 / g)\, \sin\theta$$

The final distance is

$$x_F = v_0 \cos\theta \;\; t_F = (2\, v_0^{\,2} / g)\, \sin\theta \cos\theta$$

Problem 1.38

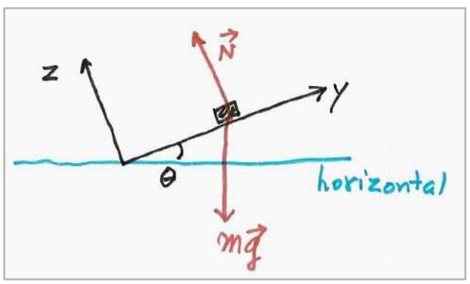
A mass slides on a ramp …



The initial position : $(x_0 , y_0 , z_0) = (0, 0, 0)$

The initial velocity : $(v_{0x} , v_{0y} , v_{0z}) = (v_{0x} , v_{0y} , 0)$

Forces



$\mathbf{F} = m\,\mathbf{g} + \mathbf{N}$

$= -mg\sin\theta\ \boldsymbol{e}_y + (N - mg\cos\theta)\ \boldsymbol{e}_z$

Equations {notation: prime ' stands for d/dt}

$z(t) = 0$ implies $N = mg\cos\theta$

$x'' = 0$ implies $x(t) = x_0 + v_{0x}\,t \quad = v_{0x}\,t$

$y'' = -g\sin\theta$ implies $y(t) = \quad v_{0y}\,t - ½\,g\,t^2 \sin\theta$

- Time to return to the x axis (F)

  $y = 0 \quad \Rightarrow \quad t_F = 2\,v_{0y}\,/(g\sin\theta)$

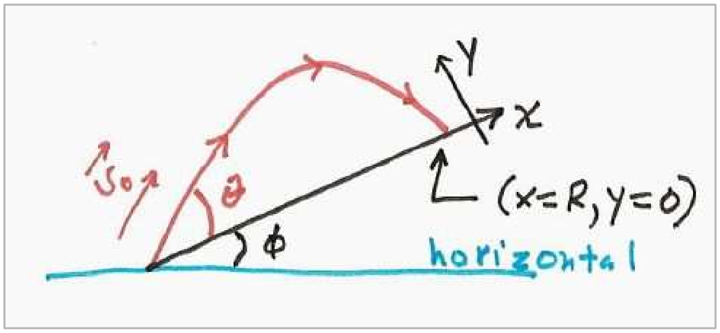
- Distance at F

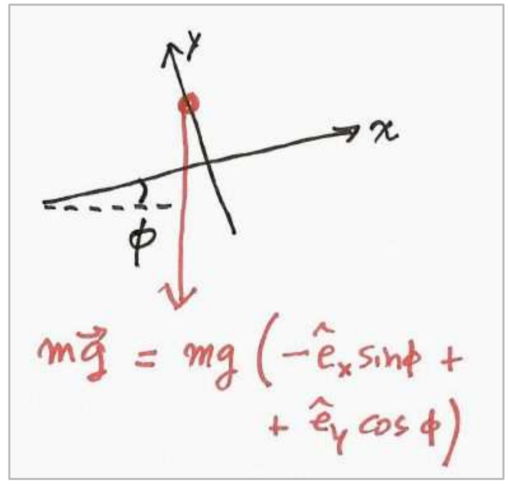
  $x_F = x(t_F) = \quad 2\,v_{0x}\,v_{0y}\,/(g\sin\theta)$

2.3

Problem 1.39

Throw a ball up a slope ...





The equations of motion and solutions are

$mz'' = 0 \quad \Rightarrow \quad \therefore \quad z(t) = 0$

$mx'' = F_x = -\, mg \sin\varphi \quad \Rightarrow \quad \therefore \quad x(t) = v_o \cos\theta\ t - \tfrac{1}{2} g \sin\varphi\ t^2$

$my'' = F_y = -\, mg \cos\varphi \quad \Rightarrow \quad \therefore \quad y(t) = v_o \sin\theta\ t - \tfrac{1}{2} g \cos\varphi\ t^2$

Range calculation. We want to calculate R; note that x = R when y = 0.

$y = 0$ implies $t_R = 2 v_0 \sin\theta \,/\, (g \cos\varphi)$

then $R = x(t_R) = v_o \cos\theta\ t_R - \tfrac{1}{2} g \sin\varphi\ t_R^{\,2}$ & now substitute t_R

Now do some algebra; the final formula is

$$\boxed{R = \frac{2 v_0^2}{g \cos^2\phi} \sin\theta \cos(\theta + \phi)}$$

R_{max} = maximum of R as a function of θ

$$\frac{dR}{d\theta} = 0 = \frac{2 v_0^2}{g \cos^2\phi}\left[\cos\theta \cos(\theta+\phi) - \sin\theta \sin(\theta+\phi)\right]$$

$$= \frac{2 v_0^2}{g^2 \cos^2\phi} \cos(2\theta + \phi)$$

$$\therefore\ 2\theta + \phi = \pi/2 \ ; \quad \theta = \frac{\pi}{4} - \frac{\phi}{2}$$

$$R_{max} = \frac{2 v_0^2}{g^2 \cos^2\phi} \sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \cos\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

after some algebra, $R_{max} = \dfrac{v_0^2}{g(1 + \sin\phi)}$

Problem 1.44

Harmonic oscillations ...

Let $\varphi(t) = A \sin \omega t + B \cos \omega t$

Derivatives:

$$\varphi'(t) = A\,\omega \cos \omega t + B\,(-\omega) \sin \omega t$$

$$\varphi''(t) = A\,(-\omega^2) \sin \omega t + B\,(-\omega^2) \cos \omega t$$

Thus

$$\varphi''(t) = -\omega^2\,(A \sin \omega t + B \cos \omega t\,) = -\omega^2\,\varphi(t)$$

The function depends on two constants, A and B, so it is the general solution to the differential equation, $\varphi'' = -\omega^2\,\varphi$.

Problem 1.51

This is a computer problem.

Hand in the computer program and the figures.

Problem 1.51

```
Remove["Global`*"]
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In[10]:=
```
g = 9.8 (* m / s^2 *)
R = 5.0 (* m *)
ang0 = 90 / 180 * Pi (* radians *)
eqs = {phi''[t] == -g / R * Sin[phi[t]],
  phi[0] == ang0, phi'[0] == 0}
Q = NDSolve[eqs, phi[t], {t, 0, 10}]
pa = Plot[phi[t] /. Q[[1]], {t, 0, 10}];
Show[pa, pb]
```

Out[10]= 9.8

Out[11]= 5.

Out[12]= $\frac{\pi}{2}$

Out[13]= $\{\text{phi}''[t] == -1.96\,\text{Sin}[\text{phi}[t]],\ \text{phi}[0] == \frac{\pi}{2},\ \text{phi}'[0] == 0\}$

Out[14]= {{phi[t] → InterpolatingFunction[Domain: {{0., 10.}} Output: scalar][t]}}

Out[16]=