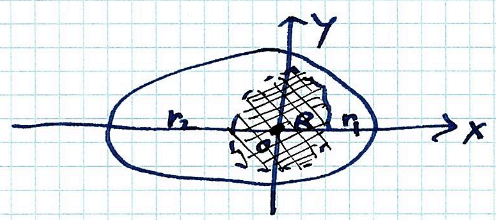
(76) Problem 8.19

Earth orbit with $r_1 = 300\,km + R$; $R = 6400\ km$

and $r_2 = 3000\ km + R$

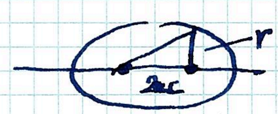


Major axis $2a = r_1 + r_2$

Eccentricity $2a = 2r_1 + 2a\varepsilon$

$$\varepsilon = \frac{a - r_1}{a} = \frac{r_2 - r_1}{r_1 + r_2}$$

- Eccentricity $\varepsilon = \dfrac{r_2 - r_1}{r_1 + r_2} = \underline{0.168}$
- Height at $x = 0$



$$r + \sqrt{(2a\varepsilon)^2 + r^2} = 2a$$

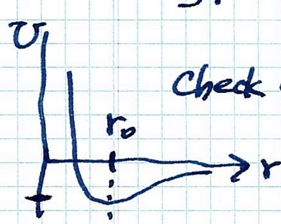
$$(2a\varepsilon)^2 + r^2 = (2a - r)^2 = 4a^2 - 4ar + r^2$$

$$r = \frac{4a^2(1 - \varepsilon^2)}{4a} = 7824\ km$$

Height at $x = 0$ $= h = r - R = \underline{1424\ km.}$

(77) Problem 8.25 $\quad F = \dfrac{-k}{r^{5/2}} \Rightarrow U = \dfrac{-2k}{3r^{3/2}}$ $\quad$ Let $m = l = k = 1$.

(a) $U_{eff}(r) = \dfrac{l^2}{2mr^2} - \dfrac{2k}{3r^{3/2}}$



Check computer plot.

$\leftarrow \frac{1}{2} - \frac{2}{3} = -0.1667$

$U'_{eff}(r_0) = 0 \Rightarrow \underline{r_0 = 1}$

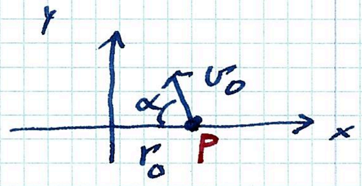
(b) Suppose $E = -0.1$.

$U_{eff}(r_{min}) = E \Rightarrow \underline{r_{min} = 0.6671}$

(c) $u''(\phi) = -u - \dfrac{m}{l^2u^2}F = -u + \dfrac{m}{l^2u^2}ku^{5/2} = -u + \dfrac{mk}{l^2}u^{1/2}$

Solve by Mathematica and plot the orbit.

(78) Problem 8.27



$\ell = \mu r^2 \dot{\phi}$ = constant

Given r_0, v_0, α.

Determine orbit parameters c, ε, δ where $r(\phi) = \dfrac{c}{1+\varepsilon\cos(\phi-\delta)}$

- $\ell^2 = \gamma \mu c = GM\mu^2 c$ and $\ell = \mu r_0 v_0 \sin\alpha$
  ↑ constant
  $\ell^2 = \mu^2 r_0^2 v_0^2 \sin^2\alpha$

Thus $c = \dfrac{\ell^2}{GM\mu^2} = \dfrac{r_0^2 v_0^2 \sin^2\alpha}{GM}$

Numerical values
$r_0 = 100 \times 10^6$ km
$v_0 = 45$ km/s
$\alpha = 50$ degrees
$GM = 13{,}34 \times 10^{19}\ m^3 s^{-2}$

$\boxed{c = 8{,}908 \times 10^{10}\ m}$

- At P, $v_r = -v_0 \cos\alpha$ and $\phi = 0$;

$$v_r = \dot{r} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \frac{-c(+\varepsilon\sin\delta)}{[1+\varepsilon\cos\delta]^2}\,\frac{\ell}{\mu r_0^2}$$

$$-v_0\cos\alpha = -\frac{\varepsilon}{c}\sin\delta\cdot\frac{\ell}{\mu} = -\frac{\varepsilon}{c}\sin\delta\; r_0 v_0 \sin\alpha$$

$$v_0\cos\alpha = \frac{\varepsilon\sin\delta\sin\alpha\; r_0 v_0}{r_0^2 v_0^2 \sin^2\alpha / GM} = \varepsilon\sin\delta\left(\frac{GM}{r_0 v_0 \sin\alpha}\right)$$

$$\therefore\ \varepsilon\sin\delta = \frac{r_0 v_0^2 \sin\alpha\cos\alpha}{GM} \equiv \beta_y$$

$\tan\delta = \dfrac{\beta_y}{\beta_x} = -6{,}845$
$\delta = 1.716$ radians
$\varepsilon = \sqrt{\beta_x^2 + \beta_y^2} = 0{,}755$

- At P, $r_0 = \dfrac{c}{1+\varepsilon\cos\delta}$

$$\therefore\ \varepsilon\cos\delta = -1 + \frac{c}{r_0} = -1 + \frac{r_0 v_0^2 \sin^2\alpha}{GM} \equiv \beta_x$$

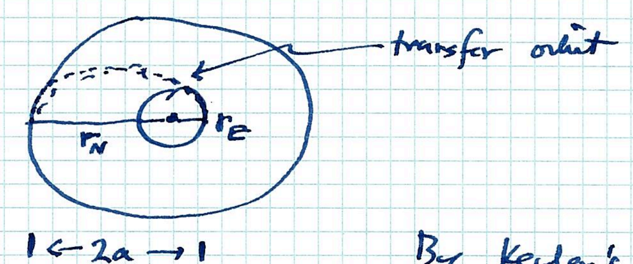
(79) Problem 8.28 Orbit equation $r(\phi) = \dfrac{c}{1+\varepsilon\cos\phi}$ where $c = \gamma\mu\ell^2$. For the same value of ℓ,

Circle ($\varepsilon = 0$) has $r_{circ.} = c$

Parabolic orbit ($\varepsilon = 1$) has $r = \dfrac{c}{1+\cos\phi}$ and $\therefore r_{min} = \dfrac{c}{2}$.

Thus $\underline{r_{min} = \dfrac{r_{circle}}{2}}$ (for the same value of ℓ)

[80] <u>Problem 8.34</u> Send a spacecraft to Neptune, on a Hohmann transfer orbit,



$$2a = r_E + r_N = 1\,AU + 30\,AU = 31\,AU$$
$$a = 15.5\,AU$$

By Kepler's 3rd law, $\tau = 2\pi\sqrt{\frac{a^3}{GM}} \propto a^{3/2}$

$\therefore\ \frac{\tau}{1\,yr} = \left(\frac{a}{1\,AU}\right)^{3/2} = 61$

The travel time is $\tau/2 = \underline{30.5 \text{ years}}$

[80x] Send a spacecraft to Mars, on a Hohmann transfer orbit, $2a = r_E + r_M = 1.0 + 1.524\,AU = 2.524\,AU$

travel time $= \frac{1}{2}\left(\frac{a}{1AU}\right)^{3/2} \cdot 1\,yr = 0.709\,yr = \underline{259 \text{ days.}}$