Ex) eg $A\vec{x}$ is

$$\begin{bmatrix} 1 & 0 \\ 4 & 3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{x_1 \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}}_{\text{column space is a plane in } \mathbb{R}^3}$$

$\vec{b} = A\vec{x}$, $\vec{b}$ lies on the plane

note: for most $\vec{b}$, there is no solution

Some terms:

The set of all linear combinations of $\vec{v}_1, \dots, \vec{v}_n$ is called the span of $\vec{v}_1, \dots, \vec{v}_n$. denoted $\text{span}(\vec{v}_1, \dots, \vec{v}_n)$

$$\text{Span}(\vec{v}_1, \dots, \vec{v}_n) = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots + \alpha_n \vec{v}_n$$

Def: The set $\{\vec{v}_1, \dots, \vec{v}_n\}$ is a spanning set for V if and only if every vector in V can be written as a linear combination of $\vec{v}_1, \ldots, \vec{v}_n$.

Ex) Which of the following are spanning sets of $\mathbb{R}^3$?

A. $\{(1,0,1)^T, (0,1,0)^T\}$

Reword: with $\begin{pmatrix}1\\0\\1\end{pmatrix}$ & $\begin{pmatrix}0\\1\\0\end{pmatrix}$ can we make <u>any</u> vector in $\mathbb{R}^3$?

$$\alpha\begin{pmatrix}1\\0\\1\end{pmatrix} + \beta\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\\\alpha\end{pmatrix}$$

component 1+3 are the same

can't ever get $\begin{pmatrix}1\\2\\-1\end{pmatrix}$

Not a spanning set ~~for~~ of $\mathbb{R}^3$.

A'. $\{(1,0,0)^T, (0,1,0)^T, (0,0,1)^T\}$

B. $\{(1,1,1)^T, (1,1,0)^T, (1,0,0)^T\}$

$$\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} + \beta\begin{pmatrix}1\\1\\0\end{pmatrix} + \gamma\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

general vector in $\mathbb{R}^3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\alpha = c$$
$$\alpha + \beta = b$$
$$\alpha + \beta + \gamma = a$$

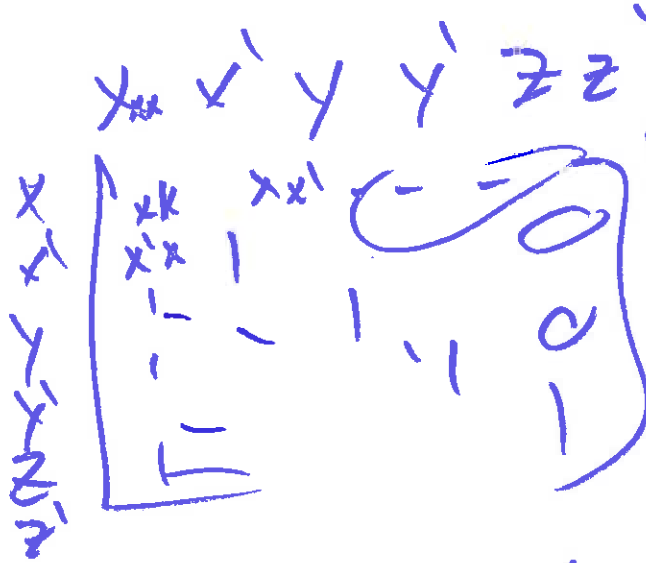
solve . . .

$$\alpha = c$$
$$\beta = (b - c)$$
$$\gamma = (a - b)$$

$x_i x_i'$

$y_i y_i'$

$z_i z_i'$



$[M_1 M_2 \cdots M_a) v_i = v_f$

$$\cancel{M} \; v_i = \quad M v_i = v_f$$

v_f