Inverses

Formula: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{1-6}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -1/5 & 2/5 \\ 3/5 & -1/5 \end{bmatrix}$

By Hand

$A^{-1} \rightarrow \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$

eliminate

$R_2 \rightarrow R_2 - 3R_1$

$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -3 & 1 \end{array}\right]$

$R_2 \rightarrow -\frac{1}{5}R_2$ eliminate

$1 - \frac{6}{5}$

$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/5 & -1/5 \end{array}\right]$

$R_1 \rightarrow R_1 - 2R_2$

$\left[\begin{array}{cc|cc} 1 & 0 & -1/5 & +2/5 \\ 0 & 1 & 3/5 & -1/5 \end{array}\right]$ So inverse is

$A^{-1} = \begin{bmatrix} -1/5 & 2/5 \\ 3/5 & -1/5 \end{bmatrix}$

Check by Multiplying

$A \cdot A^{-1}$

Inverses

What if our row ops are in different orders??

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

eliminate

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{cc|cc} -5 & 0 & 1 & -2 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

eliminate

$-\frac{6}{5} + 1$

$R_2 \to R_2 + \frac{3}{5}R_1$

$$\left[\begin{array}{cc|cc} -5 & 0 & 1 & -2 \\ 0 & 1 & 3/5 & -1/5 \end{array}\right]$$

done, same inverse

$R_1 \to -1/5\, R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1/5 & 2/5 \\ 0 & 1 & 3/5 & -1/5 \end{array}\right]$$