Phy 321 Spring 2017
HW 14.1

1. $T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$

$V = U(r) = Ae^{Br}$

$\mathcal{L} = T - V$

$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mr^2\dot{\phi}$

$F_\phi = \frac{\partial \mathcal{L}}{\partial \phi} = 0 \Rightarrow p_\phi = \text{const} = \text{angular momentum}$

$E = T + V = \frac{1}{2}m\left(\dot{r}^2 + r^2\left(\frac{p_\phi}{mr^2}\right)^2\right) + U$

Turning points $R_1, R_2 \Rightarrow$

$$\begin{cases} E = \frac{1}{2m}\frac{p_\phi^2}{R_1^2} + U(R_1) \\ E = \frac{1}{2m}\frac{p_\phi^2}{R_2^2} + U(R_2) \end{cases}$$

Subtract: $\frac{p_\phi^2}{2m}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + U(R_1) - U(R_2) = 0$

$p_\phi^2 = (-2m)\left(\frac{1}{\frac{1}{R_1^2} - \frac{1}{R_2^2}}\right)\left[U(R_1) - U(R_2)\right]$

$$p_\phi^2 = \frac{(+2M)\,R_1^2 R_2^2}{R_1^2 - R_2^2}\left[U(R_1) - U(R_2)\right]$$

$$\boxed{p_\phi = \pm\sqrt{\left(\frac{2MR_1^2R_2^2}{R_1^2-R_2^2}\right) A e^{B(R_1-R_2)}}}$$

2. the point where stick is attached to spring is at

$$x = r\sin\theta_1$$
$$y = r\cos\theta_1$$

where y axis points down

center of stick is at

$$x_{cm} = r\sin\theta_1 + \frac{l}{2}\sin\theta_2$$

$$y_{cm} = r\cos\theta_1 + \frac{l}{2}\cos\theta_2$$

$$T = T_{translation} + T_{rotation}$$

$$= \frac{1}{2}M\left(\dot{x}_{cm}^2 + \dot{y}_{cm}^2\right) + \frac{1}{2}I\,\dot{\theta}_2^2$$

$$\cancel{V = \frac{1}{2}M\left[\dot{r}\sin\theta_1 + r\cos\theta_1\dot{\theta}_1 + \frac{l}{2}\cos\theta_2\right.}$$

HW 14-3

$$T = \frac{1}{2} M \left\{ \left[\dot{r} \sin\theta_1 + r\cos\theta_1 \dot{\theta}_1 + \frac{l}{2}\cos\theta_2 \dot{\theta}_2 \right]^2 + \left[\dot{r}\cos\theta_1 - r\sin\theta_1 \dot{\theta}_1 - \frac{l}{2}\sin\theta_2 \dot{\theta}_2 \right]^2 \right\} + \frac{1}{2} \frac{Ml^2}{12} \dot{\theta}_2^2$$

$$= \frac{1}{2} M \left\{ \dot{r}^2 + (r\dot{\theta}_1)^2 + \left(\frac{l}{2}\dot{\theta}_2\right)^2 + 2\dot{r}\dot{\theta}_1 \left(\cancel{r\sin\theta_1 \cos\theta_1} \cancel{- r\sin\theta_1 \cos\theta_1} \right) + 2\dot{r}\dot{\theta}_2 \frac{l}{2} \left(\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2 \right) + 2r\frac{l}{2}\dot{\theta}_1 \dot{\theta}_2 \left(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \right) + \frac{l^2}{12}\dot{\theta}_2^2 \right\}$$

$$= \frac{1}{2} M \left\{ \dot{r}^2 + r^2\dot{\theta}_1^2 + \frac{l^2}{3}\dot{\theta}_2^2 + \dot{r}\dot{\theta}_2 l (\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2) + r\dot{\theta}_1\dot{\theta}_2 l (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) \right.$$

Use trig identities

$$\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$$
$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

$$T = \frac{1}{2}M\Big(\dot{r}^2 + r^2\dot{\theta}_1^2 + \frac{l^2}{3}\dot{\theta}_2^2$$
$$+ \dot{r}\dot{\theta}_2 l \sin(\theta_1 - \theta_2)$$
$$+ r\dot{\theta}_1\dot{\theta}_2 l \cos(\theta_1 - \theta_2)\Big)$$

$$V = \frac{1}{2}k(r-B)^2 + \text{"}mgh\text{"}$$
$$= \frac{1}{2}k(r-B)^2 - Mg\left(r\cos\theta_1 + \frac{l}{2}\cos\theta_2\right)$$

$$\mathcal{L} = T - V$$

$$p_r = \frac{\partial\mathcal{L}}{\partial\dot{r}} = M\left[\dot{r} + \frac{1}{2}\dot{\theta}_2 l \sin(\theta_1 - \theta_2)\right]$$

$$F_r = \frac{\partial\mathcal{L}}{\partial r} = M\left[r\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_1\dot{\theta}_2 l \cos(\theta_1 - \theta_2)\right]$$
$$- k(r-B) + Mg\cos\theta_1$$

$$\dot{p}_r = M\Big[\ddot{r} + \frac{l}{2}\ddot{\theta}_2 \sin(\theta_1 - \theta_2)$$
$$+ \frac{l}{2}\dot{\theta}_2 \cos(\theta_1 - \theta_2)\left[\dot{\theta}_1 - \dot{\theta}_2\right]$$

HW14.5

$$F_r = \dot{p}_r \Rightarrow$$

$$\begin{aligned}
&\ddot{r} + \frac{l}{2}\ddot{\theta}_2 \sin(\theta_1-\theta_2) + \frac{l}{2}\cos(\theta_1-\theta_2)\,\dot{\theta}_2(\underline{\dot{\theta}_1}-\dot{\theta}_2) \\
&= r\dot{\theta}_1^2 + \frac{l}{2}\cos(\theta_1-\theta_2)\,\underline{\underline{\dot{\theta}_1\cdot\dot{\theta}_2}} \\
&\quad -\frac{k}{m}(r-B) + g\cos\theta_1
\end{aligned}$$

$$\boxed{\begin{aligned}
&\ddot{r} + \frac{l}{2}\ddot{\theta}_2 \sin(\theta_1-\theta_2) - \frac{l}{2}\dot{\theta}_2^2\cos(\theta_1-\theta_2) \\
&- r\dot{\theta}_1^2 + \frac{k}{m}(r-B) - g\cos\theta_1 = 0
\end{aligned}}$$

$$p_{\theta_1} = \frac{\partial L}{\partial \dot{\theta}_1} = m\left[r^2\dot{\theta}_1 + \frac{l}{2}r\dot{\theta}_2\cos(\theta_1-\theta_2)\right]$$

$$\begin{aligned}
F_{\theta_1} = \frac{\partial L}{\partial \theta_1} = m\Big[&\frac{l}{2}\dot{r}\dot{\theta}_2\cos(\theta_1-\theta_2) \\
&- \frac{l}{2}r\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)\Big] \\
&- mgr\sin\theta_1
\end{aligned}$$

$$\begin{aligned}
\dot{p}_{\theta_1} = m\Big[&2r\dot{r}\dot{\theta}_1 + r^2\ddot{\theta}_1 \\
&+ \frac{l}{2}(\dot{r}\dot{\theta}_2 + r\ddot{\theta}_2)\cos(\theta_1-\theta_2) \\
&- \frac{l}{2}r\dot{\theta}_2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\Big]
\end{aligned}$$

HW 1~~2~~4-6

$$\dot{p}_{\theta_1} = F_{\theta_1} \Rightarrow$$

$$r^2\ddot{\theta}_1 + 2r\dot{r}\dot{\theta}_1 + \frac{l}{2}\cos(\theta_1-\theta_2)\left[\cancel{\dot{r}\dot{\theta}_2} + r\ddot{\theta}_2 \cancel{- \dot{r}\dot{\theta}_2}\right]$$

$$+ \frac{l}{2}\sin(\theta_1-\theta_2)\left\{-r\dot{\theta}_2(\cancel{\dot{\theta}_1} - \dot{\theta}_2) \cancel{+ r\dot{\theta}_1\dot{\theta}_2}\right\}$$

$$+ Mgr\sin\theta_1 = 0$$

$$\boxed{r^2\ddot{\theta}_1 + 2r\dot{r}\dot{\theta}_1 + \frac{l}{2}\cos(\theta_1-\theta_2)\left[r\ddot{\theta}_2\right] + \frac{l}{2}\sin(\theta_1-\theta_2)\left[r\dot{\theta}_2^2\right] + gr\sin\theta_1 = 0}$$

$$p_{\theta_2} = \frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = M\left[\frac{l^2}{3}\dot{\theta}_2 + \frac{l}{2}\dot{r}\sin(\theta_1-\theta_2) + \frac{l}{2}\dot{\theta}_1 r\cos(\theta_1-\theta_2)\right]$$

$$F_{\theta_2} = \frac{\partial \mathcal{L}}{\partial \theta_2} = M\left[-\frac{l}{2}\dot{r}\dot{\theta}_2\cos(\theta_1-\theta_2) + \frac{l}{2}r\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)\right]$$

$$- Mg\frac{l}{2}\sin\theta_2$$

HW 14.7

$$\dot{p}_{\theta_2} = F_{\theta_2} \Rightarrow$$

$$\frac{l^2}{3}\ddot{\theta}_2 + \frac{l}{2}\ddot{r}\sin(\theta_1-\theta_2) + \frac{l}{2}\cos(\theta_1-\theta_2)\dot{r}(\dot{\theta}_1-\dot{\theta}_2)$$

$$+ \frac{l}{2}(\ddot{\theta}_1 r + \dot{\theta}_1 \dot{r})\cos(\theta_1-\theta_2)$$

$$- \frac{l}{2}(\dot{\theta}_1 r)\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)$$

$$= -\frac{l}{2}\dot{r}\dot{\theta}_2\cos(\theta_1-\theta_2)$$

$$+ \frac{l}{2} r\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)$$

$$- \frac{gl}{2}\sin\theta_2$$

$$\Rightarrow \frac{l^2}{3}\ddot{\theta}_2 + \frac{l}{2}\sin(\theta_1-\theta_2)\left[\ddot{r} - r\dot{\theta}_1(\dot{\theta}_1 - \cancel{\dot{\theta}_2}) \cancel{- r\dot{\theta}_1\dot{\theta}_2}\right]$$

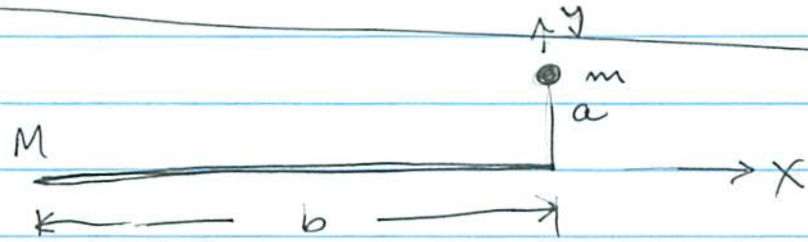
$$+ \frac{l}{2}\cos(\theta_1-\theta_2)\left[\dot{r}(\dot{\theta}_1 - \cancel{\dot{\theta}_2}) + \ddot{\theta}_1 r + \dot{\theta}_1\dot{r} \cancel{+ \dot{r}\dot{\theta}_2}\right]$$

$$+ \frac{gl}{2}\sin\theta_2 = 0$$

HW 14.8

$$\boxed{\frac{\ell^2}{3}\ddot{\theta}_2 + \frac{\ell}{2}\sin(\theta_1-\theta_2)\left[\ddot{r} - r\dot{\theta}_1^2\right] + \frac{\ell}{2}\cos(\theta_1-\theta_2)\left[r\ddot{\theta}_1 + 2\dot{r}\dot{\theta}_1\right] + \frac{g\ell}{2}\sin\theta_2 = 0}$$

3.



$$V = \text{"}\frac{-G m_1 m_2}{r}\text{"} = \int_{-b}^{0} \frac{(-G)\left(\frac{ds}{b} M\right) m}{r}$$

where r is distance from $(s,0)$ to $(0,a)$, i.e. $r = \sqrt{(s-0)^2 + (0-a)^2} = \sqrt{s^2+a^2}$

$$V = \frac{-GMm}{b}\int_{-b}^{0} \frac{ds}{\sqrt{s^2+a^2}}$$

$$\boxed{V = \frac{-GmM}{b}\log\left(\frac{b+\sqrt{a^2+b^2}}{a}\right)}$$

4. Let m = mass of Earth
M = " " Sun

<u>Before</u>: $$E = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{p_\phi^2}{2mr^2} - \frac{GmM}{r}}_{V_{eff}}$$

circular orbit so $r = R$ and $\dot{r} = 0$

V_{eff} must have minimum at R

$$\frac{p_\phi^2}{2m}\frac{-2}{R^3} - GmM\left(\frac{-1}{R^2}\right) = 0$$

$$p_\phi^2 = (mR^3)\left(\frac{GmM}{R^2}\right) = Gm^2MR$$

also $p_\phi = mv_0R$

$$\Rightarrow v_0 = \frac{p_\phi}{mR} = \sqrt{\frac{GM}{R}}$$

Could also get that from physics 1:

$$\frac{mv_0^2}{R} = F = \frac{GmM}{R^2} \Rightarrow v_0 = \sqrt{\frac{GM}{R}}$$

<u>After</u>: $M \to \frac{1}{2}M$ so

$$E = \frac{1}{2} m \dot{r}^2 + \frac{p_\phi^2}{2mr^2} - \frac{GmM}{2r}$$

p_ϕ didn't change, but E changes.

At the time of the disaster,

$\dot{r} = 0$ and $r = R$, so

$$E = \frac{p_\phi^2}{2mR^2} - \frac{GmM}{2R}$$

$$= \frac{Gm^2MR}{2mR^2} - \frac{GmM}{2R} = 0$$

So $$\boxed{0 = \frac{1}{2}m\dot{r}^2 + \frac{p_\phi^2}{2mr^2} - \frac{GmM}{2r}}$$

since $p_\phi^2 = Gm^2MR$, can also write this as

$$\boxed{0 = \frac{1}{2}m\dot{r}^2 + \frac{GmM}{2}\left(\frac{R}{r^2} - \frac{1}{r}\right)}$$

Since $E = 0$, orbit is parabola: