Homework Assignment #3   Name____grading key________

due in class Friday, September 23

Cover sheet : Staple this page in front of your solutions.

Write the requested *answers* (without calculations) on this page;
write the detailed *solutions* on your own paper.

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[11] Problem 2.2.* *Answer: the value of β is ...* **1 pt**

1.6×10^{-4} Ns/m^2

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[12] Problem 2.3.* *Answer: the Reynolds number (part b) is ...* **1 pt**

0.0108

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[13] Problem 2.10.** *Answer: the terminal speed is ...* **2 pts**

1.18×10^{-3} m/s

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[14] Problem 2.18.* *Answer: the Taylor series for ln(1+δ) is ...* **1 pt**

$\delta - \delta^2/2 + \delta^3/3$

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[15] Problem 2.26.* *Answer: the time to slow to 15 m/s is ...* **1 pt**

6.67 s

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[16] The terminal velocity of a drop of water (diameter = D) is the velocity at which **3 pts**
$F = mg - bv - cv^2 = 0$.
The parameter values for air at STP are

$b = (1.6 \times 10^{-4})\, D$ and $c = (0.25)\, D^2$, in MKS units;

also, $m = (0.52 \times 10^3 \text{ kg/m}^3)\, D^3$.
Determine v_{ter} as a function of D. Plot an accurate graph of v_{ter} versus D, from D = 0.1 mm to 3 mm. (Use a computer to make the plot.) [The result shows why water droplets in a cloud do not fall as rain.] Hand in the plot.

Answer here: Explain why water droplets in a cloud do not fall as rain.
The tiny cloud droplets fall very slowly with respect to the air.
Upward air currents will carry them aloft before they can reach the ground.

Homework Assignment #3

Problem 2.2

Stokes's law ...

Stokes's law for viscous drag, $f_{lin} = 3\,\pi\,\eta\,D\,v$.

Thus $f_{lin} = b\,v$ where $b = 3\,\pi\,\eta\,D = \beta\,D$

where $\beta = 3\,\pi\,\eta$.

For air, $\eta = 1.7 \times 10^{-5}$ Ns/m^2

Therefore, $\beta = 1.6 \times 10^{-4}$ Ns/m^2 *which agrees with eq. (2.5)*

Problem 2.3

Reynolds's number ...

(a) Given $f_{lin} = 3\,\pi\,\eta\,D\,v$ and $f_{quad} = \kappa\,\rho\,A\,v^2$.

The Reynolds number is defined by $Re = D\,v\,\rho\,/\,\eta$.

The ratio $f_{quad}\,/\,f_{lin}$ =

$$\boxed{\frac{f_{quad}}{f_{lin}} = \frac{\kappa\,\rho\,\pi\,(D/2)^2\,v^2}{3\pi\,\eta\,D\,v} = \frac{\kappa}{12}\,Re}$$

$= Re\,/48$ for $\kappa = 0.25$.

(b) For a steel ball bearing in glycerine, with the given parameter values,

$$\boxed{Re = \frac{2\text{ mm} \cdot 5\text{ cm/s} \cdot 1.3\times10^{-3}\text{ kg/cm}^3}{12\text{ Ns/m}^2}}$$

$Re = 0.0108$.

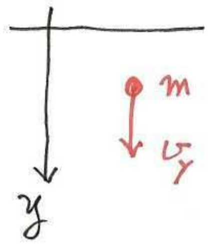
Since Re is very small, the linear resistive force is dominant.

Problem 2.10

A steel ball bearing sinking in glycerine ...

Use linear resistance, $f_{lin} = 3\pi\eta D v$.

(a) Characteristic time and terminal speed



$$m\frac{dv}{dt} = F_y = mg - \rho V g - bv$$

buoyancy force

$$\frac{dv}{dt} = g - \frac{\rho V g}{m} - \frac{b}{m}v = g(1 - \rho/\rho_{steel}) - \frac{b}{m}v = g' - \frac{b}{m}v$$

Thus $v_{ter} = \frac{mg'}{b}$ and $\tau = \frac{m}{b}$.

(b) Numerical

$m = 3.27\times10^{-5}$ kg

$b = 3\pi\eta D =$

$\tau = 1.44\times10^{-4}$ s ; $v_{ter} = 1.18\times10^{-3}$ $\frac{m}{s}$

95% of terminal speed

$$v = v_{ter}\left(1 - e^{-t/\tau}\right) = 0.95\, v_{ter}$$

$$t_{95} = 4.33\times10^{-4}\ s$$

Reynolds number at $v = v_{ter}$,

$$f_{quad} / f_{lin} = Dv\rho / (48\eta) = 5\times10^{-6}$$

is very small ; so f_{quad} is negligible.

Problem 2.18

Taylor's theorem ...

$f(x+\delta) = f(x) + f'(x)\,\delta + f''(x)\,\delta^2/2! + f'''(x)\,\delta^3/3! + \ldots$

(a) $\ln(1+\delta)$; let $f = \ln$ and $x = 1$;

$\ln(1+\delta) = \delta - \delta^2/2 + \delta^3/3 \quad - + \ldots$

(b) $\cos\delta$; let $f = \cos$ and $x = 0$;

$\cos\delta = 1 - \delta^2/2 + \delta^4/24 \quad - + \ldots$

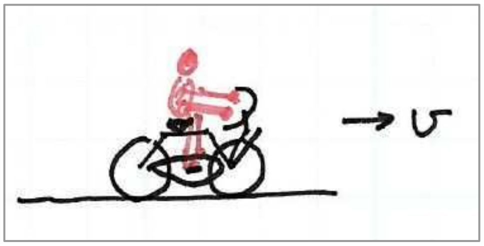
(c) $\sin\delta$; let $f = \sin$ and $x = 0$;

$\sin\delta = \delta - \delta^3/6 + \delta^5/120 \quad - + \ldots$

(d) $\exp\delta = 1 + \delta + \delta^2/2 + \delta^3/6 + \ldots + \delta^n/n! + \ldots$

Problem 2.26

A bicycle rider, coasting to a stop …



horizontal motion
with air resistance

Initial velocity = v_0 = 20 m/s and mass = 80 kg.

Use quadratic air resistance, $f_{quad} = c\,v^2$ where c = 0.20 (MKS units)

The characteristic time is $\tau = M / (c v_0) = 20$ sec.

The velocity as a function of time is $v(t) = v_0 / (1 + t/\tau)$;

therefore $t = \tau (v_0/v - 1)$

v	t
20 m/s	0 s
15 m/s	6.33 s
10 m/s	20 s
5 m/s	60 s

In[1]:=
```
Remove["Global`*"]
g = 9.8;
b = 1.6 * 10^-4 * diameter;
c = 0.25 * diameter^2;
m = 0.52 * 10^3 * diameter^3;
Q = NSolve[m * g - b * v - c * v^2 == 0, v]
v1 = v /. Q[[2]]
vter[di_] := v1 /. diameter → di
{vter[0.1*^-3], vter[3*^-3]}
```

Remove::rmnsm : There are no symbols matching "Global`*". ≫

Out[6]=
$$\left\{\left\{v \to \frac{0.00032\left(-1.-1.\sqrt{1.+1.99063\times10^{11}\,\text{diameter}^3}\right)}{\text{diameter}}\right\},\left\{v \to \frac{0.00032\left(-1.+\sqrt{1.+1.99063\times10^{11}\,\text{diameter}^3}\right)}{\text{diameter}}\right\}\right\}$$

Out[7]=
$$\frac{0.00032\left(-1.+\sqrt{1.+1.99063\times10^{11}\,\text{diameter}^3}\right)}{\text{diameter}}$$

Out[9]= {0.304055, 7.71404}

```
Plot[vter[di], {di, 0.1*^-4, 3*^-3},
 PlotRange → {{0, .003}, {0, 10}},
 AxesLabel → {"diameter [m]",
   "terminal speed [m/s]"}]
```