2.4 Rules For Matrix Operations

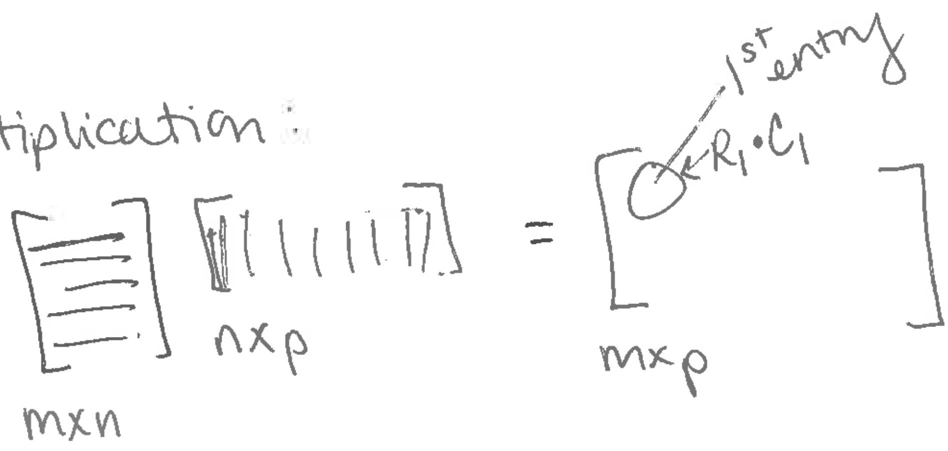
Adding: We can add two matrices if they are the same size (both matrices are $m \times n$)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1+2 & 2+7 \\ 3+-1 & 4+5 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 2 & 9 \end{bmatrix}$$

Scalar multiplication: multiply a matrix by a scalar, multiply every entry of the matrix by that number.

$$4 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

Matrix Multiplication:



Matrix Multiplication

$AB \neq BA$ matrix mult. does not usually commute

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ 1×3 $\qquad B = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ 3×1

$2 + 2(4) + 3(6)$

$$AB = \underset{1\times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}} = \underset{1\times 1}{\begin{bmatrix} 28 \end{bmatrix}}$$

$$BA = \underset{3\times 1}{\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}} \underset{1\times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \\ 6 & 12 & 18 \end{bmatrix}}$$

<u>Properties</u> A, B, C are matrices

$AB \neq BA$

$(AB)C = A(BC)$

$C(A+B) = CA + CB$

$(A+B)C = AC + BC$

2.5 Inverse Matrices

What does it mean to be an inverse?

$$AB = I$$

Then B is the inverse of A + denoted A^{-1}

* Square matrix : $n \times n$

1. If the square matrix A has an inverse, then $A^{-1}A = I$ and $AA^{-1} = I$

2. If A has an inverse, it has $\underline{\underline{n}}$ nonzero pivots (find this by doing elimination)
   * n nonzero pivots means there are no zeros along diagonal when the matrix is in upper triangular form

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$\underbrace{}_{\text{Identity}} \qquad \underbrace{}_{\text{Inverse}}$$

Check $\quad AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

on own: $A^{-1}A = I$

$\boxed{\text{Ex}}$ Solve $A\vec{x} = \vec{b}$ using inverses.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$A^{-1}A\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = \underset{2\times 2}{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 3 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} -3 \\ 2 \end{bmatrix}}$$

$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$A\vec{x} = \vec{b}$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 3 & 3 \end{array}\right]$$

use elimination to get upper triangular matrix then use back-sub.

Possibilities

for solutions to $A\vec{x} = \vec{b}$

1. no solution
2. one solution
3. infinitely many

if A^{-1} exists, there is only one solution

$$\begin{bmatrix} * & & \# \\ & * & \\ & & * \\ 0 & \text{---} & 0 \end{bmatrix} \begin{bmatrix} \\ \\ \\ 0 \end{bmatrix} \qquad \begin{bmatrix} * & & & \# \\ & * & & \\ & & * & \\ 0 & \cdots & * & 0 \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ \\ 4 \end{bmatrix}$$

$0x + 0y + \ldots = 0$

always true

$0x + 0y + \ldots \neq 4$

no solution

Ex) Find A^{-1}. $A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$

Use Guass-Jordan Elimination.

$[A \mid I]$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

Have a zero pivot, A is not invertible,

A^{-1} does not exist

* ref A^{-1} does not exist... that also means

$A\vec{x} = \vec{0}$ has nontrivial solutions

(nontrivial solutions means solutions other than $\vec{x} = \vec{0}$)

☆ A is invertible if and only if $\vec{x} = \vec{0}$ is the only solution to $A\vec{x} = \vec{0}$

Guass - Jordan Elimination to find A^{-1}

Ex) Find A^{-1} using Guass - Jordan Elimination

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

To start... Create an augmented matrix

$[A \mid I]$

2x2 Identity

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

original matrix A

Do Row Operations on $[A \mid I]$ until we get

$[I \mid A^{-1}]$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$ eliminate a_{21}

$R_2 \to R_2 - R_1$

If we eliminate an entry in Row_2, can't add a multiple of Row 2

$\boxed{\text{Not correct: } R_2 \to R_1 - R_2}$ *

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$ use R_2 to eliminate a_{12}

$R_1 \to R_1 - 2R_2$