LU decomposition

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

one way

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad R_1 \to \frac{1}{2} R_1$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 3/2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad R_2 \to \frac{2}{3} R_2$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad R_3 \to \frac{1}{2} R_3 \Rightarrow U = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Corresponding $L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & \frac{3}{2} & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Check: $LU = A$

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3/2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

yes!!

Another way

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad R_2 \to R_2 - \frac{1}{2}R_1$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad R_3 \to R_3 - \frac{2}{3}R_2$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 0 & 4/3 \end{bmatrix} \Leftarrow U'$$

corresponding

$$L' = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}$$

Check... $L'U' = A$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}}_{L'} \underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3/2 & 1 \\ 0 & 0 & 4/3 \end{bmatrix}}_{U'} = \underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}}_{A}$$