Name: Key

Please work together to solve the problems. Do not be afraid to ask questions!

1. Find the parabola $C + Dt + Et^2$ that comes closest (least squares error) to the values $\mathbf{b} = (0, 0, 1, 0, 0)$ at the times $t = -2, -1, 0, 1, 2$. First write down the five equations $A\mathbf{x} = \mathbf{b}$ in three unknowns $\mathbf{x} = (C, D, E)$ for a parabola to go through the five points. No solution becuse no such parabola exists. Solve $A^T A\hat{x} = A^T\mathbf{b}$. 4.3B in sec 4.3 (worked examples)

2. (By Calculus) Write down $E = ||A\mathbf{x} - \mathbf{b}||^2$ as a sum, of four squares –the last one is $(C + 4D - 20)^2$. Find the derivative equations $\partial E/\partial C = 0$ and $\partial E/\partial D = 0$. Divide by 2 to obtain the normal equations $A^T A\hat{x} = A^T\mathbf{b}$. #4 in 4.3

3. Suppose $\mathbf{b}$ equals 2 times the first column of A. What is the projection of $\mathbf{b}$ onto the column space of A? Is $P = I$ for sure in this case? Compute $\mathbf{p}$ and P when $\mathbf{b} = (0, 2, 4)$ and the columns of A are ~~$(0, 2, 1)$~~ $(0,1,2)$ and $(1, 2, 0)$. #14 in 4.2

4. Project $\mathbf{b}$ onto the column space of A by solving $A^T A\hat{x} = A^T\mathbf{b}$ and $\mathbf{p} = A\hat{x}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

#11 part (a) in 4.2

5. Suppose S is a six-dimensional subspace of nine-dimensional space $\mathbb{R}^9$.
   (a) What are the possible dimensions of subspaces orthogonal to S?
   (b) What are possible dimensions of the orthogonal complement $S - perp$ of S?
   (c) What is the smallest possible size of matrix A that has rowspace S?
   (d) What is the smallest possible size of a matrix B that has nullspace $S - perp$?

   4.1A in sec 4.1 (worked example)

6. Suppose V is the whole space $\mathbb{R}^4$. Then $V^\perp$ contains only the vector __________. Then $(V^\perp)^\perp$ is __________. So $(V^\perp)^\perp$ is the same as __________. #20 in 4.1

7. Why are the following statements false?
   (a) $(1, 1, 1)$ is perpendicular to $(1, 1, -2)$ so the plans $x + y + z = 0$ and $x + y - 2z = 0$ are orthogonal subspaces.
   (b) Two subspaces that meet only in the zero vector are orthogonal.

#28 in 4.1