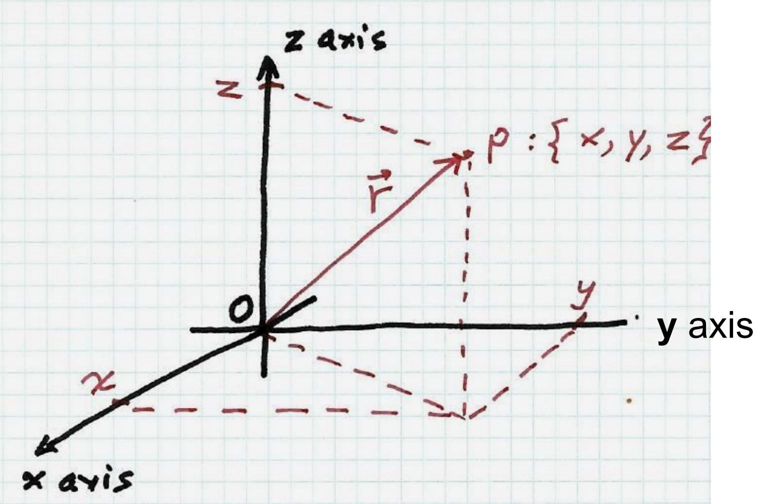
Section 1.6. Cartesian coordinates
Section 1.7. Plane polar coordinates

Read Sections 1.6 and 1.7.



$$\boldsymbol{r}(t) = x(t)\, \boldsymbol{e}_x + y(t)\, \boldsymbol{e}_y + z(t)\, \boldsymbol{e}_z$$

Hand written: $\vec{r}(t) = x(t)\, \hat{e}_x + y(t)\, \hat{e}_y + z(t)\, \hat{e}_z$

1.6. Cartesian coordinates {x,y,z}

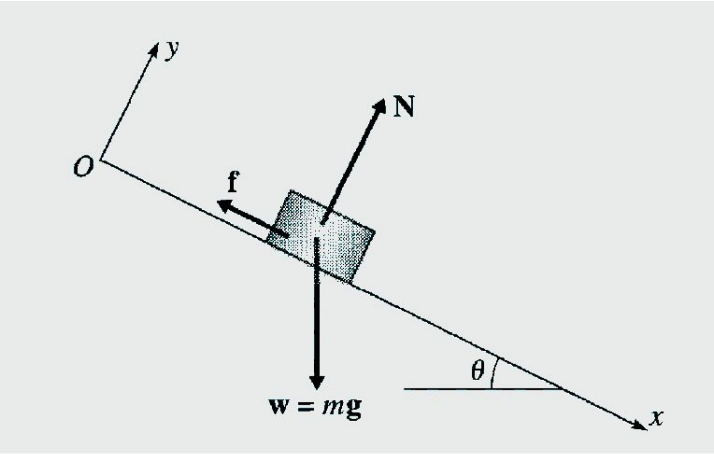
$$m\vec{a} = \vec{F} \qquad \left\{ \begin{array}{l} m a_x = F_x \\ m a_y = F_y \\ m a_z = F_z \end{array} \right.$$

$$m\ddot{\vec{r}} = \vec{F} \qquad \left\{ \begin{array}{l} m \ddot{x} = F_x \\ m \ddot{y} = F_y \\ m \ddot{z} = F_z \end{array} \right.$$

$$\boldsymbol{F} = F_x\, \boldsymbol{e}_x + F_y\, \boldsymbol{e}_y + F_z\, \boldsymbol{e}_z$$

Example 1.1. *A block sliding down an inclined plane(†)*

Figure 1.9



Determine the motion.

† Don't make the mistake of thinking that inclined planes are trivial. It was by observing balls rolling on an incline that Galileo discovered that d ∝ t^2 for constant acceleration.

Use the Cartesian coordinates x,y, shown in the figure.
Newton's second law,

$$m\, d\mathbf{v} / dt = \mathbf{F} = \boldsymbol{w} + \boldsymbol{N} + \boldsymbol{f}$$

Separate the vectors into components.

$$m\, dv_x/dt = w \sin\theta - f$$

$$m\, dv_y/dt = N - w \cos\theta$$

Now we need to use a property of the contact force (PHY 183)

$f = \mu N$ ***(coefficient of kinetic friction)***

Also, $\boldsymbol{v_y = 0}$ ***implies*** $\boldsymbol{N = mg \cos\theta}$ ***;*** therefore

$$m\, dv_x/dt = mg\, (\sin\theta - \mu \cos\theta) ;$$

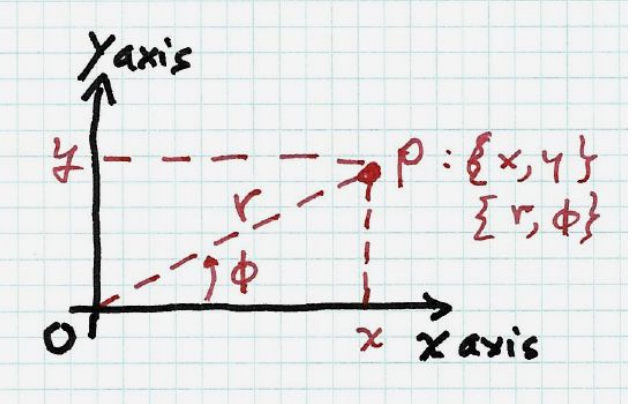
i.e., m has constant acceleration along x.

Any comments?

1.7 : Plane polar coordinates

■ Here is the figure that defines plane polar coordinates, r and φ.

Figure 1.10:
Plane polar coordinates, r,φ



■ Here are the algebraic equations for plane polar coordinates.

/1/ Express x and y in terms of polar coordinates r and φ ,

$$x = r \cos \varphi$$
$$y = r \sin \varphi$$

/2/ Express r and φ in terms of Cartesian coordinates x and y ,

$$r = \text{sqrt}(x^2 + y^2)$$
$$\varphi = \arctan(y/x)$$

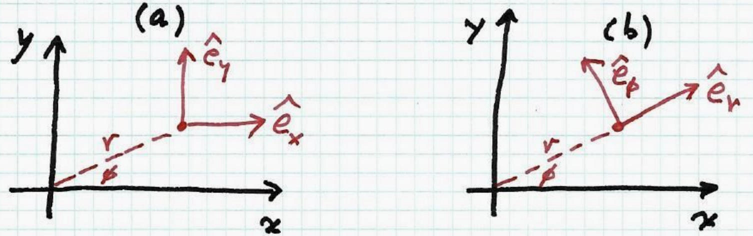
Memorize them!

Vectors in plane polar coordinates

■ Here is the figure that defines the *direction vectors* for plane polar coordinates,

Figure 1.11.

direction vectors



unit direction vectors

(a) Cartesian ; (b) Plane polar

$\hat{e}_x$ and $\hat{e}_y$ are constant ; $\hat{e}_r$ and $\hat{e}_\phi$ depend on $\vec{r}$.

more precisely, on φ

■ Here are the algebraic equations for vectors in plane polar coordinates.

$$\hat{e}_x = \hat{e}_r \cos\phi - \hat{e}_\phi \sin\phi$$

$$\hat{e}_y = \hat{e}_r \sin\phi + \hat{e}_\phi \cos\phi$$

$$\hat{e}_r = \hat{e}_x \cos\phi + \hat{e}_y \sin\phi = \vec{r}/r$$

$$\hat{e}_\phi = -\hat{e}_x \sin\phi + \hat{e}_y \cos\phi \quad \text{is } \perp \hat{e}_r$$

$\hat{e}_r$ and $\hat{e}_\phi$ depend on ϕ.

Commit these equations and figures to memory!

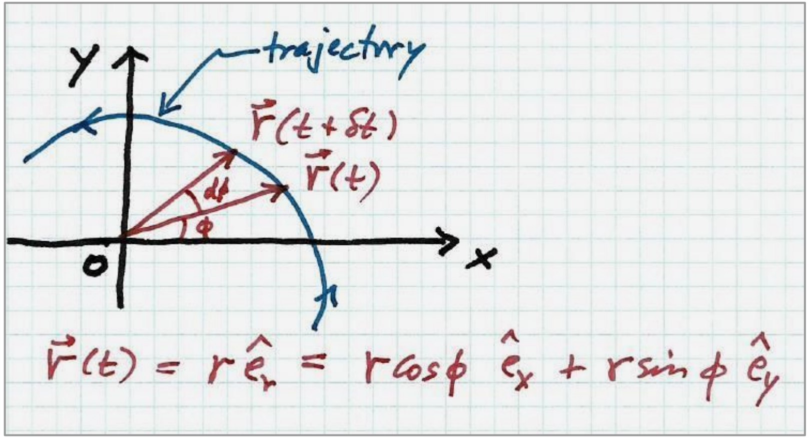
Kinematics in plane polar coordinates

A particle moves in two dimensions.

The coordinates are r(t) and φ(t).

What are the ***vectors,***

r*(t)* , ***v****(t)* and ***a****(t)* **?**



$$\vec{r}(t) = \hat{e}_x\, r\cos\phi + \hat{e}_y\, r\sin\phi = r(t)\,\hat{e}_r(t)$$

$$\vec{v} = \dot{\vec{r}} = \hat{e}_x(\dot{r}\cos\phi - r\dot{\phi}\sin\phi) + \hat{e}_y(\dot{r}\sin\phi + r\dot{\phi}\cos\phi)$$

$$= \dot{r}\,\hat{e}_r + r\dot{\phi}\,\hat{e}_\phi$$

$$\vec{a} = \dot{\vec{v}} = \hat{e}_x(\ddot{r}\cos\phi - 2\dot{r}\dot{\phi}\sin\phi - r\ddot{\phi}\sin\phi - r\dot{\phi}^2\cos\phi) + \hat{e}_y(\ddot{r}\sin\phi + 2\dot{r}\dot{\phi}\cos\phi + r\ddot{\phi}\cos\phi - r\dot{\phi}^2\sin\phi)$$

$$= (\ddot{r} - r\dot{\phi}^2)\,\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\hat{e}_\phi$$

Special case. Suppose the particle moves *on a circle*. Then …

r(t) = R (constant)

v(t) = R φ′ $\boldsymbol{e}_\varphi$ (tangent)

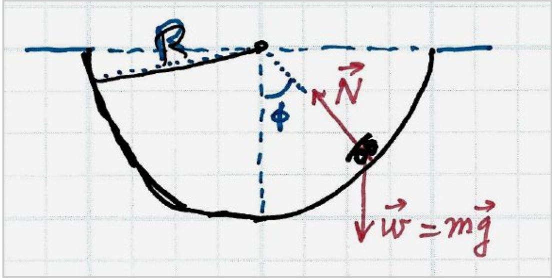
a(t) = R φ′′ $\boldsymbol{e}_\varphi$ – R (φ′)^2 $\boldsymbol{e}_r$

(tangential) (centripetal)

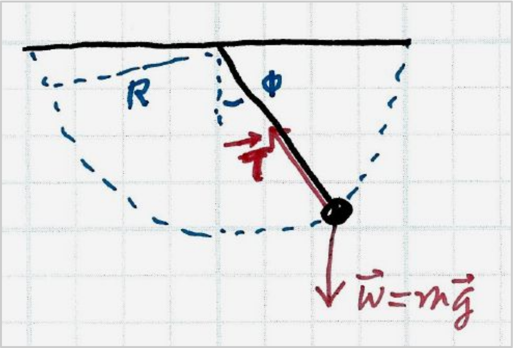
<u>Example 1.2</u>

AN OSCILLATING SKATEBOARD

See Figure 1.14



and realize that it is just like a simple pendulum



Equations in plane polar coordinates

$$F_r = mg\cos\phi - N = -mR\dot{\phi}^2 \quad (1)$$

$$F_\phi = -mg\sin\phi = mR\ddot{\phi} \quad (2)$$

Eq. (1) determines N; Eq. (2) determines φ(t).

$$\ddot{\phi} = -\frac{g}{R}\sin\phi$$

We can't solve it analytically; solve it by computer in Taylor problem 1.50.

<u>For small φ</u> we can make an approximation,

sin φ ≈ φ; so then

$$\ddot{\phi} = -\frac{g}{R}\phi$$

Small oscillations are harmonic,

$$\phi(t) = C_1\cos\omega_0 t + C_2\sin\omega_0 t$$

$$\text{where } \omega_0 = \sqrt{g/R}\,.$$

✔

Test yourself:
Calculate the *period of oscillation* for small oscillations of the the oscillating skateboard, if R = 5 m.

Test yourself:

A car drives around a circular track (radius = R) with constantly increasing speed. The angle φ as a function of time t is
$\varphi(t) = \frac{1}{2} \alpha t^2$ where α is constant.

(A) Write the coordinates x(t) and y(t).

(B) Calculate the velocity and acceleration vectors. Make a drawing that shows the velocity and acceleration vectors when the car first passes the point at $\varphi = \pi$.

Homework Assignment #2
due in class Friday, September 16
[6] Problem 1.35 *
[7] Problem 1.38 *
[8] Problem 1.39 **
[9] Problem 1.44 *
[10] Problem 1.51 *** [computer]
Use the cover sheet.

Computer problems.
Your best bet is to use *Mathematica*. It is available in many MSU microcomputer labs, e.g., 106 Farrell Hall or 1210 Anthony Hall. If you are not familiar with *Mathematica*, then see the handout.