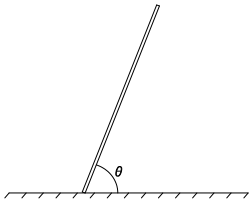
Physics 321 – Spring 2017

Homework #6, due at beginning of class Wednesday Feb 22.

1. [6 pts] A thin uniform stick of wood of length B and mass M is in the process of falling to the floor. It starts at $\theta = 90°$ with $\dot{\theta}$ vanishingly small but negative so it falls to $\theta = 0$. Assume that the end on the floor does not slip, and ignore air resistance.



   (a) Write its kinetic energy as a function of B, M, θ, and $\dot{\theta}$.

   (b) Write its potential energy as a function of B, M, g, θ.

   (c) Use energy conservation (KE + PE = constant) to obtain a relation between $\dot{\theta}$ and θ. Evaluate the constant using the initial conditions $t = 0$.

   (d) Take the derivative of your energy conservation equation with respect to time to obtain $\ddot{\theta}$ as a function of θ.

   (e) Find the horizontal component N_x of the force due to the floor as a function of B, M, g, θ.

   (f) Find the vertical component N_y of the force due to the floor as a function of B, M, g, θ.

   (g) Write the stick's angular momentum about the point of contact with the floor.

   (h) Write the torque on the stick due to gravity about the point of contact with the floor.

   (i) Use the formula $\tau = dL/dt$ where τ is the torque and L is the angular momentum, to obtain an equation of motion for $\ddot{\theta}$ as a function of θ and check that it agrees with your previous result for that.

   (j) What is the smallest coefficient of friction that will keep the stick from slipping through the entire fall? (Hint: look at the ratio N_x/N_y near $\theta = 0$.

2. [4 pts] The potential energy of a particle is given by $U(x, y, z) = a \sin(b\, x\, y\, z^2)$ where a and b are constants.

   (a) Find the magnitude of the force at the point $x = y = z = 1$.

   (b) Find the unit vector in the direction of the force at the point $x = y = z = 1$, assuming $a > 0$ and $b = \pi$.

3. [4 pts] A conservative force is acting in the two-dimensional plane (x, y). The component of force in the x-direction is $F_x = a\, x^2\, y^3$ where a is a constant.

   (a) Find the most general form possible for the potential energy $U(x, y)$.

   (b) Find the most general form possible for the y-component of the force $F_y(x, y)$.

4. [6 pts] Taylor problem 4.20

(Last updated 2/17/2017.)