Trigonometric Identities

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi \qquad \cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\cos\theta\cos\phi = \tfrac{1}{2}[\cos(\theta+\phi) + \cos(\theta-\phi)] \qquad \sin\theta\sin\phi = \tfrac{1}{2}[\cos(\theta-\phi) - \cos(\theta+\phi)]$$

$$\sin\theta\cos\phi = \tfrac{1}{2}[\sin(\theta+\phi) + \sin(\theta-\phi)]$$

$$\cos^2\theta = \tfrac{1}{2}[1+\cos 2\theta] \qquad \sin^2\theta = \tfrac{1}{2}[1-\cos 2\theta]$$

$$\cos\theta + \cos\phi = 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} \qquad \cos\theta - \cos\phi = 2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}$$

$$\sin\theta \pm \sin\phi = 2\sin\frac{\theta\pm\phi}{2}\cos\frac{\theta\mp\phi}{2}$$

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \text{[Euler's relation]}$$

$$\cos\theta = \tfrac{1}{2}(e^{i\theta} + e^{-i\theta}) \qquad \sin\theta = \tfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

Hyperbolic Functions

$$\cosh z = \tfrac{1}{2}(e^z + e^{-z}) = \cos(iz) \qquad \sinh z = \tfrac{1}{2}(e^z - e^{-z}) = -i\sin(iz)$$

$$\tanh z = \frac{\sinh z}{\cosh z} \qquad \mathrm{sech} z = \frac{1}{\cosh z}$$

$$\cosh^2 z - \sinh^2 z = 1 \qquad \mathrm{sech}^2 z + \tanh^2 z = 1$$

Series Expansions

$$f(z) = f(a) + f'(a)(z-a) + \tfrac{1}{2!}f''(a)(z-a)^2 + \tfrac{1}{3!}f'''(a)(z-a)^3 + \cdots \qquad \text{[Taylor's series]}$$

$$e^z = 1 + z + \tfrac{1}{2!}z^2 + \tfrac{1}{3!}z^3 + \cdots \qquad \ln(1+z) = z - \tfrac{1}{2}z^2 + \tfrac{1}{3}z^3 - \cdots\ [\,|z| < 1]$$

$$\cos z = 1 - \tfrac{1}{2!}z^2 + \tfrac{1}{4!}z^4 - \cdots \qquad \sin z = z - \tfrac{1}{3!}z^3 + \tfrac{1}{5!}z^5 - \cdots$$

$$\cosh z = 1 + \tfrac{1}{2!}z^2 + \tfrac{1}{4!}z^4 + \cdots \qquad \sinh z = z + \tfrac{1}{3!}z^3 + \tfrac{1}{5!}z^5 + \cdots$$

$$\tan z = z + \tfrac{1}{3}z^3 + \tfrac{2}{15}z^5 + \cdots\ [\,|z| < \pi/2] \qquad \tanh z = z - \tfrac{1}{3}z^3 + \tfrac{2}{15}z^5 - \cdots\ [\,|z| < \pi/2]$$

$$(1+z)^n = 1 + nz + \frac{n(n-1)}{2!}z^2 + \cdots\ [\,|z| < 1] \qquad \text{[binomial series]}$$

Some Derivatives

$$\frac{d}{dz}\tan z = \sec^2 z \qquad \frac{d}{dz}\tanh z = \operatorname{sech}^2 z$$

$$\frac{d}{dz}\sinh z = \cosh z \qquad \frac{d}{dz}\cosh z = \sinh z$$

Some Integrals

$$\int \frac{dx}{1+x^2} = \arctan x \qquad \int \frac{dx}{1-x^2} = \operatorname{arctanh} x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x \qquad \int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh} x$$

$$\int \tan x \, dx = -\ln\cos x \qquad \int \tanh x \, dx = \ln\cosh x$$

$$\int \frac{dx}{x+x^2} = \ln\left(\frac{x}{1+x}\right) \qquad \int \frac{x\,dx}{1+x^2} = \ln(1+x^2)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arccosh} x \qquad \int \frac{x\,dx}{\sqrt{1+x^2}} = \sqrt{1+x^2}$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \arccos(1/x) \qquad \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = \arcsin(\sqrt{x}) - \sqrt{x(1-x)}$$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \frac{x}{(1+x^2)^{1/2}} \qquad \int \ln(x)\,dx = x\ln(x) - x$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}\sqrt{1-mx^2}} = K(m),$$ complete elliptic integral of first kind

Miscellaneous Data (for use in some end-of-chapter problems)

Solar System

(mass of earth) = 5.97×10^{24} kg
(radius of earth) = 6.38×10^{6} m
(mass of moon) = 7.35×10^{22} kg
(radius of moon) = 1.74×10^{6} m
(mass of sun) = 1.99×10^{30} kg
(radius of sun) = 6.96×10^{8} m
(earth–moon distance) = 3.84×10^{8} m
(earth–sun distance) = 1.50×10^{11} m

Ideal Gases

Avogadro's number, $N_A = 6.02 \times 10^{23}$ particles/mole
Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K = 8.62×10^{-5}eV/K
Gas constant, $R = 8.31$ J/(mole·K) = 0.0821 liter·atm/(mole·K)
STP = 0°C and 1 atm
(Volume of 1 mole of gas at STP) = 22.4 liters

Conversion Factors

Area:	1 barn = 10^{-28} m^2
Energy:	1 eV = 1.60×10^{-19} J
	1 cal = 4.184 J
Length:	1 inch = 2.54 cm
	1 mile = 1609 m
Mass:	1 u (atomic mass unit) = 1.66×10^{-27} kg = 931.5 MeV/c^2
	1 lb (mass) = 0.454 kg
	1 MeV/c^2 = 1.074×10^{-3} u = 1.783×10^{-30} kg
Momentum:	1 MeV/c = 5.34×10^{-22} kg·m/s

A Few More Constants

Speed of light, $c = 3.00 \times 10^{8}$ m/s
Planck's constants: $h = 6.63 \times 10^{-34}$ J·s and $\hbar = 1.05 \times 10^{-34}$ J·s
Vacuum permeability, $\mu_o = 4\pi \times 10^{-7}$ N/A^2
Vacuum permittivity, $\epsilon_o = 8.85 \times 10^{-12}$ C^2/(N·m^2)
Coulomb force constant, $k = 1/(4\pi\epsilon_o) = 8.99 \times 10^{9}$ N·m^2/C^2

Vector Identities

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \qquad [BAC - CAB \text{ rule}]$$

Vector Calculus

$$\nabla f = \hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y} + \hat{\mathbf{z}}\frac{\partial f}{\partial z} \qquad \text{[Cartesian]}$$

$$= \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi} \qquad \text{[spherical polars]}$$

$$= \hat{\boldsymbol{\rho}}\frac{\partial f}{\partial \rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial f}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial f}{\partial z} \qquad \text{[cylindrical polars]}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{x}}\left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right) + \hat{\mathbf{y}}\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)$$

$$+ \hat{\mathbf{z}}\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right) \qquad \text{[Cartesian]}$$

$$= \hat{\mathbf{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial}{\partial \phi}A_\theta\right] + \hat{\boldsymbol{\theta}}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}A_r - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right]$$

$$+ \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}A_r\right] \qquad \text{[spherical polar]}$$

$$= \hat{\boldsymbol{\rho}}\left[\frac{1}{\rho}\frac{\partial}{\partial \phi}A_z - \frac{\partial}{\partial z}A_\phi\right] + \hat{\boldsymbol{\phi}}\left[\frac{\partial}{\partial z}A_\rho - \frac{\partial}{\partial \rho}A_z\right]$$

$$+ \hat{\mathbf{z}}\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial}{\partial \phi}A_\rho\right] \qquad \text{[cylindrical polar]}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z \qquad \text{[Cartesian]}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}A_\phi \qquad \text{[spherical polars]}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}A_\phi + \frac{\partial}{\partial z}A_z \qquad \text{[cylindrical polars]}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \qquad \text{[Cartesian]}$$

$$= \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \qquad \text{[spherical polars]}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \qquad \text{[cylindrical polars]}$$