Experiment 1: Thin Converging Lens Phys 431

This experiment is a classic exercise in geometric optics. The goal is to measure the radius of curvature and focal length of a single *converging lens*, and then to calculate the index of refraction. A major theme of this lab is that measured quantities should always include the estimated uncertainty. This includes quantities derived from measurements, in this case the index of refraction.

In the procedures for this lab, you are explicitly reminded to estimate the uncertainty several times. (In future labs these reminders will not necessarily be included.). Please see Appendix (i) and (ii) for reference material and relevant equations. Remember that you are free to observe and to ask questions to classmates as you perform the experiment. Of course, you should also seek the advice of your TA. <u>The questions, labeled **Q1**… should be addressed in your write-up in the Analysis & Discussion section.</u>

Procedure:

A. By definition, the focal length *f* is the image distance from the center of the lens for an infinitely distance object. To obtain a rough estimate for *f*, project an image of the trees outside the lab onto the white paper near the door.
Q1 Why do the trees appear upside down?

B. Use a spherometer to measure the radius of curvature for both surfaces of your lens. Please see Appendix (i). You will have to begin by finding the "zero" position, x_0, using a scratch-free spot on your bench (which is a good approximation of a flat surface). Then perform the measurement with your lens in place, x_1; the distance *h* will then be $|x_0 - x_1|$. (Always be sure to include the uncertainty. In this case you can repeat the measurements a few times to obtain an estimate for the spherometer's precision, σ_{x0}).
Q2 Having estimated σ_{x0} and σ_{x1}, can you write an expression for σ_h?

C. Arrange an object (the T on the lamp window) and screen on the optical rail with a separation greater than 4*f*. Locate the lens position which gives a sharp image on the screen. Record the object and image distances measuring from the center of the lens. (Be sure to estimate the uncertainty for these distances.) Use the thin lens equation to calculate *f*. (Also calculate σ_f). Repeat this for 4 positions of the screen increasing the object-screen separation in units of about 2 cm. Find your best value for the focal length using the last equation of Appendix (ii).
Q3 Is an iris needed or helpful to obtain a sharp image? What does this indicate?
Q4 How does your best value for *f* compare to your original rough estimate?

D. Place the light source a distance less than *f* from the lens. Try to position the screen to bring the object into focus.
Q5 Any difficulties? What is going on here?

E. Calculate the index of refraction (including uncertainty) for the glass of your lens using the lensmaker's equation. (Remember that your write-up should include comments as to whether or not your value is reasonable).

Appendix (i) Misc. Equations

Thin Lens Equation: essentially

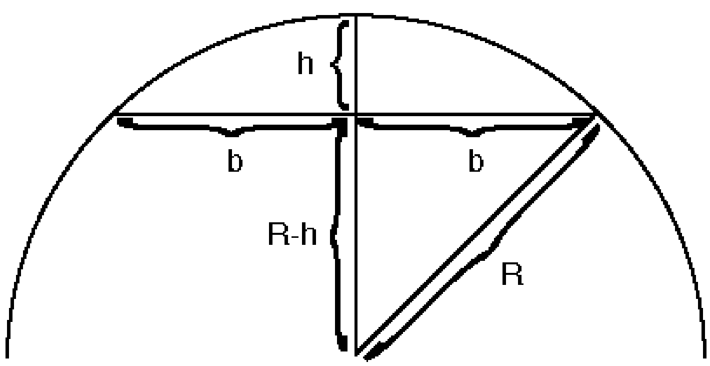
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

Lensmaker's Equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$, where we assume that the index of refraction for the surrounding medium (air) is 1.000.

Spherometer Equation:

$$R=\frac{b^2}{2h}+\frac{h}{2}$$



From Pythagoras' Theorem: $R^2=(R-h)^2+b^2$

$$R^2=R^2-2Rh+h^2+b^2$$
$$R=\frac{b^2}{2h}+\frac{h}{2}$$

Appendix (ii) Some Error Analysis

Random fluctuations in the measurement process lead to a Gaussian distribution about the true value. This distribution gives us a parameter, σ, called the "standard deviation". (Systematic errors lead to a non-Gaussian distribution.) Essentially, if many measurements are taken, 68% of the data points lie within $x_0 \pm \sigma_x$.

Now, suppose f = f(x,y) is some function of measured quantities x and y. What is the uncertainty in f, σ_f, given σ_y and σ_x? Under the assumption that the uncertainties are small compared to the absolute value of the quantities in question, the following expression always works:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x{}^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y{}^2}$$

For errors that are much smaller than the measured values (i.e. $\sigma_x << x$):

$$f = ax + by$$
$$\sigma_f = \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}$$

$$f = cxy$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

$$f = cx^a y^b$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{a\sigma_x}{x}\right)^2 + \left(\frac{b\sigma_y}{y}\right)^2}$$

$$f = ce^{bx}$$
$$\frac{\sigma_f}{f} = b\sigma_x$$

$$f = ca^{bx}$$
$$\frac{\sigma_f}{f} = (b\ln a)\sigma_x$$

Lastly, we address the situation where we make n measurements of the same quantity x, each with an uncertainty of σ_x. Intuitively, we expect that the average of all the measurements will have uncertainty smaller than σ_x. Indeed, the estimated uncertainty is reduced by $1/\sqrt{n}$:

$$\bar{x} = \frac{(x_1 + x_2 + ... + x_n)}{n} \rightarrow \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$