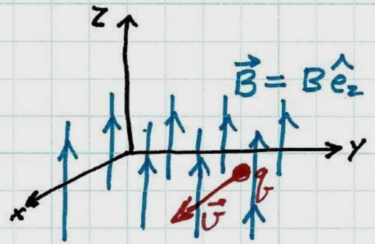
Section 2.7

*Solution for a charge q in a magnetic field **B**.*

Read Section 2.7.

Recall Monday's lecture



$m\dot{v}_z = 0$ so $v_z = v_{0z}$

$m\dot{v}_x = qBv_y$

$m\dot{v}_y = -qBv_x$

Def: $\omega = qB/m$

$\dot{v}_x = \omega v_y$

$\dot{v}_y = -\omega v_x$

consistent

η

Define a complex variable η by

$$\eta = v_x + i\,v_y$$

Now note that

$$\dot{\eta} = -i\,\omega\,\eta \qquad \text{(verify it!)}$$

The general solution of the equation of motion is $\eta(t) = A\,e^{-i\omega t}$.

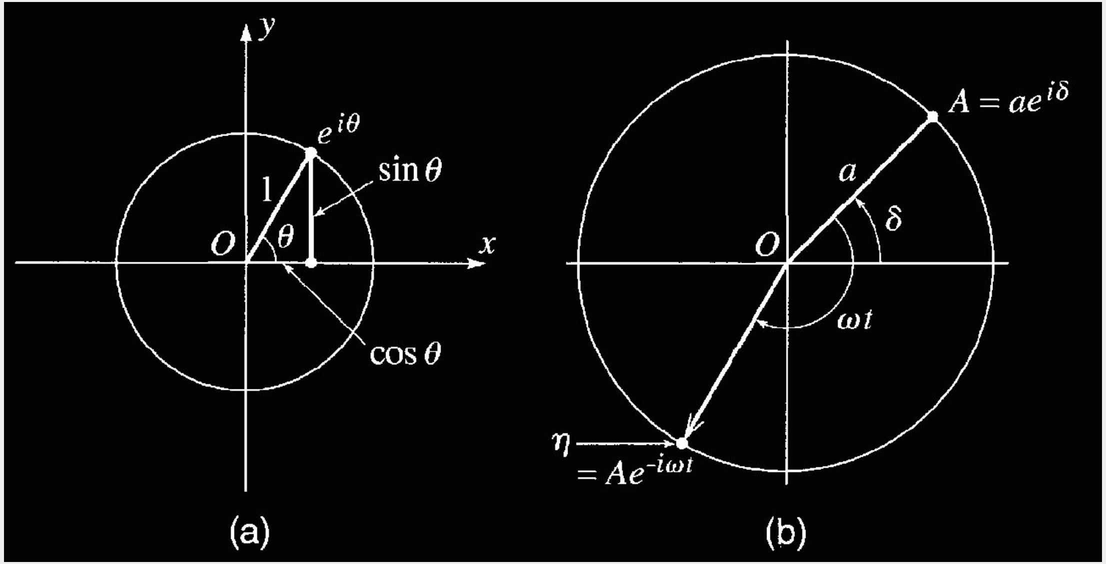
We must allow for A to be complex; e.g.,

$$A = a\,e^{i\delta} .$$

Then

$$v_x = \mathrm{Re}\,\eta = a\cos(\omega t - \delta)$$
$$v_y = \mathrm{Im}\,\eta = -a\sin(\omega t - \delta)$$

Figure 2.14 : The transverse velocity components



(a) Illustrates Euler's equation : $e^{i\theta} = \cos\theta + i \sin\theta$

(b) $\eta = A \exp(-i\,\omega t)$ and $A = a \exp(i\,\delta)$; $\eta(t)$ rotates clockwise

The trajectory, i.e., the *coordinates*

The transverse motion of a positive charge q in magnetic field **B** = B $\boldsymbol{e}_z$ ...

Define $\xi = x + iy$

Then $\dot{\xi} = \dot{x} + i\dot{y}$

$= v_x + i v_y = \eta$

$\eta = A e^{-i\omega t} \implies \xi = \frac{A}{-i\omega} e^{-i\omega t} + \text{constant}$

Or, write

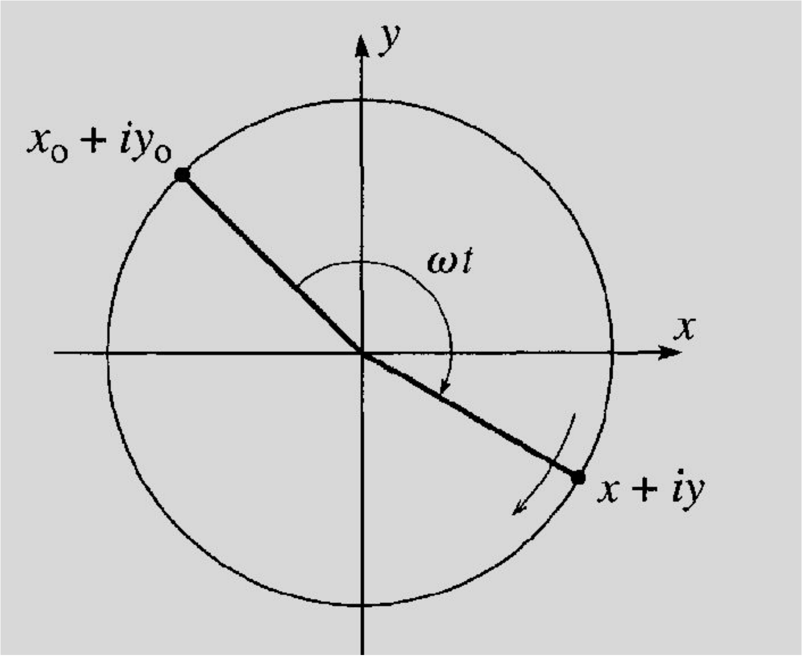
$\xi = C e^{-i\omega t} + a + ib$

W.L.O.G. set $a = 0$ and $b = 0$.

Then $x_0 + i y_0 = C$ (initial values)

$$\boxed{\begin{aligned} x + iy &= (x_0 + i y_0)(\cos\omega t - i \sin\omega t) \\ &= x_0 \cos\omega t + y_0 \sin\omega t \\ &\quad + i(y_0 \cos\omega t - x_0 \sin\omega t) \end{aligned}}$$

Figure 2.15



$x(t) = x_0 \cos\omega t + y_0 \sin\omega t$

$y(t) = y_0 \cos\omega t - x_0 \sin\omega t$

$x^2 + y^2 = x_0^2 + y_0^2$

The trajectory is a circle traversed clockwise (for q > 0).

In 3 dimensions, the general trajectory is a cylindrical helix.

Consider $y_0 = 0$.

Then

$$z(t) = v_{0z}\, t$$

$$x(t) = x_0 \cos(\omega t)$$

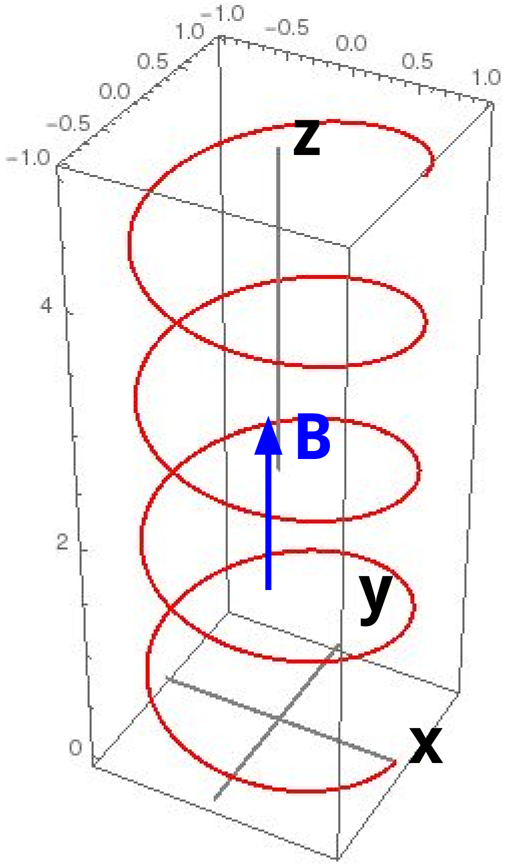
$$y(t) = -\, x_0 \sin(\omega t)$$

Radius $R = x_0$

Period $T = 2\pi / \omega$

where $\omega = qB/m$

Direction = clockwise in xy plane for positive q .



Test your understanding of magnetism:
Verify the direction from **F** = q **v x B !**

Applications of cyclotron motion

➨ ***The cyclotron***

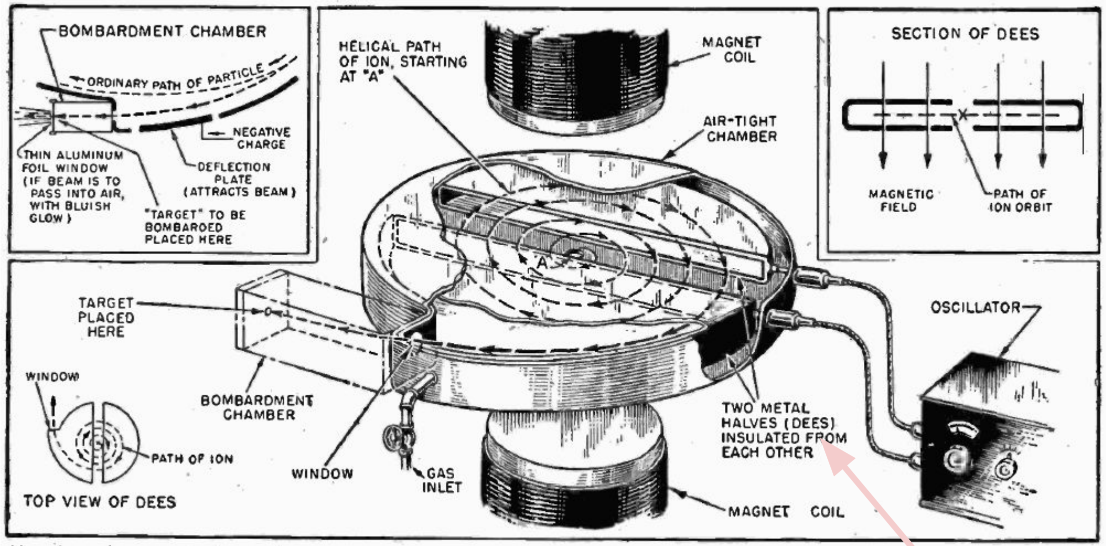
– The *cyclotron* is a type of charged particle accelerator.

– Invented by Ernest Lawrence in 1932.

– Used for scattering experiments in nuclear physics.

– The **N**ational **S**uperconducting **C**yclotron **L**aboratory at MSU used a more advanced design for the "dees".

– The particles (e.g., protons) travel on a semicircular trajectory; then the radius (and kinetic energy) increases each time they pass through the gap.



The dees" supply an electric field, which raises the particle's kinetic energy.

Magnet of the 184-inch cyclotron at Berkeley, 1946

➨ *The Large Hadron Collider at CERN*

Parameters

$2\pi R = 27$ km

proton $E = 6.5$ TeV

$= 6.5 \times 10^{12}$ e $\mathrm{Tm^2/s}$

proton $p = E/c$ $\quad (c = 3 \times 10^8$ m/s$)$

$p = eRB$

non relativistic derivation:

$mv = eRB$

$\Rightarrow \; v = R\left(\frac{eB}{m}\right) = R\omega$ ✓

$B = \dfrac{p}{eR} = 5$ T

Homework Assignment #4
due in class Friday Sept. 30
[17] Problem 2.23 *
[18] Problem 2.31 **
[19] Problem 2.41 **
[20] Problem 2.53 *
[20x] Problem 2.43 *** [computer]
Use the cover sheet.

➨ *Aurora Borealis*