Continuing Chapter 4 - Energy

Starting now, PHY 321 becomes more difficult.

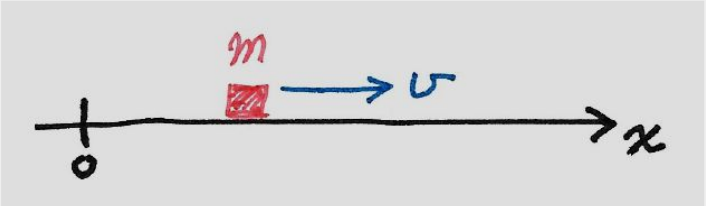
SECTION	TITLE	
4.6	Energy in linear motion	:: Monday
4.7	Curvilinear motion, and 1D systems	
4.8	Central forces	:: Wednesday
4.9	Energy for a system of 2 particles	:: Friday
4.10	Energy for many particles, and rigid bodies	

Homework Assignment 8, which includes some computer problems

Section 4.6. Energy and linear motion

We went over this last time, and you have already read Section 4.6.

We are concerned with *linear motion of a particle in a potential.*



- the coordinate x
- the equation of motion

$$m\ddot{x} = F(x) = -\,dU/dx$$

- the energies $T = \frac{1}{2} m \dot{x}^2$ and $U(x)$

Solve the equation of motion

How to determine the function x(t) ;

$$m\,x'' = F(x) = -\,dU/dx \quad \ldots$$

1. The first integral **(x″ ➡ x′)** comes from conservation of energy

$$\dot{x} = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

2. The second integral **(x′ ➡ x)** comes from the time calculation

$$dt = \frac{dx}{v} = \frac{dx}{\dot{x}}$$

$$\int_{t_0}^{t} dt' = \int_{x_0}^{x} \frac{dx'}{\pm\sqrt{\frac{2}{m}\left[E - U(x')\right]}}$$

3. Details – *the sign and the turning points and the energy* – to be worked out …

$$t - t_0 = \pm\sqrt{\frac{m}{2}} \int_{x_0}^{x} \frac{dx'}{\sqrt{E - U(x')}}$$

A trivial example: the free fall problem; we did it last time.

A less trivial example: a mass on a spring; Taylor Problem 4.28.

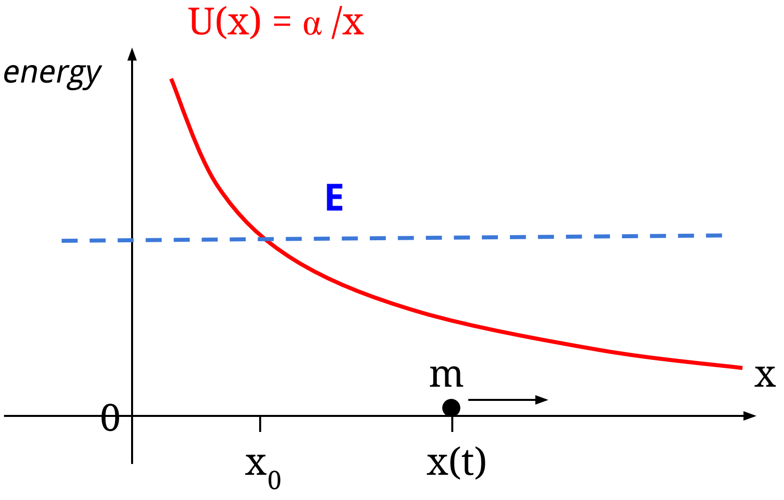
Here is a nontrivial example:

A particle moves on the x axis, with $x > 0$, and the potential energy is $U(x) = \alpha / x$. Assume α is positive, so the particles is repelled from the origin.

For example, consider a fixed charge at the origin and a moving charge (the "particle") on the positive x axis.

Suppose the particle is released from rest at x_0 . Calculate x(t).

Sketch a picture.



The problem is to calculate x(t).

Equations

$$m\ddot{x} = \alpha / x^2$$

$$x(0) = x_0 \quad \text{and} \quad \dot{x}(0) = 0$$

We can't solve them directly.
We'll use the conservation of energy to get the first integral.

1. Conservation of energy

$$E = \frac{1}{2} m \dot{x}^2 + \frac{\alpha}{x} = \frac{\alpha}{x_0}$$

$$\dot{x}^2 = \frac{2\alpha}{m}\left(\frac{1}{x_0} - \frac{1}{x}\right)$$

$$\dot{x} = \sqrt{\frac{2\alpha}{m x_0}}\left(1 - \frac{x_0}{x}\right)^{1/2}$$

2. The time calculation

$$dt = \frac{dx}{v} = \frac{dx}{\dot{x}}$$

$$\int_0^t dt' = t = \int_{x_0}^{x} \frac{dx'}{\sqrt{\frac{2\alpha}{m x_0}}\left(1 - \frac{x_0}{x'}\right)^{1/2}}$$

$$t = \sqrt{\frac{m x_0}{2\alpha}} \int_{x_0}^{x} \left(\frac{x'}{x' - x_0}\right)^{1/2} dx'$$

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \int_{1}^{x/x_0} \left(\frac{u}{u-1}\right)^{1/2} du$$

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \left\{ \left[\frac{x}{x_0}\left(\frac{x}{x_0} - 1\right)\right]^{1/2} + \underbrace{\operatorname{arcsinh}\left[\left(\frac{x}{x_0} - 1\right)^{1/2}\right]}_{\psi} \right\}$$

3. Final Result

$$t = \sqrt{\frac{m x_0^3}{2\alpha}} \left\{ \frac{1}{2}\sinh 2\psi + \psi \right\}$$

$$\sinh \psi = \left(\frac{x}{x_0} - 1\right)^{1/2} ; \quad x = x_0 \cosh^2 \psi$$

Section 4.7. Curvilinear motion and other examples of one-dimensional motion

A system is called "one-dimensional" if the configuration is determined by a single dynamical variable.

Linear motion is strictly one-dimensional; but it's not the only kind of "one-dimensional" motion in the generalized sense.

Curvilinear Motion

Consider a particle that is constrained to move on a curve. That is "one-dimensional" because only a single variable is required to specify the position of the particle.

s = arclength

Figure 4.13

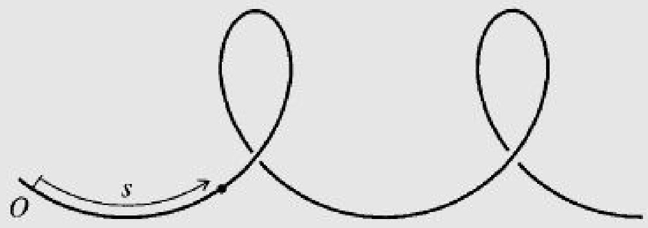


Figure 4.13 An object constrained to move on a curved track can be considered to be a one-dimensional system, with the position specified by the distance s (measured

Even this train moving on a curve is an example of *one-dimensional motion* in the generalized sense; because the configuration only depends on one variable.

Curvilinear Motion

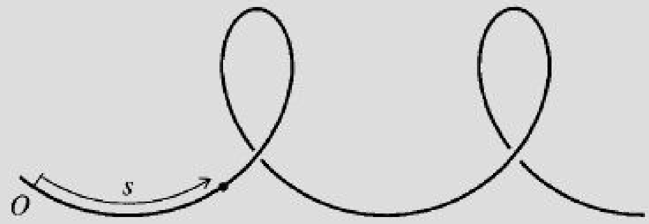
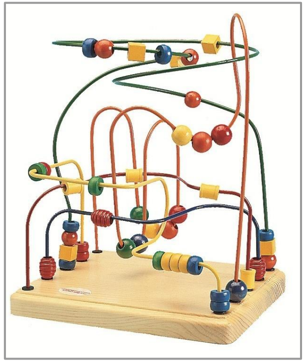


Figure 4.13 An object constrained to move on a curved track can be considered to be a one-dimensional system, with the position specified by the distance *s* (measured

- Variable is s = arclength ; $s = s(t)$
- Force equation is $m\ddot{s} = F_{tangential}$
- Energies are $T = ½ m \dot{s}^2$

  and $U(s)$ where $F_{tang.} = - dU/ds$

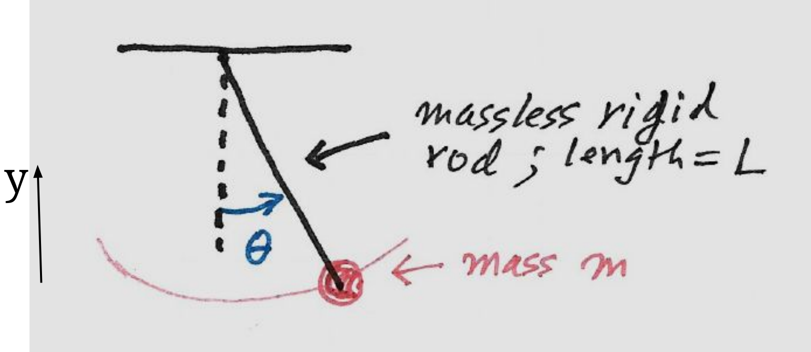
- But what keeps the particle on the curve?



Picture m as a bead threaded on a stiff wire.

Normal force
≡ force of constraint ;
≡ keeps the particle on the curve ;
≡ *does no work* ;
≡ ΔT comes from $F_{tang.}$.

The Simple Pendulum
(Taylor Problems 4.34 and 4.38)



The mass moves on a circular arc.

- Variable is θ = angle; $\theta = \theta(t)$
- Force equation ? *requires torque*
- Energies are $T = \frac{1}{2} m L^2 (\dot{\theta})^2$

and $U(\theta) = mgy = mg L (1 - \cos\theta)$

- The equation of motion.

We could write $d\boldsymbol{l}/dt$ = torque;

or, $dE/dt = 0$ implies

$$\frac{1}{2} m L^2 \, 2\dot{\theta}\ddot{\theta} + mgL \sin\theta \, \dot{\theta} = 0$$

$$\ddot{\theta} = -\frac{g}{L}\sin\theta \qquad \text{the simple pendulum}$$

This is "1-dimensional" in the generalized sense: $\exists$ *a single dynamical variable* .

Assigned in the homework.

Example 4.7

stability of a cube balanced on a cylinder

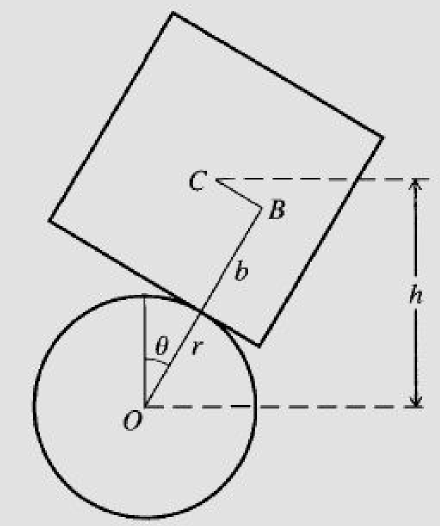
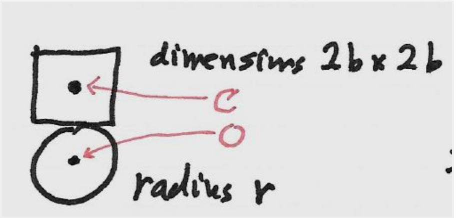


Figure 4.14 A cube, of side $2b$ and center C, is placed on a fixed horizontal cylinder of radius r and



The cylinder is fixed.

The cube is free to roll from side to side, not slipping on the cylinder.
{*center directly above center*}

Calculate U(θ).

This is a 1d problem.
Analyze the potential energy. (See the Figure.)

Let h = the height of the center of mass of the cube. Then U = m g h . Now express h in terms of the angle θ .

Example 4.7

stability of a cube balanced on a cylinder

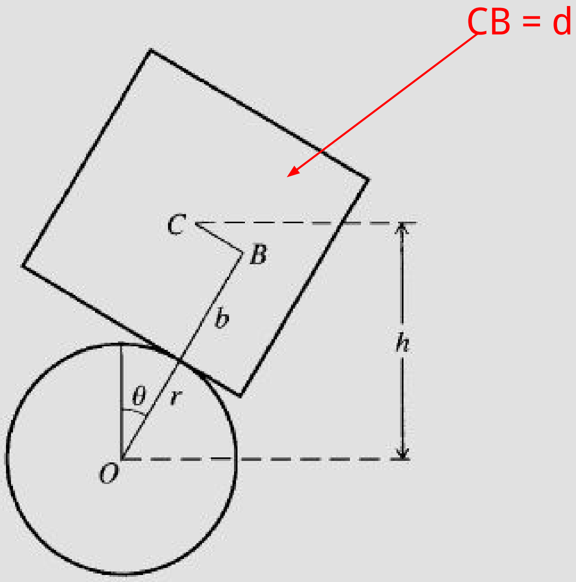


Figure 4.14 A cube, of side $2b$ and center C, is placed on a fixed horizontal cylinder of radius r and

Geometrical analysis

$r\theta = d$ (no slip) $= CB$

$h = y_C = (r+b)\cos\theta + d\sin\theta$

$U(\theta) = mgh$

$= mg\left[(r+b)\cos\theta + r\theta\sin\theta\right]$

Stability analysis 1 – **for equilibrium**

$\frac{dU}{d\theta} = 0$

$\hookrightarrow = mg\left[-(r+b)\sin\theta + r\sin\theta + r\theta\cos\theta\right]$

$U'(0) = 0$ so $\theta = 0$ is an equilibrium

Stability analysis 2 – **for stability**

$\frac{d^2U}{d\theta^2} > 0$

$\hookrightarrow = mg\left[-b\cos\theta + r\cos\theta - r\theta\sin\theta\right]$

$U''(0) = mg(r-b)$

The condition for stability is $b \leq r$.

STABILITY ANALYSIS

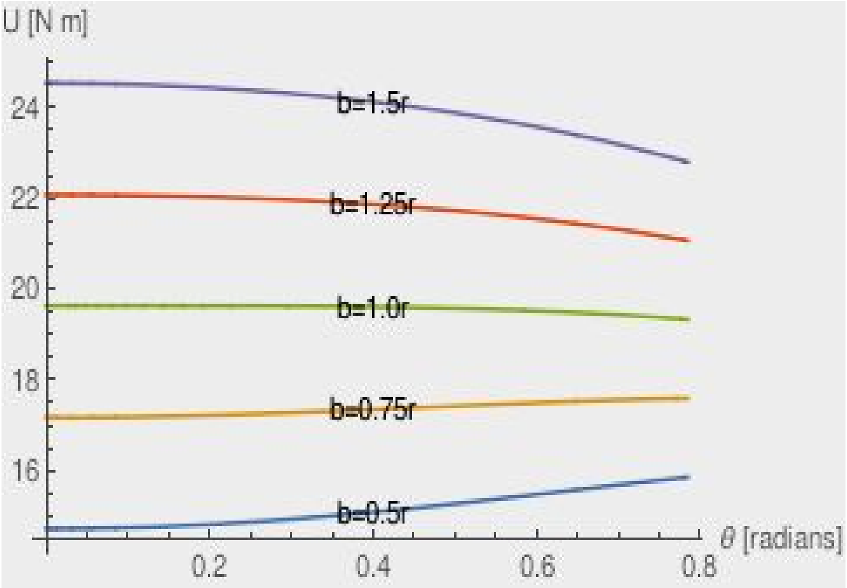
Plot U(θ) for different values of b/r .

The cube balanced at θ = 0 is stable if b ≦

r ;

i.e., it is stable if the width of the cube is smaller than the diameter of the cylinder.

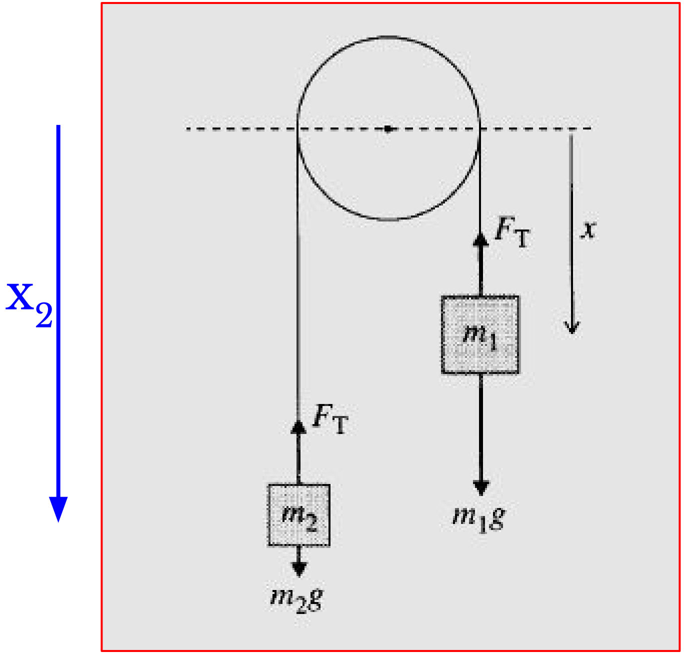
Assigned Problem 4.33.



Another example:

The Atwood machine

Figure 4.15



<u>Atwood machine</u>

The configuration depends on a single variable (x) because the length of the string is constant (L);

$L = x_2 + \pi R + x$;   or $x_2 = L - \pi R - x$

Analysis by energies:

$T = T_1 + T_2 = ½ (m_1+m_2) x'^2$

$U_1 = - m_1 g x$ *(x is downward)*

$U_2 = - m_2 g x_2 = + m_2 g x +$ *constant*

$$E = ½ (m_1 + m_2) x'^2 + (m_2 - m_1) g x$$

Energy is constant, so

$dE/dt = 0 = (m_1 + m_2) x' x'' + (m_2 - m_1) g x'$

$$(m_1 + m_2) x'' = (m_1 - m_2) g$$

Constant acceleration,

$$a = (m_1 - m_2) / (m_1 + m_2) g$$

Homework Assignment #8

due in class Friday, October 28

[37] Problem 4.26 *

[38] Problem 4.28 ** and Problem 4.29 ** [Computer]

[39] Problem 4.33 ** [Computer]

[40] Problem 4.34 **

[40x] Problem 4.37 *** [Computer]

[40xx] Problem 4.38 *** [Computer]

Use the cover page.

This is a long assignment, so start working on it now.