Find LU decomposition

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

get rid of

* Goal: use Gauss elimination row operations to make $A$ upper triangular $U$

Record Row Operations to create $L$

$R_2 \to R_2 - 2R_1 \quad (l_{21} = 2)$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1 \quad (l_{31} = 3)$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 2 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2 \quad (l_{32} = 2)$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \leftarrow U$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Use $E_{21}, E_{31}, E_{32}$ to find $L$

$A$

$E_{32}E_{31}E_{21}A$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$$

this does $R_2 - 2R_1$

matrix representation of Row operation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 2 \end{bmatrix}$$

this does $R_3 - 3R_1$

$$E_{32}E_{31}E_{21}A = U$$

$$E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}E_{32}E_{31}E_{21}A = \underbrace{E_{21}^{-1}E_{31}^{-1}}_{L}E_{32}^{-1}U$$

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad l_{21} = 2$$

$$E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \quad l_{31} = 3$$

$$E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad l_{32} = 2$$

<u>Monday:</u> Review

<u>Wednesday</u>: Exam I chap 1 & 2

<u>2.7</u>

$A^T \to$ transpose, if $A$ is $m \times n$, $A^T$ is $n \times m$

$$A = \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} \qquad A^T = \underset{3\times 2}{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}}$$

exchanges rows + columns

Properties: $(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$(A^{-1})^T = (A^T)^{-1}$

symmetric matrix $\Rightarrow S^T = S$

$S = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ $S^T = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

2x2

## Permutation matrices

exchange rows of a matrix

2x2, only 2! Permutation matrices

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ + $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

3x3 matrices $3! \to 6$ Permutation matrices

nxn matrices $n!$ Permutation matricec

☆ Sometimes we need row exchanges to produce Pivots

$\Rightarrow PA = LU$ $\qquad A = P^{-1}LU$

↑ still a permutation matrix

Ex

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -8 & 8 & 1 \\ 2 & 7 & 9 \end{bmatrix}$$

exchange Row 3 & Row 1

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ ← create P by exchanging rows 3 + R1 of Identity

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -8 & 8 & 1 \\ 2 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 9 \\ -8 & 8 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$PA = \begin{bmatrix} 2 & 7 & 9 \\ -8 & 8 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ ← now do Row operations to get U (and L)