Solution HW6 #1

We can write the two waves using the complex notation as

$$E_1(r,t) = \frac{AD}{r} e^{i(kr-\omega t)} \text{ and } E_2(x,t) = Ae^{i(kx-\omega t)},$$

with $k = \frac{2\pi}{\lambda}$.

The superposition of the two waves at a point $\vec{r} = (D, y)$ on the screen is given by

$$E_{tot}(D,y,t) = Ae^{-i\omega t}\left(\frac{D}{\sqrt{D^2+y^2}} e^{ik\sqrt{D^2+y^2}} + e^{ikD}\right) = Ae^{i(kD-\omega t)}\left(\frac{D}{\sqrt{D^2+y^2}} e^{ik\left(\sqrt{D^2+y^2}-D\right)} + 1\right).$$

We can now use the fact that $D \gg y$ and replace $\frac{D}{\sqrt{D^2+y^2}} \to 1$. However, in the exponent $k\left(\sqrt{D^2+y^2}-D\right) = 2\pi\frac{\left(\sqrt{D^2+y^2}-D\right)}{\lambda}$ we cannot use the same approximation since λ is very small.

We can expand to the lowest order in $\frac{y}{D}$ as $\sqrt{D^2+y^2}-D = D\left(\sqrt{1+\left(\frac{y}{D}\right)^2}-1\right) = \frac{y}{2D}^2$

(use Taylor's expansion $\sqrt{1+\varepsilon} \sim 1+\frac{\varepsilon}{2}$). The superposition can be written as

$$E_{tot}(D,y,t) = Ae^{i(kD-\omega t)}\left(e^{i2\pi\left(\frac{y^2}{2\lambda D}\right)} + 1\right).$$

The total intensity is

$$I = \frac{|E_{tot}|^2}{2} = \frac{A^2}{2}\left|1+e^{i\frac{\pi y^2}{\lambda D}}\right|^2.$$

You can use

$$1+e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{-i\frac{\theta}{2}}+e^{i\frac{\theta}{2}}\right) = e^{i\frac{\theta}{2}}2\cos\frac{\theta}{2}$$

to get

$$I = 4\frac{A^2}{2}\cos^2\frac{\pi y^2}{2\lambda D}.$$

2. Thin film $d = 500$ nm

Constructive interference occurs at

$$d\cos\theta = \frac{\lambda}{n_f}\left(\frac{2m+1}{4}\right)$$

$\theta = 0 \quad n_f = 1.5$

$$\lambda = \frac{4d \cdot n_f}{2m+1} = \frac{4 \cdot (500 \text{ nm})\, 1.5}{2m+1} = \frac{3000}{1,3,5,7,9}$$

$= 3000, 1000, 600, 425.6, 333.3$ nm

2 visible lines