Physics 410 -- Spring 2018

Homework #4, due Wednesday Feb. 14

1. [2] Kittel & Kroemer, Chapter 4, problem 2.

2. [2] Kittel & Kroemer, Chapter 4, problem 5.
   Hint #1: Under steady-state conditions, what is the net energy flow into an isolated body?
   Hint #2: The effective surface area of the earth that receives energy from the sun is <u>not</u> the same as the surface area that emits black-body radiation into space.

3. [2] Kittel & Kroemer, Chapter 4, problem 8.
   See hint #1 from the previous problem.

4. [3] Kittel & Kroemer, Chapter 4, problem 1.

5. [2] In class, we derived the Maxwell relation $\left(\frac{\partial \tau}{\partial V}\right)_\sigma = \left(\frac{\partial p}{\partial \sigma}\right)_V$ from the Thermodynamic Identity, by equating the crossed second derivatives of U. Derive another Maxwell relation involving derivatives of σ and p. Start from the Helmholtz Free Energy, $F = U - \tau\sigma$, and calculate its first derivatives using the Thermodynamic Identity.

(over)

6. [4] I would like you to learn a very useful tool in physics, namely, how to use the Taylor series expansion to estimate a theoretical expression in an asymptotic limit (e.g. high or low temperature), and calculate the lowest order corrections.

In class, we derived the average energy of a 1-dimensional harmonic oscillator: $U = \frac{1}{2}\coth\left(\frac{\hbar\omega}{2\tau}\right)$. We showed that $U \approx \tau$ for $\tau >> \hbar\omega$, and $U \approx \hbar\omega/2$ for $\tau << \hbar\omega$. Now consider the high-temperature (classical) limit in more detail, <u>without</u> using the *coth* function. Instead, start from the expression for the mean number of energy quanta: $\langle s\rangle = \dfrac{1}{e^{\hbar\omega/\tau} - 1}$.

(a) Evaluate $\langle s\rangle$ in the high-temperature limit, and find the next two terms in the Taylor series expansion. Your answer should be in the form $\langle s\rangle = \langle s\rangle_{asymp}(1 + ax + bx^2 + ...)$, where $x = \dfrac{\hbar\omega}{\tau}$ is a dimensionless small parameter, and you have expressions for $\langle s\rangle_{asymp}$, *a*, and *b*. If you are truly averse to doing algebra, you may use Mathematica for this problem. But be sure to put your solution in the form I specify above so you can evaluate $\langle s\rangle_{asymp}$, *a*, and *b*.

(b) Now evaluate the mean energy $U = \hbar\omega\left(\langle s\rangle + \frac{1}{2}\right)$ using your result from part (a). Your parameter $\langle s\rangle_{asymp}$ gives you the classical result, *a* shows you why the zero-point energy doesn't appear in the classical limit, and *b* gives you the next order correction.

You will need the following 2 Tayler series to do part (a), and you will need to expand the exponential to 3rd order:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \qquad\qquad \frac{1}{1+x} = 1 - x + x^2 - x^3 + ...$$