Clicker Question #1

- Assume that you have an unbiased coin. If you flip it 4 times and get heads every time, what is the probability that you will get heads if you flip it a 5^{th} time?

    A. Less than 50%
    B. Exactly 50%
    C. More than 50%

Answer: B. The coin is unbiased. Previous results have no influence over future results.

Clicker Question #2

- If 1000 people each flip a coin 4 times, what fraction of them will get 2 heads and 2 tails?

A. A majority of them (greater than 50%)
B. About 20% of them
C. About 37% of them
D. I have no clue

Answer: C. There are 6 ways to get 2 heads and 2 tails, out of 2^4=16 total configurations. So the expected fraction is 6/16 = 3/8 = 37.5%.

Clicker Question #3

- A computer uses random numbers to simulate flipping a coin one million times. That experiment is repeated one billion times to get a good estimate of the probability distribution for the number of heads, *n*. The mean value of n is clearly 500,000. What is $\langle(\Delta n)^2\rangle$?

A. 500
B. 25,000
C. 250,000
D. 500,000
E. 1,000,000

Answer: C. $\langle(\Delta n)^2\rangle$ = Npq = 250,000.

Clicker Question #4

- Same situation as the previous question. How large is the relative width of the distribution, $\delta n/\langle n\rangle$?

A. 0.0001
B. 0.001
C. 0.01
D. 0.1
E. 500

Answer: B. $\delta n = \sqrt{250{,}000} = 500$. $\delta n/n = 500/500{,}000 = 0.001$
We just showed in class that $\delta n/n = 1/\sqrt{N} = 1/1{,}000 = 0.001$

Clicker Question #5

- The number density of air molecules at standard temperature and pressure is about $3\times10^{25}\text{m}^{-3}$. Consider a room with dimensions $10\text{m}\times8\text{m}\times4\text{m}$. Let n be the number of air molecules in the West half of the room. What is $\langle n\rangle$, approximately?

A. 0.5×10^{27}
B. 0.5×10^{28}
C. 0.5×10^{29}
D. 0.5×10^{30}

Answer: B. $N = 320\text{m}^3\times3\times10^{25}\text{m}^{-3} \approx 10^{28}$, $\langle n\rangle = N/2$

Clicker Question #6

- By how much does the number of air molecules in the West half of the room fluctuate? (What is δn, the standard deviation of n?)

A. 0.5×10^{14}
B. 1×10^{14}
C. 0.5×10^{28}
D. 1×10^{28}
E. 0

Answer: A. $\delta n = (\sqrt{N})/2 = 10^{14}/2 = 0.5\times10^{14}$

Clicker Question #7

- How large are the relative fluctuations of n, i.e. what is $\delta n/\langle n\rangle$?

A. 1
B. 10^{28}
C. 10^{14}
D. 10^{-28}
E. 10^{-14}

Answer: E. $\delta n/\langle n\rangle = (0.5\times10^{14})/(0.5\times10^{28}) = 10^{-14}$

Clicker Question #8

- Are these fluctuations detectable?

A. yes
B. no
C. maybe

Answer: B. There are very few quantities in nature that humans can measure to this precision. Frequency is one.

Clicker Question #9

- What is the probability that all the air molecules will be in the West half of the room at any given time?
  (Useful info: $\log_{10}2 \approx 0.3$.)

A. 0.3×10^{-14}
B. 0.3×10^{-28}
C. $10^{-(0.3\times10^{14})}$
D. $10^{-(0.3\times10^{28})}$
E. 0.3

Answer: D. $P(N,N) = 1/(2^N) = 2^{(-10^{28})} = 10^{(-10^{28}\log 2)} = 10^{(-0.3\times10^{28})}$

Clicker Question #10

- What is the probability that exactly one-half of all the air molecules will be in the West half of the room at any given time?

A. over 90%
B. about 50%
C. about 10%
D. about 1%
E. about10^{-14}

Answer: E. Using the Gaussian approximation for the probability: $P(N,s=0) = (2/\pi N)^{1/2} = 0.8\times 10^{-14} \approx 10^{-14}$

Clicker Question #11

- Consider two isolated systems with multiplicities g_1 and g_2. What is the total multiplicity of the combined system?

A. g_1+g_2
B. g_1g_2
C. $\mathrm{Exp}(g_1+g_2)$
D. $\mathrm{Log}(g_1g_2)$
E. I have no idea

Answer: B. For each different configuration of system 1, system 2 can be in any one of its configurations.

Clicker Question #12

- Consider two spin-1/2 systems in a magnetic field, with N_1=12 and N_2=6. Initially they are isolated from each other, with s_1=6 (all spins up) and s_2=-3 (all spins down). What is the total multiplicity of the combined system?

A. 1
B. 18
C. 72
D. 2^{18}
E. I have no idea

Answer: A. Each system has only 1 possible configuration.

Clicker Question #13

- Start from the previous question. Now transfer the smallest possible amount of energy from system 2 to system 1, so that one spin is flipped in each system. Now s_1=5, s_2 = -2, so we still have s=3. What is the new multiplicity of the combined system?

A. 1
B. 18
C. 72
D. 2^{18}
E. I have no idea

Answer: C. We now have g_1=12 and g_2=6, so g_1g_2=72.

Clicker Question #14

- Same situation continued. Now put the two systems in thermal contact. We still must conserve $s = s_1+s_2 = 3$. What is the most likely configuration of the combined system?

A. $s_1 = 5$, $s_2 = -2$
B. $s_1 = 4$, $s_2 = -1$
C. $s_1 = 3$, $s_2 = 0$
D. $s_1 = 2$, $s_2 = 1$
E. $s_1 = 1$, $s_2 = 2$

Answer: D. For very large spin systems, we showed that the most probable configuration has $s_1/N_1=s_2/N_2=s/N$. Answer D obeys this with 2/12 = 1/6 = 3/18.

Clicker Question #15

- What is the probability that a hydrogen atom in equilibrium with a reservoir at room temperature will be in a particular 2s quantum state? Recall that room temperature corresponds to $k_BT \approx (1/40)\text{eV}$.

A. about 50%
B. about 1%
C. about e^{-10}
D. about e^{-400}
E. exactly zero

Answer: D. The energy difference between the ground (1s) state and the 2s state is 10.2 eV. The Boltzmann factor is $\exp(-\Delta E/k_BT) = \exp(-10.2*40) = \exp(-400)$

Clicker Question #16

- Consider a 2-level system in thermal contact with a reservoir at temperature τ. The energies of the two levels are 0 and ε. What is the partition function, Z, for this system?

A. 1
B. $e^{-\varepsilon/\tau}$
C. $1+ e^{-\varepsilon/\tau}$
D. $1+ e^{\varepsilon/\tau}$
E. $1- e^{-\varepsilon/\tau}$

Answer: C. $Z = \Sigma\, e^{-\varepsilon_s/\tau}$ (Recall that $e^0=1$)

Clicker Question #17

- Consider the same 2-level system in thermal contact with a reservoir at temperature τ. What is the probability of finding the system in the ground state?

A. 1/2
B. $e^{-\varepsilon/\tau}$
C. $1+ e^{-\varepsilon/\tau}$
D. $1/(1+ e^{-\varepsilon/\tau})$
E. $1/(1+ e^{\varepsilon/\tau})$

Answer: D. $P(s) = e^{-\varepsilon_s/\tau} / Z$

Clicker Question #18

- Consider the same 2-level system in thermal contact with a reservoir. What is the mean energy of the system in the limit of zero temperature?

A. 0
B. $\varepsilon/2$
C. ε
D. Undefined; that limit is unphysical.

Answer: A. At zero temperature, the system is in the ground state.

Clicker Question #19

- Consider the same 2-level system in thermal contact with a reservoir. What is the mean energy of the system in the limit of infinite temperature?

A. 0
B. $\varepsilon/2$
C. ε
D. Undefined; that limit is unphysical.

Answer: B. At infinite temperature, the two states are populated with equal probability. (Look at P(s), or think of the system of N spins you did on last week's homework.)

Clicker Question #20

- Consider the same 2-level system in thermal contact with a reservoir. What is the mean energy of the system at arbitrary temperature τ?

A. $\varepsilon\, e^{-\varepsilon/\tau}$
B. $\varepsilon\, e^{-\varepsilon/\tau}/(1+ e^{-\varepsilon/\tau})$
C. $\varepsilon\, /(1+ e^{\varepsilon/\tau})$
D. $\varepsilon\, /(1+ e^{-\varepsilon/\tau})$
E. More than one of those answers is correct.

Answer: E. Answers B and C are both correct. Use $U = \Sigma\varepsilon_s e^{-\varepsilon_s/\tau} / Z = \varepsilon\, e^{-\varepsilon/\tau} / Z$

Note: We did not do this question in class. But you should still know it!

Clicker Question #21

- Consider a gas of N molecules in a box of volume V_0. What is the probability of finding all N molecules in the left-hand half of the box at the same time?

A. 1/N
B. 1/N!
C. $1/2^N$
D. None of the above
E.

Answer: C. The probability is $(1/2)^N = 1/2^N$.

Clicker Question #22

- Now imagine that the box has a divider in the middle, and all N gas molecules have been put in the left side of the box. The divider is suddenly removed. By how much does the entropy of the gas increase as it expands to fill the box?

A. 2
B. 2^N
C. $2 \times \ln(N)$
D. $N \times \ln(2)$
E. None of the above

Answer: D. The multiplicity increases by the factor 2^N, so the entropy increases by $\ln(2^N) = N \times \ln(2)$. In more detail: $\sigma_f - \sigma_i = \ln(g_f) - \ln(g_i) = \ln(g_f/g_i) = \ln(2^N) = N \times \ln(2)$.