Physics 410 -- Spring 2001

Homework #5, due Wednesday Feb. 21

1. [3] This problem will give you a better understanding of the "quantum concentration" used repeatedly in your textbook. The de Broglie wavelength λ for a particle with momentum p is defined as $\lambda = h/p$, where $h = 2\pi\hbar$ is Planck's constant. If we don't know the momentum, but we know that the particle is part of a gas at temperature τ, then we can define the "thermal de Broglie wavelength" λ_{th} by assuming that the kinetic energy of the particle is equal to $\frac{3}{2}\tau$, as we derived in class. Derive an expression for the thermal de Broglie wavelength in terms of $\hbar$, τ, the particle mass m, and some numerical constants. What is the concentration n of the gas if the average spacing between particles equals λ_{th}? (Hint: n has units of inverse volume.) Compare your answer with the definition of the quantum concentration n_Q given in the book. The two formulas should differ only by a numerical constant. Evaluate the ratio n/n_Q.

2. [3] Kittel & Kroemer, Chapter 3, problem 11.
This is a quantum mechanics problem. It follows very closely the derivation in K&K pages 72-73. You may use either hard-wall boundary conditions as the book does or periodic boundary conditions as I did in class. Express the entropy in terms of the temperature τ, the particle density $n=N/L$, and a one-dimensional quantum concentration n_Q that you will define in analogy to the 3D n_Q defined in the book.

3. [2] Kittel & Kroemer, Chapter 5, problem 4.

4. [3] Kittel & Kroemer, Chapter 5, problem 1.
Hint: To see how the chemical potential varies with radius, you need to imagine yourself as a gas molecule in the centrifuge! In the rotating frame of reference, you feel an outward force (the loathsome "centrifugal force" that we <u>never</u> teach in Physics 183). Express the force in terms of the angular velocity ω, rather than v. You can convert that force to an effective potential energy using the relation between work and potential energy. Once you have your potential energy as a function of radius, just follow the standard prescription: $\mu(r) = \mu_{\text{int}} + \mu_{ext}(r)$ to find $n(r)$. To get the right sign on your potential energy term, think about the analogy with gravity: which way does the gravitational force point, and in which direction does the gravitational potential energy increase? Does $n(r)$ increase or decrease with increasing r?

(over)

5. [4] Consider two boxes filled with electrolytic solutions containing dilute concentrations of H^+ ions at temperature T = 300 K. (There must be a fixed background of negative charges in both boxes to keep the systems electrically neutral, but that doesn't affect this calculation.) The concentration of H^+ ions in box A is $n_A = 1.5 \times 10^{18} m^{-3}$, while the concentration of H^+ ions in box B is $n_A = 3.0 \times 10^{18} m^{-3}$. Now connect the boxes to each other by a narrow tube.

(a) You can prevent diffusion of H^+ ions from box B to box A by applying an electrostatic potential difference between the boxes, using a battery. What voltage should you apply to box A relative to box B to prevent diffusion of the H^+ ions? Should that voltage be positive or negative?

(b) Instead of using a battery, you decide to prevent diffusion of H^+ ions from box B to Box A by lifting box A up to a height *h* above box B. Assuming that the Earth's gravitational field is constant, with $g=9.8 m/s^2$, to what height *h* must you lift box A to prevent diffusion? Is this practical?