Physics 410 Midterm Exam #1Solutions

1. [12] Consider a system with 3 quantum states, labeled by index n=0,1, or 2, with respective energies $\varepsilon_0=0$, $\varepsilon_1=a$, and $\varepsilon_2=3a$. The system is in thermal equilibrium with a reservoir at temperature τ.

   Solution: See i-clicker questions #23-25. The class version specified the temperature $\tau = a$, whereas the exam asked for formulas valid for arbitrary τ. Also see the posted solutions to problem 2 on Homework Set #3, especially the note at the bottom of the page.

2. [12] Consider an N-particle system thermally isolated from its surroundings. The multiplicity g depends on the energy U as $g(U)=\mathrm{Exp}[(NU/U_0)^{1/2}]$, where U_0 is a constant with dimensions of energy.

   a) [4] Find an expression for the temperature τ in terms of N, U, and U_0.

   Solution: $\sigma(U, N) = \ln(g(U)) = (NU/U_0)^{1/2}$. $1/\tau = \partial\sigma/\partial U = (1/2)(N/U_0)^{1/2}U^{-1/2}$.

   $$\tau = 2(UU_0/N)^{1/2}.$$

   b) [8] Let $N = 10^{22}$ and $U_0 = 10^{-18}$ J (J=Joule). If the temperature is $\tau = 4 \times 10^{-20}$ J (a bit below room temperature), what are the energy U and entropy σ of the system?

   $U = N\tau^2/4U_0 = 4$ J $\quad\quad \sigma = (NU/U_0)^{1/2} = 2\times10^{20}$. Units are important in this problem. One way to check your algebra is to see if your answer has the right units.

3. [16] Consider a box of volume V_0 containing an ideal gas of N_0 identical molecules. Consider a section of the box with volume $V=V_0/3$.

   Solution: See posted solutions to problem 3 on Homework Set #1, and also i=clicker questions #5, 6, 9, 21, and 22. Change all 2's to 3's since we are now considering the molecules in 1/3 of the box instead of ½ the box. You may wonder how it is possible to do part (c) given that we haven't discussed multiplicities for the ideal gas. From part (a), we know that the probability to find all the molecules in the small part of the box is $(1/3)^{N_0}$. Since probabilities are just normalized multiplicities, that implies that the multiplicity for the whole box is 3^{N_0} times larger than the multiplicity for the small part.

4. [20] Consider two systems of spins in a uniform magnetic field. The energy of a state with spin excess s is U=-2smB, where m is the magnetic moment of a spin and B is the magnetic field. System 1 has $N_1=10^4$ spins, while system 2 has $N_2=2\times10^4$ spins. Initially, system 1 has half its spins up and half down, so its spin excess is $s_1=0$. Initially, system 2 has a spin excess $s_2=1500$. Use the Gaussian approximation to the multiplicity (on your formula sheet) for all parts of this problem, and ignore the first term $\left(\frac{2}{\pi N}\right)^{1/2}$.

   Solution: See the posted solutions to problem 5 on Homework Set #2.