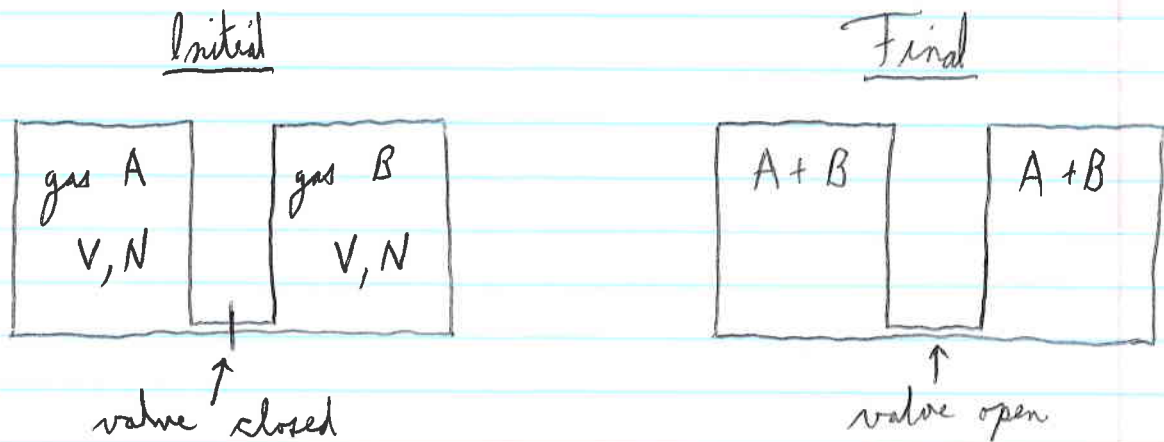


PHY 410 - Homework 7 solutions

1. K. & K. Ch 6, #6 : Entropy of mixing



$$\text{Ideal gas entropy } \sigma = N \left[\ln \frac{n_Q}{n} + \frac{5}{2} \right]$$
$$= N \left[\ln \frac{V}{N} + \text{constants} \right]$$

Gas A increases its entropy by

$$\Delta \sigma^A = \sigma_f^A - \sigma_i^A = N \left[\ln \frac{2V}{N} - \ln \frac{V}{N} \right] = N \ln \frac{2V}{V}$$
$$= N \ln 2$$

$$\Delta \sigma^B = N \ln 2 \text{ also}$$

$$\text{Total } \Delta \sigma = \Delta \sigma^A + \Delta \sigma^B = 2N \ln 2$$

If $A = B$, then the initial and final states are the same, so $\Delta \sigma = 0$.

2. K+K Ch 6, #5

At fixed N , the TI implies:

$$d\sigma = \frac{dU}{\tau} + \frac{p dV}{\tau} = \frac{1}{\tau} \left(\frac{\partial U}{\partial \tau} \right)_V d\tau + \frac{1}{\tau} \left(\frac{\partial U}{\partial V} \right)_\tau dV + \frac{p dV}{\tau}$$

An Ideal Gas has $U = c N \tau$ where the constant c depends on the number of degrees of freedom that are thermally excited. (For a monatomic gas in 3D, $c = 3/2$.)

Since U does not depend on V , $\left(\frac{\partial U}{\partial V} \right)_\tau = 0$

Plug in one of our definitions of $C_V = \left(\frac{\partial U}{\partial \tau} \right)_V$
and use the Ideal Gas Law $pV = N\tau$ to get

$$\frac{p}{\tau} = \frac{N}{V}$$

to get

$$d\sigma = C_V \frac{d\tau}{\tau} + \frac{N dV}{V}$$

Integrate both sides:

$$\sigma = C_V \ln \tau + N \ln V + \sigma_1$$

where σ_1 is a constant independent of τ, V .

The integration works only if C_V is constant over the temperature range of the integration. So this really only works for a monatomic ideal gas.

3. K. + K. Ch. 3, #6 Rotation of diatomic molecules

$$\epsilon(j) = j(j+1) \epsilon_0, \quad j = 0, 1, 2, \dots, \quad g(j) = 2j+1$$

↑
degeneracy

$$\begin{aligned} a) \quad Z_R(\tau) &= \sum_{j=0}^{\infty} g(j) e^{-\epsilon(j)/\tau} \\ &= \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon_0/\tau} \end{aligned}$$

$$\begin{aligned} b) \quad \tau \gg \epsilon_0: \quad Z_R(\tau) &\approx \int_0^{\infty} (2j+1) e^{-(j^2+j)\epsilon_0/\tau} \\ &= -\frac{\tau}{\epsilon_0} e^{-(j^2+j)\epsilon_0/\tau} \Big|_0^{\infty} = \underline{\underline{\frac{\tau}{\epsilon_0}}} \end{aligned}$$

$$c) \quad \tau \ll \epsilon_0 \quad Z_R(\tau) \approx \underline{\underline{1 + 3e^{-2\epsilon_0/\tau}}}$$

$$d) \quad \text{Use } U = \tau^2 \frac{\partial \ln Z}{\partial \tau}, \quad C_V = \left(\frac{\partial U}{\partial \tau} \right)_V$$

$$\tau \gg \epsilon_0: \quad U = \tau^2 \frac{\partial}{\partial \tau} \left(\ln \frac{\tau}{\epsilon_0} \right) = \tau^2 \cdot \frac{1}{\tau} = \underline{\underline{\tau}}$$

$$C_V = \underline{\underline{1}}$$

$$\tau \ll \epsilon_0: \quad U = \tau^2 \frac{\partial}{\partial \tau} \ln \left(1 + 3e^{-2\epsilon_0/\tau} \right) = \tau^2 \frac{3 \cdot \frac{2\epsilon_0}{\tau^2} e^{-2\epsilon_0/\tau}}{1 + 3e^{-2\epsilon_0/\tau}}$$

$$U \approx \underline{\underline{6\epsilon_0 e^{-2\epsilon_0/\tau}}}$$

$$C_V = 6\epsilon_0 \cdot \left(\frac{2\epsilon_0}{\tau^2} \right) e^{-2\epsilon_0/\tau} = \underline{\underline{12 \left(\frac{\epsilon_0}{\tau} \right)^2 e^{-2\epsilon_0/\tau}}}$$

denominator is approximately 1

Rotational Contribution to the Specific Heat of a Diatomic Ideal Gas

```

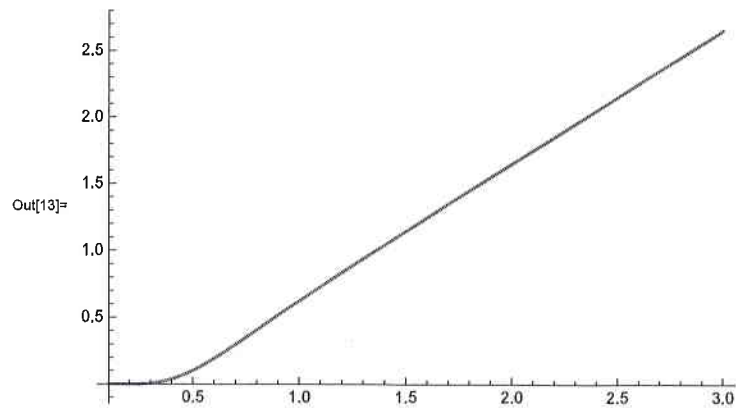
In[9]:= PartitionFunctionSum10[t_] := Sum[(2 j + 1) * Exp[-j (j + 1) / t], {j, 0, 10}]

TsquaredtimesdZdt[t_] := Sum[(2 j + 1) * j * (j + 1) * Exp[-j (j + 1) / t], {j, 0, 10}]

Energy[t_] := TsquaredtimesdZdt[t] / PartitionFunctionSum10[t];
SpecificHeat[t_] := Energy'[t];

In[13]:= Plot[Energy[t], {t, 0.1, 3}]

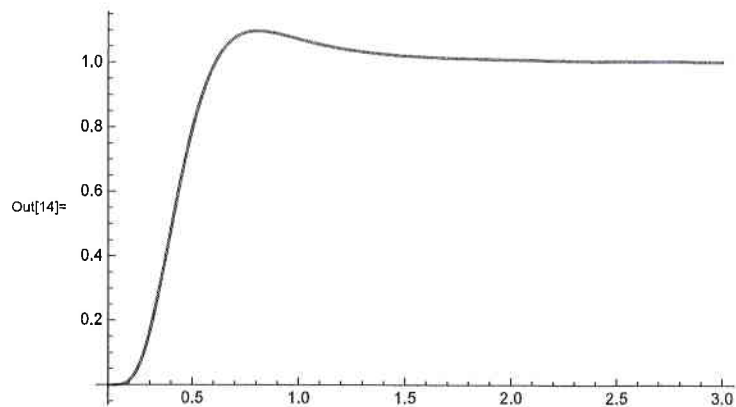
```



```

In[14]:= Plot[SpecificHeat[t], {t, 0.1, 3}, PlotRange -> All]

```



4. K + X. Ch. 6, #14 Ideal gas calculations

$$\left. \begin{aligned} 1 \text{ mole: } N &= N_A = 6.02 \cdot 10^{23} \\ \tau &= k_B T = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}} \cdot 300 \text{ K} = 4.14 \cdot 10^{-21} \text{ J} \end{aligned} \right\} N\tau = 2.49 \cdot 10^3 \text{ J}$$

a) Isothermal expansion Use table 6.3

$$\text{heat added } Q = N\tau \ln \frac{V_2}{V_1} = N\tau \ln 2 = \underline{\underline{1.73 \cdot 10^3 \text{ J}}}$$

Isentropic expansion: $Q = 0$

b) Second process: Eqn (66) in book $\tau_1 V_1^{\gamma-1} = \tau_2 V_2^{\gamma-1}$

For a monatomic ideal gas: $C_V = \frac{3}{2} N$, $C_P = \frac{5}{2} N$, $\gamma = \frac{5}{3}$

$$\frac{\tau_2}{\tau_1} = \left(\frac{V_1}{V_2} \right)^{2/3} = \left(\frac{1}{2} \right)^{2/3} = 0.630$$

$$T_2 = 0.630 \cdot T_1 = 0.630 \cdot 300 \text{ K} = \underline{\underline{189 \text{ K}}}$$

c) Irreversible expansion

$$\Delta \sigma = N \ln \frac{V_2}{V_1} = N_A \ln 2 = 4.17 \cdot 10^{23} \text{ in dimensionless units}$$

$$\begin{aligned} \Delta S &= k_B \Delta \sigma = N_A k_B \ln 2 = R \ln 2 = 8.31 \frac{\text{J}}{\text{K}} \cdot \ln 2 \\ &= \underline{\underline{5.76 \frac{\text{J}}{\text{K}}}} \end{aligned}$$

5. K. + K. Ch 6, #15

Diesel engine compression

$$a) \quad T_i = 300 \text{ K} \quad \frac{V_f}{V_i} = \frac{1}{15} \quad \gamma = 1.4$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = 300 \text{ K} \cdot 15^{0.4} = \underline{\underline{886 \text{ K}}}$$

$$b) \quad \gamma \equiv \frac{C_p}{C_v} \quad C_p = C_v + 1$$

$$\gamma C_v = C_v + 1$$

$$(\gamma - 1) C_v = 1$$

$$C_v = \frac{1}{\gamma - 1} = \frac{1}{0.4} = \underline{\underline{2.5}} = \frac{5}{2}$$

Traditional units, for 1 mole of gas:

$$C_v = \frac{5}{2} N_A k_B = \frac{5}{2} R = \frac{5}{2} \cdot 8.31 \frac{\text{J}}{\text{K}} = \underline{\underline{20.8 \frac{\text{J}}{\text{K}}}}$$

$$c) \text{ Classical Equipartition Thm: } C_v = \frac{1}{2} \times \text{number of degrees of freedom}$$

 $\Rightarrow 5$ degrees of freedom

- 3 translational
- 2 rotational