# Physics 410 -- Spring 2018

## Homework #1, due Wednesday Jan. 17

1. (a) [1] A coin is tossed 5 times. Write down all possible configurations with 3 heads. A useful notation is to represent each configuration by a row of 5 arrows, with heads and tails denoted by up and down arrows, respectively. What is the probability of getting 3 heads when tossing the coin 5 times?

   (b) [2] A coin is tossed 12 times. Draw a histogram showing the number of states as a function of the number of heads. (See Figure 1.7 in the text as an example.) Use the binomial expansion coefficients to calculate the exact multiplicity of each state.

2. [7] On pages 18-20 of your textbook, the author shows that the binomial distribution takes on a Gaussian shape for large N. You will use Mathematica to simulate the binomial distribution by generating random coin flips. If you prefer to write your own program using $C^{++}$, Python, or some other programming language, that is fine, too. But Mathematica is a powerful tool that I urge you to learn both for this course and for your own benefit.

   Mathematica is available for free to MSU students for use on your own computers: see https://techstore.msu.edu/software/wolfram-mathematica-license-student-use-0. With Mathematica running, just type in commands. Mathematica executes the commands when you hit SHIFT-ENTER. If you make a mistake you can go back to the command, edit it or re-type it, then hit SHIFT-ENTER again. Mathematica will erase the old result and execute the revised command. Use the HELP menu if you have trouble figuring out what these commands do.

   (a) [2] To get started, here are some commands you might try:

   **Random[]**
   **Random[Integer]**
   **Table[Random[Integer], {10}]**

   Here is a way to toss m coins, find the number of heads, repeat the whole trial N times, and then histogram the result. In the example below I used m=10 and N=200.

   **NumberOfHeads[m_]:=Sum[Random[Integer], {m}]**
   **ManyTrials[N_]:=Table[NumberOfHeads[10], {N}]**
   **Histogram[ManyTrials[200]]**

   Notice that when you define a function, the variable name on the left-hand-side of the definition is followed by the underscore, hence m_ instead of m. If you don't like defining all these functions, you can do everything with the single command below, <u>but don't try this until you know what you are doing.</u>

   **Histogram[Table[Sum[Random[Integer],{10}],{200}]]**

(b) [1] Have Mathematica toss 400 coins, find the number of heads, and repeat the experiment 200 times. The resulting histogram will be very bumpy.

(c) [1] Now have Mathematica repeat the 400-coin toss experiment 4000 times. (That takes about 10 seconds on my computer – it should be faster on the computers in the Microlabs.) The resulting histogram should be pretty smooth. Give the histogram a name, such as **P1=Histogram ...** so you can show it again later (see below).

(d) [1] Now graph a Gaussian function and see how well it matches your histogram. Note that the normalized Gaussian must be multiplied by the number of trials (4000) to match the histogram. I chose x-axis limits of 160 to 240, since the function is peaked at x=200.

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P2=Plot[4000*Sqrt[2/(400*Pi)]*Exp[-2*(x-200)^2/400],{x,160,240}]
Show[P1,P2]
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You are welcome to do more than I have outlined here. Your Mathematica program and output should both be included in your homework.

(e) [2] Define $n$ to be the number of heads obtained from the N coin tosses. The mean value of $n$ is called $\bar{n}$ or $< n >$. The standard deviation of $n$ is defined as $\delta n \equiv \sqrt{< (n-\bar{n})^2 >}$. What is $\delta n$ for N=12 and N=400? (You may use results we derived in class.) What is the relative uncertainty in $n$, defined as $\delta n / \bar{n}$, for the two cases? Is your result for N=400 consistent with the graphs you made in parts (c) and (d)? Discuss how likely (or unlikely) it is to obtain fewer than 30% heads in the two cases.

3. [5] Consider a gas of $N_0$ noninteracting molecules enclosed in a container of volume $V_0$. Focus attention on a subvolume V of this container and denote by N the number of molecules located within V. Each molecule is equally likely to be located anywhere within $V_0$, hence the probability of finding a given molecule in V is $p=V/V_0$.

(a) What is the mean number $\overline{N} = \langle N \rangle$ of molecules located within V? Express your answer in terms of $N_0$, $V_0$, and V. (Hint: This problem is just like the biased coin flip problem we discussed in class, where p is the probability to get "heads".)

(b) Find the relative dispersion $\overline{(N-\overline{N})^2} / \overline{N}^2$ in the number of molecules located within V. Express your answer in terms of $\overline{N}$, V and $V_0$.

(c) What does the answer to part (b) become when $V \ll V_0$? (The answer is not zero.)

(d) What value should the dispersion $\overline{(N-\overline{N})^2}$ assume when $V \rightarrow V_0$? Does the answer in part (b) agree with this?

(e) Consider the case $N_0 = 5$, and $V=V_0/4$. At any given time, what is the probability of finding all 5 molecules in V? What is the probability of finding exactly one molecule in V?