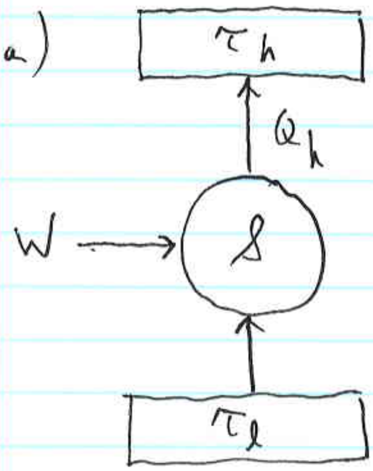
PHY 410 Homework 9 solutions

1. K. + K. Ch 8, # 1 Heat pump

a)



Energy conservation: $W + Q_l = Q_h$

Reversible: $\Delta\sigma_h = \Delta\sigma_l$

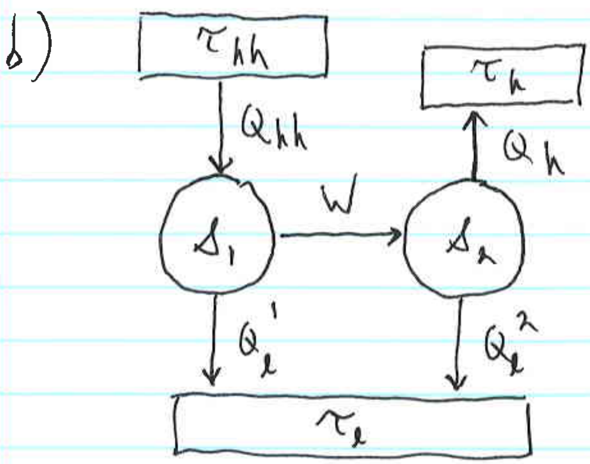
$$\frac{Q_h}{\tau_h} = \frac{Q_l}{\tau_l} \rightarrow Q_l = \frac{\tau_l}{\tau_h} Q_h$$

$$\rightarrow W = Q_h - \frac{\tau_l}{\tau_h} Q_h = Q_h\left(1 - \frac{\tau_l}{\tau_h}\right)$$

$$\frac{W}{Q_h} = \frac{\tau_h - \tau_l}{\tau_h} = \eta_c \quad \checkmark$$

If it is not reversible, then extra entropy is created inside S. Then $\sigma_h > \sigma_l \Rightarrow Q_l < \frac{\tau_l}{\tau_h} Q_h \Rightarrow \frac{W}{Q_h} > \eta_c$

b)



Engine: $\frac{W}{Q_{hh}} = \frac{\tau_{hh} - \tau_l}{\tau_{hh}}$

Heat pump: $\frac{W}{Q_h} = \frac{\tau_h - \tau_l}{\tau_h}$

Combine:

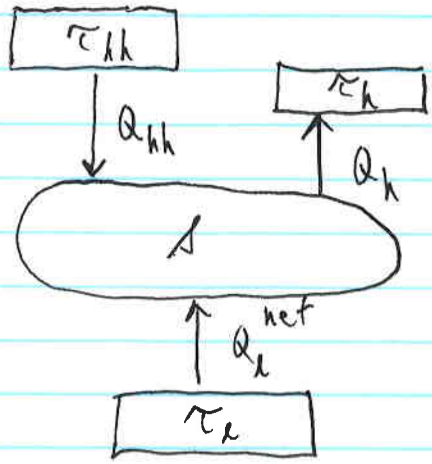
$$\frac{Q_{hh}}{Q_h} = \frac{\tau_h - \tau_l}{\tau_h} \cdot \frac{\tau_{hh}}{\tau_{hh} - \tau_l}$$

$T_{hh} = 600\,K$, $T_h = 300\,K$, $T_l = 270K$:

$$\frac{Q_{hh}}{Q_h} = \frac{30}{300} \cdot \frac{600}{330} = \frac{2}{11} = \underline{\underline{0.182}}$$

c) Energy

Using the temperatures from part b:

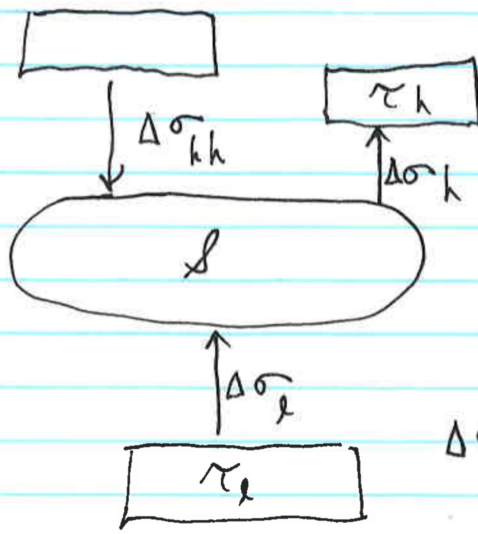


$$Q_h = \frac{11}{2} Q_{hh}$$

$$Q_\ell = Q_h - Q_{hh} = \frac{9}{2} Q_{hh}$$

There is net heat flow from the low temperature reservoir.

Entropy



$$\Delta\sigma_{hh} = \frac{Q_{hh}}{\tau_{hh}}$$

$$\Delta\sigma_h = \frac{Q_h}{\tau_h} = \frac{\frac{11}{2} Q_{hh}}{\frac{1}{2}\tau_{hh}} = 11\,\sigma_{hh}$$

$$\Delta\sigma_\ell = \frac{Q_\ell}{\tau_\ell} = \frac{\frac{9}{2} Q_{hh}}{\frac{9}{20}\tau_{hh}} = 10\,\sigma_{hh}$$

2. K. + K. Ch 8, # 26 We can get the solution from the previous problem:

$$Q_\ell = Q_h - Q_{hh} = Q_{hh}\left[\frac{\tau_h}{\tau_h - \tau_\ell}\cdot\frac{\tau_{hh} - \tau_\ell}{\tau_{hh}} - 1\right]$$

$$\frac{Q_\ell}{Q_{hh}} = \frac{\tau_h(\tau_{hh} - \tau_\ell) - \tau_{hh}(\tau_h - \tau_\ell)}{\tau_{hh}(\tau_h - \tau_\ell)} = \frac{\tau_\ell(\tau_{hh} - \tau_h)}{\tau_{hh}(\tau_h - \tau_\ell)}$$

Plug in numbers from problem 1:

$$\frac{Q_\ell}{Q_{hh}} = \frac{270 \cdot 300}{600 \cdot 30} = \underline{\underline{\frac{9}{2}}} = 4.5$$

3. K. + K. Ch. 8, #7 Light bulb in a refrigerator

From class and the book, $\frac{Q_l}{W} = \frac{\tau_l}{\tau_h - \tau_l} = \gamma_c$

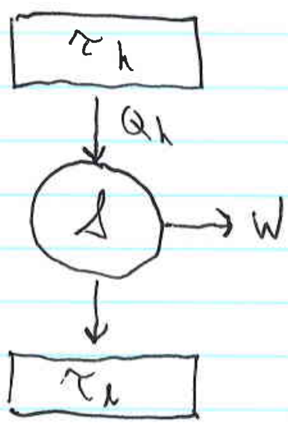
γ_c can be much larger than 1, so Q_l can be larger than W.

If $T_h = 27^\circ C = 300\,K$, $T_l = -13^\circ C = 260\,K$, then

$\gamma_c = \frac{260}{40} = 6.5$, then the net cooling power is

$\dot{Q}_l - \dot{Q}_{ext} = \underbrace{6.5 \cdot 100\,W}_{\text{refrigerator}} - \underbrace{100\,W}_{\text{light bulb}} = \underline{\underline{550\,W}}$ lots of extra cooling power

4. K. + K. Ch. 8, #5



$W = Q_h - Q_l$, $\sigma_h = \sigma_l \Rightarrow \frac{Q_h}{\tau_h} = \frac{Q_l}{\tau_l}$

$W = \left(\frac{\tau_h}{\tau_l} - 1\right) Q_l$

Let $T_h = 500^\circ C = 773\,K$, $T_l = 20^\circ C = 293\,K$

$\dot{W} = \frac{773 - 293}{293} \cdot \dot{Q}_l = \frac{480}{293} \cdot 1500\,MW = \underline{\underline{2460\,MW}}$

If $T_h = 600^\circ C = 873\,K$, then

$\dot{W} = \frac{580}{293} \cdot 1500\,MW = \underline{\underline{2970\,MW}}$

5. K. + K. Ch. 8, #6 Room air conditioner

The air conditioner works as a standard refrigerator with

$$\frac{Q_l}{W} = \gamma_c = \frac{T_l}{T_h - T_l} \quad \text{or} \quad \dot{Q}_l = \frac{T_l}{T_h - T_l} P$$

where $P \equiv \dot{W}$ is the power supplied to the air conditioner.

(a) Steady-state : $\dot{Q}_{total} = 0 = A(T_h - T_l) - \frac{T_l}{T_h - T_l} P$

$$A(T_h - T_l)^2 = T_l P$$

$$A T_l^2 - 2AT_hT_l + AT_h^2 = PT_l$$

$$T_l^2 - \left(2T_h + \frac{P}{A}\right) T_l + T_h^2 = 0$$

$$T_l = \frac{1}{2}\left[\left(2T_h + \frac{P}{A}\right) \pm \sqrt{\left(2T_h + \frac{P}{A}\right)^2 - 4T_h^2}\right]$$

take negative root because $T_l < T_h$

$$T_l = \left(T_h + \frac{P}{2A}\right) - \sqrt{\left(T_h + \frac{P}{2A}\right)^2 - T_h^2} \quad \checkmark$$

(b) $A = \frac{T_l P}{(T_h - T_l)^2} = \frac{290\,K \cdot 2000\,W}{(20\,K)^2} = \underline{1450\ \frac{W}{K}}$

Note that the authors mistakenly refer to P as the "cooling power." That expression should refer to $\dot{Q}_l = \gamma P$.

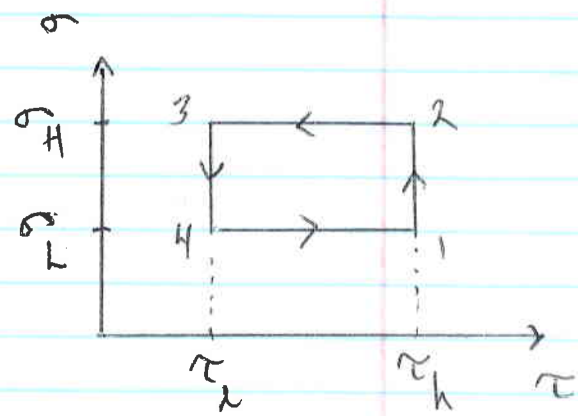
6. K. + K. Ch 8, #3 Photon Carnot Engine

$$U = \frac{\pi^2}{15\hbar^3 c^3} V\tau^4 \equiv \alpha V \tau^4 \qquad \alpha \equiv \frac{\pi^2}{15\hbar^3 c^3}$$

$$p = \frac{U}{3V} = \frac{\alpha}{3}\tau^4$$

$$\sigma = \frac{4\pi^2}{45} V \left(\frac{\tau}{\hbar c}\right)^3 = \frac{4}{3}\alpha V \tau^3$$

a) During the isentropic steps, σ = constant $\Rightarrow V\tau^3$ = constant



$$V_2 \tau_h^3 = V_3 \tau_l^3$$

$$\Rightarrow \underline{\underline{V_3 = V_2 \left(\frac{\tau_h}{\tau_l}\right)^3}}$$

$$V_1 \tau_h^3 = V_4 \tau_l^3 \Rightarrow \underline{\underline{V_4 = V_1 \left(\frac{\tau_h}{\tau_l}\right)^3}}$$

b) Isothermal expansion from V_1 to V_2

$$Q_{12} = \tau_h \Delta\sigma = \tau_h(\sigma_2 - \sigma_1) = \underline{\underline{\frac{4}{3}\alpha\tau_h^4 (V_2 - V_1)}}$$

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \frac{\alpha}{3}\tau_h^4 \int_{V_1}^{V_2} dV = \underline{\underline{\frac{\alpha}{3}\tau_h^4 (V_2 - V_1)}} \leftarrow$$

not equal

Check answers: $\Delta U = U_2 - U_1 \stackrel{?}{=} Q_{12} - W_{12}$

$$\alpha\tau_h^4 (V_2 - V_1) = \left(\frac{4}{3} - \frac{1}{3}\right)\alpha\tau_h^4 (V_2 - V_1) \quad \checkmark$$

6. c) Isentropic steps: calculate W_{23} and W_{41}

It is not easy to see how p changes with V during an isentropic expansion, so instead use this method:

$$Q = 0 \Rightarrow W = -\Delta U \qquad \text{from } \Delta U = Q - W$$

$$W_{23} = -(U_3 - U_2) = \alpha V_2 \tau_h^4 - \alpha V_3 \tau_l^4$$

$$= \alpha V_2 \tau_h^4 \left[1 - \frac{V_3}{V_2}\left(\frac{\tau_l}{\tau_h}\right)^4\right] = \underline{\underline{\alpha V_2 \tau_h^4 \left[1 - \frac{\tau_l}{\tau_h}\right]}}$$

because $\frac{V_3}{V_2} = \left(\frac{\tau_h}{\tau_l}\right)^3$

Similarly:

$$W_{41} = -(U_1 - U_4) = -\left[\alpha V_1 \tau_h^4 - \alpha V_4 \tau_l^4\right]$$

$$= -\alpha V_1 \tau_h^4 \left[1 - \frac{V_4}{V_1}\left(\frac{\tau_l}{\tau_h}\right)^4\right] = \underline{\underline{-\alpha V_1 \tau_h^4 \left[1 - \frac{\tau_l}{\tau_h}\right]}}$$

They <u>don't</u> cancel.

d) $W_{total} = W_{12} + W_{23} + W_{34} + W_{41}$

$$= \frac{\alpha}{3}\tau_h^4 (V_2 - V_1) + \alpha V_2 \tau_h^4 \left(1 - \frac{\tau_l}{\tau_h}\right) - \frac{\alpha}{3}\tau_l^4 (V_3 - V_4) - \alpha V_1 \tau_h^4 \left(1 - \frac{\tau_l}{\tau_h}\right)$$

$$= \frac{\alpha}{3}\tau_h^4 (V_2 - V_1)\left[1 - \left(\frac{\tau_l}{\tau_h}\right)^4 \frac{V_3 - V_4}{V_2 - V_1}\right] + \alpha \tau_h^4 \left(1 - \frac{\tau_l}{\tau_h}\right)(V_2 - V_1)$$

$$= \frac{\alpha}{3}\tau_h^4 (V_2 - V_1)\left(1 - \frac{\tau_l}{\tau_h}\right) + \alpha T_h^4 (V_2 - V_1)\left(1 - \frac{\tau_l}{\tau_h}\right)$$

$$W_{total} = \underline{\underline{\frac{4}{3}\alpha \tau_h^4 (V_2 - V_1)\left(1 - \frac{\tau_l}{\tau_h}\right)}}$$

$$Q_h = Q_{12} = \frac{4}{3}\alpha \tau_h^4 (V_2 - V_1) \Rightarrow \frac{W}{Q_h} = 1 - \frac{\tau_l}{\tau_h} = \frac{\tau_h - \tau_l}{\tau_h} = \eta_c \quad \checkmark$$

from part (a)