PHY 410 - Homework 1 Solutions

1. a) 5 coin flips, 3 heads   10 configurations:

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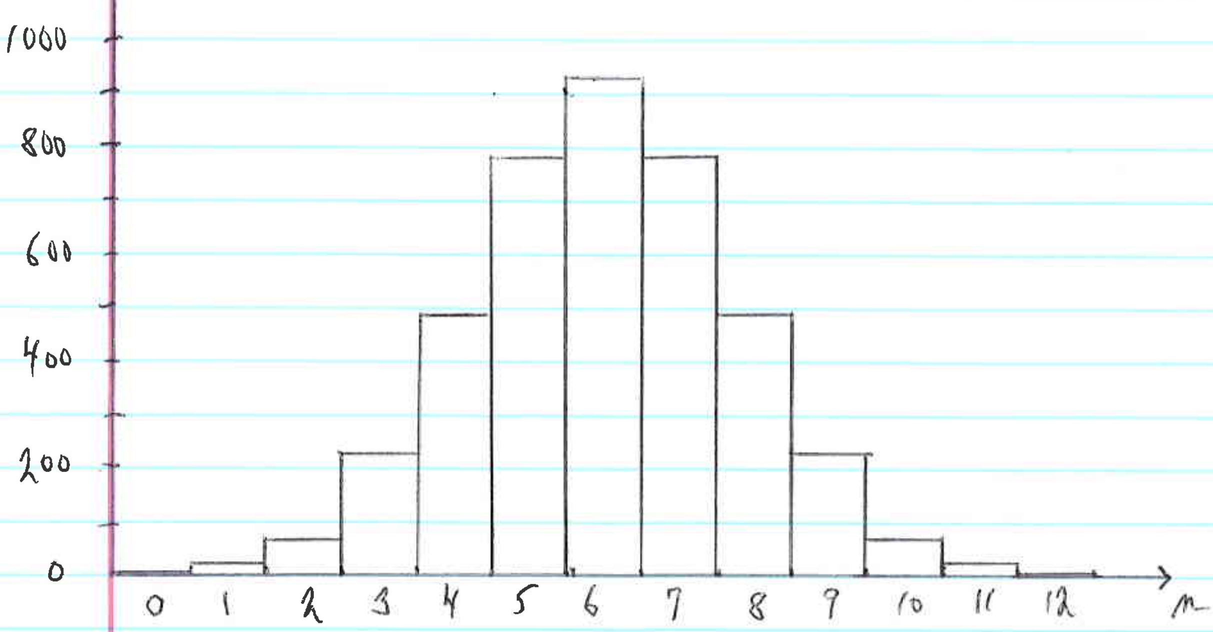
$$P = \frac{10}{2^5} = \frac{10}{32} = 0.3125$$

b) 12 coin flips   Let n = # of heads

$g(0) = 1$ $g(1) = 12$ $g(2) = \frac{12!}{2!\,10!} = 66$ $g(3) = \frac{12!}{3!\,9!} = 220$

$g(4) = \frac{12!}{4!\,8!} = 495$ $g(5) = \frac{12!}{5!\,7!} = 792$ $g(6) = \frac{12!}{6!\,6!} = 924$

$$g(12,n) = \frac{12!}{n!\,(12-n)!}$$



PHY410 : Homework 1, Problem 2

part (a) (tests)

```
Random[]
```

0.977074

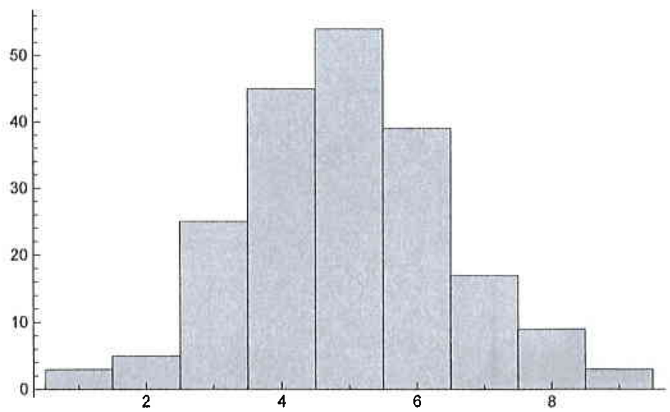
```
Random[Integer]
```

1

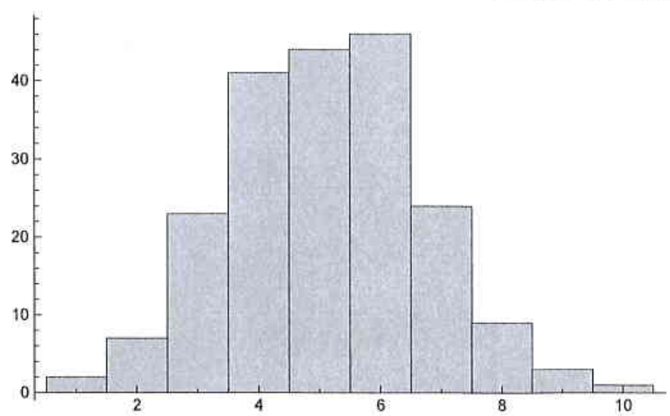
```
Table[Random[Integer], {10}]
```

{0, 1, 0, 0, 1, 1, 1, 1, 0, 0}

```
NumberOfHeads[n_] := Sum[Random[Integer], {n}]
ManyTrials[N_] := Table[NumberOfHeads[10], {N}]
Histogram[ManyTrials[200]]
```

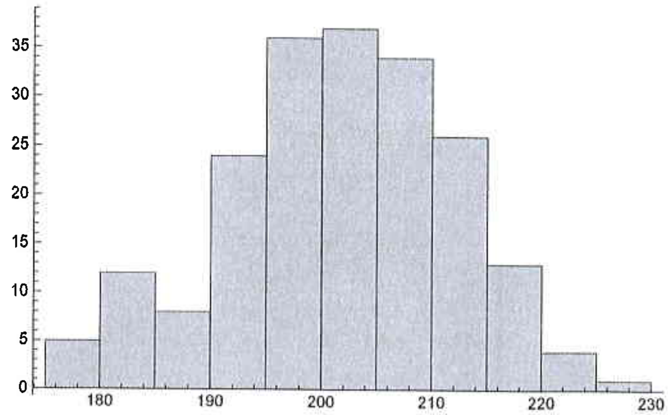



```
Histogram[Table[Sum[Random[Integer], {10}], {200}]]
```



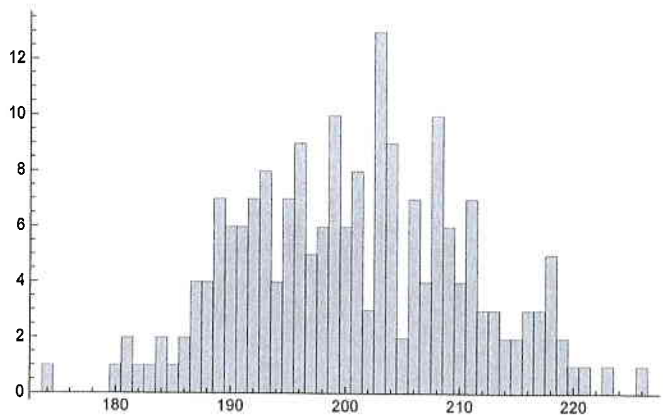
part (b)

```
Histogram[Table[Sum[Random[Integer], {400}], {200}]]
```



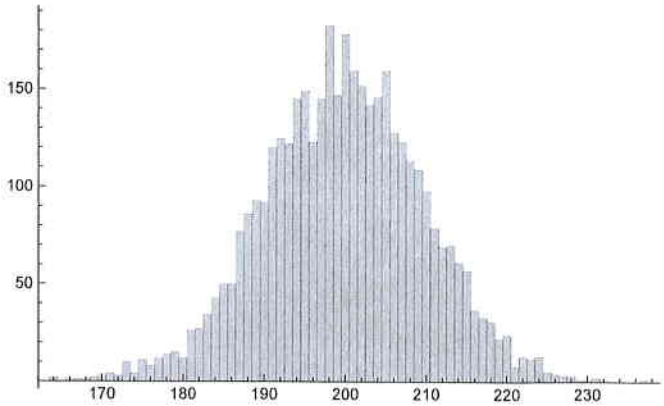
Mathematica chose a bin width of 5, but I want the bin width to be 1, otherwise the Gaussian fit in part (d) won' t work. So I asked for 60 bins :

```
Histogram[Table[Sum[Random[Integer], {400}], {200}], 60]
```



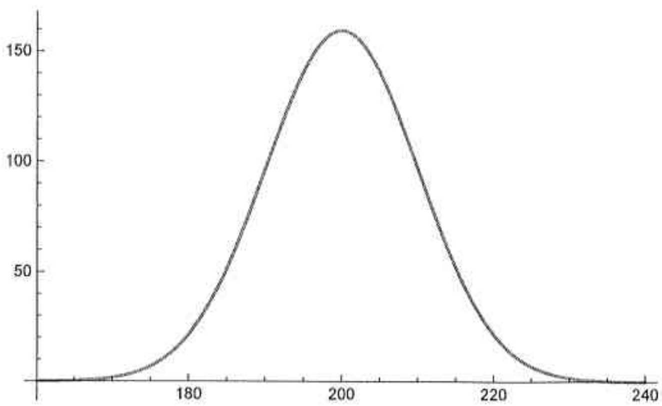
part (c)

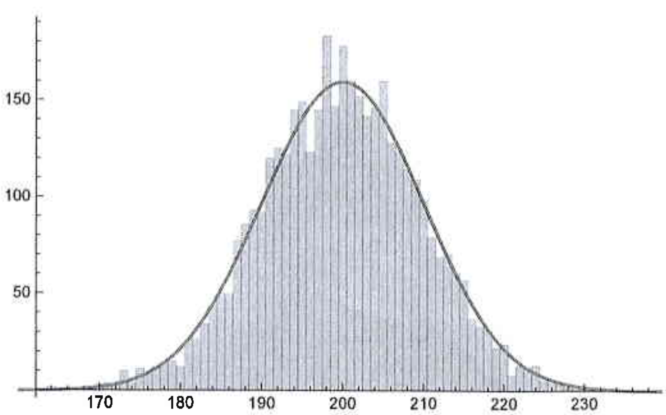
```
P1 = Histogram[Table[Sum[Random[Integer], {400}], {4000}], 80]
```



part (d)

```
P2 = Plot[4000 * Sqrt[2 / (400 * π)] * Exp[-2 * (x - 200) ^2 / 400], {x, 160, 240}]
Show[P1, P2]
```





The fit is excellent!

2. e) $\delta n = \sqrt{\langle (n-\bar{n})^2 \rangle}$ coin : $p = q = \frac{1}{2}$

From class, $\delta n = \sqrt{Npq} = \frac{1}{2}\sqrt{N}$

$$\delta n = \begin{cases} 1.7 & \text{for } N=12 \\ 10 & \text{for } N=400 \end{cases}$$

$$\bar{n} = Np = \frac{1}{2} N$$

$N=12$: $\frac{\delta n}{\bar{n}} = \frac{1.7}{6} = 0.28$ $N=400$: $\frac{\delta n}{\bar{n}} = \frac{10}{200} = 0.05$

$\delta n = 10$ is consistent with the graphs in part (d).

For $n=12$, it is not so rare to get fewer than 30% heads.

For $n=400$, it is extremely unlikely.

$30\% \times 400 = 120$, which is far to the left of the visible part of the histogram.

3. N_0 particles in V_0, N particles in sub-volume V.

This problem is like the biased coin flip, with probability $p = V/V_0$ to find a given particle in V.

In class we showed $P(N) = \frac{N_0!}{N!(N_0-N)!} p^N q^{N_0-N}$

a) In class we showed that $\langle N \rangle = p N_0 = \frac{V}{V_0} N_0$

b) In class we showed that $\langle (\Delta N)^2 \rangle = N_0 p q$

so $\frac{\langle (\Delta N)^2 \rangle}{\bar{N}^2} = \frac{N_0 p q}{(N_0 p)^2} = \frac{q}{p} \frac{1}{N_0} = \frac{1}{\bar{N}} \left(1 - \frac{V}{V_0}\right)$

c) When $V \ll V_0$, $\frac{\langle (\Delta N)^2 \rangle}{\bar{N}^2} = \frac{1}{\bar{N}}$

d) When $V \to V_0$, $\langle (\Delta N)^2 \rangle \to 0$. There are no fluctuations because the total number of particles is fixed so $N = N_0$.

e) $N_0 = 5$, $V/V_0 = 1/4$ $\quad p = 1/4 \quad q = 3/4$

Find all 5 in V: $P = p^5 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024} = 9.8 \cdot 10^{-4}$

Find exactly 1 in V:

$P = 5 p q^4 = 5 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^4 = \frac{405}{1024} \approx 0.396$