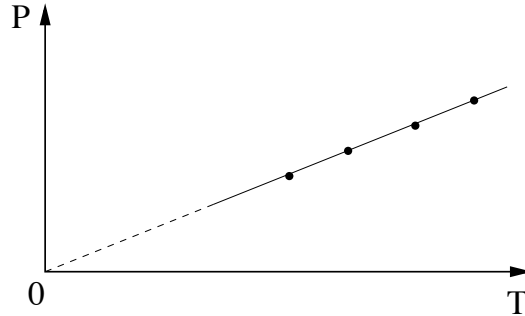


Homework #1 Solutions

Question 1a) The absolute temperature scale is based on the idea that classically, all molecular motion will cease at the absolute zero of temperature. The temperature is identified from plots of pressure vs. temperature of a gas, and extrapolating to zero pressure (i.e. no molecular motion)



1b) The first law of thermodynamics is a statement of conservation of energy for heat flow and work:

$$\Delta U = Q + W$$

The change in internal energy of a system comes about because of heat added to the system Q , plus the work done on the system. Heat and work represent energy in transit and are not thermodynamic state variables. Energy is a thermodynamic state variable. In thermodynamics the only two ways to transfer energy to a system are by means of heat (the transfer of energy that occurs spontaneously due to a difference in temperature) and work (which is all other methods of transferring energy besides heat, and requires the action of an 'agent').

Common Errors: *Some students did not mention 'work'. Work should also be included in the 1st Law.*

Question 2a) $C_{water}(@0^{\circ}C) = 4.22 kJ/kg - K$

2b) $C_{ice}(@0^{\circ}C) = 2.05 kJ/kg - K$

2c) $L_{ice} = -333.6 kJ/kg$

2d) $C_{iron}(@25^{\circ}C) = 0.450 kJ/kg - K$

2e) $\kappa_{glass} = 0.8 W/m - K$

Question 3a) The heat capacity at constant volume is the rate of change of the internal energy with temperature:

$$C_V = \frac{dU}{dT} \quad ; \quad [C_V] = \frac{kJ}{K}$$

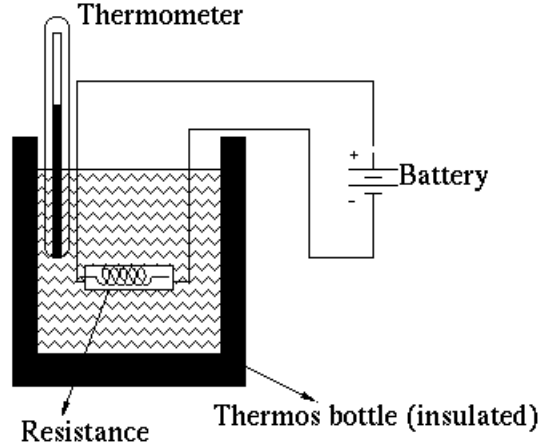
The specific heat is the heat capacity per unit mass

$$[C_V] = \frac{kJ}{kg \times K}$$

Assuming that the heat capacity doesn't change significantly over the range of temperatures we are considering, we can replace the infinitesimal quantities by finite changes. Then the amount of heat required to change the temperature of an object by ΔT at constant volume is (set $W=0$ in the first law since volume is constant):

$$Q = mc_V \Delta T$$

where m is the mass.



3b) We want to determine $c_V = \frac{Q}{m\Delta T}$. We have a thermometer to measure the change in temperature ΔT of the water. We need to control and measure the heat transfer Q . This can be done by building a simple circuit of a resistor connected to a battery and placing the resistor into the water (see figure). The resistor dissipates the electrical energy into the surrounding water. The rate of energy dissipation is given by the formula for the power dissipated as heat in a resistor

$$P = I^2 R = \frac{V^2}{R}$$

where I is the current passing through the resistance and V is the voltage of the battery. We can read the current by connecting an ammeter in series or we can measure the voltage using a voltmeter. We measure the time Δt for the temperature to rise by an amount ΔT . Then the total energy dissipated is $Q = P\Delta t$ and the heat capacity can be obtained dividing this by the change in temperature, ΔT .

Question 4a) We assume that all the kinetic energy is converted into heat Q .

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}1000kg(30m/s)^2 = 450000J = 450kJ$$

This energy has to be shared between the 4 brakes of total mass $4 \times 5kg = 20kg$.

$$\Delta T = \frac{Q}{C_{brakes}} = \frac{450kJ}{0.450kJ/kg - K \cdot 20kg} \approx 50K$$

4b) It is likely to be an overestimate because some of the heat is absorbed by the road and air (by convection) and other parts of the car (by conduction).

Question 5) First we need to decide between two possible scenarios. Is the ice going to melt completely or not? To determine this we can calculate the amount of heat that would be absorbed by the melting of ice and compare it to the heat necessary to bring all the water to the freezing point.

$$Q_{\text{melting}} = 20K \times c_{\text{ice}} \times m_{\text{ice}} + L_{\text{ice}} \times m_{\text{ice}} = 37.5kJ$$

$$Q_{\text{freezing}} = 5K \times c_{\text{water}} \times m_{\text{water}} = 21.0kJ$$

This shows that not all the ice will melt. $\Delta Q = 37.5kJ - 21.0kJ = 16.5kJ$ is the amount of missing energy, that would bring the whole system to $0^\circ C$ water. The 'missing energy' can freeze the following amount of water

$$m_{\text{ice}}(\text{final}) = \frac{\Delta Q_{\text{missing}}}{L_{\text{ice}}} = \frac{16.5kJ}{334kJ/kg} \approx 50g$$

The final configuration: 50g of ice, 1050g of water, all at $0^\circ C$.

Discussion:

a) When the temperature of the water reaches $0^\circ C$, it will not convert into ice automatically. It requires additional energy (the latent heat of fusion) to complete the phase transformation to ice. During the transformation the temperature of the ice/water mixture will remain at $0^\circ C$.

b) You should calculate whether the energy is enough for all the ice to melt or for all the water to freeze at first.

Question 6a) Work is given by the formula $dW = P(V,T)dV$. Using the ideal gas law $P(V,T) = nRT/V$ (n is the number of moles and R is the universal gas constant, $R = N_A k_B$) we can integrate the work (remember that temperature is kept constant)

$$W = \int_{V_{\text{initial}}}^{V_{\text{final}}} \frac{nRT}{V} dV = nRT \int_{V_{\text{initial}}}^{V_{\text{final}}} \frac{1}{V} dV = nRT \ln\left(\frac{V_{\text{initial}}}{V_{\text{final}}}\right) = nRT \ln(2)$$

This can be evaluated as

$$W = 1\text{mole} \times 8.314 \frac{J}{\text{mole} \cdot K} \times 293K \times \ln(2) = 1.69kJ$$

Discussion: a) the pressure here is not a constant. b) the question asks for the work done by the system, not the work done on the system.

6b) Using the first law of thermodynamics (energy conservation) together with the fact that the internal energy of an ideal gas only depends on its temperature (hence $\Delta U = 0$ in this case) we get

$$\Delta U = W + Q$$

Hence $Q = -W$ so that an amount of heat flows into the gas equal to the work done by the gas on the piston.

Discussion: Heat is not temperature, some students are confused about the two concepts.

6c) None. In this case there is no piston to which the gas particles can transfer their energy and do work on. Hence $W = 0$.

6d) None. The first law with $W = 0$ reads $\Delta U = Q$. The properties of an ideal gas say that $\Delta U = 0$ since temperature is held fixed. As a result $Q = 0$ too. Note that in a real gas, the answer will be different, because the molecules interact with each other and change their potential energy upon undergoing a free expansion.

Question 7a) The heat current will be proportional to the thermal conductivity of the glass, the area of the windows, and the temperature gradient

$$\text{Heat Loss} = \kappa_{\text{glass}} \times \text{Area} \times \frac{1}{\text{Thickness}} \times \Delta T = 0.8 \frac{W}{mK} \times 2m^2 \frac{20K}{0.005m} = 6400W$$

7b) Assuming all the electrical energy goes into heating the room (some could escape in the form of light etc.) and that the cost of electricity is \$0.12 per kW-hr we get

$$\frac{\text{Cost}}{\text{Day}} = 0.12 \frac{\$}{kW - hr} \times \frac{24hr}{day} \times 6.4kW = \$18.43/day$$

How can this cost be reduced? Double-pane insulated glass will help, along with reducing the area of the windows, and reducing the temperature gradient (turning down the thermostat, or moving to Florida).

Question 8) By equipartition of energy, we expect the average kinetic energy per particle to be $\overline{KE} = \frac{3}{2}k_B T$. For n moles of molecules this gives $N \frac{1}{2} m \overline{v^2} = \frac{3}{2} n R T$. The molar mass of helium ($N_{avg} m$ for $n=1$) is approximately 4g.

$$v_{rms}(He) = \sqrt{\frac{3RT}{4g}} = 1300 \frac{m}{s}$$