Sleeping Beauty and the Limits of Conditionalization

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1 The Sleeping Beauty Problem

Peripheral cases often play an important role in the evaluation of general theories. A theory that works well for the run-of-the-mill cases might display faults in unusual situations. The Sleeping Beauty Problem is interesting for this reason. A confluence of factors bares a tension in our intuitions. Settling this problem will teach us something about the interaction between probability and evidence more generally.

With that in mind, here is the case:

> Beauty is the subject of a bizarre experiment. On Sunday night, she will be given a drug that will cause her to sleep for three days. While she is asleep, a fair coin will be tossed. If the coin lands tails, Beauty will be briefly awoken on Monday, and awoken again on Tuesday morning. If the coin lands heads, Beauty will be awoken only on Monday morning. As a side effect of the drug, Beauty will not be able to remember her Monday waking if and when she wakes on Tuesday. Since the circumstances of the wakings are identical and Beauty cannot trust her drug-altered memory, Beauty will not be able to tell what day it is after waking up.

The Sleeping Beauty Problem concerns the probability that Beauty should assign to the proposition that the coin landed heads (HEADS) once she awakens.

According to the response that has come to dominate the literature (Elga, 2000; Dorr, 2002; Horgan, 2004), Beauty should assign a probability of ⅓ to HEADS. The main opposition proposal holds that Beauty should instead assign a probability of ½. The challenge facing the 'thirders' is to justify Beauty's change in probability assignment between Sunday night and Monday morning. On Sunday, everyone agrees that she should evaluate the probability of HEADS at ½. Thirders suggest that on Monday, she should update her assignment to ⅓. On the most plausible explanation (Horgan, 2004; Weintraub, 2004), Beauty's change of assignment is driven by a change of her self-locating evidence: she learns that she woke up *today* and appropriately updates her prior probability to reflect this new evidence.

I will do two things in this paper. First, I will present what I take to be the strongest argument for thirdism. Variations of this argument have been developed independently (Dorr, 2002), but its most thorough and consistent advocate has been Terry Horgan (2004; 2008; unpublished). I will start by presenting a version of the standard Bayesian epistemological framework, which is assumed by the argument, and use it to develop the argument. Second, I will critique one of the central tenets of the Bayesian framework, which I call the "conditionalization constraint". I will suggest that the best explanation of its virtues also implies that it is limited in significant ways. In short, I think that we are only required to obey the conditionalization constraint in those cases where ratios between probabilities reflect certain aspects of our evidence that are preserved. It is only by appreciating the limits of conditionalization that we can get a moderately plausible answer to the Sleeping Beauty Problem.

2 The Bayesian Framework

Following Horgan (Horgan, unpublished), I will hold that the sense of probability involved in this puzzle is one in which probability assignments are measures of the epistemic support that bodies of evidence provide to propositions.[1] All things being equal, strong evidence for a proposition supports a higher probability in that proposition, and strong evidence against a proposition supports a lower probability in that proposition. In this sense, probability assignments are always implicitly or explicitly relativized to bodies of evidence. The probability Beauty that should assign to HEADS is determined by the level of support Beauty's evidence provides for HEADS.

Probabilities measure evidence, but they need not always measure it with numbers. The fact that Zenyatta has a ¾ probability of winning the race means that the evidence supports her winning to degree ¾ – the evidence can be measured with the number ¾. Some bodies of evidence might only support comparative assignments ("It is more probable that Zenyatta will win than that Rachel Alexandra will") or vague assignments ("Zenyatta will probably win, but it is not definite").

What I am calling the standard Bayesian epistemological framework will set four substantive normative constraints on probability assignments. These constraints will occasionally impose enough structure on possible assignments so that they can be represented with numbers. Though not formalized in these constraints, I will also assume that all quantified probability assignments must obey the axioms of probability theory (where probabilities are numerically de-

[1]This interpretation of probability is different from, but not in opposition with, the interpretation of probability as degrees of belief. There may be multiple sensible uses of the term 'probability', but these other uses are not the ones at issue here. This interpretation is silent on matters of objectivity. There may be a fact about which probabilities are supported by which bodies of evidence, or we may instead regard ourselves as merely adopting non-representational commitments to evaluating our evidence. The answer we choose to give to this question is irrelevant to the solution to the problem.

fined) and that each probability assignment must assign the number 0 to any proposition that the body of evidence definitively rules out. These constraints have found favor throughout the history of Bayesian epistemology, and they will be employed in the next section's argument in order to quantify the probabilities of the relevant propositions, relative to Beauty's evidence.

The **first** constraint concerns the assignment of probabilities to propositions that are supported by symmetric bodies of evidence. A body of evidence is symmetric with respect to a particular partition of propositions when it provides equal amounts of evidential support for each member of the partition. Given a symmetric body of evidence for a partition of propositions that divides the possibilities into equally a priori plausible propositions, this constraint says that the probability of each proposition is equal.

A special case arises when we have no evidence for the members of a partition, and the theoretical virtues of the different members are equivalent. In such cases, the first constraint requires assigning each member of the partition an equal probability. This constraint is very controversial because of the difficulties of deciding when propositions receive equal support and when they have equivalent virtues.[2] Elga (2004) has suggested that propositions concerning one's spatial and temporal location are especially plausible candidates for symmetric support. I will assume that propositions about what time it is receive prima facie symmetric evidential support. This means that we must have some reason to think that it is Tuesday rather than Monday in order for it to be more likely to be former than the latter.

The **second** constraint concerns the probabilities supported by bodies of evidence that include information about objective chances. Though possibly objective, the present notion of probability as a measure of degrees of epistemic

[2]Not everyone would characterize the problem in this way. See White (2009) for a more in depth discussion.

support is distinct from what generally goes by the name of 'objective chances'. Objective chances are metaphysical, rather than epistemic, and relate to the propensity with which events occur. The second constraint codifies the relation between these metaphysical and epistemic senses of probability. In normal cases, the probability assignment to a proposition supported by a body of evidence will equal any objective chance that that body of evidence specifies to hold for that proposition. If a body of evidence includes the fact that P has a .5 objective chance of becoming true, that body supports P with a probability of ½.[3]

The **third** constraint says that the probability of the conjunction of two evidentially independent propositions is equal to the product of their respective probabilities.

Some bodies of evidence include both evidential symmetries and information about objective chances. For example, a coin flipped onto a table may land heads with an objective chance of .5. Our evidence may indicate nothing about whether the coin landed on the right or left side of the table. The proposition HEADS is equivalent to the disjunctive proposition (HEADS & RIGHT) ∨ (HEADS & LEFT). The proposition HEADS & RIGHT conjoins one proposition that has a known objective chance with another proposition about which we have symmetrically balanced evidence. HEADS is evidentially independent of RIGHT. When two propositions are evidentially independent relative to a body of evidence, the probability of their conjunction is equal to the product of their probabilities. In the case of the coin, the probability of HEADS & RIGHT is ¼.

Often, the first three constraints will not limit the assignments of probabilities to propositions to specific numerical values all by themselves. In such cases, it is possible that the evidence supports no precise probabilities. The **fourth constraint**, however, makes it possible to leverage numeric measures of

[3]Though Lewis was primarily concerned with subjective interpretations of probability, his discussion Lewis (1980) is the classic exposition of this idea.

probability in some assignments to numeric assignments in others. It holds only when one body of evidence includes the other, and says that the probabilities that they support are related by conditionalization.

A probability assignment is related by conditionalization to another probability assignment if the probability of every proposition in the first assignment is equal to the probability of the same proposition in the second assignment, conditional on the proposition that marks the difference between the two bodies of evidence.[4] If $E_2 = E_1 \cup \{P\}$, then $Pr_{E_2}(Q) = Pr_{E_1}(\frac{P\&Q}{Q})$.

3 The conditionalization argument

The conditionalization argument makes use of each of the four constraints discussed in the previous section in support of the thirder answer. Let "Beauty's preliminary evidence" name the body of evidence that consists in all of the evidence that Beauty has on Monday morning, except for any evidence that she receives specifically from waking up. This body of evidence will include the fact that she went to sleep on Sunday, all of the information about the experiment, the fact that she has not yet been debriefed after the experiment, and so on. It will not include that she awoke up this morning (AWOKE) or that she is now awake.

Beauty never actually consciously possesses exactly this preliminary body of evidence (Pust, 2008; Horgan, 2008) because this evidence leaves open the possibility that its possessor is currently unconscious. Assuming that Beauty can always know that she is conscious when she is contemplating probabilities, she is never in a position to assess probabilities on the basis of this evidence. But on the present framework, it is still intelligible to ask what degree of probability

[4]The probability of P conditional on Q, written Pr(P|Q), is defined as the probability of P&Q divided by the probability of Q.

this body of evidence conveys to various propositions. Beauty will acquire new evidence upon awaking, so if we can figure out what conditional probabilities are supported by the restricted set, we can figure out what probabilities Beauty should assign upon awakening.

Based on the constraints in the previous section, the probabilities of the propositions in the following partition, relative to Beauty's preliminary evidence, are each ¼.

- MONDAY & HEADS
- TUESDAY & HEADS
- MONDAY & TAILS
- TUESDAY & TAILS

Beauty's preliminary evidence provides symmetric support for MONDAY and TUESDAY, since she will be in the same epistemic situation on both days, no matter how the coin landed. With no discerning evidence, and given their obvious symmetry, the body of evidence supports the propositions MONDAY and TUESDAY with equal probabilities.

Furthermore, the objective chance that the coin landed heads is also .5. There is no 'inadmissible' information in the preliminary body of evidence, so according to the second constraint, the preliminary assignment equals the objective chance. Since the propositions MONDAY, and TUESDAY are independent of the propositions HEADS and TAILS relative to her preliminary body of evidence, the third constraint dictates that probability of MONDAY & HEADS equals the product of the probability of MONDAY and the probability of HEADS. The same goes for the other conjunctions.

Beauty's preliminary body of evidence is a subset of her body of evidence upon waking. Everything that she knew before waking, she still knows after waking. But she also learns that she awoke up on that day. So, according to the Bayesian framework, the new probability should be given by the preliminary probabilities conditional on $\overline{\text{TUESDAY \& HEADS}}$. The conditional probability of each other proposition is ⅓, so that is the probability supported by her new evidence.

4 The Conditionalization Constraint

Despite the power of this argument, I remain skeptical. Accepting thirdism is likely to commit us to deeply unintuitive verdicts about related cases.[5] If we can find plausible grounds to reject thirdism, it would be preferable to do so. I think the most dubious part of the argument lies in the conditionalization constraint.

> **Conditionalization Constraint:** The probability assignment supported by a body of evidence must equal any probability assignment supported by a subset of that body, conditional on the remaining evidence not included in that subset.

The spirit of the conditionalization constraint has played an important role in discussions of the Sleeping Beauty problem since discussion of the problem started. Elga's (2000) initial salvo against halfism relied on applying a variation of the conditionalization argument that utilized the conditionalization constraint in reverse. He assumed something about the probability of HEADS given the fact that it is Monday in order to infer a corresponding probability without that evidence. Unless we are willing to follow David Lewis (2001) in accepting that Beauty's evidence supports a ½ probability to HEADS a higher probability after she learns that it is Monday, we must either reject halfism or object to conditionalization at some point.

Much of the debate around the Sleeping Beauty problem has centered around when and how the conditionalization constraint applies in light of the complications arising from self-locating evidence, rather than whether the constraint is

[5]See (Bostrom, 2007; Titelbaum, 2013).

true as it is stated (Bostrom, 2007; Pust, 2008; Hawley, 2013). If the constraints deliver the correct probabilities for Beauty's preliminary evidence, and if her evidence upon awakening really does include her preliminary body of evidence as a subset, then the conditionalization constraint implies that Sleeping Beauty's probability in MONDAY & HEADS upon waking should equal her preliminary body of evidence's conditional probability. Since her new evidence simply rules out one possibility, the probability of MONDAY & HEADS should equal ⅓.

The conditionalization constraint could fail to hold for several reasons. First, it is possible that bodies of evidence sometimes fail to support numerically specific probabilities when conditionalization would deliver precise probabilities. It is consistent with this possibility that conditionalization places constraints on which precise probability assignments are viable without actually requiring (or permitting) us to have precise probability assignments in the first place. Perhaps new evidence can have the effect of imprecisifying a precise probability assignment.

Second, it is possible that bodies of evidence sometimes support numerically specific probabilities that are different from the conditional probabilities supported by an evidential subset. If Beauty's preliminary evidence supports a conditional probability of ⅓ in HEADS given AWOKE, but the evidential body plus AWOKE supports a probability of ½ in HEADS, then the conditionalization constraint has been violated.

I will not advocate for either option. Instead, I hope to simply cast doubt on the conditionalization constraint. My response will be based on showing how the justification for the constraint, in the cases where it applies, should not carry over to the sort of situation in which Sleeping Beauty finds herself. I think that once we see why conditionalization does hold in some cases, it will become clear that it does not hold widely enough for the conditionalization argument to work.

5 Justifying Conditionalization

I claim that the conditionalization constraint should sometimes hold, but not always. I will give an account of what makes conditionalization special in terms of feature preservation, and use this account to formulate a limited justification of the constraint.

5.1 What Makes Conditionalization Special?

In order to understand the epistemological significance of conditionalization, it is important to understand what makes it distinctive. The conditionalization constraint essentially enforces a continuity in the relative probabilities of propositions that *agree about* the additional evidence. Two propositions agree about a third when they both entail it. For instance, the propositions ALBERT WON and BETH WON agree that CODY DIDN'T WIN. It is intuitive that probability assignments should be continuous in something like this sort of way, and it explains why conditionalization is so appealing.

The conditionalization constraint preserves relations between probabilities by ensuring that the ratios of the probabilities of propositions stay fixed when a proposition that they agree about is added to the body of evidence.[6] This means

[6] To see that conditionalization preserves the ratios, suppose that P and Q are two propositions that agree about E, and let Pr(P) = xPr(Q). Then

$$\mathrm{Pr}_{+\mathrm{E}}(\mathrm{P}) = \frac{\mathrm{Pr}(\mathrm{P\&E})}{\mathrm{Pr}(\mathrm{E})} = \frac{\mathrm{Pr}(\mathrm{P})}{\mathrm{Pr}(\mathrm{E})} = \mathrm{x}\frac{\mathrm{Pr}(\mathrm{Q})}{\mathrm{Pr}(\mathrm{E})} = \mathrm{x}\frac{\mathrm{Pr}(\mathrm{Q\&E})}{\mathrm{Pr}(\mathrm{E})} = \mathrm{x}\mathrm{Pr}_{+\mathrm{E}}(\mathrm{Q}).$$

On the other hand, to see that preserving ratios requires conditionalization (for a suitably rich probability assignment), we can start by assuming that all ratios are preserved, and argue that the probabilities of P are related by conditionalization. Take an arbitrary maximally specific proposition P. Since P is maximally specific, it either entails E or $\overline{\mathrm{E}}$. If it entails $\overline{\mathrm{E}}$, it must be assigned a probability 0 in the new probability assignment, and so we can ignore it. Suppose instead that P entails E. By assumption, there is some x such that Pr(P) = xPr(E&$\overline{\mathrm{P}}$) and $\mathrm{Pr}_{+\mathrm{E}}$(P) = x$\mathrm{Pr}_{+\mathrm{E}}$(E&$\overline{\mathrm{P}}$). Since these two propositions are exhaustive of the possibilities consistent with E, their probabilities must sum to 1 relative to the new evidence, and Pr(E) relative to the old. It follows that the probabilities must be related by that if one body of evidence supports twice the probability in P that it supports in Q, then the body of evidence that results from the addition of (P∨Q∨R) would continue to support P with twice the probability that it supports for Q.

It makes sense that some of the features of the probability assignment supported by a body of evidence should be conserved with the addition of further evidence. Additional evidence should not haphazardly reshuffle the probabilities of all propositions. Probabilities that are affected should be influenced in a systematic way.

The plausibility of preserving ratios is revealed by looking at concrete examples. Suppose that Albert, Beth, and Cody ran a race, and what you know about the racers supports a ½ probability to Albert winning, a ⅓ probability to Beth winning, and a ⅙ probability to Cody winning. Then you find out that Cody didn't win (and only this). It makes sense that your new evidence would not support different ratios of probabilities for Albert and Beth winning. Cody's loss is no evidence for Albert over Beth or Beth over Albert. The ratio reflects the relative amounts of evidence you have. Therefore, if your balance of evidence does not change, you should not alter the ratio of their probabilities. Preserving their ratio requires assigning Albert a ⅗ probability of winning and Beth a ⅖ probability of winning. Any deviation from conditionalization will

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conditionalization:

$$\mathrm{Pr(P)} = \mathrm{xPr(E\&\overline{P})} = \mathrm{Pr(E)} - \mathrm{Pr(E\&\overline{P})}$$
$$\mathrm{Pr_{+E}(P)} = \mathrm{xPr_{+E}(E\&\overline{P})} = 1 - \mathrm{Pr_{+E}(E\&\overline{P})}$$
$$\mathrm{Pr_{+E}(P)} = 1 - \frac{\mathrm{P}}{\mathrm{X}}$$
$$\mathrm{Pr_{+E}(P)} + \mathrm{xPr_{+E}(P)} = \mathrm{x}$$
$$\mathrm{Pr(P)} = \mathrm{Pr(E)} - \frac{\mathrm{Pr(P)}}{\mathrm{x}}$$
$$\mathrm{xPr(P)} + \mathrm{Pr(P)} = \mathrm{xPr(E)}$$
$$\frac{\mathrm{(1+x)Pr(P)}}{\mathrm{Pr(E)}} = \mathrm{x}$$
$$\frac{\mathrm{(1+x)Pr(P)}}{\mathrm{Pr(E)}} = \mathrm{(1+x)Pr_{+E}(P)}$$
$$\frac{\mathrm{(Pr(P)}}{\mathrm{Pr(E)}} = \mathrm{Pr_{+E}(P)}$$

The values of all non-maximally specific propositions are (for finite partitions of probability space) the sums of the values of maximally specific propositions. So all of the probabilities on the expanded body of evidence must equal conditional probabilities of the smaller body of evidence.

require a change in the ratio of probabilities of two propositions which agree about the additional evidence. If such ratios should never change, then the conditionalization constraint is appropriate.

5.2 Aspect-Feature Preservation

Certain features of probability assignments reflect certain aspects of the bodies of evidence that support them. For instance, the relative probability of Albert winning and Beth winning may reflect the balance of evidence that favors Albert winning and that favors Beth winning. This aspect (the balance) of the body of evidence supports that feature (the ratio of probabilities) of the probability assignment.

Additional evidence changes the probability assignments supported by changing the aspects of the body of evidence responsible for those features. The conditionalization constraint holds where it does because of how the features supported by a body of evidence evolve with that body. If a given body of evidence supports a certain ratio of probabilities between two propositions by virtue of having a certain aspect, and that aspect doesn't change with the addition of information, then the ratio of probabilities supported shouldn't change either.

If one body directly supports one ratio of probabilities in a set of propositions by virtue of having some particular aspect, then an expanded body will directly support that same ratio to the set, unless it includes some additional evidence that alters that aspect. Usually, this only happens when the additional evidence favors one proposition in the set over others. Furthermore, it is plausible that where two propositions agree about a third, adding the third to the body of evidence will not favor one over the other. It follows that typically, probability assignments are related by conditionalization.

For example, a preliminary body of evidence might support probabilistic ratios of 3:2 between Albert and Beth each winning. It might support this ratio because it includes such facts as their relative fitness and their competitive history, and because these facts contribute to a certain holistic property of the body: a balance of evidence for each winning. It is this balance that makes it slightly more likely that Albert wins, but only slightly. When it is added that Cody did not win, this balance would not change, and hence the ratio supported by this balance would not change. Thus, the new body of evidence would continue to support the same ratio of probabilities.

6 Limitations

According to the present suggestion, the conditionalization constraint applies to cases where the ratios of probabilities in an assignment are supported by features of the body of evidence that are preserved with the addition of new evidence. This suggestion implies certain limitations to the constraint, for it will not necessarily apply in cases where a prior ratio of probabilities does not reflect any features that are preserved under the addition of new evidence.

This framework suggests the following question: just how often does a ratio of probabilities reflect an aspect of a body of evidence that is preserved with the addition of evidence about which those propositions agree? The answer, I think, is fairly often. This explains a lot of the appeal of Bayesianism. However, I don't think that it always does so.

In order to show that there are at least some cases where the feature that supports the assignment, it is helpful to pay attention to the structure of probabilistic support. In particular, I think that the distinction between direct and indirect support for probabilistic features is important. Sometimes the ratios of probabilities of propositions can be supported indirectly. When this happens, those ratios may be undermined by the addition of a proposition about which those propositions agree.

6.1 Directness

We can distinguish the way that evidential aspects support probabilistic features in terms of their directness. The evidence about the relative fitness of Albert, Beth, and Cody may support a ratio of 3:2:1 to the proposition that each won. This means that it will also support an equal absolute *difference* between Albert and Beth winning, and between Beth and Cody winning, and a value of the difference between Beth and Cody winning equal to the difference between Cody winning and no one of them winning. The way that evidential aspects support these different features are not equivalent. Some forms of support are more direct than others.

An aspect indirectly supports a feature if it supports it by virtue of supporting another feature. For instance, a probability assignment may indirectly support assigning each of three propositions a probability less than ½ by virtue of directly supporting their equivalence. Bodies of evidence will often support raw differences in probabilities by virtue of supporting the assignment of raw probability values from which those differences can be be derived. For instance, a body of evidence may support a difference of ⅓ in the probabilities of two propositions by supporting a probability of ⅓ in one and ⅔ in the other. Those specific assignments are themselves often supported by virtue of support provided for the ratios of those probabilities. The probabilities of ⅓ and ⅔ in exclusive and exhaustive propositions may be supported by virtue of support for a probabilistic ratio of 1:2. Similarly, a body may support an assignment of x and x^2 to two propositions – that is, a power relation – by virtue of support for assigning one ½ and the other ¼, which in turn arise by virtue of assigning one twice the probability of the other.

It seems that ratios of probabilities are often more directly supported than differences and power relations in probabilities.[7] In the race between Albert, Beth, and Cody, it is reasonable to hold that the evidence fairly directly supports a ratio of probabilities of 3:2:1 to the proposition that each wins. The comparative evidence for each constrains what a proper probability assignment may look like. It indirectly supports a difference of probabilities of ⅙ to Albert and Beth, and ⅙ to Beth and Cody, in light of the facts about the ratios supported and that they are the only racers. The evidence doesn't itself demand that Albert and Beth differ by ⅙. It demands that they have a certain ratio, and the only way to make all the ratios come out correct leads to them differing by ⅙.

6.2 The Problem

Now that we have distinguished indirect and direct support, it is possible to understand the limitations of the conditionalization constraint. I'll start with an example that illustrate the difficulty.

Imagine that there are two foot races tomorrow. Dara and Edgar are going to race. Each has a long history of racing, and the complex body of evidence equally supports each winning. Frank and Gina are also going to race. Neither has ever raced before, and there is no evidence available about who among those two will win. Both Dara winning and Edgar winning are supported with an equal probability, and both Frank winning and Gina winning are supported with an equal probability. This entails that Edgar winning and Frank winning will be assigned an equal probability. The way that the body

[7]This need not necessarily be the case in all situations. For instance, a body of evidence that includes the fact that the difference in objective chances between two propositions is .2 (and nothing more about their objective chances) may thereby directly support a ⅕ difference in their probabilities.

of evidence supports those equalities is quite different. Nothing about the total body of evidence itself directly supports an equal assignment of probabilities to Edgar winning and Frank winning. The probability assignment supported by the evidence gives them an equal probability, however that equal probability does not reflect any specific aspect of body of evidence. It is a result that emerges from the ways that other features of the probability assignment reflect specific aspects of the body of evidence.

If the conditionalization constraint holds where it does because of how it respects the preservation of aspects of the body of evidence by preserving relative probabilities, then conditionalization need not relate bodies of evidence where the ratios of probabilities do not reflect aspects of the bodies of evidence. This case demonstrate that this is possible. The ratios of the probabilities Edgar winning and Frank winning reflect aspects of the probability assignment that are not preserved, namely, the balance of evidence between each of the two competitors winning. That balance wasn't preserved, so we should not expect the ratios to stay fixed.

Suppose that $\overline{\text{Edgar winning \& Frank winning}}$ was learned. Assuming that the prior probabilities of Edgar winning and Frank winning were independent, the conditionalization constraint says that the new probability of Dara winning and the new probability of Gina winning should be twice as great as that of Edgar winning and of Frank winning. According to the conditionalization constraint, it makes no difference to the resulting probabilities what evidence backed each assignment in the first place.

All that matters, when it comes to using the conditionalization constraint to deduce updated probabilities, is the new evidence and the initial probability assignment. I think that this is a mistake. There need not be any way to settle what probabilities are supported by additional evidence, based solely on the probabilities supported by a given body of evidence without consideration of the evidence in that body. Probabilities measure evidence, but they abstract away from its particularities. By abstracting away from those particularities, they lose something important. How probabilities change with new information may often depend upon how those probabilities were supported in the first place.

I don't have any formal alternative to the conditionalization constraint, and I am skeptical that any alternative exists. That constraint may come as close as it is possible for a purely formal constraint to come to predicting the right probabilities for bodies of evidence. But, if so, it owes its success to non-universal commonalities in bodies of evidence. It generally works, because ratios of probabilities often reflect something that is preserved.

There are limits to the power of formal systems to capture epistemic normativity. Regardless of the existence of a convenient alternative, the conditionalization constraint should not be accepted as it stands.

7 Back to Beauty

Beauty's preliminary body of evidence supports HEADS and MONDAY in different ways, even if it supports them with the same probabilities. The probabilities of HEADS and TAILS depend on Beauty's knowledge of objective chance. The probabilities of MONDAY and TUESDAY depend on the symmetries in her temporal evidence. For this reason, it is analogous to the propositions about the races between Dora and Edgar and Frank and Gina. It may be that the combination of evidence in the preliminary body indirectly supports an equal probability of HEADS & MONDAY, TAILS & MONDAY, and TAILS & TUESDAY. However, those ratios of probabilities don't obviously represent any aspect of the evidence that survives the addition of Beauty's new information. The symmetry between MONDAY and TUESDAY is breached. It may be that the 1:1 ratio is supported by some subtle aspect of the evidence that does survive the addition of new evidence, but no one has yet spelled out what this is. Therefore, it cannot be taken for granted that the conditionalization constraint holds in this case.

If the conditionalization constraint does not hold in this case, then the conditionalization argument fails. We cannot argue from the equality of the propositions relative to the preliminary body of evidence to their equality relative to the updated body.

This response to the conditionalization argument does more than bolster anti-thirder views about the original problem. One thorny question for halfers concerns the probabilities that are supported by the body of evidence that consists in Beauty's body of evidence after waking along with the fact that it is Monday. Using the conditionalization constraint, Elga leveraged the assumption that such a body of evidence supports a ½ probability there sinto an argument for thirdism. Given that she must arrive at ½ by conditionalization, we can infer that she must start out with an equal probability in HEADS & MONDAY and TAILS & MONDAY. Some halfers have relinquished plausibility by denying the assumption that such a body of evidence really does support a ½ probability. If Beauty were to find out that it was Monday, she should then become more sure of HEADS than TAILS.

As I noted earlier, others have sought to find some way to accept a conditionalization constraint but limit its application to Beauty's case. It is somewhat harder to justify refraining from applying the constraint to this other change in evidence. However, by giving up on conditionalization as a general rule, we can embrace the so-called 'double-halfer'(Bostrom, 2007) solution without qualms. The three probability assignments (preliminary, preliminary+AWOKE, preliminary+(AWOKE & MONDAY)) need not be related to each other by conditionalization.

I have only argued that one significant argument for thirdism fails. So what should Beauty think upon waking? Given that I do not have an alternative formal system to put into place, I don't have an answer. Perhaps halfers are right, but what I have said has not ruled out thirdism. My argument is also consistent with more extreme alternatives. Perhaps Beauty's evidence only supports a vague probability assignment. Or, there may be no appropriate probability, vague or not, supported by her evidence, and Beauty ought not to believe that it has any probability. It is also possible that she ought to adopt some other kind of epistemic state to the proposition, other than a credence or a belief in a probability. These sorts of positions have been insufficiently explored to date.

8 Conclusion

The conditionalization constraint is very useful for delivering numeric probability assignments. We must be careful, however, precisely because of this utility. It is tempting to make use of tools that promise to allow us to do something that we could not do without them. However, before the conditionalization constraint can do much work, it must be suitably justified. If I am right about the true justification for conditionalization, then it cannot do the work that it has been made to do in the Sleeping Beauty Problem, and it must be deployed in a more careful fashion in solving other problems of Bayesian epistemology.

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